B Physics at Hadron Colliders
Progress and Prospects

XXVII SLAC
Summer Institute on
Particle Physics July 1999

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Outline of Presentation

- Life at the Tevatron (a good example of a hadron collider).
- CDF (a good example of a detector of $B$’s at a hadron collider).
- Measuring Lifetimes (the reason we are in this game at all).
- $B^0 - \bar{B}^0$ Mixing (developing flavor tagging algorithms).
- $B \rightarrow J/\psi K_S$ (indications of $CP$ noninvariance).
- CDF II (a good example of a second generation $B$ detector).
- Expectations for $\sin(2\beta)$ at the Tevatron in Run II (CDF II and D0$\beta$).
- The $B_s$ (lifetimes and mixing).
- Sensitivity to $B_s - \bar{B}_s$ Mixing in Run II
  (use of hadronic triggers and particle ID).
- More Fun with the $B_s$ ($\Delta\Gamma$, transversity and all of that).
- What might be done for $\alpha$ and $\gamma$.
- Forwarding Looking Projects to Watch (HERA-B, BTeV, LHCh).
Thought for the day:

Remember Hero (or Heron)

75 AD Alexandria

\[ s = \frac{1}{2} (a+b+c) \]

\[ K = \sqrt{s(s-a)(s-b)(s-c)} \]
CKM Mixing and $CP$ Violation

Weak interaction eigenstates
rotated wrt mass eigenstates:

$$
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix} =
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
$$

Unitarity implies:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

Experiments indicate that:

$$\mathcal{O}(|V_{us}|) \sim \mathcal{O}(|V_{cb}/V_{us}|) \sim \mathcal{O}(|V_{ub}/V_{cb}|)$$

Wolfenstein parameterization of Cabibbo-Kobayashi-Maskawa mixing matrix:

$$V \simeq \begin{pmatrix}
  1 - \frac{\lambda^2}{2} & \frac{\lambda}{2} & A\lambda^3(\rho - i\eta) \\
  -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
  A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4)$$

$\eta \neq 0$ implies $CP$ violation through $V_{ub}, V_{td}$; $\lambda \simeq \sin(\theta_{\text{Cabibbo}}) \simeq 0.2$
Physics with $B$'s at a Hadron Collider
Tevatron Environment

- Extended luminous region along \( p\bar{p} \) collision axis: \( \pm 30\,\text{cm} \) (rms).
- Small transverse size of luminous region: \( \sigma_x \times \sigma_y \approx 25\,\mu\text{m} \times 25\,\mu\text{m} \).
Tevatron Environment

Integrated luminosity (pb\(^{-1}\)) vs. calendar date for Run I at the Tevatron.

- CDF I design luminosity \(\sim 10^{30} \text{ cm}^{-2}\text{sec}^{-1}\)
- Run I integrated luminosity \(\sim 110 \text{ pb}^{-1}\)
- Instantaneous luminosity at the beginning of a store often \(> 2 \times 10^{31} \text{ cm}^{-2}\text{sec}^{-1}\)
- Average instantaneous luminosity increased from 2 to \(12 \times 10^{30} \text{ cm}^{-2}\text{sec}^{-1}\) during Run I.
**B Hadron Production at the Tevatron**

\[ \bar{p}p \rightarrow BX, \sqrt{s} = 1.8 \text{ TeV} \]

- **Large B production cross section:** \( \sigma_b \sim 100 \mu b \)
- **Even larger total inelastic cross section:** \( \sigma_b / \sigma_{inelastic} \sim 10^{-3} \)
- **\( \sigma_b \) largest at low \( p_T \)**
- **Usable cross section:** \( \sigma_b = 2.4 \pm 0.5 \mu b \)
  \( (p_T > 6 \text{ GeV}/c, |y| < 1) \)
- **All species accessible:** \( B^0, B^+, B_s, \Lambda_b, B_c \)

CDF measurements of the differential cross section for \( B \) production.
The momentum perpendicular to the collision axis is \( p_T \); the rapidity is
\[
y = \frac{1}{2} \ln\left(\frac{E + p_z}{E - p_z}\right) \approx -\ln[\tan(\theta/2)] \equiv \eta \quad \text{and} \quad \eta \text{ is the pseudorapidity.}
\]
Distribution of $B$ hadrons at the Tevatron.

(Left) Distribution in pseudorapidity.

(Right) Distribution of $\beta\gamma = p/m$ vs. pseudorapidity.
Inclusive Cross Section Measurements of $b$ Production.

Good agreement between different measurements.

Agreement between measurement and theory for shape of distributions.

Data a factor of $\sim 2$ to $4$ above central value of Next to Leading Order (NLO) QCD predictions.
QCD Production of Heavy Quarks

Leading order gluon fusion and $\bar{q}q$ annihilation diagrams (flavor creation)

Next to leading order gluon splitting diagrams

Next to leading order flavor excitation diagrams
Azimuthal Correlations for $\bar{b}b$ Production

- Leading order expectation is back-to-back in $\phi$
- Gluon splitting favors production closer in $\phi$

- $|\eta^*| < 0.8$
- $4 < p_t < 25$ GeV/c
- $6 < M^* < 35$ GeV/c$^2$
**Observed Correlations in Azimuth and Rapidity**

(Left) Azimuthal angle between muons from $b\bar{b}$ production observed by the D0 experiment compared to NLO QCD (HVQJET)

(Right) Central-forward rapidity correlation observed by CDF.
Collider Detector Facility (CDF) for Run I

Quadrant view of the detector, stylized to emphasize the central tracking region.
Collision axis is along the $z$ axis.
CDF I Detector Features Crucial for $B$ Physics

- Silicon Microstrip detector (SVX, SVX')
  - for detecting $b$ hadron decay point
  - 4 layer barrel geometry; $r - \phi$ readout
  - 51 cm length; 2.9 < radius < 7.9 cm (track coverage ~60%)
  - impact parameter resolution: $\sigma = [13 \oplus 40/p_T \text{GeV}/c] \mu m$

- Central Tracking Chamber (CTC) (drift chamber)
  - for reconstructing exclusive states and measuring masses
  - 3.2 meters in length; 28 < radius < 132 cm ($B = 1.4T$)
  - 84 layers in 9 superlayers (54 supply $dE/dx$ information)
  - SVX + CTC: $(\delta p_T/p_T)^2 = (0.0066)^2 \oplus (0.0009 \cdot p_T \text{GeV}/c))^2$

- Lepton detection
  - for $b \rightarrow e, \mu; \psi \rightarrow \mu^+\mu^-$
  - Electrons: $\approx 100\%$ of $\Omega$ for $|\eta| < 1$
  - Muons: 84\% of $\Omega$ for $|\eta| < 0.6$; 71\% of $\Omega$ for $0.6 < |\eta| < 1$
Run I Trigger/DAQ System for CDF

- Beam Crossing Rate $\sim 300$ kHz
- Level 1: Trigger on detector subsystems ($\sim 2000$ Hz into Level 2)
  - chambers and scintillators for muons
  - calorimeter for electrons and jets
- Level 2: Combine subsystems ($\sim 30$ Hz into Level 3)
  - based on the ALPHA processor chip
  - 2D tracks found in drift chamber
  - muon detector plus track information $\Rightarrow$ muons
  - calorimeter detector plus track information $\Rightarrow$ electrons
  - dynamic prescaling options
- Level 3: Event reconstruction ($\sim 10$ Hz to tape)
  - offline code used (less SVX), find tracks with $p_T > 1.4$ GeV/c
- $\sim 100$ million events recorded (130 kBytes/event)
- Offline: Full event reconstruction
  - include SVX information, effective tracking for $p_T > 400$ MeV/c
Run I Triggers for $B$ Physics at CDF

- Based on leptons (electrons and muons)
- Minimize $p_T$ threshold
- Seek to maximize unbiased yield within trigger/DAQ bandwidth

Typical dilepton trigger thresholds:
$B \rightarrow J/\psi \rightarrow \mu^+\mu^-$ decay modes; $\langle p_T(B \rightarrow J/\psi K) \rangle \sim 10 \text{ GeV/c}$

- $\sim 1.5 \text{ GeV/c}$ at Level 1 for two muons
- $\sim 2.0 \text{ GeV/c}$ at Level 2 for two muons

and for $(e-\mu)$ mixing measurements:

- $\sim 2.5 \text{ GeV/c}$ for a muon AND $\sim 5 \text{ GeV/c}$ for an electron at Level 2

Typical single lepton trigger thresholds:
semi-inclusive $B$ decays, mixing; $\langle p_T(B \rightarrow \ell X) \rangle \sim 20 \text{ GeV/c}$

- $E_T > 8.0 \text{ GeV}$ at Level 1 for electrons
- $p_T > 7.5 \text{ GeV/c}$ at Level 2 for electrons or muons
Mass Reconstruction Resolution

- Good mass resolution: 
  ~ 16 MeV/c^2

- Natural width for $J/\psi$
  is 87 keV

- About 243,000 $J/\psi$'s
  in sample with
  Signal/Background $\approx 7$

Mass distribution for $J/\psi \rightarrow \mu^+\mu^-$ reconstructed in CDF
(vertex constrained, silicon vertex detector information required).
Separating Prompt and Long-Lived Vertices

- About 20% of $J/\psi$'s are from $B$ decays
  (for $p_T > 5$ GeV/c),
  the rest are from prompt sources.

- Beyond 100 $\mu$m, $J/\psi$'s mainly from $B$ decays

- Width of prompt core implies vertex resolution of
  $\approx 45 \mu$m

Decay length distribution for $J/\psi \to \mu^+\mu^-$
reconstructed in CDF with the silicon vertex detector
$B \rightarrow J/\psi K^{(*)}$, $\psi(2S)K^{(*)}$ Decays

Invariant mass distributions for fully reconstructed charged (left) and neutral (right) $B$ mesons with (lower) and without (upper) a 100 $\mu$m requirement on the decay length.
$B \rightarrow J/\psi K^{(*)}, \psi(2S)K^{(*)}$ Decays

Lifetime distributions for the fully reconstructed charged (left) and neutral (right) $B$ meson signal (upper) and for the (sideband) background (lower).
Partially Reconstructed Decays

The decay length is defined in the plane transverse to the collision axis:

\[ \frac{L_{xy}}{m(B) \rho(B)} = \frac{1}{L_y} \left( \frac{\vec{V}_B - \vec{V}_p}{\vec{p}_T(D\ell)} \right) \cdot \vec{p}_T(D\ell) \]

When the B is fully reconstructed we have:

\[ c t(B) = L_{xy} \frac{m(B)}{\rho(B)} \]
When it is not we use:

\[ c t (B) = L^B_{xy} \frac{m(B)}{p_T(D\ell)} \otimes K, \]

where

\[ K = \frac{p_T(D\ell)}{p_T(B)} \]

is from Monte Carlo.

\( K \) depends on the decay mode but typically is distributed with an rms of \( \sim 0.1 \) around a mean value of \( \sim 0.8 \).
\( B \to \ell D^{(*)} X \) Decays

(a) \( \bar{B} \to D^0 \ell^- \bar{\nu} X \); \( D^0 \to K^- \pi^+ \)
Dominated by \( B^- \) decays; wrong-sign background (i.e. \( \ell^- K^+ \)) scaled by 0.5 (for clarity)
(b) \( \bar{B} \to D^{*+} \ell^- \bar{\nu} X \); \( D^{*+} \to D^0 \pi^+ \); \( D^0 \to K^- \pi^+ \)
(c) \( B \to D^{*+} \ell^- \bar{\nu} X \); \( D^{*+} \to D^0 \pi^+ \); \( D^0 \to K^- \pi^+ \pi^+ \pi^- \)
(d) \( B \to D^{*+} \ell^- \bar{\nu} X \); \( D^{*+} \to D^0 \pi^+ \); \( D^0 \to K^- \pi^+ X \) (N.B. \( \pi^0 \) missing)

(b) - (d) dominated by \( \bar{B}^0 \) decays; \( \Delta M \equiv M(D^0 \pi^+) - M(D^0) \)
Compiled results from LEP and SLC experiments ($Z \rightarrow b\bar{b}$) and from CDF ($\bar{p}p \rightarrow b\bar{b} + X$).
Results for $B^{-}$ to $B^{0}$ Lifetime Ratio

- Theoretical expectation for ratio is unity ±5%
- SLD holds the record for the most precise determination
- The competitive precision of the CDF results bodes well for $B$ physics at hadron colliders.

Compiled results from LEP and SLC experiments ($Z \rightarrow b\bar{b}$) and from CDF ($pp \rightarrow b\bar{b} + X$).
CKM Mixing in the $B^0$ System

A pure $B^0$ or $\bar{B}^0$ state evolves in time via mixing:

$$|B^0(t)\rangle \approx e^{-\frac{t}{2}}e^{-iMt}\{\cos\left(\frac{\Delta mt}{2}\right)|B^0\rangle + ie^{2i\phi M}\sin\left(\frac{\Delta mt}{2}\right)|\bar{B}^0\rangle\}$$

$$|\bar{B}^0(t)\rangle \approx e^{-\frac{t}{2}}e^{-iMt}\{\cos\left(\frac{\Delta mt}{2}\right)|\bar{B}^0\rangle + ie^{-2i\phi M}\sin\left(\frac{\Delta mt}{2}\right)|B^0\rangle\}$$

where $\Delta m = 2|M_{12}|$; $2\phi_M = \arg(M_{12})$; ($|\Gamma_{12}| \ll |M_{12}|$)
CKM Mixing in the $B^0$ System

Mixing probability:

$$|\langle \bar{B}^0 | B^0(t) \rangle|^2 \cong \Gamma e^{-\Gamma t} \sin^2 \left( \frac{\Delta m t}{2} \right) = \frac{1}{2} e^{-\Gamma t} (1 - \cos(\Delta m t))$$

Mixing asymmetry ($CP$ conserving):

$$A_{mix} = \frac{|\langle B^0 | B^0(t) \rangle|^2 - |\langle \bar{B}^0 | B^0(t) \rangle|^2}{|\langle B^0 | B^0(t) \rangle|^2 + |\langle \bar{B}^0 | B^0(t) \rangle|^2} \cong \cos(\Delta m t)$$
$B^0 - \bar{B}^0$ Mixing

Mixing probability, given $B^0$ decay at proper time ($t$):

$$P_{\text{mix}}(t) = \frac{1}{2} \cdot (1 - \cos(\Delta mt))$$
$$P_{\text{nomix}}(t) = \frac{1}{2} \cdot (1 + \cos(\Delta mt))$$

Mixing parameter, $\Delta m_{d,s}$, directly probes CKM elements ($V_{td}, V_{ts}$).

Measurement of mixing frequency requires knowledge of:

* Proper time ($ct$) of decay and flavor at decay
* Flavor 'tag' ($B$ or $\bar{B}$) at production.

Ideally flavor tag has a high efficiency ($\epsilon$) and a large (sic) dilution ($D$):

$$D = \frac{N_{\text{tag}} - N_{\text{mistag}}}{N_{\text{tag}} + N_{\text{mistag}}}$$

where $D = 1$ for a perfect tag, and $D = 0$ for a random tag.

The probability that the tag is incorrect is: $P_{\text{mistag}} = \frac{1}{2} \cdot (1 - D)$.

Power of the flavor tag goes as $\epsilon D^2$. 
The Importance of Vertex and Momentum Resolution

Recall that the decay time is determined from:

\[ c t(B) = L_{xy}^B \frac{m(B)}{p_T(B)} = \frac{L_{xy}^B}{\beta \gamma}. \]

The uncertainty on the decay time depends on the uncertainties in the determination of the vertex position and the \( B \) momentum:

\[ \frac{\sigma_t}{\tau_B} = \sqrt{\left( \frac{\delta L_{xy}^B}{L_{xy}^0} \right)^2 + \left( \frac{t \delta p_T}{\tau_B p_T} \right)^2} \text{ where } L_{xy}^0 = (p_T/m_B) \cdot c\tau_B. \]
Effectiveness of Mixing Measurement

perfect resolution

vertex resolution

momentum resolution

mistagging

vertex, momentum, mistag

plus background
Three Flavor Tagging Algorithms Employed at CDF

- Flavor ($b$ or $\bar{b}$) at decay determined by sign of lepton charge on vertex side
- Flavor at production inferred from:
  - Sign of lepton charge from $B$ hadron decay on opposite-side
  - Jet charge of opposite-side $B$ hadron
  - Same-side 'pion' tagging which exploits:
    * the charge correlations from $B^{**}$ decays (e.g. $B^{**} \rightarrow B^0\pi^+$) and/or
    * correlations in non-resonant $B - \pi$ from fragmentation.
$B^0 - \bar{B}^0$ Mixing with Dilepton Triggers (classic case)

CDF PRELIMINARY

$\Delta m_y = 5.0 \pm 0.5 \text{(stat)} \pm 0.6 \text{(syst)} \text{ ps}^{-1}$
$\chi^2 = 0.70 \pm 0.07 \text{(stat)} \pm 0.08 \text{(syst)}$

- Results of a fit to the same–sign fraction vs. proper time for data collected with an electron–muon (left) and dimuon (right) trigger.
- One lepton is associated with a secondary vertex and the other lepton is the flavor tag; unmixed events have opposite–sign lepton charges.
Jet Charge Flavor Tagging

Jet charge is the momentum weighted average of the charge inside a cone around a jet:

\[ Q_{jet} = \frac{\sum_{i=1}^{n} q_i |\vec{p}_i \cdot \hat{a}|}{\sum_{i=1}^{n} |\vec{p}_i \cdot \hat{a}|} \]

- Sum over \( i \): tracks in \((\eta - \phi)\) cone of 0.8 around opposite-side jet.
- \( \hat{a} \): opposite-side jet axis unit vector.
- Sign of \( Q_{jet} \) is the flavor tag: it is correlated with the sign of the charge of the heavy quark that produced the jet.
Jet charge tagging

Jet charge distributions for double and single vertex data:

* $f^-$ correlated with positive jet charge and vice versa.
* The flavor tagging dilution depends nearly linearly with $|Q_{jet}|$. 

CDF Preliminary

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$B^0 - \bar{B}^0$ Mixing with Soft–Lepton or Jet–Charge Tag

- The same–sign fraction as a function of proper time for events with inclusive electron and muon triggers associated with a secondary vertex.

- An additional soft lepton or the jet charge of an opposite–side jet is used for the flavor tag.

- **Soft Lepton**: $\epsilon D^2 = 0.91 \pm 0.13 \%$; **Jet Charge**: $\epsilon D^2 = 0.78 \pm 0.15$
Same-side flavor tagging for $B^0 - \bar{B}^0$ mixing

Same-side 'pion' tagging exploits:

* the charge correlations from $B^{**}$ decays (e.g. $B^{**} \rightarrow B^0\pi^+$) and/or
* correlations in non-resonant $B - \pi$ from fragmentation.

Analysis does not distinguish $B^{**}$ from fragmentation.
**$B^0 - \bar{B}^0$ Mixing with Same-Side Pion Tag**

- **Measurement of the asymmetry** $(N_{RS} - N_{WS}) / (N_{RS} + N_{WS})$ as a function of proper time for charged and neutral $B$ mesons from a single lepton trigger sample and flavor tagged using a same-side pion tag.

- **The charge of the $B$ meson is determined by reconstructing the $D^0$ or $D^{*+}$ from the $B$ decay.** $\text{SST} \, \epsilon \approx 70\%$, $\mathcal{D}_+ = 0.27 \pm 0.036$, $\mathcal{D}_0 = 0.18 \pm 0.036$. 
$B^0 - \bar{B}^0$ Mixing Measurements

CDF $\Delta m_d$ Results

- $D^*\text{lep} / \text{SST}$
  - $0.471 \pm 0.078 \pm 0.066 \pm 0.034 \text{ ps}^{-1}$
- $\text{lep} / Q^{\text{jet}, \text{lep}}$
  - $0.500 \pm 0.052 \pm 0.043 \text{ ps}^{-1}$
- $e / \mu$
  - $0.450 \pm 0.045 \pm 0.051 \text{ ps}^{-1}$
- $\mu / \mu$
  - $0.503 \pm 0.064 \pm 0.071 \text{ ps}^{-1}$
- $D^*\text{lep} / \text{lep}$
  - $0.516 \pm 0.099 \pm 0.029 \pm 0.035 \text{ ps}^{-1}$
- $D(\text{e}) / \text{lep}$
  - $0.562 \pm 0.068 \pm 0.041 \pm 0.050 \text{ ps}^{-1}$
- **Average**
  - $0.495 \pm 0.026 \pm 0.025 \text{ ps}^{-1}$

$\Delta m_d$ Results

- ALEPH
  - $0.446 \pm 0.020 \pm 0.018 \text{ ps}^{-1}$
- DELPHI
  - $0.497 \pm 0.027 \pm 0.023 \text{ ps}^{-1}$
- L3
  - $0.445 \pm 0.028 \pm 0.028 \text{ ps}^{-1}$
- OPAL
  - $0.466 \pm 0.022 \pm 0.016 \text{ ps}^{-1}$
- SLD
  - $0.526 \pm 0.043 \pm 0.031 \text{ ps}^{-1}$
- CDF
  - $0.495 \pm 0.026 \pm 0.025 \text{ ps}^{-1}$
- **Average**
  - $0.480 \pm 0.010 \pm 0.013 \text{ ps}^{-1}$
CKM Mixing and CP Violation

Decay–rate asymmetry (CP violating):

\[ A_{CP}(t) = \frac{|\langle J/\psi K_S|B^0(t)\rangle|^2 - |\langle J/\psi K_S|B^0(t)\rangle|^2}{|\langle J/\psi K_S|B^0(t)\rangle|^2 + |\langle J/\psi K_S|B^0(t)\rangle|^2} \approx \sin(2\beta) \sin(\Delta mt) \]

where

\[ \Gamma(B^0(t), B^0(t) \rightarrow J/\psi K_S) \propto e^{-\Gamma t\{1 \pm \sin(2\beta) \sin(\Delta mt)\}} \]
Getting to $\sin(2\beta)$

The observed asymmetry is reduced by the tagging dilution:

$$A_{\text{observed}} = D A_{CP}$$

The number of observed tagged events depends on the tagging efficiency, $\epsilon$:

$$N_{\text{observed}} = \epsilon N_{\text{reconstructed}}$$

The uncertainty on the $CP$ asymmetry is:

$$\sigma_{A_{CP}} = \frac{1}{D \sqrt{\frac{1 - A_{\text{observed}}^2}{N_{\text{observed}}}}} \approx \frac{1}{D \sqrt{N_{\text{observed}}}} = \frac{1}{\sqrt{\epsilon D^2 N_{\text{reconstructed}}}}$$

If one measures the time integrated asymmetry, $A_I$:

$$\sin(2\beta) = \frac{1 + x_d^2}{x_d} \cdot A_I$$

$$x_d = \frac{\Delta m_d}{\Gamma} \quad \frac{x_d}{1 + x_d^2} = 0.47$$

Uncertainty on the quantity of interest:

$$\sigma_{\sin(2\beta)} \approx 2 \cdot \frac{1}{\sqrt{(\epsilon D^2)(N_{B \to \psi K_s})}} \cdot \frac{S + B}{S}$$
Time Dependence of Mixing and CP Asymmetries

- Sensitivity for mixing maximum at $t = 0$

$$A_{\text{mix}}(t) = \frac{\mathcal{P}_{\text{unmix}} - \mathcal{P}_{\text{mix}}}{\mathcal{P}_{\text{unmix}} + \mathcal{P}_{\text{mix}}} = \cos \Delta m t$$

- Backgrounds are most important near $t = 0$

- Sensitivity for $CP$ asymmetry maximum at $\sim 2$ lifetimes

$$A_{CP}(t) \equiv \frac{\bar{B}^0(t) - B^0(t)}{\bar{B}^0(t) + B^0(t)} = \sin(2\beta) \cdot \sin \Delta m t,$$

There are advantages to be gained by performing a time dependent fit.
Results from CDF on $\sin(2\beta)$ using SST

(Left) $B^0 \rightarrow J/\psi K_S$ mass distribution with $ct > 0$ and $ct > 200\mu m$. The curve is from a likelihood fit. Fit yields $198 \pm 17$ signal events (for all $ct$). All events required to have both muons well measured in silicon vertex detector.

(Right) Mass side-band subtracted tagging asymmetry vs. the reconstructed $ct$ for $J/\psi K_S$ data. Likelihood fit (solid) and $\chi^2$ fit (dashed) are shown; inset is the likelihood scan. Result obtained: $D \sin(2\beta) = 0.31 \pm 0.18 \pm 0.03$.

For the $J/\psi K_S$ sample: $D = 0.166 \pm 0.018 \pm 0.013$. 

$\Rightarrow \sin(2\beta) = 1.8 \pm 1.1 \pm 0.3$
CDF Measurement of $\sin(2\beta)$

From $D \sin(2\beta) = 0.31 \pm 0.18 \pm 0.03$ and $D = 0.166 \pm 0.018 \pm 0.013$ we obtain $\sin(2\beta) = 1.8 \pm 1.1 \pm 0.3$. This ‘non–physical’ result can be expressed in terms of proper confidence intervals following the work by Cousins and Feldman:

This result excludes $\sin(2\beta) < -0.2$ at 95% CL. Note that independent of $D$ (provided $D > 0$), this result excludes $\sin(2\beta) < 0$ at 90% CL.

If $\sin(2\beta) = 1.0$, the median expectation for an experiment of equal sensitivity would be a 95% CL exclusion for $\sin(2\beta) < -0.89$. 