Uncertainties in some $CP$ asymmetries related to $\sin 2\beta$

Zoltan Ligeti

- Is the SM flavor sector confirmed? [ZL, hep-ph/0408267]
  ... Time dependent CPV can be larger in the SM than $(m_s/m_b) \sin 2\beta$
  ... $SU(3)$ — how far can we get with minimal assumptions?
  ... 2-body: $\phi K_S, \eta' K_S, \ldots$
  ... Brief discussion of explicit calculations
- Conclusions
CKM fits with and without assuming SM

- Consistency of SM fit often said to imply tight constraints on NP — this is wrong

SM fit: impressive agreement

NP in loops: constraints relaxed

- These measurements alone cannot exclude NP in loop processes (coincidence)
Constraining NP in mixing: the ’04 news

- NP in mixing amplitude only, $3 \times 3$ unitarity preserved: $M_{12} = M_{12}^{(SM)} r_d^2 e^{2i\theta_d}$

  $\Rightarrow \Delta m_B = r_d^2 \Delta m_B^{(SM)}$, $S_{\psi K} = \sin(2\beta + 2\theta_d)$, $S_{\rho \rho} = \sin(2\alpha - 2\theta_d)$, $\gamma(DK)$ unaffected

Constraints with $|V_{ub}|$, $\Delta m_d$, $S_{\psi K}$, $A_{SL}$

New in ’04: $\alpha$, $\gamma$, $2\beta + \gamma$, $\cos 2\beta$

- Similar to EW fit: $m_H < \text{few} \times 100 \text{ GeV}$ in SM; model independently only $\lesssim 1 \text{TeV}$
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Constraints with $|V_{ub}|, \Delta m_d, S_{\psi K}, A_{SL}$

New in ’04: $\alpha, \gamma, 2\beta + \gamma, \cos 2\beta$

• New data restrict $\theta_d, r_d^2$ significantly for the first time — still plenty of room left
Photon polarization in $B \rightarrow X \gamma$
Source of photon polarization

- In the SM, charged current is left handed, so $b \rightarrow s_L$

Photon must be left-handed to conserve $J_z$ along decay axis

Inclusive $B \rightarrow X_s \gamma$

Exclusive $B \rightarrow K^* \gamma$

Assumption: 2-body decay

Does not apply for $b \rightarrow s \gamma g$

... quark model ($s_L$ implies $J_z^{K^*} = -1$)

... higher $K^*$ Fock states

BSM right handed interaction (motivated by $\phi K_S$, etc.) can give large $b \rightarrow s \gamma_R$

- What is the SM prediction? What limits the sensitivity to new physics?
Measuring the photon polarization

- Only measurement so far is time dependent $CP$ asymmetry

$$\frac{\Gamma[\bar{B}^0(t) \to f\gamma] - \Gamma[B^0(t) \to f\gamma]}{\Gamma[\bar{B}^0(t) \to f\gamma] + \Gamma[B^0(t) \to f\gamma]} = S_{f\gamma} \sin(\Delta m t) - C_{f\gamma} \cos(\Delta m t)$$

No $\gamma_L - \gamma_R$ interference ⇒ the lore has been: $S_{K^*\gamma} = -2 \left( \frac{m_s}{m_b} \right) \sin 2\beta$

[Atwood, Gronau, Soni, PRL 79 (1997) 185]

- Babar & Belle data:

$$S_{K^*\gamma} = -0.38 \pm 0.34$$

$$S_{K_S\pi^0\gamma} = \begin{cases} -0.58^{+0.46}_{-0.38} \pm 0.11 & \text{(Belle, 0.6 GeV < } m_{K_S\pi^0} < 1.8 \text{ GeV)} \\ 0.9 \pm 1.0 \pm 0.2 & \text{(Babar, 1.1 GeV < } m_{K_S\pi^0} < 1.8 \text{ GeV)} \end{cases}$$

Need $\sim 50 \text{ ab}^{-1}$ to get $\delta(S_{K^*\gamma}) = 0.04$ experimental error

- Few other proposals, all very hard to measure:
  - photon conversion off detector, study $\gamma \to e^+e^-$ and $K^* \to K\pi$ distributions
  - $B \to K_1\gamma$, measure up-down asymmetry of $\gamma$’s relative to $K_1 \to K\pi\pi$ plane
  - $\Lambda_b \to \Lambda\gamma$ decay...

Z. Ligeti — p. 4
Right-handed photons

- Considering dominant operator in SM, $\gamma_R$ suppressed by $m_s/m_b$ to all orders in $\alpha_s$
- Can decouple $\gamma_L$ and $\gamma_R$ at the level of the Hamiltonian

\[ O_7 = \bar{s} \sigma^{\mu\nu} F_{\mu\nu} (m_b P_R + m_s P_L) b = \bar{s} \sigma^{\mu\nu} (m_b F^L_{\mu\nu} + m_s F^R_{\mu\nu}) b \]

- Dominant source of “wrong-helicity” photons in the SM is $O_2$:
  - Equal $b \rightarrow s\gamma_L, s\gamma_R$ rates at $O(\alpha_s)$; calculated to $O(\alpha_s^2/\beta_0)$
  - Inclusively only rates are calculable, get: $\Gamma^{(b\text{rem})}_{22}/\Gamma_0 \simeq 0.025$
  - Suggests: $A(b \rightarrow s\gamma_R)/A(b \rightarrow s\gamma_L) \sim \sqrt{\Gamma^{(b\text{rem})}_{22}/(2\Gamma_0)} \simeq 0.11$
  - Expect similar ratio for $A(b \rightarrow d\gamma_R)/A(b \rightarrow d\gamma_L)$ due to imperfect cancellation between (strong phases of) $c$ & $u$ loops
**Exclusive $B \rightarrow K^* \gamma$**

- Can be analyzed using SCET methods, similar to heavy to light form factors

Technically complicated: in “factorizable” part there is an operator that could contribute at leading order in $\Lambda_{QCD}/m_b$, but its $B \rightarrow K^* \gamma$ matrix element vanishes

NB: $\bar{B}^* \rightarrow \bar{K}^{(*)} \gamma_R$ occurs at leading order; yields $\bar{B}^0 \rightarrow \bar{B}^{0*} \pi_{(soft)} \rightarrow K_S \pi^0_{(soft)} \gamma_R$ with modest $m_{K}\pi$, w/o formal $\Lambda_{QCD}/m$ suppression (rather small numerically)

Subleading order: several contributions to $\bar{B}^0 \rightarrow \bar{K}^{0*} \gamma_R$, no complete study yet

- Our estimate:
  
  $$\frac{A(\bar{B}^0 \rightarrow \bar{K}^{0*} \gamma_R)}{A(\bar{B}^0 \rightarrow \bar{K}^{0*} \gamma_L)} = \mathcal{O}\left(\frac{C_2 \Lambda_{QCD}}{3C_7 m_b}\right) \sim 0.1$$

- We do not expect $S_{\rho\gamma} \ll S_{K^*\gamma}$ in SM (contrary to AGS prediction: $m_d/m_s$)
Conclusions from our analysis

- Lots of room for NP, but SM prediction is not as small and pristine as it was thought
  - Inclusive: \( \Gamma(b \to s\gamma_R)/\Gamma(b \to s\gamma_L) = \mathcal{O}(\alpha_s) \)
  - Exclusive: \( A(\bar{B} \to K^*\gamma_R)/A(\bar{B} \to K^*\gamma_L) = \mathcal{O}(\Lambda_{QCD}/m_b) \)
  - \( S_{fs\gamma} \sim \mathcal{O}(0.1) \) is possible
  - \( S_{fs\gamma} \) has significant uncertainties in SM (e.g., it depends on strong phases)
  - The suppression of \( A_R/A_L \) is not much stronger in \( b \to d\gamma \) than it is in \( b \to s\gamma \)

Comments:

- I would not average \( S_{K^*\gamma} \) and semi-inclusive \( S_{K_S\pi^0\gamma} \)
  [Understanding may improve for \( K^*\gamma \), might show stronger SM suppression]
- Not clear if \( A_R/A_L \) should increase for higher mass states; may have cancellations between different states decaying to \( K(n\pi) \) with same invariant mass
$S_{fs}$ in hadronic $b \rightarrow s$ modes

<table>
<thead>
<tr>
<th>Charmonium</th>
<th>$0.726 \pm 0.037$</th>
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<tbody>
<tr>
<td>$\phi K^0$</td>
<td>$0.34 \pm 0.20$</td>
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<tr>
<td>$\eta' K_S^0$</td>
<td>$0.43 \pm 0.11$</td>
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<tr>
<td>$f_0 K_S^0$</td>
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<tr>
<td>$\pi^0 K_S^0$</td>
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</tr>
<tr>
<td>$\omega K_S^0$</td>
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<tr>
<td>$K^+ K^- K_S^0$</td>
<td>$0.53 \pm 0.17$</td>
</tr>
<tr>
<td>$K_S^0 K^0 K_S^0$</td>
<td>$0.26 \pm 0.34$</td>
</tr>
</tbody>
</table>

Average (s-penguin): $0.43 \pm 0.07$
### Status of $\sin 2\beta_{\text{eff}}$ measurements

| Dominant process | $f_{CP}$ | SM upper bound on $| - \eta_{fCP}S_{fCP} - \sin 2\beta |$ | $\sin 2\beta_{\text{eff}}$ | $C_f$ |
|------------------|---------|---------------------------------|-----------------|-----|
| $b \to c\bar{c}s$ | $\psi K_S$ | $< 0.01$ | $+0.726 \pm 0.037$ | $+0.031 \pm 0.029$ |
| $b \to c\bar{c}d$ | $\psi \pi^0$ | $\sim 0.2$ | $+0.40 \pm 0.33$ | $+0.12 \pm 0.24$ |
| | $D^+D^-$ | $\sim 0.2$ | $+0.67 \pm 0.25$ | $+0.09 \pm 0.12$ |
| $b \to s\bar{q}q$ | $\phi K^0$ | $\sim 0.05$ | $+0.34 \pm 0.20$ | $-0.04 \pm 0.17$ |
| | $\eta' K_S$ | $\sim 0.05$ | $+0.43 \pm 0.11$ | $-0.04 \pm 0.08$ |
| | $K^+K^-K_S$ | $\sim 0.15$ | $+0.53 \pm 0.17$ | $+0.09 \pm 0.10$ |
| | $K_SK.SK_S$ | $\sim 0.15$ | $+0.26 \pm 0.34$ | $-0.41 \pm 0.21$ |
| | $\pi^0 K_S$ | $\sim 0.15$ | $+0.34 \pm 0.28$ | $+0.09 \pm 0.14$ |
| | $f^0 K_S$ | $\sim 0.15$ | $+0.39 \pm 0.26$ | $+0.14 \pm 0.22$ |
| | $\omega K_S$ | $\sim 0.15$ | $+0.55 \pm 0.31$ | $-0.48 \pm 0.25$ |

* My estimates of reasonable limits (strict bounds worse)

Results more consistent than before ICHEP’04; difference $>2\sigma$: $f^0 K_S, K_SK.SK_S$

- Largest deviations from SM: $S_{\eta' K_S} (2.6\sigma)$ and $S_{\psi K} - \langle S_{b\to s} \rangle = 0.30 \pm 0.08 (3.5\sigma)$
Aside: Model building more interesting

- The present $S_{\eta'K_S}$ and $S_{\phi K_S}$ central values can be reasonably accommodated with NP (unlike an $O(1)$ deviation from $S_{\psi K_S}$)

- $B(B \to X_s\gamma) = (3.5 \pm 0.3) \times 10^{-6}$ mainly constrains $LR$ mass insertions

Now also $B(B \to X_s\ell^+\ell^-) = (4.5 \pm 1.0) \times 10^{-6}$ agrees with the SM at $O(20\%)$ level

⇒ new constraints on $RR$ & $LL$ mass insertions
The question

How large should \( S_{fs} - S_{\psi K} \) be, so that it is definitively due to new physics?

Disclaimers: (i) The following bounds are NOT my best estimates of \( |S_{fs} - S_{\psi K}| \).
(That is not the question we were interested in)

(ii) Theory errors have no statistical interpretations; we want several times smaller experimental errors to maximize sensitivity to NP

The successes of the SM are impressive:

- Any of \( \Delta m_K, \epsilon_K^{(i)}, \sin 2\beta, \Delta m_B, B \rightarrow X_s \gamma, X_s \ell^+\ell^- \) could have shown NP

\( \Rightarrow \) Only truly convincing deviations are likely to be interesting
**CP asymmetry in $B^0 \rightarrow f_s$**

- Measuring the same angle ($\beta$) in different decays may be the best way to find NP Amplitudes with one weak phase expected to dominate:

$$A = \lambda^2 V_{cb}^* V_{cs} [P_c - P_t + T_{c\bar{c}s}] + \lambda^4 V_{ub}^* V_{us} [P_u - P_t + T_{u\bar{u}s}]$$

  dominant amplitude  suppressed by $\lambda^2$

In SM: expect $S_{f_s} \approx S_{\psi K}$ and $C_{f_s} \approx 0$ at $O(\lambda^2) \sim 5\%$ level

With NP: $S_{f_s} \neq S_{\psi K}$ and $C_{f_s} \neq 0$ possible

- $\psi K_S$: NP could enter through $B - \bar{B}$ mixing
- $\phi K_S$: NP could enter through both mixing and decay

- Main concern in SM: how to bound $|\bar{A}/A| - 1$, i.e., possible enhancement of $T_{u\bar{u}s}$?
What we are after?

- Bound CKM suppressed (second) term’s contribution:

\[
A_f \equiv A(B^0 \to f) = V_{cb}^* V_{cs}^a a^c_f + V_{ub}^* V_{us} a^u_f = V_{cb}^* V_{cs} a^c_f (1 + \xi_f)
\]

\[
\xi_f \equiv \frac{V_{ub}^* V_{us} a^u_f}{V_{cb}^* V_{cs} a^c_f}, \quad \delta_f = \arg \frac{a^u_f}{a^c_f}
\]

\[
\Rightarrow -\eta_f S_f - \sin 2\beta = 2 \cos 2\beta \sin \gamma \cos \delta_f |\xi_f|
\]

\[
C_f = -2 \sin \gamma \sin \delta_f |\xi_f|
\]

\[
C_f^2 + (\eta_f S_f + \sin 2\beta)/\cos 2\beta)^2 = 4 \sin^2 \gamma |\xi_f|^2
\]

Bounds are ellipses in \(S_f - C_f\) plane; \(C_f\)’s near 0

- Bounds on \(\xi_f\) depend on amount of hadronic physics one is willing to use

- All methods have to calculate or bound \(\xi_f\) (and \(\delta_f\) for \(C_f\))
Simplest example

- Compare: \( B_d^0 \to \pi^0 K^0 \ (\bar{b} \to q\bar{q}s) \) vs. \( B_s^0 \to \pi^0 \bar{K}^0 \ (\bar{b} \to q\bar{q}d) \)

\( SU(3) \) flavor symmetry (in this case \( U \)-spin) implies amplitude relations:

\[
A(\bar{B}_d^0 \to \pi^0 K^0) = V_{cb}^* V_{cs} (P_c - P_t + T_{c\bar{c}s}) + V_{ub}^* V_{us} (P_u - P_t + T_{u\bar{u}s}) \equiv P + T
\]

\[
A(\bar{B}_s^0 \to \pi^0 \bar{K}^0) = V_{cb}^* V_{cd} (P_c - P_t + T_{c\bar{c}s}) + V_{ub}^* V_{ud} (P_u - P_t + T_{u\bar{u}s}) = \lambda P + \lambda^{-1} T
\]

- Assume \( B_d \) decay dominated by \( P \), while \( B_s \) by \( T \):

\[
|\xi| \equiv \left| \frac{T}{P} \right| = \lambda \sqrt{\frac{\Gamma(B_s^0 \to \pi^0 \bar{K}^0)}{\Gamma(B_d^0 \to \pi^0 K^0)}}
\]

- Without assumptions:

\[
\left| \frac{\xi + \lambda^2}{1 + \xi} \right| = \lambda \sqrt{\frac{\Gamma(B_s^0 \to \pi^0 \bar{K}^0)}{\Gamma(B_d^0 \to \pi^0 K^0)}} \quad \text{hard for } |\xi|_{\text{max}} \text{ to approach } \lambda^2
\]

(would need info on phases)

Next complications: no \( B_s \) data, octet-singlet mixing, messy amplitude relations
General case

- For $\bar{b} \to q\bar{q}s$ transitions:
  \[ A_f = V_{cb}^* V_{cs} a_f^c + V_{ub}^* V_{us} a_f^u = V_{cb}^* V_{cs} a_f^c (1 + \xi_f) \]

- For $\bar{b} \to q\bar{q}d$ transitions:
  \[ A_{f'} = V_{cb}^* V_{cd} b_{f'}^c + V_{ub}^* V_{ud} b_{f'}^u = V_{ub}^* V_{ud} b_{f'}^u (1 + \lambda^2 \xi_{f'}^{-1}) \]

- $SU(3)$ gives relations among $a_f^q$ and $b_{f'}^q$: $a_f^u = \sum_{f'} x_{f'} b_{f'}^u$

The branching ratios $\mathcal{B}(f)$ constrain $a_f^c$ and $b_{f'}^u$: \[
\left| \frac{V_{ub}^*}{V_{ud}} \frac{V_{us}^*}{V_{cs}} \frac{b_{f'}^u}{a_f^c} \right| \sim \sqrt{\frac{\mathcal{B}(f')}{\mathcal{B}(f)}}
\]

- Combining $SU(3)$ and experimental data gives, conservatively:
  \[ |\xi_f| \equiv \left| \frac{V_{ub}^*}{V_{ud}} \frac{a_f^u}{a_f^c} \right| < \left| \frac{V_{us}}{V_{ud}} \right| \sum_{f'} |x_{f'}| \sqrt{\frac{\mathcal{B}(f')}{\mathcal{B}(f)}} \]

As explained, the bound is on \[ |\xi_f + (V_{us} V_{cd})/(V_{ud} V_{cs})| \], small difference if \( \lambda^2 \ll \xi_f < 1 \)
$SU(3)$ relations for $B \rightarrow P_8 P_8$

- $H \sim (\bar{b}q_i)(\bar{q}_j q_k)$ transforms as
  $3 \times 3 \times \overline{3} = 15 + \overline{6} + 3 + 3$
  $8 \times 8 = 27 + 10 + 10 + 8_S + 8_A + 1$

5 amplitudes describe 15 final states when $SU(3)$ breaking is neglected

For $\eta^{(i)}$ (singlet part), 3 more $B \rightarrow P_8 P_1$ matrix elements

$\Rightarrow$ Relations among the matrix elements

- Decomposition of $a_f^u$ and $b_f^u$, identical with that of $a_f^c$ and $b_f^c$, although the matrix elements are independent $\Rightarrow$ use: $a(f) \equiv a_f^{u,c}$ and $b(f') \equiv b_{f'}^{u,c}$
**η′K_S**: the answer

- Best bound at present comes from: \( s \equiv \sin \theta_{\eta\eta'}, \ c \equiv \cos \theta_{\eta\eta'} \)

\[
a(\eta'K^0) = \frac{s^2 - 2c^2}{2\sqrt{2}} b(\eta'\pi^0) - \frac{3sc}{2\sqrt{2}} b(\eta\pi^0) + \frac{\sqrt{3}s}{4} b(\pi^0\pi^0)
- \frac{\sqrt{3}s(s^2 + 4c^2)}{4} b(\eta'\eta') + \frac{3\sqrt{3}sc^2}{4} b(\eta\eta) + \frac{\sqrt{3}c(2c^2 - s^2)}{2\sqrt{2}} b(\eta\eta')
\]

\[
|\xi_{\eta'K_S}| < \left| \frac{V_{us}}{V_{ud}} \right| \left( 0.59 \sqrt{\frac{B(\eta'\pi^0)}{B(\eta'K^0)}} + 0.33 \sqrt{\frac{B(\eta\pi^0)}{B(\eta'K^0)}} + 0.14 \sqrt{\frac{B(\pi^0\pi^0)}{B(\eta'K^0)}} 
+ 0.53 \sqrt{\frac{B(\eta'\eta')} {B(\eta'K^0)}} + 0.38 \sqrt{\frac{B(\eta\eta)} {B(\eta'K^0)}} + 0.96 \sqrt{\frac{B(\eta\eta')} {B(\eta'K^0)}} \right)
\]

- Yields: \(|\xi_{\eta'K_S}| < 0.17\)
Using the $\eta'K^+$ mode

- Similar relations hold for charged $B$ decays ($x = \text{free param}$.)

\[
a(\eta'K^+) = \frac{(3 - x)c_s}{2} b(\eta\pi^+) + \frac{(x - 1)s^2 + 2c^2}{2} b(\eta'\pi^+) \\
+ \frac{(x - 3)s}{2\sqrt{3}} b(\pi^+\pi^0) + \frac{x s}{\sqrt{6}} b(\bar{K}^0 K^+)
\]

Experimental data $\Rightarrow |\xi_{\eta'K^+}| < 0.08$

- We have $a^c_{\eta'K^0} = a^c_{\eta'K^+}$, but $a^u_{\eta'K^0} \neq a^u_{\eta'K^+}$

\[
a^u_{\eta'K^+} \text{ has a color-allowed tree contribution} \\
a^u_{\eta'K^0} \text{ only arises from a color-suppressed tree diagram or penguins}
\]

- Assumption: $|a^u_{\eta'K^+}| \not< |a^u_{\eta'K^0}|$ (l.h.s. larger in large-$N_c$; comparable in SCET)

$\Rightarrow |\xi_{\eta'K^0}| < 0.08$
Bounds for $B \to \phi K_S$

- For $PV$ final state, more matrix elements... more complicated relations:

$$a(\phi K^0) = \frac{1}{2} [b(\bar{K}^*0 K^0) - b(K^*0 \bar{K}^0)] + \frac{1}{2} \sqrt{\frac{3}{2}} [cb(\phi \eta) - sb(\phi \eta')]$$

$$+ \frac{\sqrt{3}}{4} [cb(\omega \eta) - sb(\omega \eta')] - \frac{\sqrt{3}}{4} [cb(\rho^0 \eta) - sb(\rho^0 \eta')]$$

$$+ \frac{1}{4} b(\rho^0 \pi^0) - \frac{1}{4} b(\omega \pi^0) - \frac{1}{2\sqrt{2}} b(\phi \pi^0)$$

$\Rightarrow$ No bound on $\xi_{\phi K_S}$ using only $SU(3)$ at present (because of $\bar{K}^*0 K^0$ and $K^*0 \bar{K}^0$)

- Charged modes: $a(\phi K^+) = b(\phi \pi^+) + b(\bar{K}^*0 K^+)$ (Grossman, Isidori, Worah)

Contrary to $\eta' K_S$, $a_{\phi K^0}^u$ and $a_{\phi K^+}^u$ are of same order in $N_c$ ($u\bar{u} \to \phi$ is suppressed)

Dynamical assumption: $|a_{\phi K^+}^u| \ll |a_{\phi K^0}^u| \Rightarrow |\xi_{\phi K_S}| < 0.23$
A plea...

- Progress since 2003:
  \[ \xi_{\eta'} K_S \text{ bound: } 0.36 \text{ in '03 } \rightarrow \text{ 0.17 now } [\eta' K^+ \text{ bound: } 0.09 \rightarrow 0.08] \]
  [Due to new data: hep-ex/0403046, hep-ex/0412043]
  \[ \xi_{\phi K^+} \text{ bound: } 0.25 \text{ in '03 } \rightarrow \text{ 0.23 now } [\text{still no } \phi K_S \text{ bound based only on } SU(3)] \]

- HFAG → Rare Decays → Charmless Mesonic → \( B^+ \) table: \( K^*0 K^+ \) is one of 7 modes where CLEO rules [no Babar / Belle data; all are \( K\pi h(h) \) type final states]

- \( \phi K_S \): No bound yet on \( K^*0 K^0 \) and \( K^*0 \overline{K}^0 \)

Someone, please, look at these!
Other interesting modes

• 3-body modes: No time for $K^+K^-K_S$

  $K_SK_SK_S$

  [GLNQ; Gronau & Rosner]

  [Engelhard, Nir, Raz, ZL, to appear]

• We missed: $B \to \pi^0 K_S$ — simple amplitude relation:

  $$a(\pi^0 K_S) = \frac{1}{\sqrt{2}} b(K^+K^-) - b(\pi^0\pi^0)$$

  Follows from table shown 5 pages earlier... not noticed until asked by Babarians

  $\Rightarrow |\xi_{\pi^0K_S}| < 0.15$


• Other 2-body modes:

  E.g.: could $B \to \rho^0 K_S$ have much larger rate than $B \to \pi^0 K_S$?

  Amplitude relations involve: $B \to \rho^0\pi^0,$ $\rho^0 K^0,$ $K^\ast\pi^0,$ $K^\ast\pm K^\mp$
Calculating $S_{f_s} - \sin 2\beta$
What is known model independently?

- Model independent $\equiv$ theoretical uncertainty suppressed by small parameters
  ... so theorists argue about (small parameters) $\propto O(1)$ instead of $O(1)$ effects

- We now know a lot more about $B \to M_1 M_2$ matrix elements than a few years ago

At leading order in $\Lambda_{QCD}/m_b$ SCET and QCDF agree, except for:

- SCET proved that $\sqrt{\Lambda_{QCD} m_b}$ scale enters form factors & nonleptonics the same
- choice of input parameters (QCDF takes $\zeta_J \ll \zeta$, SCET fits to data)
- charm penguin issue

- Do not understand well yet the accuracy of $\Lambda_{QCD}/m_b$ expansion in various cases
  ... a priori not known whether $\Lambda \sim 200$ MeV or $\sim 2$ GeV ($f_\pi, m_\rho, m_K^2/m_s$)
  ... need experimental guidance to see which cases work how well

- By the time measurements get there, we should understand these issues better
QCDF predictions

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<th>$a_f^u$</th>
<th>$-\eta_{CP} S_f$</th>
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<td>$\pi K_S$</td>
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<td>$[+0.02, 0.13]$</td>
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<tr>
<td>$\rho K_S$</td>
<td>$-0.08^{+0.08}_{-0.12}$</td>
<td>$[-0.24, 0.02]$</td>
</tr>
<tr>
<td>$\eta' K_S$</td>
<td>$0.01^{+0.01}_{-0.01}$</td>
<td>$[-0.01, 0.03]$</td>
</tr>
<tr>
<td>$\eta K_S$</td>
<td>$0.10^{+0.11}_{-0.07}$</td>
<td>$[-1.45, 0.27]$</td>
</tr>
<tr>
<td>$\phi K_S$</td>
<td>$0.02^{+0.01}_{-0.01}$</td>
<td>$[+0.01, 0.04]$</td>
</tr>
<tr>
<td>$\omega K_S$</td>
<td>$0.13^{+0.08}_{-0.08}$</td>
<td>$[+0.03, 0.19]$</td>
</tr>
</tbody>
</table>


Questions: precision of input parameters; accuracy in presence of cancellations?

I’d love to see predictions for both $S_f$ and $C_f$ (and for $A_{\psi K}$)

If $C_{\omega K_S} = -0.48 \pm 0.25$ becomes significant, unclear if it could be accommodated independent of NP
Other bounds for $\eta'K_S$

- Quark model: relate $T_{\eta'K_S}$ to $B \rightarrow \pi^+\pi^-$ using naive factorization and $SU(3)$
  
  Obtain: $|\delta \sin(2\beta)| \lesssim 0.02$, [London & Soni, hep-ph/9704277]

- Solid curve: $SU(3)$ bound w/o neglecting $\lambda^2$ terms
  
  $\left| \frac{\xi + \lambda^2}{1 + \xi} \right| < X \rightarrow |\xi| < \frac{X + \lambda^2}{1 - X}$

  Dashed curve: assuming a relation between singlet and octet matrix elements

  [N.B.: I would consider the charged mode, $\eta'K^+$, bound more reliable; and it is a bit stronger]

\[\text{Diagram showing plots of } C_{\eta K} \text{ vs. } S_{\eta K} \]
Summary for $S_{\eta'K_S}$ and $S_{\phi K}$

- $S_{\eta'K_S} = 0.43 \pm 0.11$: largest single deviation from $S_{\psi K}$ at present (2.5σ)

Conservative SM bound: $|\xi_{\eta'K_S}| < 0.17$ (< 0.08 using $\eta'K^+$ and large $N_c$)

$S_{\eta'K_S}$ at its present central value with < half the error would signal NP

Would not only exclude SM, but MFV and universal SUSY models such as GMSB

- $S_{\phi K} = 0.34 \pm 0.20$: significant effect still possible, need to further decrease errors

No bound yet based only on $SU(3)$; w/ some dynamical assumption, $|\xi_{\phi K_S}| < 0.23$

$S_{\phi K_S}$ at its present central value with smaller error would be a sign of NP

- There is a lot to learn from more precise measurements
Conclusions

• Consistency of SM fit does not imply similarly tight constraints on NP

• Right-handed photon polarization in $B \to X\gamma$ is only suppressed by $\alpha_s$ and $\Lambda_{QCD}/m_b$; $S_{K^*\gamma}, S_{K_S\pi^0\gamma} \sim 0.1$ possible in SM, significantly larger implies NP

• Our bounds on $|\sin 2\beta - S_{f_s}|$ are weaker than estimates based on explicit calculations, but have the advantage of being model independent

• $SU(3)$ breaking effects could be significant, but the bounds are probably still very conservative — with more data the bounds will improve

• Present $S_{n'K_S}$ and $S_{\phi K_S}$ central values with $5\sigma$ significance would be convincing signals of NP
Backup slides
Three-body decays

\[ K^+ K^- K_S \]
$K^+K^-K_S$ does not have definite $CP$, so more complicated than 2-body decays

Consider only $b \to sg$ penguin diagrams: $I = 0$; initial $B \in \frac{1}{2} \Rightarrow K^I\overline{K}^J K^L \in \frac{1}{2}$

Only 2 isospin amplitudes $[(K^IK^L) \in \{0, 1\}]$, no interference in total rates

Denote: $A_{IJL}(p_1, p_2, p_3) \equiv A[B \to K^I(p_1)\overline{K}^J(p_2)K^L(p_3)]$ \quad $I, J, L = \{+, -, 0, S\}$

Then: $A_{00+} = A_{+-0}$, $A_{+00} = A_{0-+}$, $A_{000} = A_{+--}$

Can write $H$ as: $H \propto (B^iK_i) \left[ x (K^jK_j)_{l=\text{even}} + \sqrt{1 - x^2} (K^jK_j)_{l=\text{odd}} \right]$}

$B \to K^+K^-K^0$: $CP(K^+K^-) = +1 \Rightarrow CP(K^+K^-K_S) = (-1)^l$

$\Rightarrow x^2$ gives the $CP$-even fraction

$B \to K^0\overline{K}^0K^+$: $l = \text{even}$ is $K_SK_S + K_LK_L$, $l = \text{odd}$ is $K_SK_L$

$\Rightarrow x^2 = 2\Gamma_{+SS}/\Gamma_{+00} = 2\Gamma_{+SS}/\Gamma_{+-0} = 0.97 \pm 0.15 \pm 0.07$
BELLE analysis (cont’d)

● Once we know the $CP$-even fraction, $x^2$, and it is near 1, we’re in good shape:

$$S_{KKK} = \frac{S_{KKK}^{\text{exp}}}{2x^2 - 1}$$

... $S_{KKK}$ is the would-be $S$, if $K^+K^-K_S$ had a definite $CP$  ⇒ DONE!

● N.B.: predictions of isospin symmetry that enter this analysis are not yet tested

$$B_{+-+} = (3.30 \pm 0.18 \pm 0.32) \times 10^{-5}$$

$$B_{+-0} = (2.93 \pm 0.34 \pm 0.41) \times 10^{-5}$$

$$B_{+SS} = (1.34 \pm 0.19 \pm 0.15) \times 10^{-5}$$

$$B_{SSS} = (0.43^{+0.16}_{-0.14} \pm 0.75) \times 10^{-5}$$

Isospin does not imply $B_{+-+} \approx B_{+-0}$; a test requires measuring a rate with $K_L$, e.g.: $B_{+-S} = \frac{1}{2} B_{+SL} + B_{+SS}$
When the $b \to u\bar{u}s$ part of $H$ is included, $H \in \{0, 1\}$

With $K^I \bar{K}^J K^L \in \{\frac{1}{2}, \frac{3}{2}\}$ — 5 independent isospin amplitudes \([(K^I K^L) \in \{0, 1\}]$

Only a single amplitude relation remains:

$$A_{000} + A_{+-+} + A_{+00} + A_{00+} + A_{+-0} + A_{0-+} = 0$$

Both relations used before:

$$\Gamma_{+-S}(l = \text{even})/\Gamma_{+-S} = \Gamma_{+00}(l = \text{even})/\Gamma_{+00}$$

$$\Gamma_{+00} = \Gamma_{+-0}$$

are corrected by terms proportional to the ratio of $I \in 1$ and $I \in 0$ contributions

... At present no constraint on these from data

N.B.: Large isospin violation: $\mathcal{B}(\phi \to K^+ K^-) \approx 49\%$ and $\mathcal{B}(\phi \to K_S K_L) \approx 34\%$ can be understood arising due to phase space; contribution to $x^2$ is only $\mathcal{O}(4\%)$
U-spin analysis

- Even if the $I = 1$ part in $H$ were negligible, so determination of $CP$-even fraction in $K^+K^-K_S$ very precise, it would not imply $-S_{KKK} = \sin 2\phi_1$ to same precision

The $b \to \bar{u}\bar{d}s$ tree has $I = 0$ part, which would not affect extraction of $CP$-even fraction, but would shift $S_{KKK}$ from $\sin 2\phi_1$

- $U$-spin ($d \leftrightarrow s$) relates $B^+ \to h_i^+ h_j^- h_k^+$ modes: $B^+$ is singlet, $(K^+, \pi^+)$ is doublet

$b \to (\bar{u}u + \bar{d}d + \bar{s}s)q$ penguin and $b \to u\bar{u}q$ tree amplitudes ($q = d, s$) are $\Delta U = 1/2$

⇒ $2$ $U$-spin amplitudes for $B^+ \to h_i^+ h_j^- h_k^+$

$U$-spin relation: $a(K^+K^-K^+) = b(\pi^+\pi^-\pi^+)$ [same accuracy as $SU(3)$]

Experimental data: $\Rightarrow |\xi_{KKK}| = \left| \frac{V_{us}}{V_{ud}} \right| \sqrt{ \frac{B(B^+ \to \pi^+\pi^-\pi^+)}{B(B^+ \to K^+K^-K^+)} } \approx 0.13 (\pm 0.06)?$