MEASUREMENT OF
ELASTIC AND INELASTIC PARITY-VIOLATING
ELECTRON SCATTERING FROM THE PROTON

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Abstract

We propose to measure the parity-violating asymmetry in elastic electron scattering from the proton at both high and low $Q^2$: 2.0 and 0.5 GeV$^2$. The inelastic excitation of the $\Delta$ will also be measured at $Q^2 = 2$ GeV$^2$. The measurements will be made in SLAC End Station A utilizing both $(e,e')$ and recoil $(e,p)$ detection. The primary measurement, elastic scattering, will be quite sensitive to the possible role of strange quarks in the proton. Since the $Q^2$ dependence of the strange quark form factors is completely unknown, we maximize the possibility of observing them by making measurements at two widely separated $Q^2$ points. Even if their contribution is much smaller than suggested by Jaffe,$^1$ our measurements will be quite sensitive to it. Unlike lower energy experiments, there is essentially no contamination from the poorly known axial form factor and its radiative corrections. Measuring the purely isovector excitation of the $\Delta$ resonance could provide a significant improvement in our knowledge of the electron axial vector–quark current isovector coupling and may be sensitive to higher-order electroweak processes. For both the elastic and inelastic cases, the Standard Model predictions are unambiguous and any deviation from them would signal the presence of extremely interesting additional physics, either in the hadronic structure or in the electroweak interaction.

I. Introduction

Measurement of the parity-violating asymmetry in electron scattering is a powerful way to study the neutral current weak interaction. The first parity-violating electron scattering experiment performed at SLAC$^2$ not only provided an important experimental test of the Standard Model, but also established the basic strategy for these kinds of experiments. There is a two-fold motivation for parity experiments: one is to test the structure of neutral weak currents and the other is to use the $Z^0$ as a probe of hadronic structure. Of course, these are not orthogonal concepts and a careful theoretical analysis is required for any actual experiment.

Assuming the validity of the Standard Model, the electroweak interaction offers a tool for exploring new aspects of hadronic structure via parity-violating electron scattering.$^3$ In particular, it has been pointed out that it can be used to explore strange quark matrix elements in the nucleon. This, in turn, provides vital information for establishing a connection between the heavy constituent quarks of the quark model and the light current quarks of QCD. The basic assumption is that the constituent quarks are current quarks “dressed” in a complicated fashion with $q\bar{q}$ pairs and gluons. For this reason, one expects non-zero strange matrix elements even though the nucleon is comprised of only $u$ and $d$ valence constituent quarks.

Existing evidence for strange matrix elements comes from the pion-nucleon $\sigma$ term, polarized muon–polarized proton scattering and elastic neutrino–proton scattering. The pion-nucleon $\sigma$ term$^4$ can be interpreted as measuring the scalar matrix element $m_{\sigma}(p|\bar{s}s|p)$ and indicates a surprisingly large value: $334\pm132$ MeV. Unexpected results from the EMC collaboration$^5$ on the spin-dependent structure function of the proton appear to violate the Ellis-Jaffe sum rule.$^6$ As one of several possible “solutions” to this “spin crisis”, it has been suggested that the axial current strange quark matrix element $(p|\bar{s}\gamma^\mu\gamma_5s|p)$ may be sizable in ordinary nucleons and could significantly contribute to the form factors.$^7$
The EMC results are not inconsistent with the more direct information on the axial contribution that comes from neutrino scattering\([8]\). There is additional evidence for nonzero strange matrix elements from nonleptonic hyperon decays\([9]\).

The above evidence is, however, rather indirect. Parity-violating elastic electron scattering, on the other hand, directly probes the strange vector current matrix element \(\langle p\bar{s}\gamma_\mu s|p\rangle\) assuming only that isospin is a good symmetry and that the effects of charm and heavier flavors are negligible. Such experiments have been approved at Bates\([11]\) and are proposed for CEBAF\([12][13]\). These experiments will sample \(Q^2\) from 0.1 to about 1.3 GeV\(^2\) to study the neutral weak form factors and the strangeness in the proton. At SLAC one can access higher \(Q^2\) where the strangeness contribution might be much greater. Furthermore, if there is a large strange quark contribution, it would be of great benefit to measure it at more than one laboratory. Let us denote the combination of the Standard Model and no strange quark content to the proton by NSQSM. Any result which deviated from the NSQSM would certainly need confirmation. However, measurements at SLAC could be made much earlier than those at CEBAF.

SLAC is the only facility where a polarized high energy electron beam is available that allows one to achieve high \(Q^2\) (> 1.5 GeV\(^2\)) while still working at small scattering angle (< 7°). At small scattering angles, the e-p elastic cross sections are sufficiently large to produce a reasonable counting rate. Furthermore, because the Standard Model asymmetry increases linearly with \(Q^2\), we would measure asymmetries in excess of \(10^{-4}\) at the higher \(Q^2\) even before considering any strangeness contribution. Such an asymmetry is comparable to the previous SLAC measurement\([8]\) and significantly larger than that expected for the lower \(Q^2\) Bates and CEBAF experiments where the asymmetries are a few times \(10^{-5}\) or less. However, our low \(Q^2\) point would also have an asymmetry at the \(10^{-5}\) level.

An important additional advantage of small scattering angle is that the axial-vector weak current contribution is almost completely suppressed. The experiment proposed here can then give clean information on the vector weak form factors at high \(Q^2\). The measurements will be extremely sensitive to the strange quark contributions, especially to \(G_M^{(s)}\), the strange magnetic form factor contribution.

The measurements at the \(\Delta\) region will be taken at the same time as the elastic scattering measurements with no additional beam time. The asymmetry at the \(\Delta\) is about the same as that of the elastic scattering, while the cross section at the \(\Delta\) is slightly higher. Therefore, when we measure the asymmetry for elastic scattering, we will obtain the \(\Delta\) "for free" to the same or better statistical accuracy. Since the strange quark current is isoscalar, the purely isovector \(N \rightarrow \Delta\) transition allows the determination of a particular coupling constant (the axial electron–vector quark isovector coupling). At tree level, this should be equal to a similar quantity extracted from neutrino-quark scattering. On the other hand, higher-order electroweak processes, which we may be sensitive to, affect the electron and neutrino results differently.

We plan to make measurements of \((e,e')\) at a scattering angle of 6.5° for \(Q^2\) of 2.0 GeV\(^2\). We will simultaneously detect recoil protons at 68° to obtain the low \(Q^2\) point (the corresponding electron angle is 3.1° ). The beam energy will be 12.9 GeV. We will need 35 days of beam time to measure the asymmetries with 15% (25%) uncertainties at the high (low) \(Q^2\) (assuming the Standard Model asymmetries which, since they don't include strangeness, should provide a lower limit).

The polarized electron beam and polarimeter are under development for E142 and E143 and will be ready before this experiment. A 30 cm liquid hydrogen target will be used for
this experiment. The same target has been used for several previous SLAC experiments.\textsuperscript{[14]}.

We plan to use the proposed 16 GeV/c spectrometer\textsuperscript{[18]} to detect electrons and the 1.6 GeV spectrometer, augmented by a pair of front quadrupoles, for recoil protons. We will maximize the use of existing equipment in order to make this a relatively low-cost experiment. It would play an essential role in establishing the influence of strange quarks in the proton.

II. Elastic Scattering

Within the context of the Standard Model, parity-violating electron-proton scattering can be used to study the structure of the nucleon. The leptonic couplings are completely determined while the hadronic couplings, expressed as form factors which describe the nucleon structure, are the objects to be studied by experiments.

The electromagnetic vector current for the proton can be expressed in terms of its two electromagnetic form factors, $F^v_1$ and $F^v_2$:

$$< p | J^v_\mu | p > = \bar{u}_p \left( \gamma_\mu F^v_1(Q^2) + \frac{i \sigma_\mu \nu q^\nu}{2 M_p} F^v_2(Q^2) \right) u_p$$

where $u_p$ is the proton spinor.

The existence of the neutral weak boson, the Z$^0$, provides a new current for the proton:

$$< p | J^Z_\mu | p > = \bar{u}_p \left( \gamma_\mu F^Z_1(Q^2) + \frac{i \sigma_\mu \nu q^\nu}{2 M_p} F^Z_2(Q^2) + \gamma_\mu \gamma_5 G^Z_A(Q^2) \right) u_p$$

where $F^Z_1$ and $F^Z_2$ are the neutral weak vector form factors (analogous to the electromagnetic ones), and $G^Z_A$ is the neutral weak axial vector form factor. The weak form factors $F^Z_1$ and $F^Z_2$ are fundamental quantities that are presently completely unknown (except for the constraint that $F^Z_1(0) = 0$).

The asymmetry for the scattering of longitudinally polarized electrons from an unpolarized proton target may be used to measure these weak form factors. It will be convenient to define Sachs form factors via:

$$G^E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$
$$G^M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

where $\tau = Q^2 / 4M_p^2$.

In lowest order, the asymmetry is determined by the interference terms between the electromagnetic and the neutral weak amplitudes. Assuming strong isospin, and allowing for the presence of strange quarks, the weak electric and magnetic form factors are:\textsuperscript{[18]}

$$G^E_0(Q^2) = (1 - 4 \sin^2 \theta_W) G^E_1(Q^2) - G^E_2(Q^2) - G^{(s)}_E(Q^2)$$
$$G^M_0(Q^2) = (1 - 4 \sin^2 \theta_W) G^M_1(Q^2) - G^M_2(Q^2) - G^{(s)}_M(Q^2)$$

(3)
The elastic parity-violating asymmetry then takes the form:[16]

$$A = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-} = \frac{G_FM_p^2}{\sqrt{2}\pi\alpha} \left[(1 - 4\sin^2\theta_W) - \left(\frac{1}{\epsilon(G_E^p)^2 + \tau(G_M^p)^2}\right) \left\{ \frac{\epsilon G_E^p(G_E^p + G_M^{(s)}) + \tau G_M^p(G_M^p + G^{(s)})}{(G_E^p)^2 + \tau(G_M^p)^2} \right\} \right],$$

where $\epsilon^{-1} = 1 + \left(2|q|^2/Q^2\right)\tan^2\theta_e/2$ is the longitudinal polarisation of the virtual photon, the superscripts $p,n$ and $(s)$ denote the proton, neutron and strange form factors and the subscripts $E,M$ and $A$ indicate electric, magnetic and axial form factors. The expression simplifies considerably since, at SLAC kinematics, $\epsilon \geq 0.99$. We have:

$$A = \frac{G_FM_p^2}{\sqrt{2}\pi\alpha} \left[(1 - 4\sin^2\theta_W) - \frac{G_E^p(G_E^p + G^{(s)}) + \tau G_M^p(G_M^p + G^{(s)})}{(G_E^p)^2 + \tau(G_M^p)^2} \right].$$

The neglected axial contribution is at the 1% level. Furthermore, in the absence of strangeness, the longitudinal $(G_E^p)$ piece contributes only 5% to the asymmetry so that, a priori, a SLAC measurement is primarily sensitive to transverse components.

Jaffe[15] has derived forms for $G_E^{(s)}$ and $G_M^{(s)}$ based on an analysis of the proton elastic form factors.[17] In particular, he attributes the pole near 1 GeV to the $\phi$ meson. One of his results (Fit 7.1) is shown by the solid curves in Figure 1. At low $Q^2$, the form factors are characterized by two numbers: the strangeness radius $r_s$ and magnetic moment $\mu_s$. These are the goals of the lower $Q^2$ measurements. Of equal interest is the $Q^2$ dependence. Jaffe's analysis results in a fairly "radical" dependence in the sense that the strange form factors fall rather slowly with $Q^2$. However, his results should be considered as an expansion about $Q^2 = 0$[18] and must be modified to attain the $1/(Q^2)^4$ asymptotic behavior expected from quark counting rules. Indeed, a central physics issue is the $Q^2$ dependence of these form factors and when, or if, they attain their asymptotic $1/(Q^2)^4$ behavior. Although the proton elastic form factors go like $1/(Q^2)^2$ at fairly low $Q^2$ (around 6 GeV$^2$), it is by no means obvious that the strange form factors should approach their asymptotic behavior so early since, by their very nature, they involve $s\bar{s}$ loop diagrams where the applicability of perturbative concepts may be less valid. Therefore, we feel it is important that the proposed experiment sample two widely separated $Q^2$ points.

To exhibit the influence of different $Q^2$ dependencies, we followed the suggestion of Donnelly and Musolf[16] and gave the strange form factors a more normal, although ad hoc, dipole-like $Q^2$ dependence (but keeping the same values of $r_s$ and $\mu_s$). In particular, $G_E^{(s)}$ has the same form as $G_E^p$ (see eq. 8 below) and $G_M^{(s)}$ is a pure dipole. These are indicated by the dot-dashed curves in Figure 1. We also considered a pure $1/(Q^2)^4$ form as shown by the dashed curves. Finally, there is a Skyrme model prediction for the $Q^2 = 0$ values $r_s$ and $\mu_s$.[19] They find $(\mu_s, r_s^2) = (-0.13, 0.10 \text{ fm}^2)$ compared with Jaffe's values of $(-0.43, 0.16 \text{ fm}^2)$. These authors didn't give the $Q^2$ dependence.

If the electromagnetic form factors are known with sufficient precision, $G_E^{(s)}$ and $G_M^{(s)}$ may be determined by measuring $G_E^{(s)}(M)$. The electromagnetic form factors of the proton and
the magnetic form factor of the neutron have been measured to good precision, while the electric form factor of the neutron is poorly measured. More precise measurements of these form factors are either underway or planned. However, at high $Q^2$, the contribution of $G_E^n$ to the asymmetry is small, as will be shown below.

Figure 2 displays the asymmetry for several cases. The solid line includes the contribution from Jaffe's $G_E^{(s)}$ and $G_M^{(s)}$ while the dash-double-dot line excludes the contribution from $G_E^n$. The dot-dashed and long-dashed lines used the modified $G_E^{(s)}$ and $G_M^{(s)}$ discussed above. Finally, the short-dashed line is the Standard Model result. The dipole parameterization was used for $G_E^n$, $G_M^n$ and $G_M^n$:

$$G_E^n(Q^2) = G_M^n(Q^2)/\mu_p = G_M^n(Q^2)/\mu_n = \frac{1}{(1 + Q^2/M_V^2)^2}$$

with $M_V = 0.84$ GeV, $\mu_p = 2.79$ and $\mu_n = -1.91$. For $G_E^n$ we used following parameterization:[20]

$$G_E^n(Q^2) = -\mu_n \sigma G_E^n(Q^2) \xi(Q^2)$$

with $\xi(Q^2) = 1/(1 + 5.87)$. Because the contribution of $G_E^n$ to the asymmetry is small, use of other parameterizations will not change the asymmetry significantly. Note that even if the strange form factors have the considerably more rapid fall-off with $Q^2$ given by the dot-dashed curve, the proposed experiment can still readily distinguish this from the NSQSM result. This can be seen somewhat more clearly in Figure 3 where we show the percentage deviation from the NSQSM case.

A proper interpretation of precision measurements of $p(e, e')$ requires that one take electroweak radiative corrections into account. According to the calculations of Musolf and Holstein,[21] the weak electric, magnetic and axial form factors will be modified by radiative corrections. The radiative correction factors, which enter as $1 + R$, for $G_E^{p,M}$, $G_M^{p,M}$
Figure 2. Parity-violating asymmetry vs. $Q^2$ at $E_0 = 12.9$ GeV. Solid curve: $G_E^{(s)}$ and $G_M^{(s)}$ from Jaffe.\(^1\) Dot-dashed and long-dashed curves: $G_E^{(s)}$ and $G_M^{(s)}$ with modified $Q^2$ dependence (see text). Short-dashed curve: Standard Model result with $G_E^{(s)} = G_M^{(s)} = 0$. Dash-double-dot curve: $G_E^{(s)} = G_M^{(s)} = G_E^s = 0$. Representative data points for 35 days of running are shown at $Q^2 = 0.5$ and 2.0 GeV$^2$. The open diamond would be the $(e,p)$ result for the nominal solid angle of the 1.6 GeV spectrometer. For comparison, the open circle shows the projection for $(e,e')$ at $E_0 = 16.2$ GeV and $\theta_s = 5.5^\circ$.

Figure 3. Percentage change from the no-strange-quarks case for each of the models shown in Figure 2. Curves have the same meaning as in Figure 2. Representative data points for 35 days of running are shown at $Q^2 = 0.5$ and 2.0 GeV$^2$. The open diamond would be the $(e,p)$ result for the nominal solid angle of the 1.6 GeV spectrometer. For comparison, the open circle shows the projection for $(e,e')$ at $E_0 = 16.2$ GeV and $\theta_s = 5.5^\circ$. 

(6)
and $G_{E,M}^{(a)}$ are $R_{V}^{p} = -0.33$, $R_{V}^{o} = 0.015$ and $R_{V}^{(0)} = -0.015$ respectively for top quark and Higgs masses of $M_t = 120$ GeV and $M_H = 100$ GeV. The axial correction is considered to be both large and theoretically uncertain: $R_A = -0.24 \pm 0.22$.\(^{[21]}\)

The radiative corrections reduce the tree-level asymmetry by a small amount: 1.8\%-3.3\%. We note that the hadronic uncertainty in $R_{V}^{p}$ ($\delta R_{V}^{p} = \pm 0.01$) has been neglected. However, this hadronic uncertainty in $R_{V}^{o}$ was estimated only from the hadronic correction to the $Z^0 - \gamma$ two-boson-exchange graph, while contributions from other intermediate states were not included, so that the error in $R_{V}^{o}$ could be larger. Regardless, electroweak radiative corrections are likely to be well below the statistical uncertainty of the proposed measurement. Furthermore, by working at forward electron angles, we are insensitive to the axial radiative correction that has the largest theoretical uncertainty.

III. Inelastic Scattering

Data on inelastic $p(\vec{e}, e')$ would be acquired simultaneously with the elastic data. Perhaps the most interesting resonance, and the only one for which theoretical calculations are presently available, is the $\Delta$. To the extent that the $\Delta$ region consists of only the isovector resonance, there can be no contribution from the purely isoscalar current of strange quarks.\(^{[18]}\) Of course, any isoscalar background beneath the $\Delta$ could be affected by strange quarks and, in any event, this background must be understood for a proper interpretation of the results. Given this caveat, the $N \rightarrow \Delta$ transition may be used to measure the Standard Model couplings, in particular the electron axial vector–hadron vector isovector one. This may be seen by considering the phenomenological Lagrangian:\(^{[22]}\)

\[
\mathcal{L} = \frac{G_F}{\sqrt{2}} \left[ \bar{\epsilon} \gamma_\mu \gamma_5 e \left( \frac{\tilde{\alpha}}{2} (\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d) + \frac{\tilde{\gamma}}{2} (\bar{u} \gamma_\mu u + \bar{d} \gamma_\mu d) \right) \\
+ \bar{e} \gamma_\mu \left( \frac{\tilde{\beta}}{2} (\bar{u} \gamma_5 \gamma_\mu u - \bar{d} \gamma_5 \gamma_\mu d) + \frac{\tilde{\delta}}{2} (\bar{u} \gamma_5 \gamma_\mu u + \bar{d} \gamma_5 \gamma_\mu d) \right) \right] \tag{9}
\]

where $\tilde{\alpha}$ and $\tilde{\gamma}$ are the isovector and isoscalar couplings of the axial electron current to the vector quark current while $\tilde{\beta}$ and $\tilde{\delta}$ are the corresponding isospin components for a vector electron current coupling to an axial quark current. There could also be additional contributions from strange quarks. The Standard Model specifies all of the above couplings in terms of $\sin^2 \theta_W$ and $\rho = M_W / M_Z \cos^2 \theta_W$. The values of the coupling constants in the Standard Model are given by:

<table>
<thead>
<tr>
<th></th>
<th>Standard Model</th>
<th>$\sin^2 \theta_W = 0.23$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\alpha}$</td>
<td>$-\rho(1 - 2 \sin^2 \theta_W)$</td>
<td>$-0.54$</td>
</tr>
<tr>
<td>$\tilde{\beta}$</td>
<td>$-\rho(1 - 4 \sin^2 \theta_W)$</td>
<td>$-0.08$</td>
</tr>
<tr>
<td>$\tilde{\gamma}$</td>
<td>$\frac{2}{3} \rho \sin^2 \theta_W$</td>
<td>$0.15$</td>
</tr>
<tr>
<td>$\tilde{\delta}$</td>
<td>$0.$</td>
<td>$0.00$</td>
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(7)
The parity-violating asymmetry of the $N \rightarrow \Delta$ transition takes the form: \[^{[23][24][25]}\]

$$A = \frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \left[ \tilde{\alpha} + \tilde{\beta} F(Q^2, s) \right] ,$$

where $F(Q^2, s)$ is a weak structure function of the $N \rightarrow \Delta$ transition. \[^{[24]}\] To the extent that non-resonant background underneath the $\Delta$ is neglected (or at least understood), the $N \rightarrow \Delta$ transition isolates the isovector couplings. In the high-energy limit ($Q^2 < 2M E$) Cahn and Gilman\[^{[25]}\] showed that the structure function is $\ll 1$ so that the $\tilde{\beta}$ term is negligible (also, $\tilde{\beta} < \tilde{\alpha}$) so that an $N \rightarrow \Delta$ measurement is almost a pure $\tilde{\alpha}$ measurement.

We note that, of the two axial electron-vector quark couplings, $\tilde{\alpha}$ and $\tilde{\gamma}$, it is $\tilde{\alpha}$ which is the least well known. Atomic parity-violation\[^{[26]}\] (APV) and the Bates $^{12}$C($e,e'$) experiment\[^{[27]}\] were almost purely $\tilde{\gamma}$ ones; the best limit on $\tilde{\alpha}$ comes from the original SLAC parity experiment of Prescott et al.\[^{[2]}\] and is $-0.67 \pm 0.19$, a 30% result. Figure 4 shows the limits in the $\tilde{\alpha}-\tilde{\gamma}$ plane from the SLAC and APV experiments.

![Figure 4. 90% confidence limits contours in the $\tilde{\alpha}-\tilde{\gamma}$ plane from the SLAC and APV experiments.](image)

Better information on $\tilde{\alpha}$ would allow a more precise test of an equality between the coupling measured in reactions induced by charged leptons and ones induced by neutrinos. \[^{[22]}\] However, these equalities are modified by higher-order electroweak processes since some, such as box diagrams containing one $Z^0$ and one photon, are absent for neutrino reactions. \[^{[28]}\] At tree level one should have:

$$\left( \frac{\tilde{\gamma}}{\tilde{\alpha}} \right)_{\nu_q} = \left( \frac{\tilde{\gamma}}{\tilde{\alpha}} \right)_{e_q}$$

The current limit\[^{[29]}\] for the right-hand-side is $-0.213 \pm 0.060$ which we could improve by a factor of 2. Given the present successes of the Standard Model, such an improvement would be primarily testing the role of higher-order processes.

Since the $N \rightarrow \Delta$ cross section is comparable to the elastic, essentially the same statistics will be obtained. Figure 5 shows the ($e,e'$) cross section\[^{[30]}\] on the proton for $E_0 = 12.9$ GeV
Figure 5. The (e,e') cross section vs. energy transfer $\omega$ at $E_0 = 12.9$ GeV and $\theta_e = 6.5^\circ$. The individual resonances are indicated by short-dashed curves while the deep inelastic contribution is given by the dot-dashed curve. The inset shows the (e,\pi^-) cross section.

and $\theta_e = 6.5^\circ$. The SM asymmetry is comparable to the elastic so that similar precision (15%) would result.

Finally, we note that it appears that the non-resonant contribution may indeed be small, at or below the 10% level. This is based on a comparison of the Cahn-Gilman pure $\Delta$ calculation to that of Li, Henley and Hwang\cite{31} who computed the PV asymmetry in pion electroproduction including all Born diagrams, $\rho$ and $\omega$ meson contributions as well as the $\Delta$. Even though the latter results are for low energies ($\sim 800$ MeV) where the the high-energy limit ($Q^2 \ll 2ME$) required by Cahn and Gilman does not really apply, the two calculations are within 10% of each other over the $Q^2$ range 0.04–0.53 GeV$^2$. While not definitive, it does give confidence in the interpretability of our proposed measurement. This is further strengthened by the calculations of Pollock\cite{32} who finds that, at 4 GeV, the background terms contribute less than 5% to the total asymmetry and the axial ($\tilde{A}$) term only 3%. The latter contribution is also a decreasing function of energy, going like $1/\sqrt{E_0}$.

We emphasize that, for both the elastic and inelastic measurements, any deviation from the no-strange-quarks/tree-level Standard Model would be extremely interesting, whether due to strange quarks or not. Our measurement would be sensitive enough to reveal even modest deviation. It must also be viewed in the context of the lower energy parity measurements planned for Bates and CEBAF. A systematic analysis of all of the data promises to significantly increase our understanding of hadronic structure.

IV. Experimental Plan

Our initial plan is to measure, in End Station A, two $Q^2$ points simultaneously: one at low $Q^2$ (0.5 GeV$^2$) and another at high $Q^2$ (2.0 GeV$^2$). We propose using a beam energy of 12.9 GeV and detecting electrons at 6.5$^\circ$ and recoil protons at 67.9 degrees.

We must use a multiple of 3.237 GeV beam energy to ensure longitudinal polarization in the End Station. We have chosen 12.9 GeV as a compromise between statistics, running
The elastic (e,p) cross section at \( Q^2 = 0.5 \text{ GeV}^2 \) is 110 nb/sr which results in an average rate of 37,870 Hz and a per-pulse rate of 316 for a solid angle of 9.0 msr and 13.5 cm target length acceptance. The (e,p) kinematics are indicated in Figure 6.

To achieve an absolute statistical uncertainty \( \Delta A_m \) in the measured asymmetry, the total number of counts needed is:

\[
N_t = \frac{1}{(P_t \Delta A_m)^2}
\]  

(12)
Figure 6. Proton kinetic energy (left-hand scale) and $Q^2$ (right-hand scale) vs. proton angle for elastic $(e,p)$ scattering at 12.9 GeV. The symbol indicates the proposed kinematics.

A 16% measurement of the Standard Model asymmetry implies $\Delta A_m = 2 \times 10^{-5}$ and $N_t$ of order $5 \times 10^{10}$ at the high $Q^2$ point. This results in a 22% measurement for the low $Q^2$ $(e,p)$ point (even though the asymmetry is smaller the solid angle is considerably larger). If there is indeed as large a strangeness contribution as predicted by Jaffe, the asymmetry will be much larger. In this case, we will implement an optional run plan and measure asymmetries from $Q^2$ of 1.5 to 3.5 GeV$^2$ with 5% uncertainties. We would utilize beam energies of 12.9 and 16.2 GeV. The total time request would remain the same.

V. Experimental Apparatus

Major systems for the parity-violation experiment include electron and proton spectrometers, detectors, a liquid hydrogen target, a polarized electron source, an electron polarimeter and, if necessary, the feedback systems described below. Fortunately, a good deal of the equipment is, or will be, available for use in other experiments. Therefore, this experiment will entail relatively low cost.

V.1 Spectrometers

For our measurements, we would like to have an electron spectrometer with reasonably large solid angle ($\geq 0.5$ msr) and sufficient resolution ($\leq 0.25\%$) to resolve the elastic peak from the inelastic background. It should be able to reach small scattering angle ($\leq 7.0^\circ$). Both momentum and angular foci are necessary so as to maintain good momentum resolution in the presence of kinematic broadening resulting from the finite angular acceptance.

We intend to use the 16 GeV/c spectrometer proposed for the color transparency/$z > 1$ experiment\cite{84}. The small angle mode of this spectrometer achieves the desired qualities. Its layout is shown in Figure 7. The properties of this device are:

- Momentum acceptance: $\pm 5\%$
- Maximum momentum: 16 GeV/c
- Minimum angle: 6.5°
- Resolution: ≤ 0.18% (D/M=0.67)
- Solid angle: 0.80 msr
  - Vertical angle acceptance: ±20 mr
  - Horizontal angle acceptance: ±10 mr
- Target length acceptance (at 6.5°): 35 cm

Figure 7. Layout of the proposed 16 GeV/c spectrometer.

The primary criterion is that there is sufficient separation between elastic scattering and pion threshold. We have performed a Monte-Carlo calculation of the electron trajectories in the 16 GeV spectrometer. Figure 8 shows the z − y positions at the detector location for elastic scattering and pion threshold (modelled as a discrete state). The strength at pion threshold was set equal to the elastic in order to make the resolution more obvious; we emphasize that the actual strength in the first 18 MeV above pion threshold is only 4% of the elastic. We took the beam energy spread to be flat over ±0.2% (this is conservative) and ±20 mrad scattering angle acceptance. At high energy, energy loss in the target is almost entirely radiative (bremsstrahlung). This was modelled using the Heitler equation. The optics of the spectrometer were treated using second-order TRANSPORT whose accuracy was verified by a RAYTRACE calculation. We assumed that the detector elements could be positioned within ±1.0 mm. As the figure shows, there is adequate separation between the elastic and inelastic events. There is a similar separation between the Δ and higher resonances so that the Δ will be adequately resolved.

The 1.6 GeV/c spectrometer will be used for recoil protons. While its target length acceptance of 12 cm and 0.1% momentum resolution are adequate, the 3 msr solid angle would result in a 38% statistical error bar (for the NSQSM case). It would, therefore, be advantageous to increase the solid angle by means of front quadrupoles as was done in the NE11 experiment. A solid angle of 9 msr can be achieved, leading to a much better statistical precision of 22%.
Figure 8. Monte-Carlo simulation of scattered electron trajectories in a detector oriented perpendicular to the central ray. The dots (plusses) indicate elastic (inelastic) events.

There is some recent experience at SLAC with (e,p) elastic scattering in this momentum range. During NE18, recoil protons were detected in the 1.6 GeV spectrometer. Although the beam energy was only 2.4 GeV, the $Q^2$ of 0.40 GeV$^2$ and proton angle of 63.5° are comparable to our conditions. The (e,p) cross section from that run is shown in Figure 9.[27] The protons were detected in a segmented scintillator array (which also saw positive pions). It is clear that the elastic peak is easily resolved. The solid line represents the sum of the inelastic (e,p) and (e,$\pi^+$) cross sections.[30]

It may also be possible to use the 8 GeV/c spectrometer in the “reverse-quad” mode that was employed in experiments E140, NE17 and NE18. This tune achieved 3.5 mrad and a 20 cm target acceptance (for a 30 cm target the average solid angle would be about 2.5 mrad). We would obtain a 27% statistical error bar with this apparatus.

V.II Detectors

Measurement of small asymmetries is usually accomplished by integration rather than particle counting techniques. This allows high rates to be easily handled and avoids the potential problem of helicity-correlated deadtime effects. We intend to construct UVT Lucite Cherenkov detectors for the electron spectrometer. These are preferred over shower counters since transverse leakage would be a problem in the latter. Although a Lucite detector cannot identify electrons from pions, one expects extremely little $\pi^-$ contribution at these kinematics as the inset in Figure 5 shows.

Unlike a scintillator with a large Landau tail in its energy loss response, the nearly Gaussian response of a Lucite detector results in essentially no pulses whose height greatly exceeds the mean value. This alleviates the problem of false asymmetries caused by electrons from one helicity state producing a few more very large pulse heights than electrons from the other helicity state and thereby degrading the statistics. This type of detector was used successfully in the Bates parity experiment.[27]
Figure 9. Elastic $(e,p)$ cross section measured during NE18. The recoil protons were detected in the 1.6 GeV spectrometer. The elastic peak is clearly resolved. The solid line represents the sum of the inelastic $(e,p)$ and $(e,\pi^+)$ cross sections.

The elastic $(e,p)$ rates for the low $Q^2$ point are in the 100/pulse range and the protons have energies around 250 MeV. These would produce large signals in plastic scintillators. To avoid large fluctuations in light output due to energy straggling it will be necessary to have sufficiently thick scintillators to stop these protons. Taking the $\pm 5\%$ momentum acceptance into account, Figure 10 indicates that 30 cm of NE-102 plastic will stop all protons entering the 1.6 GeV spectrometer. We will build up this thickness using a sequence of $\sim 2$ cm pieces. Only the region of the elastic peak would be instrumented.

Figure 10. Proton kinetic energy as a function of thickness in NE-102 plastic scintillator.

(14)
V.III Target

A 30 cm long liquid hydrogen target has been used in several previous SLAC experiments\cite{14} and we plan to use the same target. Our power requirement is less than some of the previous experiments. We do not anticipate any problems with the target. Previous experience indicates that density fluctuations due to beam heating are not significant although any helicity-dependence to the target thickness will need careful examination. Valuable additional information will soon be available from the SAMPLE experiment\cite{11} at Bates where a 40 cm hydrogen target and a 40 $\mu$A beam will be used.

V.IV Polarized Electron Source and Polarization Measurement

The SLAC polarized electron source currently under development for experiments E142 and E143 is expected to reach 80% polarization.\cite{18} We will use the polarized source with moderate average beam current (about 10 $\mu$A). The beam polarization will be monitored by a Moller polarimeter which is also under development for E142 and E143.

VI. Systematic Errors

VI.1 Introduction

We first note that we are proposing a relatively “easy” parity experiment: our low $Q^2$ asymmetries are $\geq 10^{-5}$ which is almost 20 times the asymmetry successfully measured at Bates\cite{27} while the high $Q^2$ asymmetry is nearly 200 times larger. Furthermore, that experiment achieved a systematic error on the asymmetry of only $2 \times 10^{-8}$. For comparison, the recent Mainz experiment\cite{38} measured $A = (-3.5 \pm 0.7 \pm 0.2) \times 10^{-8}$ (statistical and systematic) while the original SLAC measurement also observed $\sim 10^{-4}$ asymmetries with systematic uncertainties of the order of 10%. As we discuss in detail below, it should be relatively straightforward, given both the experience at SLAC and Bates, to obtain systematic uncertainties at the $10^{-8}$ level.

In this type of asymmetry measurement, only one parameter, the electron helicity is changed and that change occurs rapidly, e.g., 120 Hz. The systematic errors associated with knowledge of absolute beam energy, absolute solid angle etc. cancel when the asymmetry ratio is formed. With careful design, systematics as low as 0.02 ppm have been achieved; this would be an 0.02% systematic effect for the asymmetries considered here. Clearly, we don’t have to work quite that hard.

A major source of systematic error is the fact that the beam parameters (energy, angle, etc.) do not remain constant under helicity reversal. One must therefore:

- Minimize the helicity-dependent difference in beam parameters. Feedback systems were successfully employed at both Bates and SLAC to stabilize the beam parameters.
- Correct for residual beam parameter differences. Accurate beam position monitors can provide sufficient information to correct for helicity-correlated differences in beam position, energy and angle.

It is important to realize that it is the beam properties relative to each helicity state, not the absolute ones, that must be monitored precisely. We would like the relative precision
in beam energy and angle to be such as to make the related cross section differences, i.e.,
the effect on the asymmetry essentially negligible. For that purpose, we choose a 1% effect
on $A$. Then, since the uncertainty $\delta A$ is given by:

$$\delta A = \frac{\sqrt{1 - A^2}}{2} \sqrt{\left(\frac{\delta \sigma_+}{\sigma_+}\right)^2 + \left(\frac{\delta \sigma_-}{\sigma_-}\right)^2},$$

(13)

we need to know the derivatives of $\sigma(e,e')$ with respect to $\theta_e$ and $E_0$. These are:

$$\frac{\delta \sigma}{\delta E_0} = 24.4 \text{ nb/sr/GeV}, \quad \frac{\delta \sigma}{\delta \theta_e} = 3.6 \text{ nb/sr/mrad},$$

at $E_0 = 12.9 \text{ GeV}$ and $\theta_e = 6.5^\circ$. Thus, to have a 1% effect from both $E_0$ and $\theta_e$ we must
know their helicity-dependent difference to: $|\delta E_0| = 2.7 \times 10^{-6} \text{ GeV}$ and $|\delta \theta_e| = 18.1 \times 10^{-6}
\text{ mrad}.$

At first glance, these seem like extremely difficult criteria. However, they are determined
by many difference measurements (where many systematic errors cancel) of beam position
monitors (BPMs). In such a measurement, the accuracy with which one knows the centroid
increases as the square root of the number of measurements. The earlier SLAC experiment
employed BPMs that gave $10 \mu m$ measurements per pulse, the accuracy being limited by
electronic noise. If one measures in a region where the dispersion is $1\% / \text{cm}$, then our
position requirement is $0.15 \mu m$ which, at $120 \text{ Hz}$, is achieved in $37 \text{ seconds}$. Similarly, if
we choose BPM positions as in the original SLAC parity experiment ($48 \text{ m}$ separation), we
achieve their result of $0.3 \mu rad$ per pulse angular resolution. Our criterion of $0.02 \mu rad$ is
met in $5 \text{ seconds}.$

The situation for the recoil proton measurement is essentially the same:

$$\frac{\delta \sigma}{\delta E_0} = 43.0 \text{ nb/sr/GeV}, \quad \frac{\delta \sigma}{\delta \theta_e} = 9.9 \text{ nb/sr/mrad},$$

Thus, similar results will be achieved for both the $(e,e')$ and $(e,p)$ measurements. Of course,
due to the smaller asymmetry in the $(e,p)$ case, the systematics will constitute a larger
percentage error. However, time limitations already impose a higher statistical uncertainty
so that the larger contribution of systematics is acceptable.

VI.II PITA Effect

One outcome of the Bates experience was that beam intensity differences were dominated
by intensity differences in the laser beam at the source. These are converted into energy
differences by beam loading in the accelerator and, since the beam transport is not perfectly
achromatic, also into position and angle differences. The resulting effect is known as the
Polarization Induced Transport Asymmetry (PITA) effect.

This is potentially a quite serious source of systematic effects. However, a feedback system
which regulated the voltage on the Pockels cell was successfully developed at Bates.\cite{40} This
system was able to maintain false asymmetries at the 1 ppm level for long time scales. The
effect of the system is indicated in Figure 11 which shows the false asymmetry with and
without feedback. Since the source conditions at SLAC are similar to those at Bates, such
a system could readily be implemented. We would first conduct studies to determine the
size of the PITA effect to see whether a feedback system is dictated.
Figure 11. Helicity correlated false asymmetry due to PITA effect. Top: Without feedback. Bottom: with feedback. The horizontal lines indicate the time-averaged value which, when the feedback was activated, was 0.9 ± 1.0 ppm.

VI.III Calibration and Correction

If the monitoring system reads non-zero helicity correlated differences, then the measured asymmetry $A_m$ must be corrected to obtain the physics asymmetry. This is accomplished as follows. The physics asymmetry may be written as:

$$A_p = A_m - \sum_i a_i \delta M_i,$$

where the $\delta M_i$'s are the helicity correlated monitor differences and the $a_i$ are the (calibrated) coefficients which are also $a_i = \frac{\partial \sigma}{\partial \delta M_i}$.

At Bates, the $a_i$ were calibrated by pulsing a set of steering coils in a known manner. These would change the beam on target in such a way as to provide controlled energy or angle changes. The resulting asymmetry and cross section was then recorded. This allows $\frac{\partial \sigma}{\partial C_i}$, where $C_i$ is the $i^{th}$ coil, to be determined. Then we have:

$$\frac{\partial \sigma}{\partial C_i} = \sum_j \frac{\partial \sigma}{\partial M_j} \frac{\partial M_j}{\partial C_i} = \sum_j a_j \frac{\partial M_j}{\partial C_i},$$

and the $a_j$ are found by matrix inversion. A crucial feature of the Bates implementation was that coil pulsing occurred simultaneously with data taking (by introducing a 'blank' pulse at 47 Hz). This allows systematic effects to be determined under conditions identical to those with which the data are taken. A typical result for the measured and corrected asymmetries is shown in Figure 12. In this case, the correction was for the helicity correlated energy difference.
VI. IV Additional Systematic Effects

Due to helicity-correlated changes in beam position, there could be helicity-correlated changes in target length. If the spectrometer is appropriately collimated, these will not affect the count rate directly. They will, however, affect the radiative correction for each helicity state. If the average offset from dead center is denoted by $y_0$ and the helicity-correlated difference in beam position by $\delta y$, then the helicity-correlated change in target length $\delta L$ is:

$$\delta L = \frac{y_0 \delta y}{R}$$

where $R$ is the radius of curvature of the entrance window. This generates a change in the asymmetry due to the radiative correction $R_c$:

$$\delta A = \left| \frac{1}{R_c} \frac{\partial R_c}{\partial L} \right| \delta L$$

To keep this at the 1% level, we must have $y_0 \delta y \leq 1.2 \times 10^{-4}$ cm$^2$ for $R = 1$ cm. This implies that we must know both $y_0$ and $\delta y$ to 0.1 mm which is the per pulse accuracy of the BPMs. Thus, this source of false asymmetry should present no problems.

A horizontal beam position shift will also change the solid angle to first order (vertical shifts affect it only in second order). If $\delta x$ is the horizontal shift, then $\delta \Omega/\Omega = (\delta x/D) \sin \theta_s$ where $D$ is the distance from the target to the solid angle defining aperture. If $D$ were as small
as 1 m, then keeping this effect to 1% would require $\delta z \leq 14 \mu m$, again at the level of the single pulse resolution of the BPMs.

A transverse component of the beam polarization can in principle produce an asymmetry via the electromagnetic process of Mott scattering. However, this asymmetry goes to zero as $\theta_s \to 0$ and decreases like $1/E_0$ so that at our kinematics it is $3 \times 10^{-10}$ which is totally negligible.

VI. V Verification

The primary means of verification would be to take half of our production data with a half-wave plate in the source to change the sign of the electron polarization. The resulting two data sets should have the same magnitude, but opposite sign, physics asymmetries. Any residual difference is a measure of the false asymmetry.

There are at least three additional ways to check for false asymmetries, all of which were successfully employed in the original SLAC parity experiment.

- Make a measurement using the unpolarized beam from the regular SLAC source.
- Make measurements by varying the laser polarization at the source. In the original experiment, this was done by rotating the calcite prism. At 45°, it produces unpolarized electrons.
- Make measurements at a nearby energy where the precession angle is an odd multiple of $\pi/2$ so that the beam in the End Station is transversely polarized.

There is a serious time issue here. Using each of the above methods to measure the elastic asymmetry to the same statistical accuracy as for the physics run would require at least four times as much beam time. On the other hand, they provide truly definitive information. We favor the second method above since it produces conditions closest to those under which the actual data are taken. Therefore, we propose to perform this check in the deep inelastic region. By tuning the spectrometer to this region, we would achieve a counting rate 12 times that of the elastic (see $(e,e')$ cross section plot in Figure 5). This would allow us to take comparable statistics to the elastic in a few days and give us high confidence in the measured elastic asymmetries.

VII. Summary of Beam Request

In preparing our time estimates we assumed the following parameters:

- Electron solid angle: 0.8 msr
- Proton solid angle: 9.0 msr
- Average current: 10 $\mu$A
- Liquid hydrogen target length & thickness: 30 cm & 2.1 gm/cm$^2$
- Luminosity (e): $8.0 \times 10^{37}$/cm$^2$/s (30 cm target acceptance)
- Luminosity (p): $3.6 \times 10^{37}$/cm$^2$/s (13.5 cm target acceptance).
- Radiative correction factor: 1.7.
Electron beam polarization: 80%

A significant amount of time will be necessary prior to production data taking to ensure proper operation of beam position monitors, target system, polarized source, spectrometer and detectors. The outcome of the above tests would be to demonstrate that the entire system is working at the necessary level to make $10^{-6}$ asymmetry measurements. We allocated 6 weeks (40 days) for these tests. Contingent upon the successful completion of these tests, we require a total of 35 days (with a 43% contingency for machine efficiency etc.) of dedicated production running. The time requirements are summarized below:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Setup, check-out and calibrations</td>
<td>40 days</td>
</tr>
<tr>
<td>Data Acquisition</td>
<td>35 days</td>
</tr>
<tr>
<td>Contingency (43%)</td>
<td>15 days</td>
</tr>
<tr>
<td>Total</td>
<td>90 days</td>
</tr>
</tbody>
</table>

(*) Assumes $G_E^{(a)} = G_M^{(a)} = 0$, includes radiative correction.
References

R.D. McKeown and E.J. Beise, Comm. Nucl. Part. Phys. 20, 105 (1990);
[34] SLAC proposal E145