Decelerating Gradient for FFTB Dielectric Tube Experiment

The background reference is ARDB-368. There it is shown that the decelerating gradient a distance $s$ behind a point charge $q$ is

$$G(s) \approx \frac{q c Z_0}{\pi R^2} \nu_0(s/R).$$  \hfill (1.1)

In this expression the dielectric rod has an inner hole of radius $R$ and an infinite outer radius. When $s \leq R$ the following approximation holds

$$\nu_0(s/R) \approx 1 - \frac{s}{R} \frac{2\varepsilon_r}{\sqrt{\varepsilon_r - 1}} + \frac{s^2}{R^2} \left( \frac{2\varepsilon_r^2}{\varepsilon_r - 1} - \frac{\varepsilon_r}{2\sqrt{\varepsilon_r - 1}} \right)$$  \hfill (1.2)

where the relative dielectric constant $\varepsilon_r$ is frequency independent. When this approximation does not hold, $\nu_0$ must be calculated numerically. The result is shown in Figure 1 below.

Assume a Gaussian bunch with rms length $\sigma$. The decelerating field at a location $s$ within the bunch is

$$V(s) = \frac{q c Z_0}{\pi R^2} \frac{1}{\sqrt{2\pi} \sigma} \int_{-\infty}^{s} \nu_0 \left( \left( s - s' \right)/R \right) \exp \left( -\frac{s'^2}{2\sigma^2} \right) ds'$$  \hfill (1.3)

Making a change of variables

$$V(s/R) = \frac{q c Z_0}{\pi R^2} \frac{1}{\sqrt{2\pi} \sigma/R} \int_{-\infty}^{\infty} \nu_0(u) \exp \left( -\frac{u^2}{2\sigma^2} \right) \exp \left( -\frac{u}{2(\sigma/R)} + \frac{us/R}{(\sigma/R)^2} \right) du$$  \hfill (1.4)

and perform the integrals numerically.

The result can be shown as a universal set of curves for different ratios of $\sigma/R$ because that is the only R dependence except for the leading factor of $R^{-2}$. The results are shown below for $R = 100 \mu m$. These results can be used to calculate the performance for other parameters. For example, for $\sigma = 10 \mu m$ and $R = 200 \mu m$, $\sigma/R = 0.05$, and the maximum gradient will be

$$G_{\text{max}} = 7 \text{GeV/m} \left( \frac{100 \mu m}{200 \mu m} \right)^2 = 1.75 \text{GeV/m}. \hfill (1.5)$$

Figure 1: Cherenkov radiation wakefield
The particles experiencing that gradient will trail the center of the bunch by 14 µm.

Figure 2: Decelerating gradient vs position in bunch for R = 100 µm

Figure 3: Maximum decelerating gradient and location of that gradient within the bunch

The particles experiencing that gradient will trail the center of the bunch by 14 µm.

1 Bob Siemann & Alex Chao, ARDB Note 368, "Wakefields in a Dielectric Tube with a Frequency Dependent Dielectric Constant"