Correction of Chromatic Resolution in Streak Camera Measurements
Continuation of ARDB-158

In technical note ARDB-158 the chromatic contribution to the resolution of SLAC's Hamamatsu streak camera was found to be consistent with other measurements. This led to understanding the narrow bandwidths that Robert Holtzapple found necessary to achieve good resolution in linac bunch length measurements.

Experiment E-157 will use the streak camera as the primary diagnostic, and it is likely that as much light intensity as possible will be necessary to measure the energy profile of the beam tail. (There is an open question of preventing degradation of the resolution due to space charge effects from the beam core, but that is another discussion.) One way to increase the bandwidth would be the use of a reflective objective lens that Hamamatsu is reported to be selling soon. This note discusses another approach that will give a factor of ten in bandwidth while using the present objective - use of a structure constructed with diffraction gratings and with a negative value of group velocity dispersion, \( \frac{d v_g}{d \lambda} \).

This technique is well known in the terawatt laser business where compressors are used together with a frequency chirp to compress pulses. There the chirp is obtained by stretching the pulse with gratings, and the compression can be extraordinary. The recent Physics Today article of Mourou et al states compression in the range \(10^3\) to \(10^5\) are achieved.\(^1\) The gain in this case will be not nearly as impressive, but an order of magnitude increase in light intensity looks possible.

The compressor is a set of parallel gratings spaced a distance \(L\) apart (see Figure 1 below). The red and blue light enter on parallel paths, and, since the gratings are parallel, they leave of parallel, but displaced, paths. A light ray incident at angle \(\theta_i\) is refracted to an angle \(\theta_r\) given by

\[
\sin \theta_r = \frac{n \lambda}{d} - \sin \theta_i
\]

where \(n\) is the diffraction order, \(d\) is the grating spacing, and \(\lambda\) is the wavelength. For \(\lambda = 500\text{nm}\) and \(\theta_i < 30^\circ\), there is only one diffraction order for \(d < 2\lambda = 1\ \mu\text{m}\). Choose these values for \(n\) and \(d\) in the examples that follow. This equation assumes greater than \(90^\circ\) diffraction, \(\theta_r\) negative means less than \(90^\circ\). For \(\lambda = 500\text{nm}\) and \(\theta_i < 30^\circ\), there is only one diffraction order for \(d < 2\lambda = 1\ \mu\text{m}\). Choose these values for \(n\) and \(d\) in the examples that follow. The path length depends on wavelength as

\[
L(\lambda) = \frac{L}{\cos \theta_r},
\]

and the effective, normalized group velocity is

\[
\frac{v_g(\lambda)}{c} = \frac{1}{n(\lambda)} = \cos \theta_r.
\]

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Figure 2 below shows the group velocity and compares it with the group velocity of SF4 glass. The derivatives have the opposite sign, and the compressor can compensate for the chromatic resolution of the camera objective. The approximate length of the compressor, L, is given by

$$\frac{dt}{d\lambda} = \frac{D}{v_G^2} \frac{dv_G}{d\lambda} + \frac{L}{v_C^2} \frac{dv_C}{d\lambda} = 0,$$

where $v_G$ ($v_C$) is the group velocity in glass (compressor). Using the approximate values $D = 45$ mm (see ARDB-158), $dv_G/d\lambda = 4 \times 10^{-4}$ c/nm, and $v_G = 0.53$ c gives

$$L = \frac{4 \times 10^4 d}{\tan \theta_r \sec^2 \theta_r}.$$

Using this value for $L$ gives the results in Figure 3 for resolution vs bandwidth at different incident angles ($\theta_i$). The length calculated with the result above is not quite optimum (see Figure 4), but it is close enough.

My conclusion is that it is possible to increase the bandwidth by roughly an order of magnitude using a diffraction grating compressor.
Figure 3: Resolution without and with diffraction grating compressor and $\theta_i = 5^\circ$, $15^\circ$ and $25^\circ$.

Figure 4: Resolution with $\theta_i = 10^\circ$ vs compressor length