Lecture 2: Time-Dependent Measurements with Flavor and CP Samples

- Reconstruction of B meson samples
- Lifetimes and $B^0$ oscillation measurements with flavor eigenstate decay
Experimental Technique for B Factories

\[ B_{\text{tag}} = \bar{B}^0(t) \]
\[ \Rightarrow B_{\text{rec}} = B^0(t) \]

\[ \Upsilon(4S) \text{ produces coherent } B \text{ pair: } t \rightarrow \Delta t \]

Time-integrated asymmetries are zero

\[ B_{\text{rec}}^0 = B_{\text{flav}}^0 \] (flavor eigenstates) \rightarrow \text{ lifetime, mixing analyses}

\[ B_{\text{rec}}^0 = B_{\text{CP}}^0 \] (CP eigenstates) \rightarrow \text{ CP analysis}

Exclusive B Meson Reconstruction
Main Variables for B Reconstruction

For exclusive B reconstruction, two nearly uncorrelated* kinematic variables are used:

\[ \Delta E = E_B^* - E_{beam}^* \]

Signal at \( \Delta E \sim 0 \)

\[ m_{ES} = \sqrt{(E_{beam}^*)^2 - (p_B^*)^2} \]

Signal at \( m_{ES} \sim m_B \)

For \( (E_B^*, p_B^*), E_{beam}^* \) B candidate (energy, 3-momentum) and beam energy in \( \Upsilon(4S) \) frame

Resolutions

\[ \sigma_{\Delta E}^2 = \sigma_{beam}^2 + \sigma_E^2 \sim \sigma_E^2 \]

\[ \sigma_{m_{ES}}^2 = \sigma_{beam}^2 + \left( \frac{p}{m_B} \right)^2 \sigma_p^2 \sim \sigma_{beam}^2 \]

\[ \sigma_{\Delta E} \sim 10 - 40 \text{ MeV} \]

\[ \sigma_{m_{ES}} \sim 2.6 \text{ MeV/c}^2 \]

* If \( \sigma_E \) were zero, the variables would be fully correlated; however, \( \sigma_E \) is typically at least 5 times larger than \( \sigma_{beam} \) and so dominates \( \Delta E \)
Example for Hadronic B Decays

Signal Region:

\[ [m_{ES}, \Delta E] = [m_B \pm 3\sigma_{m_{ES}}, 0 \pm 3\sigma_{\Delta E}] \]

Sideband Region:

Defined outside signal region in order to estimate backgrounds

\[ B^0 \rightarrow J/\psi K_S \]
Continuum Background Suppression

- Separate 2-jet continuum from spherical BB events via event shape variables

Ratio of second-to-zero\textsuperscript{th} order Fox-Wolfram moments ($R_2$):

![Graph showing $R_2$ distribution with B decays and continuum regions]

Angle of thrust axis of “rest of the event” wrt $B$ candidate direction ($\theta_T$)

![Graph showing $\cos\theta_T$ distribution with B decays and continuum regions]
Inclusive Open Charm States

- Select intermediate mesons using either mass or mass difference:

- After selection, candidates are constrained to nominal masses.
Inclusive Charmonium Signals

\[ J/\psi \rightarrow e^+e^- \]

\[ J/\psi \rightarrow \mu^+\mu^- \]

\[ \psi(2S) \rightarrow J/\psi \pi^+\pi^- \]

\[ J/\psi \rightarrow e^+e^- \]

\[ J/\psi \rightarrow \mu^+\mu^- \]

\[ \psi(2S) \rightarrow J/\psi \pi^+\pi^- \]

\[ J/\psi \rightarrow e^+e^- \]

\[ J/\psi \rightarrow \mu^+\mu^- \]

\[ \psi(2S) \rightarrow J/\psi \pi^+\pi^- \]
Flavor Eigenstate Neutral B Sample

Charmed decay modes \( \sum BF(\bar{D}^0) \sim 28\% \) \( \sum BF(D^-) \sim 12\% \)

\[ D^*^- \rightarrow \bar{D}^0 \pi^- , \bar{D}^0 \rightarrow K^+ \pi^-, K^+ \rho^- , K^+ \pi^+ \pi^- \pi^- , K_S^0 \pi^+ \pi^- \]

\[ D^- \rightarrow K^- \pi^+ \pi^+ , K_S^0 \pi^- \]

B decay modes \( \sum BF(B^0) \sim 4.1\% \)

\[ B^0 \rightarrow D^{(*)-} h^+(h^+ = \pi^+, \rho^+, a_1^+) \]

Self-Tagging Modes

\[ B^0 \rightarrow J/\psi K^{*0} (\rightarrow K^+ \pi^-) \]

**BABAR**

Preliminary

\( N_{\text{tag}} = 23618 \)

Purity = 84%

\( m_{ES} \text{ [GeV/c}^2\text{]} \)

\( N_{\text{tag}} = 1757 \)

Purity = 96%

\( m_{ES} \text{ [GeV/c}^2\text{]} \)
Flavor Eigenstate Charged B Sample

Charm decay modes \( \sum BF(\bar{D}^0) \sim 28\% \)
\[
\bar{D}^* \rightarrow \bar{D}^0\pi^0, \bar{D}^0 \rightarrow K^+\pi^-, K^+\rho^-, K^+\pi^+\pi^-\pi^-, K_S^0\pi^+\pi^-
\]

B decay modes \( \sum BF(B^0) \sim 1.2\% \)
\[
B^+ \rightarrow \bar{D}(^{(*)})^0\pi^+
\]

\[
B^+ \rightarrow J/\psi K^{(*)}, \psi(2S)K^+, \chi_{c1}K^+, \eta_cK^+
\]

**Self-Tagging Modes**

**B\( ^{ABAR} \)**

Preliminary

\( N_{\text{tag}} = 15915 \)

Purity = 87%

**B\( ^{ABAR} \)**

Preliminary

\( N_{\text{tag}} = 6245 \)

Purity = 94%
Golden Sample: \((cc)K_s\) CP Eigenstates

\[ J / \psi K_s^0 \rightarrow \pi^+ \pi^- \]
\[ N_{\text{cand}} = 974 \]
\[ \text{Purity} = 97\% \]

\[ \psi(2S)K_s^0 \rightarrow \pi^+ \pi^- \]
\[ N_{\text{cand}} = 150 \]
\[ \text{Purity} = 97\% \]

\[ J / \psi K_s^0 \rightarrow \pi^0 \pi^0 \]
\[ N_{\text{cand}} = 170 \]
\[ \text{Purity} = 89\% \]

\[ \chi_{c1} K_s^0 \rightarrow \pi^+ \pi^- \]
\[ N_{\text{cand}} = 80 \]
\[ \text{Purity} = 95\% \]

Candidates & purity for \(m_{ES} > 5.27\) GeV/c\(^2\)

BABAR
81.3 fb\(^{-1}\)
Selecting candidates for $B \rightarrow J/\psi K_L$

- $K_L$ detected by nuclear interactions in EMC or IFR
  - EMC neutral clusters with energy between 0.2 and 2.0 GeV
    - Veto clusters forming $\pi^0$s with any other photon ($E_\gamma > 30$ MeV)
    - Remove clusters ($E > 1$ GeV) containing two distinct bumps
  - IFR neutral clusters are 2 or more RPC layers that are unmatched to any projected charged track
  - Only able to determine angle of $K_L$ wrt interaction point, not energy

- $B$ candidates formed from mass-constrained $J/\psi \rightarrow \ell^+\ell^-$ and $K_L$ candidates
  - Since there should be missing momentum along $K_L$ direction, cut on difference between observed and expected
  - Use cuts on $J/\psi \rightarrow \ell^+\ell^-$ helicity angle ($\sin^2 \theta_h$ for signal) and $B$ candidate polar angle ($\sin^2 \theta$ wrt to $z$-axis)
Final Candidate Selection

- Apply constraint of known $m_B$ mass & $K_L$ direction $d_{K_L}$ to determine momentum $p_{KL}$

\[ m_B^2 = \left( E_{J/\psi} + \sqrt{m_{K_L}^2 + p_{K_L}^2} \right)^2 - \left( \vec{p}_{J/\psi} + p_{K_L} \vec{d}_{K_L} \right)^2 \]

- Search for signal in the one remaining variable expressed as:

\[ \Delta E = E_{J/\psi}^* + E_{K_L}^* - E_{beam}^* \]

\[ p_B^* = |p_{J/\psi}^* + p_{K_L}^*| \]
**CP Eigenstate Sample: \( B \rightarrow J/\psi K_L \)**

Resolution of about 10 MeV for \(\Delta E\) after \(m_B\) constraint

**Signal**

\(J/\psi\) Background Estimated with MC

**Fake \(J/\psi\) Background**

Estimated with sidebands

\(N_{\text{cand}} = 988\)

Purity = 55%
Sample $B^0 \rightarrow J/\psi K_L$ Event
One More Mode for CP Sample

\[ J/\psi K^{*0} \rightarrow K_s^0 \pi^0 \]

\[ N_{\text{cand}} = 147 \]

Purity 81%

\[ D_\perp = 1 - 2 \cdot R_\perp, \]

\[ R_\perp \equiv |A_\perp|^2 \text{ fraction of CP-odd} \]

BABAR PRL 87, 241801 (2001)

\[ R_\perp = (16.0 \pm 3.2_{(\text{stat})} \pm 1.4_{(\text{syst})})\% \]

BELLE hep-ex/0205021

\[ R_\perp = (19.1 \pm 2.3_{(\text{stat})} \pm 2.6_{(\text{syst})})\% \]

Conclude: \( J/\psi \) mostly CP even
Angular Analysis at BABAR

Projections of fits for amplitudes and rescattering phases:

\[ |A_0|^2, |A_\parallel|^2, |A_\perp|^2, \phi_\perp, \phi_\parallel \]
Time-Dependent Analysis Strategies

Factorize the analysis into building blocks

**Measurements**

- $B^+ / B^0$ Lifetimes
- $B^0 \bar{B}^0$ Mixing

**Analysis Ingredient**

- a) Reconstruction of $B$ mesons in flavor eigenstates
- b) $B$ vertex reconstruction
- c) Flavor Tagging + a + b
- d) Reconstruction of neutral $B$ mesons in $CP$ eigenstates + a + b + c

Increasing complexity
Measurement of $B^0$ and $B^+$ Lifetime

$\beta_\gamma = 0.55$

$\sigma_z \sim 110 \mu m$

$\gamma(4s)$

1. Fully reconstruct one $B$ meson in flavor eigenstate ($B_{REC}$)

2. Reconstruct the decay vertex

3. Reconstruct inclusively the vertex of the “other” $B$ meson ($B_{TAG}$)

4. Compute the proper time difference $\Delta t$

5. Fit the $\Delta t$ spectra

$\Delta t \cong \Delta z/\gamma \beta c$
**B-Lifetimes: Time Distributions**

**LEP/CDF**

Production point of $B$ meson is known with good accuracy.

Control of the resolution function at negative times.

**B Factories**

Proper time difference obtained from distance $\Delta z$ between the two $B$ vertices.

Fit the parameters of the resolution function together with the lifetime.
1. Reconstruct $B_{\text{rec}}$ vertex from $B_{\text{rec}}$ daughters

2. Reconstruct $B_{\text{tag}}$ direction from $B_{\text{rec}}$ vertex & momentum, beam spot, and $\gamma(4S)$ momentum = pseudotrack

3. Reconstruct $B_{\text{tag}}$ vertex from pseudotrack plus consistent set of tag tracks

4. Convert from $\Delta z$ to $\Delta t$, accounting for (small) $B$ momentum in $\gamma(4S)$ frame

Result: High efficiency (97%) and $\sigma(\Delta z)_{\text{rms}} \sim 180\mu m$ versus $\langle |\Delta z| \rangle \sim \beta\gamma c\tau = 260\mu m$
Conversion from $\Delta z$ to $\Delta t$

Proper time difference: $\Delta t = t_{rec} - t_{tag} = m_B \left[ \frac{z_{rec}}{p_{z,rec}} - \frac{z_{tag}}{p_{z,tag}} \right]$

where $t_{rec}$ and $t_{tag}$ are in different frames

Boost Approximation
Neglect $p_B^*$, take $p_{z,rec} = p_{z,tag} \approx \langle p_{Y(4S),z} \rangle$:

$$\Delta t = \frac{\Delta z}{\beta \gamma c}$$

Since one $B$ is fully reconstructed and two $B$ mesons are correlated:

$$\Delta z = \beta \gamma_{rec}^* c (t_{rec} - t_{tag}) + \gamma \beta_{rec}^* \gamma_{rec}^* \cos \theta^*_{rec} c (t_{rec} + t_{tag})$$

Improved Boost Approximation
Since $\langle \cos \theta^*_{rec} \rangle = 0$,

$$\Delta t = \frac{\Delta z}{\beta \gamma_{rec}^* c}$$

0.2% effect

Do not know $(t_{rec} + t_{tag})$, but can compute $\Delta t$ average event-by-event:

$$\langle t_{rec} + t_{tag} \rangle_{\Delta t} = \tau_B + |\Delta t|$$

Average $\tau_B$ Approximation

$$\Delta z = \beta \gamma_{rec}^* c \Delta t + \gamma \beta_{rec}^* \gamma_{rec}^* \cos \theta^*_{rec} c (\tau_{B^0} + |\Delta t|)$$

Improves resolution by 5% in quadrature
Actual $\Delta t$ Signal Resolution Function

Event-by-event $\sigma(\Delta t)$ from vertex errors

$B^0 \rightarrow D^{(*)-} \pi^+, \rho^+, a_1^+$

$\sigma(\Delta z)$ [cm]

$\frac{(\Delta t_{\text{meas}} - \Delta t_{\text{true}})}{\sigma(\Delta t)}$

Asymmetric RF: Tracks from long-lived charm in tag vertex

Empirical Models with parameters fit to data: Gaussian convolved with an exponential [lifetime] or Triple Gaussian [mixing, CP]
Effect of Charm Tracks on $\Delta t$

$\Delta t > 0$

$\Delta t_{\text{meas}} - \Delta t_{\text{true}} < 0$

Underlying principle: tag vertex dominates resolution

$\Delta t < 0$

$\Delta t_{\text{meas}} - \Delta t_{\text{true}} < 0$
Gaussian-Exponential $\Delta t$ Resolution Model

Motivated by inclusion of charm decay products in determination of $B^{\text{tag}}$ vertex, creating a small bias in the mean residual $\Delta t$ distribution

$$R(\delta t; \hat{\alpha}) = f \times G(0, S\sigma_{\Delta t}) + (1 - f) \times G(0, S\sigma_{\Delta t}) \otimes E(\kappa\sigma_{\Delta t})$$

where $\delta t = \Delta t - \Delta t_{\text{true}}$

Parameters: $\hat{\alpha} = \{0.69 \pm 0.07, 1.21 \pm 0.07, 1.04 \pm 0.24\}$

- $f =$ fraction in core Gaussian component
- $S =$ scale factor for estimated event-by-event errors
- $\kappa =$ effective lifetime for charm bias component

Outlier component (measurement error not applicable) added explicitly to PDF used in likelihood fit with one additional free parameter $f_{\text{outlier}}$

$$f_{\text{sig, outlier}} = 0.2^{+0.2}_{-0.3}\%, f_{\text{bk, outlier}} = 0.2^{+0.2}_{-0.3}\%$$
Method for Extracting Lifetime

Global unbinned maximum likelihood technique

- Includes probability-density functions (PDFs) for signal & backgrounds
- Incorporates model for $\Delta t$ resolution function for signal & background

**Primary advantages:**

- Incorporates all correlations between parameters describing dataset
- Extracts maximum statistical precision for desired result

**Cautions:**

- Need to build reasonable model that incorporates physical correlations
- Need to thoroughly test the model with Monte Carlo simulation to verify complete understanding
### Likelihood Function for Lifetime Fits

**Signal model:**

\[
H_{\text{sig},i} (\Delta t, \sigma_{\Delta t}, \tau_B, \hat{\theta}, \hat{\theta}_B, f^0_{\text{sig}, \text{outlier}}, f_0^0, f^0_{\text{bk}, \text{outlier}}, f^0_{\text{bk}, 0}, \tau^0_{\text{bk}}) = e^{-|\Delta t_i|/\tau_B} \otimes R(\Delta t_i; \hat{\theta}_B)
\]

Two single-sided exponentials convolved with signal Gaussian resolution function

Db \]

\[
H_{\text{bk},i} (\Delta t, \sigma_{\Delta t}, f_{\text{bk}, 0}, \tau_{\text{bk}}, \hat{\theta}) = \left[ f_{\text{bk}, 0} \delta(\Delta t_i) + (1 - f_{\text{bk}, 0}) e^{(\Delta t_i)/2\tau_{\text{bk}}} \right] \otimes R(\Delta t_i; \hat{\theta}_{\text{bk}})
\]

Prompt and lifetime components convolved with separate background Gaussian resolution function

**Outlier model:**

\[
\alpha_{\text{outlier},i} (\Delta t) = \mathcal{N}(0, 10 \text{ ps})
\]

Gaussian with zero mean and fixed 10 ps width

**Probability Density Function (PDF):**

\[
P_i = p_{\text{sig},i} \left[ (1 - p_{\text{sig},i}) f^0_{\text{sig}, \text{outlier}} + p_{\text{sig},i} f^0_{\text{sig}, \text{outlier}} \right] + p_{\text{bk},i} \left[ (1 - p_{\text{bk},i}) f^0_{\text{bk}, \text{outlier}} + p_{\text{bk},i} f^0_{\text{bk}, \text{outlier}} \right]
\]

Assign probabilities for individual events to be signal \((p_{\text{sig},i})\) or background \((1 - p_{\text{sig},i})\), based on observed \(m_{ES}\) value and a separate global fit to the \(m_{ES}\) distribution for the sample

**Likelihood Function**

\[
\ln L = \sum_{i_0} \ln P_{i_0}(\Delta t_i, \sigma_{\Delta t}, \tau_B, \hat{\theta}, \hat{\theta}_B, f^0_{\text{sig}, \text{outlier}}, f_0^0, f^0_{\text{bk}, \text{outlier}}, f^0_{\text{bk}, 0}, \tau^0_{bk}) + \sum_{i_+} \ln P_{i_+}(\Delta t_i, \sigma_{\Delta t}, \tau_B, \hat{\theta}, \hat{\theta}_B, f^+_\text{sig}, \text{outlier}, f^+_\text{sig}, \text{outlier}, f^+_\text{bk}, \text{outlier}, f^+_\text{bk}, 0, \tau^+_\text{bk})
\]
Likelihood Function for Lifetime Fits

**Signal model:**
Two single-sided exponentials convolved with signal Gaussian resolution function

**Background model:**
Prompt and lifetime components convolved with separate background Gaussian resolution function

**Outlier model:**
Gaussian with zero mean and fixed 10 ps width

**Probability Density Function (PDF):**
Assign probabilities for individual events to be signal \( p_{\text{sig},i} \) or background \( 1 - p_{\text{sig},i} \), based on observed \( m_{\text{ES}} \) value and a separate global fit to the \( m_{\text{ES}} \) distribution for the sample

**Likelihood Function:**
Sum PDFs for charged and neutral samples for a combined fit with a total of 19 free parameters
Likelihood Function for Lifetime Fits

**Signal model:**

\[ \mathcal{H}_{\text{sig},i}(\Delta t_i, \sigma_{\Delta t,i}; \tau_B, \hat{\alpha}) = \frac{e^{-|\Delta t_i|/\tau_B}}{2\tau_B} \otimes R(\Delta t_i; \hat{\alpha}) \]

\[ \mathcal{H}_{\text{bk},i}(\Delta t_i, \sigma_{\Delta t,i}; f_{\text{bk},0}, \tau_{\text{bk}}, \hat{\beta}) = \left[ f_{\text{bk},0} \delta(\Delta t_i) + (1 - f_{\text{bk},0}) \frac{e^{-|\Delta t_i|/\tau_{\text{bk}}}}{2\tau_{\text{bk}}} \right] \otimes R(\Delta t_i; \hat{\beta}) \]

**Outlier model:**

\[ \mathcal{O}_{\text{outlier},i}(\Delta t_i) = G(0, 10 \text{ ps}) \]

**Probability Density Function (PDF):**

\[ P_i = \rho_{\text{sig},i} \left[ (1 - f_{\text{sig, outlier}}) \mathcal{H}_{\text{sig},i} + f_{\text{sig, outlier}} \mathcal{O}_{\text{outlier},i} \right] + \]

\[ (1 - \rho_{\text{sig},i}) \left[ (1 - f_{\text{bk, outlier}}) \mathcal{H}_{\text{bk},i} + f_{\text{bk, outlier}} \mathcal{O}_{\text{outlier},i} \right] \]

**Likelihood Function**

\[ \ln \mathcal{L} = \sum_{i=0} \ln P_{i0}(\Delta t_i, \sigma_{\Delta t,i}; p_{\text{sig},i}, \tau_B, \hat{\alpha}, \hat{\beta}_0, f_{\text{sig, outlier}}^0, f_{\text{bk, outlier}}^0, f_{\text{bk},0}, \tau_{\text{bk}}^0) + \]

\[ \sum_{i=+} \ln P_{i+}(\Delta t_i, \sigma_{\Delta t,i}; p_{\text{sig},i}, \tau_B, \hat{\alpha}, \hat{\beta}_+ f_{\text{sig, outlier}}^+, f_{\text{bk, outlier}}^+, f_{\text{bk},0}, \tau_{\text{bk}}^+) \]
**Signal and Background Probabilities**

**Neutral B Mesons**
- Signal: $6967 \pm 95$
- Purity $\approx 90\%$

**Charged B Mesons**
- Signal: $7266 \pm 94$
- Purity $\approx 93\%$

ARGUS function:

$$A(m_{ES}; m_0, \xi) = A_B m_{ES} \sqrt{1 - x_{ES}^2} e^{\xi(1 - x_{ES}^2)}, \text{ where } x_{ES} = m_{ES} / m_0$$

**Background properties from sideband events**

- $p_{sig,i} \sim 0$
- $p_{sig,i} \sim 0.9$

**BABAR**
- $20.6 \text{ fb}^{-1}$

Aug 5-7, 2002  
D. MacFarlane at SSI 2002
### B-Lifetime Measurements

\[
\begin{align*}
\tau_{B^0} &= 1.546 \pm 0.032 \pm 0.022 \text{ ps} \\
\tau_{B^+} &= 1.673 \pm 0.032 \pm 0.023 \text{ ps} \\
\tau_{B^+} / \tau_{B^0} &= 1.082 \pm 0.026 \pm 0.012
\end{align*}
\]

(BABAR PRL 87, 201803 (2001)

(error PDG2000 ~ 0.03 ps, stat+syst)

- Good agreement with previous lifetime measurements
- Excellent control of the time resolution function (parameterization, tails)

Proof of principle for time-dependent analysis at B Factories

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Aug 5-7, 2002

D. MacFarlane at SSI 2002
**B-Lifetime Measurements**

- \( \tau_{B^0} = 1.554 \pm 0.030 \pm 0.019 \text{ ps} \)
- \( \tau_{B^+} = 1.695 \pm 0.026 \pm 0.015 \text{ ps} \)
- \( \tau_{B^+} / \tau_{B^0} = 1.091 \pm 0.023 \pm 0.014 \)

- Good agreement with previous lifetime measurements
- Excellent control of the time resolution function (parameterization, tails)

(error PDG2000 ~ 0.03 ps, stat+syst)

- **Proof of principle for time-dependent analysis at B Factories**

Belle PRL 88, 171801 (2002)
Measurement of $B^0\overline{B^0}$ Mixing

3. Reconstruct Inclusively the vertex of the “other” $B$ meson ($B_{\text{tag}}$)

4. Determine the flavor of $B_{\text{tag}}$ to separate Mixed and Unmixed events

5. Compute the proper time difference $\Delta t$

6. Fit the $\Delta t$ spectra of mixed and unmixed events

1. Fully reconstruct one $B$ meson in flavor eigenstate ($B_{\text{rec}}$)

2. Reconstruct the decay vertex

$\gamma(4s) \quad \beta_\gamma = 0.55$

$\sigma_z \sim 110 \mu m$

$\Delta t \approx \Delta z/\gamma \beta c$

$\Delta t$ spectra of mixed and unmixed events
Methods for B Flavor Tagging

Many different physics processes can be used

Primary lepton

\[ B^0 \rightarrow D^{*-} \ell^+ \nu \]

Secondary lepton

\[ B^0 \rightarrow D \pi^+, \bar{D}^- \rightarrow K^* \ell^- \nu \]

Kaon(s)

\[ B^0 \rightarrow \bar{D}X, \bar{D} \rightarrow K^+ X \]

Soft pions from \( D^* \) decays

\[ B^0 \rightarrow D^{*-} X^+, \bar{D}^{*-} \rightarrow \bar{D}^0 \pi^- \]

Fast charged tracks

\[ \ell^+, \pi^+, \rho^+ \]

\[ \ell^- \]

\[ W^+, W^-, \bar{c}, \bar{s} \]

\[ b, K^+ \]
Flavor Tagging for Mixing Study

- Use charge correlations with decay products to define two physics categories
  - “Lepton” $B^0 \to D^* \ell^+$
  - “Kaon” $B^0 \to D^0 \bar{X}, D \to K^+ X$

- $p_{\ell}^* > 1.0[1.1]$ GeV/c for $e[\mu]$
- No conflicting Lepton tag, $\sum_{\text{kaons}} q_i \neq 0$

- Multivariable techniques used to combine PID, kinematic variables, correlations, event information
  - e.g., primary lepton without PID, soft pions from $D^*$ decays $B^0 \to D^- X^+, D^* \to D^0 \pi^-$
  - Multivariable analysis with neural network techniques: “NT1”, “NT2” categories

![Graph showing BABAR distributions for different categories](image)
Electron ID at BABAR

- Match track to EMC cluster
- $0.89 < \frac{E_{EMC}}{p} < 1.2$
- EM shower shape requirements
- DCH $dE/dx$ and DIRC Cherenkov angle consistent with electron hypothesis

$\text{eff } e = 91\%, \pi \text{ misid}= 0.13\%$
Electron ID at Belle

- Match track to ECL cluster
- $E_{ECL}/p$ ratio requirement
- EM shower shape requirements, track-cluster matching
- DCH dE/dx, TOF, ACC consistent with electron hypothesis

eff $e=94\%$, $\pi$ misid$=0.5\%$
Muon ID at BABAR

- # int. lengths in IFR >2.2
- Difference in measured and expected int. length <1
- Match bet’n extrapolated track and IFR hits
- Requirements on average and spread of # of IFR hits per layer

\textbf{eff} \ \mu=75\%, \ \pi \ \text{misid}=2.5\%
Muon ID at Belle

- Difference in measured and expected range
- Match bet’n extrapolated track and KLM hits
- Likelihood based selection

\[ \text{eff } \mu = 90\%, \ \pi \text{ misid } = 1\% \]
Some Inputs to NN Tagger

Sum of energy within 90° of estimated $W$ direction

$$B^0 \rightarrow D^{*-} \ell^+$$

Opening angle between input track and thrust axis of recoil $B$

$$B^0 \rightarrow D^{*-} X^+$$
$$D^{*-} \rightarrow \bar{D}^{0} \pi^-$$

Opening angle between input track and missing momentum vector

$$B^0 \rightarrow D^{*-} \ell^+$$

CMS momentum of input track

$$B^0 \rightarrow D^{*-} \ell^+$$
Flavor Tagging Performance in Data

The large sample of fully reconstructed events provides the precise determination of the tagging parameters required in the $CP$ fit.

<table>
<thead>
<tr>
<th>Tagging category</th>
<th>Fraction of tagged events $\varepsilon$ (%)</th>
<th>Wrong tag fraction $w$ (%)</th>
<th>Mistag fraction difference $\Delta w$ (%)</th>
<th>$Q = \varepsilon(1-2w)^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton</td>
<td>10.9 ± 0.3</td>
<td>9.0 ± 1.4</td>
<td>0.9 ± 2.2</td>
<td>7.4 ± 0.5</td>
</tr>
<tr>
<td>Kaon</td>
<td>35.8 ± 1.0</td>
<td>17.6 ± 1.0</td>
<td>-1.9 ± 1.5</td>
<td>15.0 ± 0.9</td>
</tr>
<tr>
<td>NT1</td>
<td>7.7 ± 0.2</td>
<td>22.0 ± 2.1</td>
<td>5.6 ± 3.2</td>
<td>2.5 ± 0.4</td>
</tr>
<tr>
<td>NT2</td>
<td>13.8 ± 0.3</td>
<td>35.1 ± 1.9</td>
<td>-5.9 ± 2.7</td>
<td>1.2 ± 0.3</td>
</tr>
<tr>
<td>ALL</td>
<td>68.4 ± 0.7</td>
<td></td>
<td></td>
<td>26.1 ± 1.2</td>
</tr>
</tbody>
</table>

Highest “efficiency”

Smallest mistag fraction

Error on $\sin 2\beta$ and $\Delta m_d$ depend on the “quality factor” $Q$ approx. as:

$$\sigma(\sin 2\beta) \sim \frac{1}{\sqrt{Q}}$$

BABAR

29.7 fb$^{-1}$

Aug 5-7, 2002

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**B-Mixing Analysis: Time Distributions**

- **UnMixed**
- **Mixed**

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**Decay Time Difference (reco-tag) (ps)**

- UnMixed
- Mixed

---

**Decay Time Difference (reco-tag) (ps)**

- UnMixed
- Mixed

---

**$f_{\text{mixing}, \pm}(\Delta t) = \left\{ \begin{array}{c} e^{-|\Delta t|/\tau_B} \\ 4\tau_B \\ (1 \pm (1 - 2\omega) \cos \Delta m_d \Delta t) \end{array} \right\} \otimes R(\Delta t)$**

"$f_{\text{mixing}, +}$" $\iff$ unmixed \((B^0_{\text{flav}} \bar{B}^0_{\text{tag}} \text{ or } \bar{B}^0_{\text{flav}} B^0_{\text{tag}})\)

"$f_{\text{mixing}, -}$" $\iff$ mixed \((B^0_{\text{flav}} B^0_{\text{tag}} \text{ or } \bar{B}^0_{\text{flav}} \bar{B}^0_{\text{tag}})\)

- $\omega$ is the flavor mistag probability
- $R(\Delta t)$ is the time resolution function
**Triple-Gaussian $\Delta t$ Resolution Model**

\[
R(\delta t; \hat{a}) = f_{\text{core}} \times G(\mu_{\text{core}}, \sigma_{\text{core}}) + f_{\text{tail}} \times G(\mu_{\text{tail}}, \sigma_{\text{tail}}) + f_{\text{outlier}} \times G(\mu_{\text{outlier}}, \sigma_{\text{outlier}})
\]

- Core Gaussian
- Tail Gaussian
- Outlier Gaussian

\[
f_{\text{core}} = (1 - f_{\text{tail}} - f_{\text{outlier}})
\]

**Bias**

\[
\begin{align*}
\mu_{\text{core}} &= b_{\text{core}} \sigma_{\Delta t} \\
\mu_{\text{tail}} &= b_{\text{tail}} \sigma_{\Delta t} \\
\mu_{\text{outlier}} &= 0
\end{align*}
\]

**Widths**

\[
\begin{align*}
\sigma_{\text{core}} &= S_{\text{core}} \sigma_{\Delta t} \\
\sigma_{\text{tail}} &= S_{\text{tail}} \sigma_{\Delta t} \\
\sigma_{\text{outlier}} &= 8 \text{ ps}
\end{align*}
\]

**Parameters:**

\[
\hat{a} = \{ f_{\text{tail}}, f_{\text{outlier}}, S_{\text{core}}, b_{\text{core}, c}, b_{\text{tail}} \}
\]

- $f$ = fractions in tail and outlier
- $S$ = scale factor for estimated event-by-event errors
- $b$ = bias factor due to inclusion of charm products in tag vertex

**Event-by-event uncertainties from vertex fits $\sigma(\Delta t)$**
Correlation: $\sigma(\Delta t)$ and residual $\Delta t$ bias

- $D$ flight direction
- Charm tracks
- $z_{tag}$
- Prompt $B$ tracks
- $\sigma(z_{tag})$

$\sigma(\Delta t)$ smallest, $\Delta t$ bias zero
$z$ axis

- Monte Carlo

$\sigma(\Delta t)$ largest, $\Delta t$ bias largest
Yet another correlation!

Mystery:
Tests of likelihood fit with full Monte Carlo shows bias of $+0.007 \pm 0.003$ for $\Delta m_d$ (about 40% of statistical error!)

$p_t$ spectrum of wrong-sign kaons softer than those in correct-tag processes

But: $\sigma(\Delta t) \propto 1/\sum p_t^2$

Systematic difference in $p_t$ leads to correlation

Mistag fraction versus vertex error for kaon tags
Mistag rate scaled by $\sqrt{\sum p_t^2}$

Resolved!
## Fit Results for Signal Resolution Parameters

<table>
<thead>
<tr>
<th></th>
<th>Run 1</th>
<th>Run 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{\text{core}}$</td>
<td>$1.37 \pm 0.09$</td>
<td>$1.18 \pm 0.11$</td>
</tr>
<tr>
<td>$b_{\text{core \ lepton}}$</td>
<td>$0.06 \pm 0.13$</td>
<td>$-0.04 \pm 0.16$</td>
</tr>
<tr>
<td>$b_{\text{core \ kaon}}$</td>
<td>$-0.22 \pm 0.08$</td>
<td>$-0.25 \pm 0.09$</td>
</tr>
<tr>
<td>$b_{\text{core \ NT1}}$</td>
<td>$-0.07 \pm 0.15$</td>
<td>$-0.45 \pm 0.21$</td>
</tr>
<tr>
<td>$b_{\text{core \ NT2}}$</td>
<td>$-0.46 \pm 0.12$</td>
<td>$-0.20 \pm 0.16$</td>
</tr>
<tr>
<td>$b_{\text{tail}}$</td>
<td>$-5.0 \pm 4.2$</td>
<td>$-7.5 \pm 2.4$</td>
</tr>
<tr>
<td>$f_{\text{tail}}$</td>
<td>$0.014 \pm 0.020$</td>
<td>$0.015 \pm 0.010$</td>
</tr>
<tr>
<td>$f_{\text{outlier}}$</td>
<td>$0.008 \pm 0.004$</td>
<td>$0.000 \pm 0.014$</td>
</tr>
</tbody>
</table>

Non-zero bias for kaons
Additions to the Likelihood Function

1. Allow for difference in $B^0$ vs $\bar{B}^0$ mistag rates

$$\langle w \rangle = \frac{1}{2}(w + \bar{w}) \quad \Delta w = (w - \bar{w})$$

$$D = 1 - 2w \quad \bar{D} = 1 - 2\bar{w}$$

$$\langle D \rangle = \frac{1}{2}(D + \bar{D}) \quad \Delta D = D - \bar{D}$$

PDFs now depend on tag state as well

$$f_{mixing,\pm, tag= B^0}(\Delta t) \propto \left\{ \left( 1 + \frac{1}{2} \Delta D \right) \pm \langle D \rangle \cos \Delta m_d \Delta t \right\}$$

$$\quad f_{mixing,\pm, tag= \bar{B}^0}(\Delta t) \propto \left\{ \left( 1 - \frac{1}{2} \Delta D \right) \pm \langle D \rangle \cos \Delta m_d \Delta t \right\}$$

2. Allow for prompt and non-prompt background components

Adds fractions for each tagging category, effective lifetime for non-prompt component, and “effective dilutions” for both

4+1+8 parameters
Blind Analyses

- Analyses were done “blind” to eliminate possible experimenters’ bias
  - In general, measurements of a quantity “X” are done with likelihood fits - blinding achieved by replacing “X” with “X+R” in likelihood fits
    - R is drawn from a Gaussian with a width a few times the expected error
    - Random number sequence is “seeded” with a “blinding string”
    - The reported statistical error is unaffected
    - Allows all systematic studies to be done while still blind
**$\Delta m_d$ Likelihood Fit**

Combined unbinned maximum likelihood fit to $\Delta t$ spectra of mixed and unmixed events in the $B$ flavor sample

<table>
<thead>
<tr>
<th>Fit Parameters</th>
<th>#</th>
<th>Main Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m_d$</td>
<td>1</td>
<td>Signal</td>
</tr>
<tr>
<td>Mistag fractions for $B^0$ and $\bar{B}^0$ tags</td>
<td>8</td>
<td>Signal</td>
</tr>
<tr>
<td>Signal resolution function</td>
<td>2x8 *</td>
<td>Signal</td>
</tr>
<tr>
<td>Description of background $\Delta t$</td>
<td>5+8</td>
<td>Sidebands</td>
</tr>
<tr>
<td>Background $\Delta t$ resolution</td>
<td>2x3 *</td>
<td>Sidebands</td>
</tr>
<tr>
<td>$B$ lifetime from PDG 2002</td>
<td>0</td>
<td>$\tau_B = 1.548$ ps</td>
</tr>
<tr>
<td>Total parameters</td>
<td>44</td>
<td></td>
</tr>
</tbody>
</table>

* 2 running periods

✓ All $\Delta t$ parameters extracted from data
✓ Correct estimate of the error and correlations
### Likelihood Function for Mixing Fits

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Signal model:</strong></td>
<td>PDF with $\Delta m_d$ [1] for mixed and unmixed events convolved with triple Gaussian signal resolution function [8] for 2 periods of alignment. Dilutions and dilution differences [8] are also incorporated.</td>
</tr>
<tr>
<td><strong>Background model:</strong></td>
<td>Prompt and lifetime components [5] for mixed and unmixed samples convolved with a common background double Gaussian resolution function [3] for 2 periods of alignment. Separate dilutions and dilution differences incorporated [8].</td>
</tr>
<tr>
<td><strong>Outlier model:</strong></td>
<td>Incorporated into resolution functions as Gaussian, with zero mean and fixed 8 ps width</td>
</tr>
<tr>
<td><strong>Probability Density Function (PDF):</strong></td>
<td>Assign probabilities for individual events to be signal ($p_{\text{sig},i}$) or background ($1 - p_{\text{sig},i}$), based on observed $m_{ES}$ value and a separate global fit to the $m_{ES}$ distribution for the sample</td>
</tr>
<tr>
<td><strong>Likelihood Function:</strong></td>
<td>Sum PDFs for mixed and unmixed samples for a combined fit with a total of 44 free parameters</td>
</tr>
</tbody>
</table>
Final Tagged Mixing Sample

Lepton

\[ N_{\text{signal}} = 1097 \pm 34 \]
\[ \text{Purity} = (96.0 \pm 0.7)\% \]

Kaon

\[ N_{\text{signal}} = 3156 \pm 63 \]
\[ \text{Purity} = (84.6 \pm 0.7)\% \]

Background properties from sideband events

NT1

\[ N_{\text{signal}} = 798 \pm 31 \]
\[ \text{Purity} = (88.9 \pm 1.2)\% \]

NT2

\[ N_{\text{signal}} = 1293 \pm 43 \]
\[ \text{Purity} = (79.4 \pm 1.3)\% \]
Mixing with Hadronic Sample

Precision measurement consistent with world average

$$\Delta m_d = (0.516 \pm 0.016_{\text{stat}} \pm 0.010_{\text{syst}}) \text{ ps}^{-1}$$

BABAR PRL 88, 221802 (2002)
Mixing Asymmetry with Hadronic Sample

Unfolded raw asymmetry

\[ A_{\text{mixing}}(\Delta t) \approx (1 - 2\omega) \cos \Delta m_{B_d} \Delta t \]

Folded raw asymmetry

\[ \sim 1 - 2\omega \]

\[ \sim \pi / \Delta m_d \]

\(B\bar{A}B\bar{A}\)

29.7 fb\(^{-1}\)
# Systematic Errors on $\Delta m_d$

<table>
<thead>
<tr>
<th>Description of background events</th>
<th>$\sigma[\Delta m_d]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Background fractions, sideband extrapolation</td>
<td>0.004</td>
</tr>
<tr>
<td>Background $\Delta t$ structure and resolution</td>
<td></td>
</tr>
<tr>
<td>Peaking B+ background</td>
<td></td>
</tr>
<tr>
<td>$\Delta t$ resolution and detector effects</td>
<td>0.005</td>
</tr>
<tr>
<td>Silicon detector residual misalignment</td>
<td></td>
</tr>
<tr>
<td>$\Delta t$ resolution model</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta t)$ requirement</td>
<td></td>
</tr>
<tr>
<td>Fit bias correction and MC statistics</td>
<td>0.003</td>
</tr>
<tr>
<td>Fixed lifetime from PDG2000 *</td>
<td>0.006</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.010</strong></td>
</tr>
</tbody>
</table>

* Now improved with PDG2002
Mixing in Hadronic Modes at Belle

\[ \Delta m_d = (0.528 \pm 0.017_{\text{stat}} \pm 0.011_{\text{syst}}) \text{ ps}^{-1} \]
Measurement of $\sin^2 \beta$

Tag B
$\sigma_z \sim 110 \, \mu m$

Reco B
$\sigma_z \sim 65 \, \mu m$

$\gamma(4s)$

$\beta \gamma = 0.56$

$\Delta t \cong \Delta z/\gamma \beta c$

3. Reconstruct Inclusively the vertex of the “other” $B$ meson ($B_{\text{tag}}$)

4. Determine the flavor of $B_{\text{tag}}$ to separate $B^0$ and $\bar{B}^0$

5. Compute the proper time difference $\Delta t$

6. Fit the $\Delta t$ spectra of $B^0$ and $\bar{B}^0$ tagged events

1. Fully reconstruct one $B$ meson in $CP$ eigenstate ($B_{\text{rec}}$)

2. Reconstruct the decay vertex

Aug 5-7, 2002  D. MacFarlane at SSI 2002
Time-Dependent CP Asymmetries

Time-dependence of $B^0 - \bar{B}^0$ mixing

$$A_{\text{mixing}}(\Delta t) = \frac{N(\text{unmixed}) - N(\text{mixed})}{N(\text{unmixed}) + N(\text{mixed})} \approx \cos \Delta m_d \Delta t$$

Time-dependence of CP-violating asymmetry in $B^0_{CP} \rightarrow J/\psi K^0_S$

$$A_{CP}(\Delta t) = \frac{N(B_{\text{tag}} = B^0) - N(B_{\text{tag}} = \bar{B}^0)}{N(B_{\text{tag}} = B^0) + N(B_{\text{tag}} = \bar{B}^0)} \approx \sin 2\beta \sin \Delta m_d \Delta t$$

(Assuming no confusion of $B_{\text{rec}}$ state)

Use the large statistics $B_{\text{flav}}$ data sample to determine the mistag probabilities and the parameters of the time-resolution function.

Aug 5-7, 2002 D. MacFarlane at SSI 2002
Summary

- **Precision measurements of lifetimes and B0 oscillations have been performed at the B Factories**
  - Require the development of all techniques for B reconstruction, determination of vertex separation, tagging of the recoil B state at decay, unbinned maximum likelihood fitting and validation procedures
  - Results are in excellent agreement with previous results and represent some of the single most-precise measurements available

\[ \Delta m_d = 0.516 \pm 0.016 \text{ (stat)} \pm 0.010 \text{ (syst)} \text{ ps}^{-1} \]

**BABAR** PRL 88, 221802 (2002)

\[ \Delta m_d = 0.528 \pm 0.017 \text{ (stat)} \pm 0.011 \text{ (syst)} \text{ ps}^{-1} \]

**BELLE** preprint-02/020, hep-ex/0207022, to appear in PLB

- **Tomorrow**
  - Apply tools to measurement of CP violation in neutral B decays
Bibliography: Lecture 2

1. [lifetime] BABAR Collab., B.Aubert et al., PRL 87, 201803 (2001)
5. [mixing] BABAR Collab., B.Aubert et al., hep-ex/0201020, to appear in PRD