Perturbation Measurements

This is a report of perturbation (bead-pull) measurements made in the first W-band structure. The structure was mounted in the configuration to measure reflection coefficient. Power was brought into the bottom of the structure, and it was terminated at the other end.

![Figure 1: Sketch of measurement](image)

The perturbation was a 15 - 20 μm fiber stretching in the x-direction. The fiber was aligned with respect to the vacuum slot in the structure using a video microscope and some positioning arms with web hinges for fine adjustment. The parallelism of the vacuum slot and fiber travel was measured. It was found that the structure was tilted by 2.6 mrad (15 μm in 7395 μm).

$S_{11}$ was measured, and it was decided to concentrate on measurements of the mode with resonant frequency $f = 91.177 \text{ GHz}$. The rest of the measurements were at this frequency.

![Figure 2: $S_{11}$ scan through the resonance made after the fiber was removed. Bead-pull measurements were made at $f = 91.177 \text{ GHz}$](image)

Changes in $S_{11}$ with perturbation position is measured. Therefore, a measurement must be made with the perturbation missing. This measurement was made ~ 1.2 hours after the first bead-pull measurement. The result was that the offset phasor can be written as

$$\text{Offset} = Ae^{j\theta}; A = 0.13581 \text{ mV}; \theta = -9.95 \times 10^{-3} (-0.57^\circ)$$
The means values reported in the equation above were the result of 75 measurements. The rms variation of the amplitude and phase of those measurements were \( \sigma_A = 0.4 \, \mu \text{V} \) and \( \sigma_\theta = 0.10^\circ \).

Measurements were made at three different offsets (in y) from the center of the vacuum gap. These measurements were at \( \Delta y = -300 \, \mu \text{m}, 0, +300 \, \mu \text{m} \). One measurement at \( \Delta y = 0 \) was performed at the beginning of the measurement series, and this measurement was repeated at the end of the series just before measurement of the Offset. Polar plots of the last measurement at \( \Delta y = 0 \) is shown in the figure below.

Figure 3: Polar plots of the perturbation measurement for \( \Delta y = 0 \). The plot on the left shows the measurement uncorrected for the Offset which is indicated as the blue circle. The Offset is subtracted for the plot on the right. The * indicates the beginning of the scan at \( z = -1.7 \) mm.

The reflection coefficient depends on position of the perturbation as

\[
\Delta \Gamma(z) = \exp \left( -2 j \Delta \phi \frac{z}{d} + j \theta_0 \right) \sum_{p=-\infty}^{\infty} F_p \exp \left( -j (2 \pi p \frac{z}{d} + \theta_p) \right)
\]

where \( \Delta \phi \) is the phase advance per cell and \( d = 1.094 \, \text{mm} \) is the period of the structure. The phasor produced by the perturbation can be fit. The quality of the fit was measured by the average value of \( |\Delta \Gamma(z) - F(z)| \) divided by the average value of \( |\Delta \Gamma(z)| \). The fit quality is independent of the maximum value of \( |p| \) for \( |p| \geq 1 \) when \( \Delta y = 0 \). The fit for \( |p| = 2 \) is shown in figures 5 & 6, and the fit results are in the table for the last run and the first run in the measurement series.

<table>
<thead>
<tr>
<th></th>
<th>Last Run</th>
<th>First Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \phi )</td>
<td>66.3(^\circ)</td>
<td>66.1(^\circ)</td>
</tr>
<tr>
<td>( F_0 )</td>
<td>1.48\times10^{-5}</td>
<td>1.471\times10^{-5}</td>
</tr>
<tr>
<td>( F_1/F_0 )</td>
<td>0.0028</td>
<td>0.0051</td>
</tr>
<tr>
<td>( F_{-1}/F_0 )</td>
<td>0.3159</td>
<td>0.3211</td>
</tr>
<tr>
<td>( F_2/F_0 )</td>
<td>0.0011</td>
<td>0.00004</td>
</tr>
<tr>
<td>( F_{-2}/F_0 )</td>
<td>0.0080</td>
<td>0.0096</td>
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</tbody>
</table>
The results agree on the phase advance per cell, on the magnitude of the perturbation and that the dominant space harmonics are for $p = 0$ and $p = -1$.

![Fit Quality for Thread Centered in Gap](image1)

**Figure 4:** Fit quality for different values of maximum $|p|$ for the last run

![Phase advance/cell](image2)

**run10r** ($o = \text{measured, } - = \text{fit}$)

Phase advance/cell = 1.157 (66.28 deg)

![Magnitude vs. Position](image3)

**Figure 5:** Fit to the phase of the perturbation
Similar analyses have been performed for the $\Delta y = \pm 300 \, \mu m$. The goodness of fits indicates that terms up to $|p|_{\text{max}} = 3$ are needed. The data for one of the runs with the superimposed fit is shown in figure 8. The behavior is strikingly different from that when the tread is centered. The phasor is approximately constant for long periods of time and then makes rapid changes. Fitting the data gives a phase advance per cell of $\Delta \phi = 66.7^\circ$ which compares well with the $66.3^\circ$ and $66.1^\circ$ measured with the thread centered. The amplitudes of the space
Figure 8: Polar plot of data and fit for $\Delta y = 300 \mu$m. The fit has $|p|_{\text{max}} = 3$.

Harmonics for $\Delta y = +300 \mu$m and $\Delta y = -300$ are not in good agreement between the two runs. This may be due to the $|\Delta y|$'s not being equal.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta y = 300 \mu$m</th>
<th>$\Delta y = -300 \mu$m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \varphi$</td>
<td>66.7°</td>
<td>66.7°</td>
</tr>
<tr>
<td>$F_0$</td>
<td>$1.21 \times 10^{-5}$</td>
<td>$1.29 \times 10^{-5}$</td>
</tr>
<tr>
<td>$F_1/F_0$</td>
<td>0.0531</td>
<td>0.0164</td>
</tr>
<tr>
<td>$F_{-1}/F_0$</td>
<td>0.8533</td>
<td>0.6957</td>
</tr>
<tr>
<td>$F_2/F_0$</td>
<td>0.0292</td>
<td>0.0285</td>
</tr>
<tr>
<td>$F_{-2}/F_0$</td>
<td>0.2467</td>
<td>0.1558</td>
</tr>
<tr>
<td>$F_3/F_0$</td>
<td>0.0242</td>
<td>0.0218</td>
</tr>
<tr>
<td>$F_{-3}/F_0$</td>
<td>0.0449</td>
<td>0.0263</td>
</tr>
</tbody>
</table>
Appendix: Application of the Non-Resonant Perturbation Theory of C. W. Steele

Charles Steele derives a formula for non-resonant perturbation theory that relates the change in the reflection coefficient to the induced electric and magnetic dipole moments. His equation (16) is

$$2P_i \left( \Gamma_p - \Gamma_a \right) = - j \omega \left[ E_a \cdot p - \mu_a H_a \cdot m \right]$$

where

- $P_i$ is the incident power which is assumed to be the same in the perturbed and unperturbed cases;
- $\Gamma_p$ and $\Gamma_a$ are the reflection coefficients in the perturbed and unperturbed cases, respectively;
- $\omega$ is the frequency;
- $\mu_a$ is the permeability in the unperturbed case;
- $E_a$ and $H_a$ are the electric and magnetic fields without perturbation; and
- $p$ and $m$ are the electric and magnetic dipole moments set up by the perturbing object.

The dipole moments can be rewritten in terms of the polarization, $P$, and magnetization, $M$, which are the dipole moments/unit volume

$$2P_i \left( \Gamma_p - \Gamma_a \right) = - j \omega \int_{V_p} \left[ E_a \cdot P - \mu_a H_a \cdot M \right] dV$$

The coordinate system used for the bead-pull is that $z$ is the longitudinal direction (the beam direction) and $x$ and $y$ are the transverse directions. The vacuum slot runs in the $x$-direction, and the fiber integrates over the $x$ dependence of the fields. The fiber is a dielectric, so $m = 0$. The electric dipole moment has components in the $x$, $y$, and $z$-directions. For the $z$-component of the electric field

$$\int_{V_p} E_{az} P_z dV = \varepsilon_0 \left( \frac{\varepsilon}{\varepsilon_0} - 1 \right) \frac{16}{3} R^3 \frac{1}{L_p} \int_{L_p} E_{az}^2 dx$$

The integral is over the length of the fiber, $L_p$, and $R$ is the fiber radius. The proportionality constant comes from Collins' expression for polarizability of a disk.\(^2\) and integral gives the square of the electric field averaged over the length of the fiber. The expression for the $y$-direction is the same

$$\int_{V_p} E_{ay} P_y dV = \varepsilon_0 \left( \frac{\varepsilon}{\varepsilon_0} - 1 \right) \frac{16}{3} R^3 \frac{1}{L_p} \int_{L_p} E_{ay}^2 dx$$

In the $x$-direction

$$\int_{V_p} E_{ax} P_x dV = \varepsilon_0 \left( \frac{\varepsilon}{\varepsilon_0} - 1 \right) \pi R^2 \int_{L_p} E_{ax}^2 dx$$

The result for the change in reflection coefficient is

\(^1\)C. W. Steele, IEEE MTT MTT-14, 70 (1966)
\[ \Delta \Gamma = \Gamma_p - \Gamma_a = -\frac{j\omega\varepsilon_0}{2P_1} \left( \frac{\varepsilon}{\varepsilon_0} - 1 \right) R^2 \left\{ \pi \int_{L_p} E_{ax}^2 \, dx + \frac{16R}{3L_p} \int_{L_p} (E_{az}^2 + E_{ay}^2) \, dx \right\} \]

Assuming a periodic structure, the field expressed in terms of space harmonics is

\[ E(x, y, z) = \sum_{n=-\infty}^{\infty} E_{pn}(x, y) \exp[-j(\beta z + 2\pi n z/d)] \]

Substituting \( z = z_1 + md \), where \( z_1 \) is the \( z \) coordinate within a cell, into this gives \( \beta = \Delta \phi / d \) where \( \Delta \phi \) is the phase advance per cell. Therefore,

\[ E(x, y, z) = \sum_{n=-\infty}^{\infty} E_{pn}(x, y) \exp[-j\frac{z}{d}(\Delta \phi + 2\pi n)] \]

Each of the field components advances with the same phase advance per cell, so even though they are not in phase with each other. Therefore, a measurement of the phase advance per cell does not depend on the alignment of the thread. Inserting this expression into that for \( \Delta \Gamma \) gives

\[ \Delta \Gamma \propto \sum_{n,m} E_{an} \cdot E_{am} \exp\left[-j\frac{z}{d}(2\Delta \phi + 2\pi(n + m))\right] \]

\[ = \exp\left[-2j\frac{\Delta \phi}{d} \right] \sum_{n,m} E_{an} \cdot E_{am} \exp\left[-j2\pi(n + m)\frac{z}{d}\right] \]

The phase of \( \Delta \Gamma \) as a function of \( z \) depends on the alignment of the thread since that will determine the number and strength of the space harmonics, but a measurement of the phase advance per cell, \( \Delta \phi \), does not depend on the alignment of the thread.