Sensitivities of one-prong tau branching fractions to tau neutrino mass, mixing, and anomalous charged current couplings

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Workshop on the $\tau$-charm Factory

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Introduction

We analyse the sensitivity to new physics of the $\tau$ partial widths for the following decays:

- $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$
- $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$
- $\tau^- \rightarrow \pi^- \nu_\tau$
- $\tau^- \rightarrow K^- \nu_\tau$

We consider the effects of the following:

- mass $m_{\nu_3}$ of the third generation neutrino $\nu_3$
- mixing of $\nu_3$ with a 4th generation neutrino $\nu_4$ (mass $> M_Z/2$) (kinematically forbidden $\Rightarrow$ suppression of decay rate)
- anomalous weak charged current magnetic dipole coupling ($\kappa$)
- anomalous weak charged current electric dipole coupling ($\bar{\kappa}$)
- Michel parameter $\eta$
Theoretical Predictions ($m_{\nu_\tau}$ and $\sin^2 \theta$)

\[ B_{\ell \text{th.}} = \frac{G_F^2 m_{\tau}^5}{192\pi^3} \left( 1 - 8x - 12x^2 \ln x + 8x^3 - x^4 \right) \]
\[ \times \left[ \left( 1 - \frac{\alpha(m_{\tau})}{2\pi} \left( \pi^2 - \frac{25}{4} \right) \right) \left( 1 + \frac{3}{5} \frac{m_{\tau}^2}{m_W^2} \right) \right] \]
\[ \times \left[ 1 - \sin^2 \theta \right] \left[ 1 - 8y(1 - x)^2 + \cdots \right] \]  

1. $x = m_\ell^2/m_{\tau}^2$, $y = m_{\nu_3}/m_{\tau}$
2. QED coupling, $\alpha(m_{\tau}) \simeq 1/133.3$
3. W mass, $m_W = 80.41 \pm 0.10$ GeV
4. ellipsis denotes negligible higher order terms

1st square brackets: radiative corrections
2nd square brackets: mixing of $\nu_3$ and $\nu_4$ ($\nu_{\tau} = \cos \theta |\nu_3\rangle + \sin \theta |\nu_4\rangle$)
3rd square brackets: phase-space suppression for $m_{\nu_3} > 0$
**Theoretical Predictions (\(m_{\nu_{\tau}} \text{ and } \sin^2 \theta\))**

**Branching fractions for \(\tau^- \rightarrow h^- \nu_{\tau}\), with \(h = \pi/K\)**

\[
B_{h_{\text{th.}}}^3 = \frac{G^2 m_{\tau}^3}{16 \pi} \tau_{\text{F}} f_h^2 |V_{\alpha \beta}|^2 (1 - x)^2 \left(1 + \frac{2\alpha}{\pi} \ln \frac{m_2}{m_{\tau}} + \cdots\right) \\
[1 - \sin^2 \theta] \left[1 - y \left(\frac{2 + x - y}{1 - x}\right) \left(1 - \frac{y(2 + 2x - y)}{(1 - x)^2}\right)^{\frac{1}{2}}\right]
\]

(2)

- \(x = m_h^2 / m_{\tau}^2\),
- \(f_{\pi} |V_{ud}| = (127.4 \pm 0.1)\text{MeV} \) (from \(\pi^- \rightarrow \mu^- \bar{\nu}_\mu\))
- \(f_K |V_{us}| = (35.18 \pm 0.05)\text{MeV} \) (from \(K^- \rightarrow \mu^- \bar{\nu}_\mu\))
- ellipsis represents missing terms estimated to be \(O(\pm 0.01)\)

**1st term in square brackets:** mixing with a fourth generation neutrino

**2nd term in square brackets:** phase-space suppression for \(m_{\nu_{\tau}} > 0\)

Additional constraint from non-threshold determinations of tau-mass, e.g. CLEO analysis of \(\tau^+ \tau^- \rightarrow (\pi^+ n \pi^0 \bar{\nu}_\tau)(\pi^- m \pi^0 \nu_{\tau}) \) (with \(n \leq 2, m \leq 2, 1 \leq n + m \leq 3\))

\[m_{\tau} = (1777.8 \pm 0.7 \pm 1.7) + \frac{[m_{\nu_{\tau}}(\text{MeV})]^2}{1400 \text{MeV}}\]
Theoretical Predictions \((\kappa, \tilde{\kappa}, \eta)\)

Anomalous dipole moment couplings described by effective Lagrangian

\[
\mathcal{L} = \frac{g_{\pi}}{\sqrt{2}} \left[ \gamma_{\mu} + \frac{i\sigma_{\mu\nu}}{2m_{\tau}} (\kappa_{\tau} - i\tilde{\kappa}_{\tau} \gamma_{5}) \right] P_{\mu} \nu_{\tau} W^{\mu} + \text{(Hermitian conjugate)},
\]

Theoretical predictions for branching fractions \(B_{\ell}\) for \(\tau^{-} \rightarrow \ell^{-}\bar{\nu}_\ell \nu_{\tau}(X_{EM})\), with \(\ell^- = e^-, \mu^-\) and \(X_{EM} = \gamma, \gamma\gamma, e^+e^-, \ldots\)

\[
B_{\ell}^{\text{th.}} = \frac{G_{F}^{2} m_{\tau}^{5} r_{\tau}}{192\pi^{3}} \left(1 - 8x - 12x^{2}\ln x + 8x^{3} - x^{4}\right) \\
\times \left(1 - \frac{\alpha(m_{\tau})}{2\pi} \left(\pi^{2} - \frac{25}{4}\right)\right) \left(1 + \frac{3}{5} \frac{m_{\tau}^{2}}{m_{W}^{2}}\right) [1 + \Delta_{\ell}]. \tag{3}
\]

Effects of new physics parametrised by \(\Delta_{\ell}\)

\[
\begin{align*}
\Delta_{\ell}^{\kappa} &= \frac{\kappa}{2} + \frac{\kappa^{2}}{10} \tag{4} \\
\Delta_{\ell}^{\tilde{\kappa}} &= \frac{\tilde{\kappa}^{2}}{10} \tag{5} \\
\Delta_{\ell}^{\eta} &= 4\eta_{\tau}\ell\sqrt{x} \tag{6}
\end{align*}
\]

Both leptonic tau decay modes probe are sensitive to \(\kappa\) and \(\tilde{\kappa}\)

Only \(\tau^{-} \rightarrow \mu^{-}\bar{\nu}_\mu \nu_{\tau}\) sensitive to \(\eta\) (relative suppression of \(m_{e}/m_{\mu}\) for \(\tau^{-} \rightarrow e^{-}\bar{\nu}_e \nu_{\tau}\))

Semi-leptonic tau branching fractions are not sensitive
 Fits

Three sets of fits are performed

- **Case 1**
  Use current world averages of the experimental measurements.

- **Case 2**
  Use estimated errors on measurements at a $\tau cF$
  (assume no improvement in the tau lifetime error)

- **Case 3**
  Use estimated errors on measurements at a $\tau cF$
  (and suppose CLEO/b-factories reduce tau lifetime error by $\times 2$)

For Cases 2 and 3 the central values are unknown
⇒ adjust branching fractions to their SM central values, so predictions are not arbitrarily biased by current experimental central values.
Input Parameters

<table>
<thead>
<tr>
<th>Value</th>
<th>Future Error</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_\tau$ (MeV)</td>
<td>$1776.96^{+0.31}_{-0.27}$ (BES)</td>
<td>$0.1$ (Marbella)</td>
<td>$0.1$ (Marbella)</td>
<td></td>
</tr>
<tr>
<td>$\tau_\tau$ (fs)</td>
<td>$290.5 \pm 1.0$ (TAU98)</td>
<td>$1.0$ (TAU98)</td>
<td>$0.5$ (our hope)</td>
<td></td>
</tr>
<tr>
<td>$B_\pi$ (%)</td>
<td>$17.81 \pm 0.06$ (TAU98)</td>
<td>$0.018$ (Marbella)</td>
<td>$0.018$ (Marbella)</td>
<td></td>
</tr>
<tr>
<td>$B_\mu$ (%)</td>
<td>$17.36 \pm 0.06$ (TAU98)</td>
<td>$0.017$ (Marbella)</td>
<td>$0.017$ (Marbella)</td>
<td></td>
</tr>
<tr>
<td>$B_\rho$ (%)</td>
<td>$11.08 \pm 0.13$ (TAU98)</td>
<td>$0.011$ (Marbella)</td>
<td>$0.011$ (Marbella)</td>
<td></td>
</tr>
<tr>
<td>$B_K$ (%)</td>
<td>$0.695 \pm 0.026$ (TAU98)</td>
<td>$0.003$ (Marbella)</td>
<td>$0.003$ (Marbella)</td>
<td></td>
</tr>
</tbody>
</table>

Fits for $m_{\nu_\tau}$ and $\sin^2 \theta$

- Combined likelihood fits to all four tau decay channels
- Include constraint from CLEO tau mass determination

Fits for $\kappa$, $\bar{\kappa}$, and $\eta_{T\mu}$

- Combined likelihood fits two leptonic tau decay channels
- Semi-leptonic decays are not sensitive

Each parameter is analysed separately

- conservatively assume in each case that the other four parameters are zero
### Results

**Constraints on \( m_{\nu_3} \), \( \sin^2 \theta \), \( \kappa \), \( \bar{\kappa} \), and \( \eta_{\tau\mu} \) (95% C.L.)**

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(now)</td>
<td>(( \tau CF, \sigma_{\tau} = 1.0 \text{ fs} ))</td>
<td>(( \tau CF, \sigma_{\tau} = 0.5 \text{ fs} ))</td>
</tr>
<tr>
<td>( m_{\nu_3} &lt; 36 \text{ MeV} )</td>
<td>( m_{\nu_3} &lt; 34 \text{ MeV} )</td>
<td>( m_{\nu_3} &lt; 28 \text{ MeV} )</td>
</tr>
<tr>
<td>( \sin^2 \theta &lt; 0.0053 )</td>
<td>( \sin^2 \theta &lt; 0.0039 )</td>
<td>( \sin^2 \theta &lt; 0.0024 )</td>
</tr>
<tr>
<td>(-0.011 &lt; \kappa &lt; 0.017 )</td>
<td>(-0.011 &lt; \kappa &lt; 0.009 )</td>
<td>(-0.006 &lt; \kappa &lt; 0.005 )</td>
</tr>
<tr>
<td>(</td>
<td>\bar{\kappa}</td>
<td>&lt; 0.26 )</td>
</tr>
<tr>
<td>(-0.030 &lt; \eta_{\tau\mu} &lt; 0.052 )</td>
<td>(-0.030 &lt; \eta_{\tau\mu} &lt; 0.029 )</td>
<td>(-0.017 &lt; \eta_{\tau\mu} &lt; 0.016 )</td>
</tr>
</tbody>
</table>

For Cases 2 and 3 the limiting error is from tau lifetime
(arbitrarily setting all other errors to zero yields negligible improvement)
Discussion ($m_{\nu_\tau}$)

Limit on $m_{\nu_3}$ can be interpreted as limit on $m_{\nu_\tau}$
($\sin^2 \theta$ is small as is mixing of $m_{\nu_3}$ with lighter neutrinos)

Our constraint is less stringent than the best direct constraint
($m_{\nu_\tau} < 18.2$ MeV at 95% C.L. from ALEPH) but it is

- statistically independent
- insensitive to fortuitous or pathological events at kinematic limits
- almost independent of absolute energy scale of the detectors
- independent of details of resonant structure of multi-hadron $\tau$ decays

Our constraint on $m_{\nu_\tau}$ improves only slightly with the $\tau cF$ input

Our method is not competitive with other $\tau cF$ analyses
for which expected $\tau cF$ sensitivity is $O(2$ MeV)
Discussion \((\sin^2 \theta)\)

Our upper limit on \(\sin^2 \theta\) is already the most stringent experimental constraint on mixing of the third and fourth neutrino generations.

This constraint will improve by a factor of up to two using future \(\tau cF\) data (depending on the improvement in the error on \(\tau_\tau\)).

We anticipate that this technique will continue to provide the most stringent constraints in the foreseeable future.
Discussion ($\kappa$, $\bar{\kappa}$)

Our results on $\kappa$ and $\bar{\kappa}$ are currently the most precise

Constraint on $\bar{\kappa}$ is less stringent compared to $\kappa$ due to lack of linear terms

Anomalous magnetic moments due to compositeness are expected to be of order $m_\tau/\Lambda$ where $\Lambda$ is the compositeness scale

$\Rightarrow \tau$ appears to be a point-like Dirac particle up to

$$\Lambda \approx m_\tau/0.017 = 105 \text{ GeV}$$

Results are comparable to those from anomalous weak neutral current couplings and better than those from anomalous EM couplings

The results for $\kappa$ and $\bar{\kappa}$ will improve with $\tau c F$ data and will probe the point-like nature of the tau up to a scale of

$$\Lambda = O(180 \text{ GeV})$$ (for no improvement in $\tau_\tau$)

$$\Lambda = O(300 \text{ GeV})$$ (for $\times 2$ improvement on $\tau_\tau$ error)
Discussion ($\eta_{\tau\mu}$)

This $\eta_{\tau\mu}$ is currently the most precise

Compare to $\eta_{\tau\mu} = -0.04 \pm 0.20$ from momentum spectrum of muons from $\tau$ decays

$\eta_{\tau\mu}$ sensitive to type II charged Higgs

$$\eta_{\tau\mu} = -\left( \frac{m_\tau m_\mu}{2} \right) \left( \frac{\tan \beta}{m_H} \right)^2 \quad (7)$$

- $\tan \beta$ - ratio of VEV's of two Higgs fields
- $m_H$ - mass of the charged Higgs

Our fit yields $\eta_{\tau\mu} > -0.0232 \Rightarrow$

$$m_H < (2.01 \tan \beta) \text{ GeV (95\%C.L.)}$$

For $\tau$F: $\eta_{\tau\mu} > -0.014 \Rightarrow$

$$m_H < (2.55 \tan \beta) \text{ GeV (95\%C.L.)}$$

$\sim 25\%$ reduction in maximum value of $\tan \beta$ for given $m_H$ compared to today
<table>
<thead>
<tr>
<th>$\tau e^+ e^-$</th>
<th>$\tau \mu^+ \mu^-$</th>
<th>$\tau \rightarrow \pi^- \nu_\tau$</th>
<th>$\tau \rightarrow K^- \nu_\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Today</td>
<td>Slight improvement (but complementary)</td>
<td>World-best</td>
<td>World-best</td>
</tr>
<tr>
<td>$\sin^2 \theta$</td>
<td>$\kappa$</td>
<td>$\hat{\kappa}$</td>
<td>$\hat{\kappa}$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$\tau$</td>
<td>$\tau$</td>
<td>$\tau$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$\tau$</td>
<td>$\tau$</td>
<td>$\tau$</td>
</tr>
<tr>
<td>$\tau$ compositeness</td>
<td>$\Lambda &gt; 105 \text{ GeV}$</td>
<td>$\Lambda &gt; 105 \text{ GeV}$</td>
<td>$\Lambda &gt; 105 \text{ GeV}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$m_H &lt; (2,1) \text{ GeV}$</td>
<td>$m_H &lt; (2,1) \text{ GeV}$</td>
<td>$m_H &lt; (2,1) \text{ GeV}$</td>
</tr>
</tbody>
</table>

Ultimate limitation is the error on the tau lifetime.
Tau Neutrino Mass from Decay Rates of Charmed Pseudoscalar Mesons

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March 1999, SLAC Workshop on the \( \tau \)-charm Factory
The Idea

- Charmed pseudoscalars $D$ and $D_s$ can (just) decay to $\tau \nu_\tau$

- Sensitive to neutrino mass

- Theoretical errors ($f_P$; $P=D,D_s$) can be taken out using ratio of muon and tau decay rates

- $\tau c F$ a good source of charmed pseudoscalars (P. Kim)


$$x_P = \frac{\text{BR}(P \to \tau^+ \nu_\tau)}{\text{BR}(P \to \mu^+ \nu_\mu)}$$

$$x_P = \sqrt{M_P^4 - 2M_P^2(m_\tau^2 + m_\nu^2)} + (m_\tau^2 - m_\nu^2)^2 \frac{M_P^2(m_\tau^2 + m_\nu^2) - (m_\tau^2 - m_\nu^2)^2}{(M_P^2 - m_\mu^2)^2 m_\mu^2}$$
Fig. 4. Curves of $x_D$ and $x_F$ as functions of $m$, calculated using (20) and the values given in the text for the relevant masses.
<table>
<thead>
<tr>
<th>xP (Mev)</th>
<th>xP</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>9.72019</td>
</tr>
<tr>
<td>15</td>
<td>2.60433</td>
</tr>
<tr>
<td>14</td>
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<td>13</td>
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<td>12</td>
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<td>11</td>
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<tr>
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<tr>
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<tr>
<td>2</td>
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<td>6.6389</td>
</tr>
<tr>
<td>0</td>
<td>9.74183</td>
</tr>
</tbody>
</table>
How Well Could We Do?

Not much of the basic input information is known, so let's estimate freely assuming no theoretical errors and no errors on pseudoscalar masses:

- Expect $0.9 \times 10^7 \ D_s \ pairs \ per \ year \ and \ 2.0 \times 10^7 \ D^\pm \ pairs \ per \ year \ at \ 10\text{fb}^{-1}$.
- Know $\text{BR}(D_s \rightarrow \mu\nu) \sim 4 \times 10^{-3}$ (large errors).
- Expect $\text{BR}(D \rightarrow \mu\nu) \sim 7 \times 10^{-4}$

Suppose we have $10^7 \ D_s$'s and and get 40,000 $D_s$ decaying to muons. Assume the statistical error on this quantity dominates the error on $x_{D_s}$.

- 5 parts per mil, and an absolute error on $x$ of .05 - i.e. about 25 MeV at one sigma.

Suppose we have $2 \times 10^7 \ D$'s. Then we get 14000 $D$'s decaying to muons. Assume again this error dominates (less of a good approximation)

- 8.5 parts per mil, and an absolute error on $x$ of .022 - i.e. about 13 MeV at one sigma.
Conclusions

- Leptonic charmed meson decays can give us some statistically independent information about a possible tau neutrino mass complementary to what comes from kinematics.

- There's a lot we still don't know about charmed mesons, but most of the theoretical uncertainties (form factors) can be eliminated by taking ratios.

- Perhaps not the greatest way to get information on a neutrino mass, but it's one more piece!

- Beware highly unsuppressed $D \rightarrow \mu \nu \bar{\nu}$ (Rizzo)
  - Bguts on make this harder? (P. Kim)
First Steps in Tau Neutrino Mass Determinations from One-Prong Tau Decay Kinematics

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Workshop on the $\tau$-charm Factory

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\textsuperscript{2} CERN, Geneva, Switzerland
\textsuperscript{3} EPFL, Lausanne, Switzerland
The Idea

- Measure the momentum spectra of pions from \( \tau \) decays almost at rest (JZK).
- Sensitive to neutrino mass
- Theoretical errors should be easily controllable
- Complementary to multi-pion invariant mass analyses and others.

\[ 5 \times 10^6 \ \tau \rightarrow \pi \nu \] decays
- Baseline resolutions (Marbella)
- \( \sim 5 \) MeV neutrino mass sensitivity at 1 \( \sigma \) (no beam, stat. only)
- More work needed, but this looks promising.
  (But see more detailed talk of A. Stahl)