Abstract

Fabrication errors in traveling wave structures result in non-axisymmetric RF fields that couple to the rf drive at the fundamental mode frequency. We calculate the excitation of the dipole mode and the integrated effect on the beam, using the thin iris and small hole approximation.

1 INTRODUCTION

An advantage commonly claimed for structures with circular symmetry is that the accelerating kick is independent of transverse position, and no transverse deflection results from the fundamental accelerating mode. Here we calculate the correction to this statement resulting from misalignment of the accelerating cells during the fabrication process. The resulting tolerance on fabrication errors is qualitatively different from the well-known limit deriving from the loss in no-load voltage from cell detuning [1]. While the effect is small for conventional accelerator applications, it could be significant for applications requiring low-emittance beams, and for structures of small dimension, where mechanical alignment is difficult.

We consider an n-cell travelling wave structure as a stack of coupled cavities, as illustrated in Fig.1. To calculate the cell-to-cell coupling in the general case,

\[ e_\lambda(t) = \int d^3\vec{r} \tilde{E}(\vec{r}, t) \cdot \tilde{H}_\lambda(\vec{r}), \]

for which Maxwell's Equations may be expressed as

\[ \left( \frac{\partial^2}{\partial t^2} + \omega_\lambda^2 \right) e_\lambda = -\omega_\lambda c \int \left( \hat{n} \times \tilde{E} \right) \cdot \tilde{H}_\lambda \, dA, \]

with \( \omega_\lambda = k_\lambda c. \) The magnetic field amplitude

\[ h_\lambda(t) = \int d^3\vec{r} \tilde{H}(\vec{r}, t) \cdot \tilde{H}_\lambda(\vec{r}). \]

may be computed from \( \partial h_\lambda / \partial t = -\omega_\lambda e_\lambda / Z_0, \) with \( Z_0 \approx 377 \Omega. \) When coupling is magnetic it is convenient to transform the coupling integral to the form,

\[ \left( \frac{\partial^2}{\partial t^2} + \omega_\lambda^2 \right) e_\lambda = \omega_\lambda Z_0 \tilde{H}_\lambda(0) \cdot \int \hat{r} \left( \hat{n} \cdot \frac{\partial \tilde{H}}{\partial \vec{r}} \right) dA. \]

For illustration, consider the case of two right-cylindrical pillboxes operated in the TM010 mode. One has, for the unperturbed mode,

\[ \tilde{E}_{01} = \beta_0 J_0(\beta_0 r) E_{01}, \quad \tilde{H}_{01} = -\hat{n} J_1(\beta_0 r) E_{01}, \]

where \( \beta_0 = \omega_{01} / c = j_{01} / R, \) the cavity radius is \( R, \) the length is \( L, \) and \( j_{01} \approx 2.405. \) The normalization is \( E_{01} = \left( \pi LR^2 j_1^2(\beta_0) \right)^{-1/2}, \) with \( j_1(j_{01}) \approx 0.5192. \) In the presence of a centered aperture of radius \( a << R, \) the electric field lines are deformed, some terminating on the iris edge, with the result that in the iris there is a tangential electric field [1]

\[ \tilde{E}_\perp = \frac{1}{\pi} \frac{\vec{r}}{\sqrt{\vec{r}^2 - a^2}} \Delta E_c, \]

where \( \Delta E_c = (e_{01}^1 - e_{01}^2) E_{01} \) is the discontinuity in normal electric field across the closed iris, and \( e_{01}^k \) is the amplitude of mode \( \lambda, \) in cell \( #k. \) Evaluating the port integral, one finds

\[ \left( \frac{\partial^2}{\partial t^2} + \omega_{01}^2 \right) e_{01}^1 = -\frac{1}{2} \kappa_{01} \omega_{01}^2 \left( e_{01}^1 - e_{01}^2 \right), \]

Figure: 1 An ideal, perfectly aligned n-cell structure.

\[ #1 \quad #2 \quad \ldots \quad #k \quad \ldots \quad #n \]

\#1 #2  •••  #k  •••  #n
\[
\left( \frac{\partial^2}{\partial z^2} + \omega_0^2 \right) e^2_{0i} = -\frac{1}{2} \kappa_{01} \omega_0^2 \left( e^2_{0i} - e^1_{0i} \right),
\]
with coupling constant
\[
\kappa_{01} = \frac{4}{3 \pi \varepsilon_0^2 c^2} \frac{a^3}{LR} = 1.575 \frac{a^3}{LR}.
\]

In the general case of multiple cells, one has a chain of coupled oscillators. Including the perturbation due to wall-losses, one finds that the \( k \)-th interior cell satisfies
\[
\left( \frac{\partial^2}{\partial z^2} + \omega_{0k} \omega_{0k} \right) e^2_{0i} = \frac{1}{2} \omega_{0k}^2 \left( \kappa_{0k} e^2_{0i} - \kappa_{0k} e^{k+1}_{0i} \right).
\]

For a perfectly tuned structure driven at the design frequency \( \omega \), the mode amplitude, takes the form, \( k_{0i} = \tilde{e}_{0i} e^{-\gamma(k-1)} \), with \( \gamma = j \theta + \Gamma \), \( \theta \) the design phase-advance per cell, and \( \Gamma \) the damping decrement per cell. The accelerating or "no-load" voltage may be expressed as \( V_{\text{NL}} = \Re \tilde{V}_{0i} e^{j \pi x} \), with \( t \) the arrival time at the first cell, and
\[
\tilde{V}_{0i} = \left( E_0 \right) e^{j \theta / 2} \sum_k e^{j(k-1)\theta} e^k_{0i},
\]
with \( T = \sin(\phi / 2)/(\phi / 2) \) and \( \phi = \omega L / c \) the transit angle at the drive frequency. By design, travelling-wave structures are operated at synchronism \( \theta = \phi \), so that
\[
\tilde{V}_{0i} = \left( E_0 \right) e^{j \theta / 2} \tilde{e}_{0i} \left( 1 - e^{-nT} \right) \left( 1 - e^{-T} \right)^{-1}.
\]
For a constant impedance structure \( n \Gamma = \tau \) is the attenuation parameter for the structure.

Next, let us perform a similar calculation for the TM\(_{110}\) mode in this geometry. Considering the \( x \)-coupled polarization, the mode basis functions are
\[
\tilde{E}_{1i} = \tilde{z} E_{1i} J_{1}(\beta_{11} r) \cos \phi,
\]
\[
\tilde{H}_{1i} = E_{1i} \left( \frac{\tilde{r} J_{1}(\beta_{11} r)}{\beta_{11}} \sin \phi + \hat{\omega} J_{1}(\beta_{11} r) \cos \phi \right).
\]

where \( \beta_{11} = \omega_{11} / c = j_{11} / R \), \( j_{11} = 3.832 \) and the normalization is \( E_{1i} = \left( \tilde{z} LR^2 J_{1}(j_{11}) \right)^{-1/2} \), where \( J_{1}(j_{11}) = 0.4028 \). Near the axis \( \tilde{H}_{1i} = \tilde{z} \tilde{E}_{1i} \).

To compute the cell-to-cell coupling of the TM\(_{110}\) mode one requires the normal component of the magnetic field in the iris [2],
\[
\tilde{H}_{z} = \frac{\tilde{z} \hat{\omega}}{\sqrt{a^2 - r^2}} \Delta H_y \sin \phi,
\]
where \( \Delta H_y = \frac{1}{2} E_{1i} \left( h_{1i} - h_{1i}^2 \right) \) is the discontinuity in tangential magnetic field across the conducting boundary in the absence of the port. Evaluating the port integral
\[
\tilde{H}_k(0) \sum_{\text{port}} \int \tilde{r} \left( \tilde{n} \cdot \tilde{H} \right) dA = \frac{1}{2} a^3 E_{1i} \Delta H_y,
\]
and, eliminating \( h_{1k}^2 \) in favor of \( e^k_{1} \), one finds,
\[
\left( \frac{\partial^2}{\partial z^2} + \omega_{11}^2 \right) e^1_{1i} = \frac{1}{2} \omega_{11}^2 \kappa_{11} e^1_{1i} - e^2_{1i},
\]
\[
\left( \frac{\partial^2}{\partial z^2} + \omega_{11}^2 \right) e^2_{1i} = \frac{1}{2} \omega_{11}^2 \kappa_{11} e^2_{1i} - e^1_{1i},
\]
where the coupling constant is
\[
\kappa_{11} = \frac{4}{3 \pi \varepsilon_0^2 c^2} \frac{a^3}{LR}.
\]

In the normal development of the subject of dipole modes, it would be natural at this point to extend the formulation to include excitation by the beam in a multi-cell structure. However, our interest is not beam-excitation of dipole modes, but excitation by the externally driven fundamental mode.

### 2 CELL MISALIGNMENT

We consider next a multi-cell structure with misalignments, parameterized as illustrated in Fig. 2. Depending on the fabrication technique, additional constraints may apply. For example, for the Mark III structure [1], all \( c_i \) to within the straightness of the pipe, and the \( x_i \) are roughly independent. For the DDS structure [3], all \( c_i \), \( x_i \) to the machining accuracy of the cups, while the cell-to-cell offsets are roughly independent. Note that each of these approximate equalities places the emphasis on post-machining assembly, where the largest errors occur in practice.

![Figure 2](attachment:image.png)

To compute the dipole mode excitation in cell \( k \), let us first fix attention on the downstream port. We will work to lowest order in misalignments, neglecting quadrupole and higher modes, and neglecting too the perturbation to the fundamental mode from the dipole.
mode. In the absence of a port the discontinuity in tangential magnetic field from the TM$_{01}$ mode alone is

$$\Delta H_y = \frac{1}{2} E_{01} p_{01} \left( \frac{x - c_k}{h_{01}} - \frac{x - c_{k+1}}{h_{01}} \right).$$

It is the zeroth Fourier component in iris-centered azimuth of this term that determines the normal magnetic field threading the port, and this is just the average over the coupling iris, \( \langle \Delta H_y \rangle = \frac{1}{2} E_{01} p_{01} (e_k h_{01}^k - \delta h_{01}^{k+1}) \), with \( e_k = x_{k+1} - c_k \), and \( \delta_k = x_k - c_k \). The resulting drive term for the \( k \)th-cell TM$_{11}$ mode is then

$$\left( \frac{\partial^2}{\partial x^2} + \omega_{11}^2 \right) e_{11}^{k \text{ upstream}, 01} = -\omega_{11}^2 \eta (e_k h_{01}^k - \delta h_{01}^{k+1}),$$

where the coupling constant is

$$\eta = \frac{1}{2} a^3 E_{01} \frac{d_{11}}{\alpha^2} - 1.73 \frac{a^3}{L^2 R}.$$

Including all other perturbations one has

$$\left( \frac{\partial^2}{\partial x^2} + \omega_{11-k}^2 + \omega_{11-k}^2 \right) e_{11}^{k} = \frac{1}{2} \omega_{11-k}^2 (\kappa_{11-k} e_{11}^{k-1} + \kappa_{11-k} e_{11}^{k+1}) - \omega_{11-k}^2 D_k,$$

with

$$D_k = \eta_+ (e_k e_{11}^{k-1} - \delta e_{11}^{k+1}) + \eta_- (\delta e_{11}^{k-1} - e_k e_{11}^{k-1}).$$

As described in the introduction the terms \( \tilde{e}_{11}^k \) are known; thus the amplitudes \( \tilde{e}_{11}^k \) may be obtained in the frequency domain, with the numerical solution of a tri-diagonal matrix equation. The impulse to an electron entering the first cell at time \( t \) can then be computed from

$$\Delta p_x = \Re \tilde{P}_{11} e^{i\omega t},$$

where \( \omega \) is the frequency of the drive and the integral of vertical magnetic field yields

$$\tilde{P}_{11} = \frac{i}{2} \frac{e^{i\omega}}{\omega_{11}^2} E_{01} L T e^{i\omega / 2} \sum_k e_{11k} e^{i(k-1)\omega}.$$

To make some analytic progress, note that it is typically the case that the fundamental mode drive frequency lies in a stop-band for the dipole modes and thus to a good approximation, adjacent TM$_{11}$ excitations are decoupled. Thus \( e_{11k} \) \( \approx \frac{\alpha_{11-k}}{\Delta_x} D_k \), with

$$\Delta_x = \omega_{11}^2 - \omega_{11-k}^2 + j \omega_{11-k} \omega / Q_k.$$ 

For illustration let us simplify to the case of a constant impedance structure. Supressing subscripts, and substituting \( e_{01} \) \( \approx \tilde{e}_{01} e^{-\gamma(k-1)} \), we have

$$e_{11}^{k} e^{i(k-1)\omega} \approx \frac{\alpha_{11-k}}{\Delta_x} \eta \{ \varepsilon_k + \delta_{k+1} e^{-\gamma} - \varepsilon_{k-1} e^{-\gamma} \}.$$

Performing the sum, we have,

$$\frac{1}{mc} \tilde{P}_{11} = \frac{i}{2} e^{i\omega} \sum_{k=1}^{n} e_{11k} e^{i(k-1)\omega} \frac{\alpha_{11-k}}{\Delta_x} \eta \{ \varepsilon_k + \delta_{k+1} e^{-\gamma} - \varepsilon_{k-1} e^{-\gamma} \},$$

where,

$$n \xi = \sum_{k=1}^{n} e_{11k} + \sum_{k=2}^{n} \delta_{k-1} e^{-\gamma} - \sum_{k=1}^{n} \delta_{k+1} e^{-\gamma} - \sum_{k=1}^{n} e_{11k} - 1 \{ (e^{-\gamma} - 1) \sum_{k=2}^{n} (x_k - c_k) + (c_n - c_1) \}.$$

This phasor \( \xi \) is, in magnitude, roughly an average of misalignments over the structure. However, remarkably, for cup-type cells and assuming the first and last cells are centered on the beam-axis, there is no deflection to lowest order in misalignments. For the Mark III-type assembly errors add. We may extract from this result a simple rule of thumb for the maximum deflection \( \Delta \rho \) for a short \( n \Gamma \ll 1 \) structure imparting maximum energy \( \Delta E \),

$$\Delta \rho = \frac{1}{c} \Delta E \frac{\Delta \rho}{\Delta E},$$

with

$$\frac{1}{c} = 1.6 \frac{a^3}{L^2 R}.$$

In this form the result permits simple, practical estimates. For example, taking an 11.4GHz structure, operated in 2\pi/3-mode, with \( a/\lambda \approx 0.16 \), we find \( L \approx 0.88 \text{cm}, \ ) \( R \approx 1 \text{cm}, \ ) \( a \approx 0.42 \text{cm}, \ ) \text{ and } d \approx 15 \text{cm}. \text{ Thus a } 1 \mu \text{m average offset between iris and cell center, in a } 100 \text{MeV accelerating section, can produce a maximum kick of } 7\times 10^{-6} \text{MeV/c, or a } 0.6 \mu \text{rad deflection for a 1GeV beam. This kick is reduced to zero as the beam approaches the accelerating crest.}

3 CONCLUSIONS

We have quantified a new alignment tolerance on accelerating structures, arising from cross-talk between the fundamental and dipole modes, occurring when cylindrical symmetry is broken due to fabrication errors.

REFERENCES