What do we know about leptonic CP violation?

- The CKM "model" provides, at present, an adequate parameterization of $\mathcal{CP}$ in the quark sector.

→ Is that all there is? Is CKM the sole source of $\mathcal{CP}$? Probably not!
  
  - Baryogenesis  
  - Strong CP problem $\sim \frac{\Theta}{3\pi} \mathcal{F}$
  
  $\Rightarrow$ new physics

* But at least we believe we know who of the important players are: $u,d,s,c,b,e$
Figure 1: The unitarity triangle.

Figure 2: The current constraints and the favoured unitarity triangle. The constraint coming from $B^0$ oscillations is a limit at 95% of Confidence Level, while the others represent a ±1σ variation of the experimental and theoretical parameters entering the formulae in the text.
Figure 7: The $\beta - \bar{\beta}$ allowed region. The contours at 68% and 95% C.L. are shown. The continuous lines correspond to the constraints obtained from the measurements of $\frac{\Delta m_{21}}{\Delta m_{32}}$, $\Delta m_{42}$, and $c_K$. The dotted curve corresponds to the 95% C.L. limit obtained from the experimental limit on $\Delta m_{32}$.

\[
\sin 2\beta \bigg|_{68.3} = 0.73 \pm 0.08 \\
\sin 2\beta \bigg|_{95.4} = 0.74 \pm 0.41
\]

Figure 8: The $\sin 2\beta$ and $\sin 2\alpha$ probability density distributions. The dark-shaded and the clear-shaded intervals correspond to 68% and 95% C.L. regions respectively.
Figure 2: The $\sin2\alpha$ and $\sin2\beta$ distributions have been obtained using the constraints corresponding to the values of the parameters listed in Table 1. The contours at 68% and 95% are shown. The $\sin2\alpha$ and $\sin2\beta$ distributions are also shown. The dark-shaded and the clear-shaded intervals correspond, respectively, to 68% and 95% confidence level regions.
Figure 9: The left and the right plots show the probability density distributions for $\Delta m_\text{B}$ and $|V_{ud}|/|V_{cb}|$ respectively. The dark-shaded and the clear shaded intervals correspond to 68% and 95% C.L. regions respectively.

Table 3: The $\Delta m_\text{B}$ and $|V_{ud}|/|V_{cb}|$ measured values are compared with those obtained using the fitting procedure after having removed them from the fit.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Measured value</th>
<th>Fitted value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m_\text{B}$</td>
<td>$&gt; 12.4$ ps$^{-1}$ at 95% C.L.</td>
<td>[9.5 - 17] ps$^{-1}$ 68% C.L.</td>
</tr>
<tr>
<td>$</td>
<td>V_{ud}</td>
<td>/</td>
</tr>
</tbody>
</table>

From these results the important impact of these two measurements in the determination of the allowed region for $\rho$ and $\eta$ is clear. Furthermore the expected probability distribution for $\Delta m_\text{B}$ shows that present analyses are exploring the one sigma region.

Conclusions

Important improvements have been obtained in the last two years in the analyses of $B^0 - \bar{B}^0$ oscillations. Combining LEP results with those from SLD and CDF, $\Delta m_d$ frequency is presently known with a 3.4% relative error ($\Delta m_d = 0.477 \pm 0.017$ ps$^{-1}$). The sensitivity on $\Delta m_\text{B}$ is at 13.8 ps$^{-1}$ and, the actual LEP/SLD/CDF combined limit, of 12.4 ps$^{-1}$ at 95% of C.L., is exploring the region where $\Delta m_\text{B}$ is expected to be according to the analysis [4]. The measurement of $\Delta m_\text{B}$ is still a challenge for LEP collaborations, $|V_{ub}|$ has been
\( (\varepsilon'/\varepsilon)/(\eta/0.4) \)
- In the case of leptons, we don't yet know the players or how many there are:
  e.g., how many $\nu$'s are there?

  $\rightarrow \nu_2 \rightarrow 3$ light LH $\nu$'s only

  $\rightarrow$ Solar/Atmos./LSND $\rightarrow$ 3 diff. $\Delta m^2$
  $\therefore$ at least 4 masses
  $\nu_e, \nu_\mu, \nu_\tau, \nu_4$, an isosinglet $\nu$

  $\rightarrow$ Majorana or Dirac?

  $\rightarrow$ See-saw? Heavy neutrals (vector-like/isosinglets)?
  $m_{\nu} \sim m_0 / m_N$

- With extra neutral fields $[\nu_S / N_i]$ there are many possible opportunities to generate
  CP for leptons divorced from quarks
How do we find $\delta^\prime$ in the leptonic sector?

Usual suspects

- $edm$'s of charged leptons
- $\delta^\prime$ decays of $\tau$, e.g.,
  $\tau \rightarrow k \pi \nu$

- More $\delta^\prime$ in $\gamma$ oscillation

CPT: $P \nu_a \rightarrow \nu_p = P \bar{\nu}_p \rightarrow \bar{\nu}_a$

$\Rightarrow \quad \text{C.P.} : \quad P \nu_a \rightarrow \nu_p = P \nu_p \rightarrow \nu_a = P \bar{\nu}_a \rightarrow \bar{\nu}_p$

$\delta^\prime$ arises here due to complex phase(s) in the leptonic CKM matrix

- $\; V_{ckm} \equiv U_{\nu L}^\dagger U_{\ell L}$

- 3 $\ell$'s $\leftrightarrow$ 3 Dirac $\nu$'s
- $V_{ckm}$ has 3 angles and 1 phase $\{\nu$'s massive + non-degenerate\}
However, you get more phases if:

- 3 $e^\prime$s $\oplus$ 3 Majorana $\nu$s: 3 $\nu$s $\oplus$ 3 phases
- 3 $e^\prime$s $\oplus$ 3 LH $\nu$s $\oplus$ 1 isosinglet

$\rightarrow$ 6 phases etc

\[ V^2 \text{ is first 3 columns of an (3+n) x (3+n) matrix} \] many $\nu$s + phases possible

Simplest case:

\[ \begin{align*}
A &= \frac{P_{\nu_e} \nu_e - P_{\nu_\mu} \nu_\mu}{P_{\nu_e} \nu_e + P_{\nu_\mu} \nu_\mu} \quad \text{for} \quad d = m_2 - m_1 \sim 5 \times 10^{-6} \text{ eV}^2 \\
B &= \frac{m_3 - m_1}{d} > 3 \times 10^{-5} \text{ eV}^2
\end{align*} \]

Oscillations observed when $L \sim E/D^2$ but not $\nu_e$.

To see $\nu_e$, need $L \sim E/D^2$.

Extremely long half-times "seen" $L \sim E/D^2$.

Hard to say the least... but rewarding.

$\rightarrow$ Direct $\nu_e$ test of mixing matrix.
Scaling games with dipole moments

\[ a_x \sim C \left( \frac{m_x}{\Lambda} \right)^n \]
\[ d_x \sim C' \left( \frac{m_x}{\Lambda} \right)^{n+1} \]

- \[ n = ? \]
- \[ n = 2 \text{ in many cases: SM (SUSY) low-scale Q.G., etc.} \]

- Imagine \[
\begin{align*}
\text{(i)} & \quad d_x, a_x \text{ not far below present bounds} \\
\text{(ii)} & \quad C, C' \text{ generation independent}
\end{align*}
\]

\[ |d_x| < 3 \times 10^{-16} \text{ e-cm} \quad |a_x| \lesssim 0.05 \text{ now} \]

- As we vary \( n \), what does new physics nearby for \( e^- \) tell us about \( e^+ \mu^- \)?

Recall

\[ |\Delta a_x^{e^-} | \lesssim 4 \times 10^{-12} \]
\[ |\Delta a_x^{e^-} | \lesssim 8 \times 10^{-9} \]

\[ |d_e^{e^-} | \lesssim 8 \times 10^{-23} \text{ e-cm} \quad |d_{\mu}^{e^-} | \lesssim 4 \times 10^{-19} \text{ e-cm} \]
\[ n \geq 5 \text{ for } o_n \]

\[ n \leq 3 \text{ or for } d_n + 1 \]

\[
\begin{align*}
\text{(i)} & \quad 6 \\
\text{(ii)} & \quad 4.10^{-11} \\
\text{(iii)} & \quad 4.9.10^{-8}
\end{align*}
\]

\[
\begin{align*}
\text{(3)} & \quad 4.7.10^{-12} \\
\text{(2)} & \quad 2.10^{-9} \\
\text{(1)} & \quad 3.10^{-5} \\
\text{(n)} & \quad 1 \quad 8.9.10^{-20} \\
\end{align*}
\]

\[
\begin{align*}
\bar{a} & \quad \bar{a} \\
\bar{r} & \quad \bar{r} \\
\bar{d} & \quad \bar{d}
\end{align*}
\]
models which satisfy present constraints from e+e- bounds will never give large $a^\gamma / d\gamma$ if simple scaling holds with $n=1$ or $2$ - lots of models

\textbf{OUTS?}

- Operator Approach to edm's + mdm's
  \{ e.g. each gen. has a different \} \new operator

$$\Delta \mathcal{L} = \frac{1}{\Lambda^2} \left\{ \left[ C_{\tau \nu} \bar{\tau} \nu P_{\tau\nu} \tau \phi + \bar{\tau}_{\tau \nu} \tau \nu P_{\tau\nu} \tau \phi \right] B_{\tau \nu} + \left[ C_{\tau \nu} \bar{\tau} \nu P_{\tau\nu} \tau \phi \right] W_{\tau \nu} \right\} + \text{h.c.}$$

$c$'s = 1 data implies

$\Lambda_e > 58 - 77 \, \text{PeV}$ \text{ a long way to go}

$\Lambda_\tau > 7 - 9 \, \text{TeV}$, $\Lambda_e > 0.4 - 0.5 \, \text{TeV}$

$\phi \Rightarrow \frac{\chi}{\sqrt{2}}, \quad B_{\tau \nu} = c_w A_{\tau \nu} - s_w Z_{\tau \nu}, \quad W_{\tau \nu} = c_w Z_{\tau \nu} + s_w A_{\tau \nu}$
- $B_{\nu \nu}$ terms induce $a_{\tau, \nu} \frac{y_{i, \nu}}{v}$ and $c_{\tau} \frac{y_{i, \nu}}{v}$ only.

- $W_{\nu \nu}$ terms induce $a_{\tau, \nu, w} \frac{y_{i, \nu, w}}{v}$ and $d_{\tau, \nu, w}$ and new 4-point interactions due to gauge invariance.

Test (?!) in $Z$ radiative decays?

SM

Anomalous

Production

How big are these contributions??

- $B_{\nu \nu} \rightarrow \{ 1 \}$ only, $W_{\nu \nu} \rightarrow \{ 1, 3 \}$

- Rate ?, $E_\gamma$, $\cos \theta_\gamma$, $M_\gamma$ ? No time!
\[ \text{OPAL: } B(\tau \rightarrow K^0\nu\nu\nu) = \]
\[ (3.0 \pm 0.4 \pm 0.5) \cdot 10^{-3} \quad E_\gamma > 20 \text{MeV} \]
\[ (2.7 \pm 0.6) \cdot 10^{-3} \quad > 37 \text{MeV} \]

\[ \text{Mark II: } (2.3 \pm 1.1) \cdot 10^{-3} \quad E_\gamma > 37 \text{MeV} \]

\{ SM MC (E_\gamma > 20 \text{MeV}) \text{ is } 2.82 \cdot 10^{-3} \]

\{ Distributions agree with SM expectations so far \]

::: More Ways to Circumvent m_\tau^\text{n} Scaling 

\(-7\) factor of m_\tau^\text{n} arise both from couplings in loops as well as the need to flip helicity in the loop to generate a \(\sigma_{\mu\nu}\) term 

::: Try to get some other heavy fermion to flip helicity in a 1-loop graph 

Imagine \( \delta_2^x = e \frac{\delta}{m_\tau} \approx 2.2(5) \cdot 10^{-16} \text{ e-cm} \)

\(\Rightarrow\) what \(\delta's\) can we get? \(\Rightarrow\) present limit
Basic Idea; at 1-loop \( \delta \sim \frac{a}{\pi} \frac{m_{\tau}}{m_{\Xi}} \)

- multi-Higgs \( \delta \sim 10^{-4} \) ("tan\( \beta \)" type enhancements)

1-loop \( \sim \frac{t}{\pi} \left( \frac{m_{\Xi}}{\tau} \right)^3 \), 2-loop \( \sim (\frac{\tau}{\Xi})^3 (\frac{\Xi}{\tau}) \)

- non-MSSM SUSY

\( \chi_i \) phase in \( \Xi \Xi \chi_i \); couplings

\( \chi_i \) plays role of \( F \)

\( \delta \sim \text{few} \cdot 10^{-4} \)

- LQ's or \( R \) violation

\( \tau \), \( t \), etc

\( \delta \sim 10^{-3} \)

- \( d_e : d_{\mu} : d_{\tau} \sim \text{suggestive} \)

- \( m_e : m_{\mu} : m_{\tau} \sim m_{\mu} m_{\tau} : m_e m_{\tau} \)

is 'faster' than \( m_e \)
Left-Right: $W-W'$ mixing, heavy RH neutrinos

Model

$\frac{\tau}{N} \frac{\tau}{W'}$

- $N$ plays role of $F$
- $\tau$ highly suppressed by mixing angles

$\delta \sim 10^{-6}$ at best

Ad-Hoc

- Heavy $\nu^+$

Charged Higgs

$\delta \sim \text{few} \cdot 10^{-3}$

I conclude from this survey that

$\delta^2 \sim 10^{-18}$ e- cm$^2$ is "best": you can hope

for unless NP is very bizarre...

Perhaps a more discrete survey would be

useful? 
\( \chi^0 \) in \( \tau \) decays: e.g., \( \tau \rightarrow K \pi \nu \)

- Here, constraints from the quark sector play the most important role.

\[
M = \sqrt{2} G_F \left[ (1+x) \overline{\chi} \chi \tau^\nu \right] \sum \mathcal{J}_\mu + \sum \mathcal{J}_\nu \frac{m_k^2}{\mu^2} \chi \mathcal{F}_3 (\tau^\nu) \phi \]

\[
\mathcal{J}_\mu = \sqrt{2} \chi \overline{\phi} \frac{m_k^2}{m_\phi m_\tau} \mathcal{F}_3 (\phi) \]

- Figure of merit: \( \text{Im} \xi \), \( \xi = \frac{m_k}{m_\phi m_\tau} \frac{\gamma}{1+x} \)

- Scalars? MHD or LQG's

\[
\mathcal{L}_{\text{MHD}} = \sqrt{2} G_F \mathcal{L}_{\phi} \frac{m_\phi m_\tau}{m_k} \left[ \overline{\chi} \tau^\nu (\overline{\phi} \mathcal{L}_3 \phi) + \frac{m_k}{m_\phi m_\tau} \overline{\chi} \tau^\nu (\overline{\phi} \mathcal{L}_2 \phi) \right] + \text{hc}
\]
\[ \text{Im } \xi_{\text{mHB}} \propto - \frac{\tau}{m_H^2} \left\{ \text{Im} (\tau^2) + \frac{\mu_u}{m_s} \text{Im} (\tau^2) \right\} \]

\[ \propto -3.2 \times 10^{-4} \left( \frac{\mu_u}{m_H} \right)^2 \left[ \text{Im} (\tau^2) + \frac{1}{50} \text{Im} (\tau^2) \right] \]

\[ \text{Grossman: } | \text{Im } \xi_{\text{mHB}} | \lesssim 0.27 \text{ for } m_H = 60 \text{ GeV} \]

\[ \text{from } B \to \tau \nu e X \]

\[ L_{\text{HQ}} \sim -\frac{\lambda_{25} \lambda_{15}^*}{2 M^2} (\bar{s}_L u_R)(\bar{t}_L \tau_R) + \ldots \]

\[ \text{Davies et al.} \]

\[ \text{Im } \xi_{\text{LQ}} \propto -\frac{\tau}{m_{\text{qq}}^2} \frac{\text{Im} (\lambda_{25} \lambda_{15}^*)}{4 \sqrt{2} G_F s_w M^2} \]

\[ \overline{D} \to D \text{ mixing } \to |\text{Im } \xi_{\text{LQ}}| \lesssim 0.14 \text{ if } \]

\[ \text{LQ couplings to } q^i s + d^i s \text{ of same generation are similar in size} \]

\[ \text{Choi et al. } \propto 10^3 \tau \text{'s needed to probe to this level whether polarized or not} \]
Summary

- We may know less about the players in the lepton sector than the quark sector

- $\chi^2$ probed by
  
  edm's, decays, $\nu$ osc. asymmetries

  small but 'pay direct' involves input from quark sector

  probes phases in mixing matrix directly

  in lepton sector

- Keep looking - surprises do happen.