On the Effect of Beam Ionization in E-157
ARDB Tech Note 3/15/99

In the E-157 Plasma Wakefield Accelerator experiment, the goal is to measure energy change along the beam by measuring the beam deflection in a bending magnet. By subtracting the beam deflections measured with the plasma on and off, the energy change caused by the plasma can be probed. By the “plasma off” condition we mean the situation in which the ionizing ArF laser is turned off but the Li and He vapor are still present. The question arises as to errors in the measurement due to transverse beam deflections in the plasma off condition. Specifically, this note addresses the following questions: How much ionization of the plasma will there be in the plasma off condition and what will be the effect of this remnant plasma on deflection of the beam?

Deflection Estimate

To estimate the errors introduced by beam ionization, consider the contributions to beam deflection with plasma “on” and “off”:

Plasma off: \[ d = d(\text{correlated energy spread}) + d(\text{misalignment}) \times (1 + k_{\beta}'s\times \text{sink}_{\beta}'l) \]

Plasma on: \[ d = d(\text{correlated energy spread}) + d(\text{misalignment}) \times (1 + k_{\beta}s\times \text{sink}_{\beta}l) + d(\text{energy gain}) \]

Where here \( k_{\beta} \) is the wavenumber for betatron oscillations in the plasma, primes refer to conditions in the plasma off case, \( l \) is the length of plasma (approximately 1m) and \( s \) is the distance to the diagnostic (approximately 10m). The betatron wavenumber in the plasma on case is constant except for the head of the beam and has a value of \( l/n\pi \approx 10m^{-1} \).

Impact ionization of the plasma off case will produce a plasma density rising toward the tail of the beam to a value \( n_o' \) and giving a \( k_{\beta}' = \sqrt{n_o'/n_o} k_{\beta} \). Expanding \( \text{sink}_{\beta}'l \) for small arguments gives the deflection error induced by the ionization in the plasma off condition:

\[ \frac{dy_f}{dy_o} = \frac{n_o'}{n_o} (n\pi)^{2/3} \frac{s}{l} \]

where \( dy_o \) is the initial displacement of a particle from the axis. In order to avoid deflection and spot size increases, we would like the right hand side to be smaller than one. Nominally, since \( s/l \approx 10 \) and \( (n\pi)^{2/3} \approx 100 \), this corresponds to an ionized density \( n_o' < .001 n_o \approx 2 \times 10^{11} \text{ cm}^{-3} \). For an initial misalignment of 10 microns and a final deflection tolerance of 100 microns (30 MeV energy resolution), we can allow \( dy_f/dy_o \approx 10 \) or \( n_o' \approx .01 n_o \approx 2 \times 10^{12} \text{ cm}^{-3} \).
Sources of Ionization:

With the ArF laser off, the possible sources of ionization of the plasma are 1) impact ionization by the beam, 2) photo-ionization by synchrotron radiation in the beamline, 3) photo-ionization by Cerenkov radiation in the gas and OTR in the upstream diagnostic foil, and 4) secondary ionization (impact ionization) by electrons from the other three processes. Impact ionization appears to be the dominant contributor. Marc Hill is exploring the use of a Monte Carlo code GEANT from CERN to model these effects and the showers that can be produced when they combine. Here we make simple analytic estimates of each of the contributions to ionization below. The neglect of showers is justified since the path lengths are small compared to the radiation length in the gas as described in (2) below.

1) Impact ionization. The density rise due to impact ionization can be estimated as $n_i = n_n N \sigma/A$, where $n_n$ is the neutral gas density (approximately $2 \times 10^{15}$ cm$^{-3}$ in the lithium vapor), $N$ is the number of beam particles ($\sim 4 \times 10^{10}$), $\sigma$ is the cross-section for impact ionization and $A$ is the cross-sectional area of the beam. For our parameters and a beam spot size of 50 microns, this is roughly $5 \times 10^{11}$ cm$^{-3}$ $\sigma/10^{-18}$ cm$^{2}$. The ionization cross-section for Li is not well documented, but based on the similarity of the cross-sections for several gases, a value of $\sigma=10^{-19}$ cm$^{2}$ is a reasonable guess at 30 GeV. This gives an ionized density of $5 \times 10^{11}$ cm$^{-3}$ which is marginally consistent with the requirement in the section above. Thus spot sizes smaller than 25 microns may ionize too much gas to allow the subtraction diagnostic technique to be useful. Since the width of the plasma wake at the last bin is 50 microns, the spot size should not be larger than that. These considerations taken together constrain the beam size to the 25 to 50 micron range.\(^1\)

Note that we have not added the contribution to beam deflection from ionized helium in the Li oven. This may contribute an additional deflection of the same order (since the gas density times length product is similar).

2) X-ray Photo-ionization. Bremstrahlung from upstream apertures and Li nuclei, synchrotron radiation from quads and betatron oscillations in the gas/plasma will produce x-rays that can ionize the Li gas. We can estimate the photo-ionization by the x-rays in the beam path in two ways: (i) by scaling the uv photo-ionization data from the UCLA experiment or (ii) from first principles. In the UCLA experiment, a flux of uv photons of $100\text{mJ/cm}^2$ produced 10% ionization in a Li gas of density $2 \times 10^{15}$ cm$^{-3}$. We expect the cross-section for photo-ionization to be smaller for x-rays than uv photons by at least an order of magnitude [and possibly as much as five

\(^1\) For a normalized emittance of $50\pi$ mm-mrad, the spot size of the 25 micron beam will expand to 700 microns after the 10m drift space, while the 50 micron beam will expand to less than 200 microns. Both are acceptable, although the latter case may enable more accurate energy resolution while the former may capture more charge in the last bin.
orders, see Phys. Rev. D, 56 141(1996)]. In addition there are .001 as many photons in a mJ of x-rays compared to a mJ of uv. Thus for every mJ of x-rays in the beam path, the amount of ionization would scale as .1%/mJ/cm²[from before] x (1cm/50µ)² /2π[area of beam pipe] x (1/10) [reduced cross-section] x.001 [relative number of x-rays/same energy of uv photons] < 0.1% ionization. Based on Dave Whittum’s recent tech. note on betatron radiation in the plasma (plasma on), we may expect less than .4mJ of x-rays from this effect. The x-rays from the upstream effects may be of the same order but in a larger spot [Feb. run x-ray estimates may be helpful here]. The bremsstrahlung from collisions with Li gas nuclei can be estimated from the radiation length in Li Xo~70 g/cm², which at this density (10⁻⁸ g/cm³) becomes 70,000 km. This gives an energy loss over 1 meter of 500eV per incident electron or .003 mJ for the beam. Combined with the above arguments this gives an ionized density of less than 6x10¹¹ cm⁻³. Thus even with this overestimated cross-section, the photo-ionization should be below acceptable limits. Another estimate can be made from Fig. 23.4 of the Phys. Rev. D article above which shows ionization losses at approximately 5 MeV per radiation length (in lead). Thus scaling to the radiation length of the Li gas, each electron loses .07 eV to ionization over a meter or enough energy for approximately .01 ionizations/electron. Thus one would estimate that a beam of 4x10¹⁰ electrons could produce up to 4x10⁸ ions over a length of 100cm and cross-section of 2π(50µ)² for an ionized density of 2x10⁸ cm⁻³. Thus we expect photo-ionization from x-rays to be negligible.

3) **UV Photo-ionization.** UV photons produced by the upstream OTR radiator and by Cerenkov radiation in the Li/He gas can also contribute to ionization. An estimate of the number of uv photons per electron from OTR is of order .01. Cerenkov radiation produces 30 photons /m in air, so we estimate that .003 photons will be produced in the lower density Li. Taking the energy of these at 10eV and the spot size of 50µ gives a uv fluence of .005 mJ/cm² or an ionized density (by comparison to the ArF laser data) of 10¹⁰ cm⁻³. Again this is a negligible effect.

4) **Secondary Ionization.** The electrons produced by impact ionization (or the other mechanisms) can in turn further ionize the neutral gas. To quantify this note that the number of secondaries produced per incident beam electron is approximately

\[ \frac{1}{\lambda_{mfp}} = n_e \sigma l \sim (2 \times 10^{15} \text{ cm}^{-3})(10^{-18} \text{ cm}^2)(100 \text{ cm}) = .2 \]

Thus there is approximately one secondary for each five beam particles. Since the secondaries may be at low energies near the peak ionization cross-section, their cross-section for ionizing more of the gas may be 100 times higher (order 10⁻¹⁶ cm²). Data for the distribution of secondaries produced by beams up to a few keV show that they are roughly uniformly distributed from 2 to a few tens of eV, falling sharply after 100 eV [Green and Sawada, J. Atmos. Terr. Phys. 34, 1719 (1972); R. Garvey et al., JAP 48, 4353 (1977)]. Their angular distribution, though not necessarily isotropic, will clearly take them out of the cylindrical column defined by the beam in typically a few beam radii. Even if they were collimated forward in a cone angle defined by the center of mass.
energy (an angle of order $1/\gamma_{\text{cm}} \sim 1/170$ for hard collisions), they would still have a path length $l_s$ in the beam of less than 1 cm. Combining these factors gives an upper estimate of the number of tertiary electrons per secondary electron $N_t$. By scaling the previous result above, we find $N_t \sim 0.2$ (previous result) $\times \frac{\sigma_s}{\sigma}$ (increase in cross-section) $\times \frac{l_s}{1\text{m}}$ (decrease in path length). For $\frac{\sigma_s}{\sigma} \sim 100$, $l_s \sim 50\mu \text{m}$ to 1 cm, $N_t$ becomes of order 0.001 to 0.1. Thus secondary ionization within the beam pipe will be completely negligible. Secondary ionization of the gas surrounding the beam pipe can be larger and slightly affect the wake but not the focusing force on the beam. This has been modeled previously in a PIC code by D. Gordon. The resulting additional ionization peaks off the beam axis and increases the plasma density by less than 1%.

In summary, direct impact ionization of neutral Li is the primary contributor to plasma production in the “plasma off” condition. Greater than 0.1% ionization will cause plasma lens-like deflections of the beam and begin to interfere with the accuracy of the energy change diagnostic. The beam ionizes less than 0.1% of the gas, provided that the beam spot size is kept larger than 25 microns.