D-Branes, Helices, and Proton Decay

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Proton decay is investigated by methods of category theory. The investigation leads to the conclusion that proton decay is forbidden.

1 Introduction

Recently the new description of D-branes was proposed [1–3]. This description is based on methods of category theory [4]. In the present paper we apply these methods to investigation of proton decay.

2 The triangulated category

The triangulated category contains the following data [4]:

1) Distinguished triangles

\[
\begin{array}{c}
\text{C} \\
\text{A} \rightarrow \text{B}
\end{array}
\]

(\text{C} = \text{Cone}(f))

(where vertices are complexes of coherent sheaves),

2) Octahedral diagrams

\[
\begin{array}{c}
\text{F} \leftarrow \text{E} \\
\text{F} \rightarrow \text{E}
\end{array}
\]

(1)

(where distinguished triangles are marked by •).

These data satisfy Verdier axioms.

3 Helices

Let us consider the special class of distinguished triangles

\[
\begin{array}{c}
\text{V}_X^i \\
\text{V}_X^j \\
\text{V}_X^i \rightarrow \text{V}_X^j
\end{array}
\]

(2)

where \( V_X^i \) and \( V_X^j \) are coherent sheaves over the Calabi–Yau manifold \( X \), which are constructed by mutations of helices [5–7].
A collection of coherent sheaves \( \{ \mathcal{R}_W^i \} \) over the weighted projective space \( W \) is called a helix if the following condition is satisfied: The Euler matrix
\[
\chi(\mathcal{R}_W^i, \mathcal{R}_W^j) = \int_W \text{ch}(\mathcal{R}_W^i \otimes \mathcal{R}_W^j) \text{td}(T_W)
\]
is an upper-triangular matrix with ones on the diagonal.

There exists a mutated helix \( \{ \mathcal{S}_W^j \} \) over the weighted projective space \( W \) if the following orthogonality relation holds
\[
\int_W \text{ch}(\mathcal{R}_W^i) \text{ch}(\mathcal{S}_W^j) \text{td}(T_W) = \delta_{ij}.
\]

Coherent sheaves \( V_X^j \) are obtained by the restriction of \( \mathcal{S}_W^j \) to the Calabi–Yau manifold \( X \).

We interpret vertices of distinguished triangles (2) as B-type D-branes if criteria for \( \Pi \)-stability are satisfied [2]. Edges of triangles (2) are interpreted as superstrings.

4 \( \Pi \)-stability

In order to investigate \( \Pi \)-stability of the D-brane Cone(\( f \)) against decay into the D-branes \( V_X^i \) and \( V_X^j \) we need to compute the central charges of \( V_X^i \) and \( V_X^j \).

The central charge of \( V_X^i \) is determined by [2]
\[
Z(V_X^i) = \sum_k Q_k^i \Pi^k = \int_X e^{-B-iJ} \text{ch}(V_X^i) \sqrt{T_X},
\]
where \( Q_k^i \in H^3(Y, \mathbb{Z}) \) are the RR charges [8] (\( Y \) is the mirror of \( X \)), \( \Pi^k \) is the Kähler period vector (which describes the Kähler moduli space of \( X \) [9]), \( B + iJ \) is the complexified Kähler form, \( T_X \) is the tangent sheaf over \( X \).

The grade associated with the central charge (3) is defined by
\[
\varphi(V_X^i) = -\frac{1}{\pi} \arg Z(V_X^i)
\]

The D-brane Cone(\( f \)) is \( \Pi \)-stable if
\[
\varphi(V_X^j) - \varphi(V_X^i) < 0.
\]

The application of criteria for \( \Pi \)-stability to distinguished triangles enclosed in the octahedral diagram (1) leads to the following rule of D-brane decays [2]:

\( \star \) If \( C \) is stable against decay into \( A \) and \( B \), but that \( B \) itself is unstable with respect to a decay into \( E \) and \( F \), than \( C \) will always be unstable with respect to decay into \( F \) and some bound state \( G \) of \( A \) and \( E \).

5 Proton decay

In a grand unified theory [10] proton decay is described by the quark-lepton diagram
Assuming that quarks, leptons and $X$-bosons are solitonic excitations in a proton, we can construct the octahedral diagram

$$
\begin{array}{c}
\bar{d} \\
\downarrow^{[1]} \\
\bullet \\
\uparrow \\
\bar{d}
\end{array}
\begin{array}{c}
\bar{d} \\
\downarrow^{[1]} \\
\bullet \\
\uparrow \\
\bar{d}
\end{array}
$$

$$
\begin{array}{c}
\bar{d} \\
\downarrow^{[1]} \\
\bullet \\
\uparrow \\
\bar{d}
\end{array}
\begin{array}{c}
\bar{d} \\
\downarrow^{[1]} \\
\bullet \\
\uparrow \\
\bar{d}
\end{array}
$$

which induces proton decay.

Let us consider the distinguished triangle

$$
\begin{array}{c}
\bar{d} \\
\downarrow^{[1]} \\
\bullet \\
\uparrow \\
\bar{d}
\end{array}
\begin{array}{c}
\bar{d} \\
\downarrow^{[1]} \\
\bullet \\
\uparrow \\
\bar{d}
\end{array}
$$

enclosed in the octahedron (4). Taking into account the allowed region (shown in white) for $u$-quark and $d$-quark masses [11]

we conclude that in the triangle (5) $u$ is stable with respect to a decay into $\bar{d}$ and $\bar{d}$. This conclusion is incompatible to the rule of decays $\star$ (where $B$ is unstable with respect to a decay into $E$ and $F$). Therefore proton decay is forbidden.