MAGNETIC CONTRIBUTION TO THE PROTON-NEUTRON MASS DIFFERENCE
AND THE ROLE OF THE NUCLEON PSEUDORESONANCE *

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ABSTRACT

It is found that the case for an appreciable "magnetic" n-p mass difference as advanced recently by Cornwall and Patil is seriously weakened when the dependence of such a calculation on strong interaction dynamics is properly taken into account. The unobservable nucleon "pseudoresonance" in the renormalization function \( M_p \approx 1230 \text{ MeV}, \Gamma_p \approx 160 \text{ MeV} \) is shown to play a dominant role in this connection; in the absence of the Roper pole-mechanism of Cornwall and Patil it still produces a "magnetic" mass difference, \( M_n - M_p \approx 0.9 \text{ MeV} \).

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I. INTRODUCTION

Recently, Cornwall and Patil\textsuperscript{1} asserted that there could be a quite appreciable "magnetic" n-p mass difference were proper account taken of the Roper (P_{11}) resonance at 1400 MeV. Their calculation of $M_n - M_p \approx 2.4$ MeV without the corrections for Coulomb self-energy is based on three assumptions, (a) that unsubtracted dispersion relations hold for the nucleon proper self-energy part,\textsuperscript{2} (b) the intermediate states are dominated by the N\gamma state, (c) the "magnetic" NN\gamma proper vertex functions are dominated by the Roper resonance. Although Cornwall and Patil\textsuperscript{1} note that assumption (a) is closely related to the hypothesis of vanishing nucleon wave-function renormalization, nowhere in their actual calculation do they make use of this connection. (Indeed, their expression for the imaginary part of the "magnetic" proper self-energy [Eq. (8) of Ref. 1] is written in terms of the improper magnetic vertex function [the form factors $F_{n,p}^{m,n}(S)$] instead of the proper vertex.) In this paper, we find, after making use of this connection and refining the considerations of Ref. 1 somewhat, that the resultant sensitivity of this type of calculation to strong-interaction dynamics (by way of the necessary introduction of the nucleon "pseudoresonance")\textsuperscript{3} must seriously weaken the case for so naive a model of the n-p mass difference.

II. THE CALCULATION

Inspection of, say, the $pp\gamma$ proper vertex in Bincer's\textsuperscript{4} form,

\[
\bar{e}u_p(p)\gamma_\mu(p,p+\ell) = \bar{e}u_p(p) \left\{ [\gamma_\mu - i\sigma_{\mu\nu}\frac{p}{2M} F^{\nu}_{2}(W) + \ell_\mu F^{\nu}_{3}(W)] Z(W) \Lambda_+ (\ell + \ell) \\
+ [\gamma_\mu - i\sigma_{\mu\nu}\frac{p}{2M} F^{\nu}_{2}(-W) + \ell_\mu F^{\nu}_{3}(-W)] Z(-W) \Lambda_- (\ell + \ell) \right\},
\]

(1)
where
\[ \left[ S_F^I (\pm W) \right]^{-1} = Z (\pm W) (\pm W-M) \] (2)

indicates that an initial refinement of the calculation of Ref. 1, would consist in the simple replacement
\[ F_M^{n,p} (\pm W) \rightarrow \Gamma_M^{n,p} (\pm W) = F_M^{n,p} (\pm W) Z (\pm W) \] (3)
in the dispersion integral for \((M_n - M_p)\) magnetic in the W-representation. Thus,

\[ (M_n - M_p)_{\text{magnetic}} = - \frac{\alpha}{16\pi M^2} \int_{\infty}^{W-M} \frac{dW}{W-M} \left( \frac{W^2-M^2}{W^3} \right)^3 \left| F_M^n (W) \right|^2 - \left| F_M^p (W) \right|^2 \left| Z (W) \right|^2 \\
+ \frac{\alpha}{16\pi M^2} \int_{\infty}^{W+M} \frac{dW}{W+M} \left( \frac{W^2-M^2}{W^3} \right)^3 \left| F_M^n (-W) \right|^2 - \left| F_M^p (-W) \right|^2 \left| Z (-W) \right|^2. \] (4)

Now, from Ida's\(^3\) analysis of the nucleon renormalization function \(Z(W)\), we know that \(|Z(-W)|^2 \ll 1\) for \(W \geq M\) except in the neighborhood of the weak reflection of the physically unobservable "pseudoresonance" in \(Z(W)\) at \(W = M_p = 1230\) MeV. The resultant strong damping of the contribution to \((M_n - M_p)_{\text{magnetic}}\) from the "negative branch" leads us to regard it as a perturbation on our calculation and it is neglected henceforth. A calculation of the contribution to \((M_n - M_p)_{\text{magnetic}}\) from the "positive branch" in the "narrow-resonance" limit\(^5\) now shows the effect of the real Roper pole to be enhanced by a factor \(|Z(M_R)|^2 \approx 2\); the inclusion of the contribution from the narrow \(\Gamma_p (M_p) = 160\) MeV nucleon pseudoresonance will only serve to further amplify this result. This suggests that we ought to examine more closely the assumption of Cornwall and Patil\(^1\) that

\[ F_M^{n,p} (s) = \mu_n, p \left( \frac{M_n^2 - M_p^2 + i \Gamma_n M_R}{s - M_n^2 + i \Gamma_n M_R} \right). \] (5)
If we put Bincer's representation for the improper (proton) vertex into "Gordon" form, so that,

\[ \bar{e}_p(p) \Gamma_\mu(p, p+\ell) \equiv \bar{e}_p(p) \left[ \frac{1}{M+W} (2p_\mu + \ell_\mu) - \frac{1}{M+W} i\sigma_{\mu\nu} \ell_\nu (1+F_2(W)) \right. \]

\[ + \ell_\mu F_3(W) \Lambda (\omega + \ell) + \left[ \frac{1}{M-W} (2p_\mu + \ell_\mu) - \frac{1}{M-W} i\sigma_{\mu\nu} \ell_\nu (1+F_2(-W)) \right. \]

\[ + \ell_\mu F_3(-W) \Lambda (\omega + \ell) \right] , \] (6)

then it seems equally reasonable to take, for example,

\[ F^P_M(W) = \frac{2M}{W+M} \left[ 1 + \kappa^P \left( \frac{M-M_R+1}{M-W+1} \right) \right] , \] (7)

that is we embed the Roper resonance in \( F_2(W) \) only. Substituting our expression (7) for \( F^P_M(W) \) and an analogous one for \( F^n_M(W) \) along with a one-resonance-pole fit to \( Z(W) \),

\[ Z(W) \propto \frac{M-M_p+1}{W-M_p+1} \] (8)

into the one-branch-approximation to \((M_n-M_p)_{\text{magnetic}}\), we find

\[ (M_n-M_p)_{\text{magnetic}} \propto \frac{\alpha (M_p^2 - M_n^2)^2}{16M^2M_p^3} \frac{\Gamma_R^2}{(M_p^2-M)^2} \left[ \frac{(M_p-M)^2 + \frac{\Gamma_R^2}{4} \left( \frac{\Gamma_R}{\Gamma_p} \right)^2}{(M_p+M)^2 \left( \frac{\Gamma_R}{\Gamma_p} \right)^2} \right] \]

\[ \times \left[ \left| F^P_M(M_p) \right|^2 - \left| F^n_M(M_p) \right|^2 \right] \]

\[ + \frac{\alpha (M_R^2 - M_n^2)^2}{4M^3} \left| Z(M_R) \right|^2 \frac{(\kappa^P - \kappa^n) \Gamma_R}{(M_R+M)} \] (9)
in the narrow resonance limit. Numerically,

\[(M_n - M_p)^{\text{magnetic}} = (1.9 \pm 0.6) = 2.5 \text{ MeV,}\]  

(10)

with the enhanced Roper term accounting for less than 25% of the result. Thus there is the likelihood that the correct value of \(M_n - M_p\) is to emerge in this model as the difference between two dynamically sensitive numbers. On the other hand it is amusing to see that if we neglect the effect of a Roper resonance entirely, then

\[(M_n - M_p)^{\text{magnetic}} \approx 0.9 \text{ MeV,}\]  

(11)

with this contribution wholly a consequence of the pseudo-resonance.

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REFERENCES AND FOOTNOTES

5. The "narrow-resonance" limit is characterized by the approximation,

\[ |s - M^2_R - iM_R \Gamma_R|^{-2} \rightarrow (M_R \Gamma_R)^{-1} \pi \delta(s - M^2_R). \]

In our reconstruction of the calculation of Ref. 1 we find \( M_n - M_p \approx 2.7 \text{ MeV} \) in this limit; since this result differs from that quoted in Ref. 1 by only 13%, the introduction of a width-function \( \Gamma_R(s) \) reflecting the p-wave character of the Roper resonance does not seem warranted.
6. This fit automatically satisfies the additional requirements, \( Z(M) = 1, \)

\( Z(\infty) = Z = 0. \)
7. We neglect the terms proportional to \((\kappa^p)^2 - (\kappa^n)^2\).