Pulsed Temperature Rise

The pulsed temperature rise for a short wavelength accelerator with a 100 nS long pulse and a gradient of 1 GeV/m is plotted below.

These curves were arrived at using relationships for cylindrical waveguides. Use the following notation:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>iris radius</td>
</tr>
<tr>
<td>b</td>
<td>outer radius</td>
</tr>
<tr>
<td>G</td>
<td>gradient</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>wavelength</td>
</tr>
<tr>
<td>( T_g )</td>
<td>normalized time constant</td>
</tr>
<tr>
<td>( \Delta T )</td>
<td>temperature rise</td>
</tr>
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<td>temperature rise</td>
</tr>
<tr>
<td>( \beta_g )</td>
<td>group velocity</td>
</tr>
<tr>
<td>( T_p )</td>
<td>pulse length</td>
</tr>
<tr>
<td>( s_{at} )</td>
<td>normalized elastance</td>
</tr>
<tr>
<td>( W )</td>
<td>stored energy</td>
</tr>
<tr>
<td>( T_d )</td>
<td>dissipated power</td>
</tr>
<tr>
<td>( A )</td>
<td>surface area</td>
</tr>
<tr>
<td>( d )</td>
<td>diffusion length</td>
</tr>
</tbody>
</table>

The stored energy is

\[
w = G^2 \lambda^2 \left[ \frac{(a/\lambda)^2}{s_{at}(1-\beta_g)} \right]
\]

where \( s_{at} \), the normalized elastance, is given by*

\[
s_{at}(V - m/C) = 5.7 \times 10^{10} \beta_g^{0.4}
\]

The dissipated power is

\[
P_d = \frac{2wc^{1.5}}{T_g \lambda^{1.5}}
\]

where \( T_g \), the normalized time constant, for the structure is*
The temperature rise is given by

\[ \Delta T = \frac{P_d T_p}{C A d}. \]

The surface area can be approximated as

\[ A \approx 2\pi b = 2\pi\lambda \frac{b}{a} \frac{a}{\lambda} \]

where*

\[ \frac{b}{a} = 1.04 - 0.29 \ln \beta_g + 0.068(\ln \beta_g)^2 \]

and \(a/\lambda\) comes from inverting the equation*

\[ \beta_g = \exp \left[ 3.1 - \frac{2.4}{\sqrt{a/\lambda}} - 0.9 \frac{a}{\lambda} \right] \]

The diffusion length \(d\) is

\[ d = \frac{\sqrt{KT_p}}{C} \]

Putting these equations together gives the temperature rise plotted at the beginning of this note.

These temperature rises are unacceptably large. There are a number of possible cures:

1. Lower the gradient since the temperature rise is proportional to \(G^2\). This defeats the purpose of this research.
2. Reduce the pulse length since the temperature rise is proportional to \(T_p^{1/2}\). This is a weak dependence.
3. Lower the operating temperature. The temperature rise is

\[ \Delta T \propto \frac{\rho}{\sqrt{C K}} \]

where \(\rho\) is the resistivity, \(C\) is the heat capacity and \(K\) is the thermal conductivity. Compare room temperature with LN\(_2\) temperature. The resistivity is proportional to temperature for this temperature range, so

\[ \frac{\rho(77\ K)}{\rho(300\ K)} = \frac{77}{300} = 0.257 \]

The American Institute of Physics Handbook (Third Edition) gives data on thermal conductivity and heat capacity. For the thermal conductivity (page 4-154)

\[ \frac{K(77\ K)}{K(300\ K)} = \frac{610\ W/m-K}{401\ W/m-K} = 1.521 \]

For the heat capacity (page 4-106)

\[ T_g(\sec) = 2.36 \times 10^8 (1 + 1.25\beta_g^{1.5}) \]
\[
\frac{C_p(77 \text{ K})}{C_p(300 \text{ K})} = \frac{3.00 \text{ cal/mole K}}{5.95 \text{ cal/mole K}} = 0.504
\]

Combining these numbers:

\[
\frac{\Delta T(77 \text{ K})}{\Delta T(300 \text{ K})} = \sqrt{\frac{0.257}{1.521 \times 0.504}} = 0.58
\]

4. Using a long wavelength.

* R. Palmer, SLAC-PUB-4295