METHODS OF DRIVING LONG LINEAR ACCELERATORS

Prepared by the Phasing and Drive Line Sub-committee for Project M.

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Master RF Drive and Trigger System
Proposed by William Brobeck and Associates
June, 1953

a. General Description

This system would be built in duplicate in order to reduce "outage" because failure of any of these components removes excitation to a significant fraction of the accelerator. Thermal variation of the phase velocity in the master RF and trigger lines can be kept to a minimum by using the TEM mode in coaxial lines with styrofoam dielectric. The attenuation in this type of line is of the order of one db per hundred feet. Booster amplifiers will be required every 2500 feet in each of these lines. Transfer from one line to the other is accomplished by coax relays operated from the main control room. The trigger pulse for each sector should be bridged off of the master trigger coax by means of a non-inductive isolating resistor. The value of this resistor will be different for each sector in order to compensate for the attenuation along the coaxial line. The trigger cable from each bridging junction of the master trigger coax to the sector pulser can be RG 8/U. Precise timing of the sector pulsers can be achieved by suitable choice of the length of this cable.

Phase of the RF pulse is accomplished by adjustment of the phase shifter in the input guide of the sector RF booster amplifier.

b. Master Oscillator

The master oscillator consists of a temperature controlled crystal oscillator operating at a relatively low frequency--11.73 mc., with five frequency triples producing 2850 mc. The final amplifier of the master oscillator chassis would be a 50 watt Klystro. The crystal
to the proper drive level by adjustment of the attenuator in the input of each final amplifier.

The sector pulser produces two output pulses—one for driving all of the final amplifier pulser's of the sector and the other delayed about 100 microseconds for pulsing any Klystron cubicle between beam pulses for test purposes. The RF drive for the cubicle being tested would be obtained from a portable signal generator.

In Stage I each sector uses only 12 Klystron final amplifiers; thus each drives two sections of the accelerator. The phase adjustment for the second sector is made by means of a "U" section of guide. "U" sections of the proper length with shims of the proper thickness would be selected to give the desired phase shift. A phase shifter capable of operating at this power level has been developed at Stanford and could be used in place of the "U" section if desired.

In Stage II eleven more Klystron cubicles are added to each sector so that each 10 ft. section of accelerator has its own Klystron final amplifier. The master pulser repetition rate is changed from 360 to 60 pps and the rectifier transformers of the high voltage power supplies are changed from delta-delta to delta-wye. The high voltage power requirements for Stage II are, therefore, almost the same as Stage I.

b. Sector Pulser

The sector pulser is much like the master pulser so far as pulse height, duration and rise and fall time are concerned. However, in addition to the main driving pulse, there is a delayed pulse required which can be used to test a Klystron cubicle by pulsing it out with the beam.
oscillator and the frequency multipliers would operate CW in order to achieve a high degree of frequency stability.

c. Master Pulser

The master pulser is triggered by the 60 cps line. It produces a 15 kw, one microsecond pulse at a repetition rate of 300 pps for Stage I or 60 pps for Stage II. The jitter time of the pulse must be less than 0.1 microsecond. It should have an auxiliary output, isolated from the main output, which can be used to drive experimental equipment in the counting area.

d. RF Booster Amplifiers

The RF booster amplifier modulators are essentially the same as the main modulators except that Varian VAS7C tubes are used in the place of the Stanford klystrons. Since the beam voltage is only 90 kv a lower ratio pulse transformer (7:1) and a higher impedance pulse line are required. The pulse length should be somewhat longer than that of the final amplifier to reduce the timing accuracy requirements.

Sector RF Drive and Trigger System - Drawing D-4

a. General Description

The sector driver amplifier excites a terminated waveguide from which the klystron final amplifiers receive their input signal. The signal is extracted by means of four hole directional couplers which provide a high degree of discrimination against the phase error produced by any reflected wave in the drive guide. The coupling factor of the directional couplers vary with their position in order to compensate for attenuation in the guide and thus provide the same amount of RF power (10 kw) for each final amplifier. This is further reduced
Comparison of Two RF Feed Methods for Project H

Proposed by R. E. Neal

November 8, 1953

Two methods of driving the klystrons for Project H are discussed below. Both of these methods make use of only a single drive source and hence, would require high gain klystrons to keep the drive requirements within reasonable limits.

Method 1. A single drive line is used running the length of the accelerator. It is fed from a source located at the center of the accelerator. Equal amounts of power are coupled from the main waveguide to drive each of the high power klystrons.

Method 2. (Due to W. Gallagher). A single drive source located at the center of the accelerator is again required. The source power is split repeatedly until the number of feeds is equal to the number of klystrons powering the accelerator. The positions of the tee junctions are chosen so that all line lengths are identical and equal to \( \frac{L}{2} \) where \( L \) is the accelerator length. This eliminates differential phase shift due to frequency and temperature deviations except for second order effects.

Comparing the data of tables 1 and 2 (pertaining to method 1 and method 2, respectively) it is noted that the drive power required in the second method is approximately 7 times that required in the first method. The total length of waveguide varies from 2.5 \( L \) to 5 \( L \) in the various cases illustrating method 2 while it is equal to \( L \) for all cases in method 1.

Method 2 requires \( n-1 \) identical tee junctions whereas method 1 requires \( n \) directional couplers, having \( n/2 \) different coupling
factors \( n = \text{total number of klystrons} \).

Method 2 might be feasible provided high gain klystrons are developed. For example, if the required drive power per klystron is 10 watts, the total drive power in the last example of Table 2 could be provided by a single 12 megawatt driver klystron or two 6 megawatt klystrons operating in parallel. It might be possible to operate the waveguide in air after the first several power divisions. Temperature regulation of the guide may not be required provided the ambient temperature is reasonably constant throughout the tunnel.

The value of the attenuation factor \( X \) is quite important in both of these methods. However, method 2 is more sensitive to \( X \) than method 1. For example, an increase in \( X \) from .006 to .007 dB/ft. increases the total drive power requirement in method 1 by a factor 2.75 whereas the corresponding factor in method 2 is 3.25. With either of these methods, an effort should be made to keep the attenuation factor as low as possible by suitable choice of transmission line type, dimensions, material, etc.
**METHOD I**

![Drive source diagram](image)

Symbols:
- \( L \) = Total length of accelerator
- \( L \) = Length between feeds.
- \( n \) = Total number of high power klystrons
- \( P_k \) = Peak drive power required per klystron
- \( P_t \) = Total drive power from source.
- \( X \) = Attenuation in waveguide in db/ft.
- \( B = 10 \times X / L \)

Total source power required:

\[
P_t = 2P_k \left( 1 + B + B^2 \right)^{\frac{n}{2}}
= 2P_k \left( B + \frac{1}{B} \right)
\]

Total length of waveguide (excluding transverse sections) = \( L \)

*Table I* (based on \( X = 0.003 \) db/ft.)

<table>
<thead>
<tr>
<th>( L ) (ft)</th>
<th>( n )</th>
<th>( \hat{R} ) (ft)</th>
<th>( B )</th>
<th>( P_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,240</td>
<td>128</td>
<td>30</td>
<td>1.117</td>
<td>83,160</td>
</tr>
<tr>
<td>10,240</td>
<td>256</td>
<td>40</td>
<td>1.0567</td>
<td>41,700</td>
</tr>
<tr>
<td>10,240</td>
<td>512</td>
<td>30</td>
<td>1.023</td>
<td>34,200</td>
</tr>
<tr>
<td>10,240</td>
<td>1024</td>
<td>10</td>
<td>1.0138</td>
<td>176,000</td>
</tr>
</tbody>
</table>
METHOD 2

Symbols:
- L = Total length of accelerator
- \( \beta \) = Length between feeds
- n = Total number of high power klystrons
- \( P_k \) = Peak drive power required per klystron
- \( P_{\text{t}} \) = Total drive power from source
- X = Attenuation in waveguide in dB/ft.
- \( \beta \) = 10 \( \times \) \( \beta \)/10

Total length drive line to each klystron = \( \frac{L}{n} \)
Total attenuation in drive line to each klystron = \( \frac{LX}{2} \) dB

(Y) Drive power per klystron required at source = \( 10 \log \frac{LX}{2} \) \( P_k \)

\( P_{\text{t}} \) = Total drive power required from source = \( nP_k 10 \log \frac{LX}{2} \) = \( nP_k \beta \)

(Z) Total length of waveguide required (excluding traverses) = \( \frac{L \times \beta n}{2 \times \text{dB} \times \text{ft}} \)
Total number of identical tee junctions required = \( n-1 \)

Table 2 (based on \( X = 0.002 \) dB/ft.)

<table>
<thead>
<tr>
<th>L(\text{ft})</th>
<th>n</th>
<th>( \beta )</th>
<th>( \frac{LX}{2} )</th>
<th>Y</th>
<th>( P_{\text{t}} )</th>
<th>( P_k )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,240</td>
<td>123</td>
<td>20</td>
<td>30.7 dB</td>
<td>1175 P_k</td>
<td>100,000 P_k</td>
<td>3.5 L</td>
<td></td>
</tr>
<tr>
<td>10,240</td>
<td>225</td>
<td>20</td>
<td>30.7 dB</td>
<td>1175 P_k</td>
<td>300,000 P_k</td>
<td>4.5 L</td>
<td></td>
</tr>
<tr>
<td>10,240</td>
<td>512</td>
<td>20</td>
<td>30.7 dB</td>
<td>1175 P_k</td>
<td>600,000 P_k</td>
<td>4.5 L</td>
<td></td>
</tr>
<tr>
<td>10,240</td>
<td>1024</td>
<td>10</td>
<td>30.7 dB</td>
<td>1175 P_k</td>
<td>1,200,000 P_k</td>
<td>5 L</td>
<td></td>
</tr>
</tbody>
</table>
The ratio of the total drive power required by method 2 to the total drive power required by method 1 is:

\[
\frac{P_{2}}{P_{1}} = \frac{a_{2} \frac{E_{2}}{E_{1}}^{2/3}}{a_{1} \frac{E_{1}}{E_{1}}^{2/3}} = 1.15 \times L_{1}
\]

Thus, method 2 required relatively more drive power as the accelerator length increases or as the waveguide attenuation increases.
Method 2 as discussed previously has the following advantages:

(a) The feed lines to all klystrons are equal in length, thus greatly reducing phase shifts due to frequency and temperature variations.

(b) All couplers are identical in design and coupling factor (3 dB, i.e., simple tee junctions)

The disadvantages of method 2 are:

(a) Higher drive power is required than in the case of method 1.

(b) The total length of waveguide required is 3.5 to 5 times the klystron length (in the examples given in Table 3).

Disadvantage (b) above is not serious since the cost of waveguide is small compared to the total cost of the drive system. Disadvantage (a), however, limits the usefulness of this method unless the waveguide attenuation factor can be kept to a very low value or unless klystrons of very high gain are developed.

A simple modification of method 2 can be made which retains both of the advantages given above while eliminating or greatly reducing disadvantage (a). Disadvantage (b) is not affected. This change consists of using additional klystrons in one or more of the levels of the drive chain. The scheme illustrated in
the sketch permits a large reduction of drive power. The power required from klystron \( K_2 \) (or \( K_3 \)) is reduced by a factor of \( 2 \times 10^{\frac{LX}{40}} \) compared to the power required from \( K_1 \) when it is used alone as in the original version of this method. For the parameters of Table 2 (\( L = 10,243 \) ft, \( X = 508 \text{ ohms} \text{ ft} \)) this reduction factor is 70. All the values under \( P_R \) in Table 2 can be reduced by this factor, giving 2100 \( P_R \), 4200 \( P_R \), 5200 \( P_R \), and 11,040 \( P_R \) from top to bottom. The drive power now required from \( K_1 \) is \( 2 P_R \times 10^{\frac{LX}{40}} = 70 P_R \).

A better balance of klystron powers can be obtained with the following arrangements (however, more tubes are required)

[Diagram]

(remainder of chain same as before)

In this case, the reduction factor in power from \( K_2 \) (or \( K_3 \)) compared to \( K_2 \) used alone is \( 4 \times 10^{\frac{LX}{30}} = 600 \) using the values under \( P_R \) in Table 2 become 197 \( P_R \), 374 \( P_R \), 743 \( P_R \), and 1,406 \( P_R \) from top to bottom. The drive power required from \( K_1 \) now becomes 600 \( P_R \).

Further remark:

1) Method 2 as discussed in either the original or modified forms would be troubled with reflections from the input cavities of the high power klystrons since the impedance match will not be perfect for all values of beam voltage. The use of pads or ferrite isolators in the input line to each klystron is desirable, the latter being preferred in order to conserve drive power. Similar devices are also desirable in method 1.
2. When the drive source is in the center of the accelerator as in both methods 1 and 2, the drive pulse will arrive at both ends of the accelerator simultaneously. However, the electrons will arrive at the end of the machine about 10 microseconds after being emitted from the gun (for a 10,000 ft. length). This means that the drive pulse must be 10 microseconds longer than the desired R.F. pulse length (i.e., 12 microseconds for a 2 Microsecond R.F. pulse). Some allowance may also be needed for deterioration of the rise and fall times of the drive pulse in passing through the long drive line. A consequence of the long drive pulse length is that the first section of the accelerator will be utilizing the front of the drive pulse while the last section of the accelerator will use the end of the drive pulse. Intermediate sections will utilize their proportional parts of the drive pulse. This means that it will be necessary in some cases to pay close attention to keeping stable frequency and phase characteristics over the entire drive pulse to minimize electron energy spread.

The need for an extended drive pulse exists even if the driver is located at the gun end of the accelerator provided that ordinary 1¼" x 3" waveguide is used for the drive line. The group velocity in this guide is about 2/3 the velocity of light causing the R.F. wave to require about 5 microseconds longer to reach the end of the machine than the electron beam. This means that a drive pulse length of about 7 microseconds would be needed.
Addendum No. 2 to R.F. Drive Memorandum

In the previous addendum two modifications of drive method No. 2 were proposed to overcome the principal disadvantage of the original version of this method, viz., the high drive power requirement. Another disadvantage which this method shares with other methods having sources located at the center of the accelerator was discussed in remark No. 2 (page 8). This is the requirement of a long pulse length to allow for the difference in arrival times of the r.f. and electron pulses at the opposite ends of the accelerator. In the example given, an r.f. pulse length of 12 microseconds was required with good frequency and phase stability necessary over the entire pulse length to prevent electron energy spread.

The required driver pulse length may be considerably reduced in the following way: the drive source (K₁ in the sketch on page 7) can be a cw rather than a pulsed tube. This source then supplies the power to cw booster amplifiers at the location of K₂, K₃, K₄, and K₅. The booster amplifiers in turn drive the four pulsed tubes (K₂, K₃, K₄, K₅). Otherwise the scheme is the same as illustrated on page 7. The power output requirement from the cw tubes is not excessive. For example, suppose that 1000 watt cw tubes with 35 to 40 db gain are available. One such tube could be used as the drive source and four as the boosters described above. The pulsed tubes, K₂, K₃, ..., K₅, as well as the accelerator klystrons could then be 5-22 mw tubes designed to have a gain of 40 db.

By the use of the cw source and boosters as described above the need for 10 + 2 = 12 microseconds pulse length for the drive source is reduced to 2.5 + 2 = 4.5 microseconds since the pulsed sources each now drive only one-quarter of the klystrons along the accelerator length. The above example is given for illustration only. The number of levels supplied by the cw source and boosters can be adjusted to suit the specifications of
the various available tube types. In general, the larger the number of levels of the drive system supplied with cw power, the shorter the pulse length of the pulsed tubes can be made; however, the total required number of tubes in the drive system increases as this is done.
Remarks on the Drive Line for Project M

Proposed by W. J. Gallagher

November 17, 1950

1. From the gun end of the accelerator, the phase at any point \( z \) down the accelerator is

\[ \phi_z = \frac{\omega_z}{v_p} \tag{1} \]

which becomes at the operating condition

\[ \phi_z = \frac{\omega_z}{c} . \]

A major difficulty appears to arise when the velocity of light occurs at different frequencies (due to temperature change, for example). Then the specified phase at any point changes by amount

\[ \delta \phi_z = \frac{\delta z}{c} \left( \omega_0 + \frac{\delta \omega}{c} \right) . \]

That is, if the entire accelerator were a rigid continuous tube the change in running frequency due to thermal expansion would be compensated by the expansion of the tube, and each point would still have the correct phase.

For example, if the specified phase and frequency at a point \( z_0 \) is \( \phi_0 \) and \( \omega_0 \), respectively, then

\[ \phi_0 = \frac{\omega_0 z_0}{c} . \]

If, because of temperature change, the point \( z_0 \) expands to \( z_0 + \delta z \) and a new running frequency \( \omega_0 + \delta \omega \) is required, the new phase condition is

\[ \phi_0 + \delta \phi = \frac{\left( \omega_0 + \delta \omega \right)
(\omega_0 + \delta \omega)}{c} \]

\[ = \frac{\omega_0^2}{c} \left( 1 - g \delta T \right) \left( 1 + g \delta T \right) = \phi_0 - \left( g \delta T \right)^2 \]

since \( \omega_0 + \delta \omega = \omega_0 \left( 1 - g \delta T \right) \)

see paragraph 3.

and \( \omega_0 + \delta \omega = \omega_0 \left( 1 + g \delta T \right) \)

That is, the phase adjustment does not depend upon the place along the accelerator and is vanishingly small.

If sections are allowed to expand into expansion joints and the two
ends of the accelerator are approximately fixed, then a temperature change \(\delta T\) will require a frequency adjustment \(\omega_0 + \delta \omega\) which at a point \(\theta_0\) will require a phase adjustment computed from
\[
\delta \phi = \frac{(\omega_0 + \delta \omega) \pi \delta_0}{c} = \frac{\omega_0 \pi \delta_0}{c} (1 + q \delta T)
\]
or of amount
\[
\delta \phi = \frac{\pi \omega_0}{c} q \delta T.
\]

To ensure the phase excursions are less than 0.1 radian at 10,000 feet requires that the temperature variations are less than 0.0005°C. This is the same requirement as is necessary for a constant frequency machine.

Suppose the 10,000 foot accelerator was restrained from expanding with temperature change. The expansion is for a 1°C temperature change
\[
\delta L = \alpha \delta T
\]

\[
= 12 \times 10^{-6} \times 1.6 \times 10^{-5} \times 5 = 2.6 \text{ inches}
\]

If the expansion is prevented the pressure developed where Young's modulus
\[
\frac{\delta P L}{\delta T} = 9.6 \times 10^6
\]

\[
p = \frac{\delta P L}{\delta T} = \frac{9.6 \times 10^6}{12 \times 10^6} = 1600 \text{ lb. per sq. in.}
\]

The point is that the pressure is sufficient to overcome the support friction if such a mounting were proposed.

To compute the allowable frequency or temperature variations, we can compute the permissible energy loss due to the displacing. The fractional expression for the fractional energy loss due to systematic errors is rather complicated but is closely approximated by
\[
\frac{\delta V}{V} = \frac{1}{6} \phi \frac{\delta \phi}{\phi} \frac{3 \chi^2}{\chi^2} \quad (2)
\]

The frequency error due to temperature change can be calculated from the effect on the dimensions,
\[
\frac{\delta f}{f} = \left[ \frac{\partial f}{\partial a} (2a) + \frac{\partial f}{\partial b} (2b) + \frac{\partial f}{\partial c} (2c) + \frac{\partial f}{\partial d} (2d) \right] \sigma \quad (3)
\]
where \( \gamma \) is the coefficient of thermal expansion of copper. The partials are known experimentally for the Mark III:

\[
\begin{align*}
\frac{\partial^2 \gamma}{\partial \alpha^2} (2a) &= (+0.89) (522) \rightarrow 214 \\
\frac{\partial^2 \gamma}{\partial \alpha \partial \beta} (2c) &= (0.003)(326) \rightarrow -3300 \\
\frac{\partial^2 \gamma}{\partial \alpha^2} (3c) &= (+0.074) (120) \rightarrow 33 \\
\frac{\partial^2 \gamma}{\partial \beta \partial \mu} (\eta) &= (-0.104) (268) \rightarrow -28
\end{align*}
\]

\( g = 1.3 \times 10^{-5} \) per degree C.

so that

\[
\frac{\partial \gamma}{\partial \alpha} = -0.066 \text{ Kc/sec per degree C.}
\]

3. This should also be compatible from the expression

\[
\frac{\partial^2 \gamma}{\partial \alpha^2} = \frac{\partial^2 \lambda}{\partial \alpha^2} = \frac{d^2 \lambda}{\partial \alpha^2}
\]

so

\[
\frac{d^2 \lambda}{\partial \alpha^2} = g = 2060 \times 1.6 \times 10^{-5} = 0.66 \text{ Kc/sec per degree C.}
\]

A third way of looking at the dephasing due to temperature changes is by differentiating Eq. (1)

\[
\delta \omega_n = \frac{2}{c^2} \delta \omega
\]

and combining with the expansion coefficient

\[
\delta \omega = -\gamma \delta T
\]

to obtain

\[
\delta \omega_n = \frac{\alpha}{c^2} \delta T
\]

7. Because of ambient temperature level changes and variations before steady state is reached the drive frequency will probably have to be changed. This will change the specified phase at any point \( \alpha \) along the accelerator by amount

\[
\delta \omega_n = \frac{\alpha}{c^2} \delta \omega_n
\]

To prevent dephasing by more than a tenth radian over two miles the frequency stability of the driver must be

\[
\delta \omega = \frac{c}{\delta \omega_n} \frac{3 \times 10^{10}}{3 \times 10^9} (\frac{1}{3}) = 10^6
\]

or

\[
\delta f < 1.860 \text{ cps.}
\]
In earlier discussions it appeared that the "long line effect" (that is, critical dependence of the precise phase at a specified point down a long line on the frequency) would dephase the entire machine whenever the drive frequency was changed. It was proposed to employ a branched transmission system where every feed point was equidistant from the drivers. Regardless of the dispersions of the propagation system every section would then be driven at the same phase, neglecting differential effects. From the above current on the specified phase along the accelerator it is clear that when the drive frequency is raised the phase should be advanced at any point along the accelerator, and oppositely. The branched system doesn't do this.

The best obvious way to do this is with a dispersionless velocity of light transmission system, as it will always exactly meet the phasing condition,

$$\phi = \frac{R}{c} \omega.$$ 

If the beam and the wave have the same phase at the gun end of the accelerator they should have the same phase all the way down the two mile path. This is evident on a Brillouin diagram, where it is clear that at any frequency the drive line is at the velocity of light and it is only a matter of tuning around to get the accelerator intersection

![Diagram showing the relationship between MWGD, Cour, and beam Auger.](image)

at \( v = c \). The coax is always correctly adjusted for driving the system.

9. **Coax Drive Line.**

If we presume the drive line to be coax (air, or vacuum) line we can estimate the necessary design. Presumably we will use near 77 ohm
\((h/a = 3.60)\) line for minimum attenuation.

The propagation constants

\[ i = \sqrt{(R + joL)(G + joC)} = \alpha + j\beta \]

may be approximated as

\[ \alpha = \frac{R}{2} \sqrt{\frac{C}{L} + \frac{G}{2} \sqrt{\frac{L}{C}} \frac{R}{2Z_0}} \]

\[ \beta = \frac{2ULa}{R} \]

where \( G \to 0 \) and

\[ R = \frac{Z_0}{2} \left( \frac{1}{\alpha} + \frac{1}{\beta} \right) \]

The skin resistance of copper at 15 cm is \( 0.018 \) ohms/cm.

The maximum power transmissible without exceeding a field strength in \( E_m \) is

\[ P = \left( \frac{\frac{2ULa}{\eta}}{\varepsilon_0} \right)^2 Z_0 \]

where \( a \) is the radius of the center conductor.

A difficulty is not evident. Now much drive power is required depends on the attenuation of the drive line (and the amount to be delivered), but the attenuation of the drive line depends on its dimensions which are set by the amount of drive power it is required to transmit.

Because of the advantages of a linear feed, where the driver is at the gun end, we will consider this system. The system requires a minimum pulse length and programs the drive power down the guide in precisely the desired manner.

Suppose an amount of power \( P_0 \) is to be delivered to \( n \) stations, in parcels of amount \( p_i \), where the transmission attenuation between stations is \( \alpha \). Then

\[ P_0 = p_1 \left( \frac{\alpha}{1} + \frac{\alpha^2}{1} + \alpha^3 + \cdots \right)^n \]

\[ = p_1 \frac{\alpha}{1 - \alpha^n} \]

\[ = p_1 \frac{\alpha}{1 - \alpha} \]

\[ = p_1 \frac{\alpha}{1 - \frac{1}{\alpha}} \]
For example, if 1024 stations 10 feet apart are considered, where each tube gets an input \( p_i \), if the drive line were copper pipe and \( \alpha = 1 \text{ cm} \), then

\[
R_s = .015 \text{ ohms/sq} \\
R = .00806 \text{ ohms/cm} \\
\alpha = .002 \text{ nepers/cm}
\]

and \( \frac{p_O}{p_i} = 89,000 \).

This seems fantastic, but when it is considered that the attenuation to the last station is 50 db this becomes realistic. The practical result is that such a scheme as this cannot be considered without some amplifiers along the drive line. To enlarge the coax would raise the possibility of loading.

10. The couplers of such a drive line as the above have the values, for the \( n \)th coupler,

\[
\text{db}_n = 10 \log \left[ \frac{\frac{p_O}{p_1} e^{-n\alpha L} - 1 - e^{-n\alpha L}}{1 - e^{-\alpha L}} + 1 \right].
\]

This follows from the progression

\[
\frac{p_O e^{-\alpha L}}{p_1} \left( \frac{p_O e^{-\alpha L}}{p_1} - \frac{p_i}{p_1} \right) e^{-\alpha L} \ldots = \left[ \frac{p_O e^{-n\alpha L} - p_i e^{-(n-1)\alpha L}}{p_1} \right] \ldots
\]

the general term of which contains a geometrical progression which can be summed over \( n \) terms conveniently. This results in pretty crazy couplers. For the example in section 9, the first one is a 49 db coupler through which about 20 lb is expected to pass. However, these are not fatal defects.

11. The problem of loading in the coax is a more involved discussion and has run to a dozen pages, which will not be reproduced here. Copies are available from me. The general conclusion is that loading conditions do exist for a couple of types of modes and we will have to be conscious of this. This conclusion is based on coax of 75 ohms for 20 mw transmission.

12. Undoubtedly a problem would be the pulse deformation due to the slight dispersiveness arising from the supports required for the center conductor. For a 1 usec pulse at 3000 Mc the bandwidth must be non-dispersive over
approximately ± 8 Mc/sec to prevent serious distortion over a short transmission path. For a given length of line the amount of distortion can be kept to an acceptable degree if the line is non-dispersive over a wide enough band. For longer pulses the problem is less serious.

The possibility of increasing the size of the coax to guarantee a maximum loss was investigated. The attenuation of coax line depends only on the material, characteristic impedance and inner conductor diameter. For 77 ohm copper line,

\[
\alpha = \frac{R}{2Z_0} \frac{R_s}{2Z_0} \frac{1.002}{\ln \frac{a}{2Z_0}} \text{ nepers/m}
\]

where \(a\) is in centimeters. For 3000 meters of line

\[
\alpha = \frac{50}{a} \text{ db}
\]

from which it is evident that the losses cannot be sufficiently reduced unless the line is about a foot or two in diameter.

In any calculation for very long lines the additional attenuation arising from the connectors should be included.
Phase Correction and Transmission Line Problem
Proposed by D. J. Goerz
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Several methods have been proposed dealing with problem of transmitting a stable frequency down the length of the accelerator; most of the methods proposed consider transmission at 2356 Mc. As an alternative I would like to suggest the use of a low frequency which is then multiplied at each klystron.

There are several schemes for multiplication from a low frequency, for example, from 5 Mc to 2356 Mc, such as direct multiplication, use of a frequency discriminator or a phase discriminator. A frequency discriminator with \((f_0 - f_1)^{-1}\) control circuit would give phase stabilization type of control, but such a \((f_0 - f_1)^{-1}\) control is neither physically realizable nor defined analytically in the operating region, i.e., a \((f_0 - f_1)^{-1}\) phase discriminator is both physically realizable and defined analytically in the operating region.

With phase stabilization it is possible to realize the transference of frequency stability from one frequency region to another with any desired precision.

Conventional multiplication usually yields a high frequency spectrum that is non-monochromatic but has sidebands arising from lower frequency modulations that remain because of the finite selectivity of the circuits. Furthermore, since multipliers with a gain of less than one (silicon diodes) introduce additional noise into the spectrum, it is not desirable to multiply by more than a factor of 10 in these diodes. With the system described it is possible to have a high multiplication ration (75), excellent stability and large control range. Phase correction can be made by varying the phase of a low frequency oscillator.
The advantages of the system:

The system combines the automatic phase shift device with a means for transmitting a stable-frequency down the length of the accelerator. Since the signal is transmitted at low frequency (<40 Mc) there is no need for expensive directional couplers and associated vacuum equipment. It is possible to build a line with zero dispersion at this frequency, drive frequency can be adjusted without affecting the phase of individual sections. Phase adjustments can be made at the low frequency, this can consist of either a mechanical or electrical phase shifter.

Method of operation:

A 5 Mc signal is generated from a crystal, and is transmitted down the length of the accelerator. A second signal is generated at approximately 200 kc which is also transmitted the length of the accelerator. The 40 Mc signal is then fed to a crystal and a tuned section of line acts as a filter for the 40 Mc harmonics, the output signal is mixed with the output from a klystron and the difference frequency amplified in an i-f amplifier. The i-f is then compared with the low frequency signal and output phase error signal is used to control the voltage on the klystron oscillator. The frequency is then controlled by tuning the 200 kc oscillator. The phase is corrected by changing the phase of the 200 kc signal. This can be done by an electrical phase shift or a mechanical phase shifter. The phase information can be obtained from the electron beam or any of the other appropriate means.

The frequency stability of such a system is of the order 1 part in 10^12 cps. With such a system it is necessary to have continuous phase comparison due to internal drift with time of about 30° per minute.