Reference for Field Perturbation Measurements

In a non resonant perturbation measurement, the change in the reflection coefficient, $\Delta S_{11}$, is given by

$$\Delta S_{11} = -\frac{j\omega}{2P_i} \left\{ \varepsilon_0 E_0^2 \left( \alpha_{Ep} \cos^2 \theta_E + \alpha_{En} \sin^2 \theta_E \right) + \mu_0 H_0^2 \left( \alpha_{Mp} \cos^2 \theta_M + \alpha_{Mn} \sin^2 \theta_M \right) \right\}$$

where $P_i$ is the incident power and $\omega$ is the (angular) RF frequency;
$\alpha_{Ep}$ and $\alpha_{En}$ are the electric polarizabilities for electric field parallel and perpendicular to the axis of the perturbation, respectively;
$\alpha_{Mp}$ and $\alpha_{Mp}$ are the magnetic polarizabilities for magnetic field parallel and perpendicular to the axis of the perturbation, respectively. Note that there is a sign difference between the equation above and eq. (35) of Steele. His negative sign is incorporated into the polarizabilities.

$\theta_E$ and $\theta_M$ are the angles between the axis of the perturbing object and the impressed electric and magnetic fields, respectively.

In a resonant perturbation measurement, the change in the frequency, $\Delta \omega$, is given by

$$\frac{\Delta \omega}{\omega_0} = \frac{\omega - \omega_0}{\omega_0} = -\frac{1}{4U} \left\{ \varepsilon_0 E_0^2 \left( \alpha_{Ep} \cos^2 \theta_E + \alpha_{En} \sin^2 \theta_E \right) + \mu_0 H_0^2 \left( \alpha_{Mp} \cos^2 \theta_M + \alpha_{Mn} \sin^2 \theta_M \right) \right\}$$

where $U$ is the stored energy.

Polarizabilities can be calculated using the results in Section 7.3 and Table 12.1 of Collin. They can be expressed in terms of the volume, $V$, of the object and two functions $L_p$ and $L_n$. The electric polarizabilities are

$$\alpha_{Ep} = \frac{V}{L_p + 1/\chi_E}; \quad \alpha_{En} = \frac{V}{L_n + 1/\chi_E}$$

where $\chi_E$ is the electric susceptibility

$$\chi_E = \frac{\varepsilon - \varepsilon_0}{\varepsilon_0}$$

The electric polarizability of a conductor is obtained by the limit $\chi_E \rightarrow \infty$.

The magnetic polarizabilities are

$$\alpha_{Mp} = \frac{V}{L_p + 1/\chi_M}; \quad \alpha_{Mn} = \frac{V}{L_n + 1/\chi_M}$$

where $\chi_M$ is the magnetic susceptibility

$$\chi_M = \frac{\mu - \mu_0}{\mu_0}$$

The magnetic polarizability of a conductor is obtained by the limit $\chi_M \to -1$.

The values of the functions $L_p$ and $L_n$ for a sphere, needle, and disk are given in Table 1 below.

**Table 1: Functions for Calculating Polarizabilities**

<table>
<thead>
<tr>
<th>Sphere of radius $a$</th>
<th>$V = \frac{4}{3}\pi a^3$</th>
<th>$L_p = L_n = \frac{1}{3}$</th>
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</table>

**A needle with length $= 2a$ and diameter/length $= \beta$**  
($p \Rightarrow$ parallel to the long axis of the needle)

| $V = \frac{4}{3}\pi a^3 \beta^2$ | $E = \sqrt{1 - \beta^2}$ | $L_p = \frac{\beta^2}{2E^3} \left[ \ln \left( \frac{1 + E}{1 - E} \right) - 2E \right]$ | $L_n = \frac{1 - L_p}{2}$ |

**A disk of radius $= a$ and thickness/diameter $= \beta$**  
($n \Rightarrow$ perpendicular to the disk)

| $V = \frac{4}{3}\pi a^3 \beta$ | $E = \sqrt{1 - \beta^2}$ | $L_n = \frac{\beta}{E^3} \left[ \frac{E}{\beta} - \tan^{-1} \left( \frac{E}{\beta} \right) \right]$ | $L_p = \frac{1 - L_n}{2}$ |

* Needle and disk approximated as spheroids.

MATLAB functions have been written to calculate these polarizabilities. They have dimensions and susceptibility as arguments.

- `sphere(a,chi)` polarizability for a sphere
- `needle(a,beta,chi)` polarizability of a needle
- `disk(a,beta,chi)` polarizability of a disk

Use $\chi = \infty$ for the electric polarizability of a conductor and $\chi = -1$ for the magnetic polarizability of a conductor.
Polarizabilities of Conducting Needles and Disks. To be compared with figure 10.8 and 10.9 of Ginzton and figures 2 and 3 of Maier and Slater.