Extracting $V_{ub}$ Using $b \rightarrow s\gamma$

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Goal - Measure $V_{ub}$

Number of ways to measure $V_{ub}$:

- **Exclusive measurements** - $B \rightarrow \pi \ell \bar{\nu}$ and $B \rightarrow \rho \ell \bar{\nu}$
- **Inclusive measurements** - $b \rightarrow u \ell \bar{\nu}$ with cuts on
  - Electron energy
  - Hadronic invariant mass
  - Dilepton invariant mass
  - Mixed cuts

Each method has advantages and disadvantages

All should be done!
In principle, easy to extract $V_{ub}$:

Total $b \rightarrow \ell \bar{\nu}$ rate

\[
\Gamma = \frac{m_b^5 G_F^2}{192\pi^3} |V_{ub}|^2 \left[ C_1(\alpha_s) + \frac{\lambda_1}{2m_b^2} - \frac{9\lambda_2}{2m_b^2} + \mathcal{O} \left( \frac{\Lambda_{QCD}^3}{m_b^3}, \alpha_s^3 \right) \right]
\]

**Big problem** - $b \rightarrow c$ background!

Can cut on electron spectrum

Introduces new scale $\Delta E$

Leads to difficulties
New scale $\Rightarrow$ New problems

Scales important for $b \rightarrow u$ decay are $m_b$ and $\Lambda_{\text{QCD}}$

Calculation of total rate done in series in $\frac{\Lambda_{\text{QCD}}}{m_b}$

For the cut rate, new scale $\Delta E$: have terms like $\frac{\Lambda_{\text{QCD}}}{\Delta E} \sim \mathcal{O}(1)$

Also, have series in $\alpha_s(m_b)$, including terms like

$$\alpha_s(m_b) \log^2 \left(1 - \frac{2E_e}{m_b} \right) = \alpha_s(m_b) \log^2 \left(\frac{2\Delta E}{m_b} \right) \sim \mathcal{O}(1)$$

Both the non-perturbative and perturbative series **breakdown**!
Non-perturbative series (Fermi motion)

We calculate inclusive decay using partons \((b, u, \text{etc})\)

This leads to **partonic maximum energy** \(x \equiv \frac{2E_e}{m_b} \to 1\)

\(B\) meson decaying, with **hadronic maximum energy** \(x \to 1 + \bar{\Lambda}\)

Difference is result of **Fermi motion** of \(b\) inside \(B\)

Can show from first principles that

\[
\frac{d\Gamma}{dE} = \int_{2E-m_b}^{\bar{\Lambda}} dk_+ f(k_+) \frac{d\Gamma_p}{dE}(m_b^*) + O\left(\frac{\Lambda}{m_b}\right)
\]

with \(m_b^* = m_b + k_+\)

\(f(k_+)\) universal \(\Rightarrow\) also appears in \(b \to s\gamma\)
Removing $f(k_+)$

We could extract $f(k_+)$ in $b \rightarrow s\gamma$ and then use in $b \rightarrow u$

Better, skip middle step, by taking ratio of (moments of) rates

$$\left| \frac{V_{ub}}{V_{tb}V_{ts}^*} \right|^2 = \frac{3\alpha}{\pi} C_F^2 (1 + H_{\text{mix}}^\gamma) \int_{x_B^c}^1 dx_B \frac{d\Gamma}{dx_B} \left( \int_{x_B^c}^1 du_B W(u_B) \frac{d\Gamma^\gamma}{du_B} \right)^{-1}$$

where

$$W(u_B) = u_B^2 \int_{x_B^c}^{u_B} dx_B \left\{ 1 - 3(1 - x_B)^2 \right. \right.$$  

$$\left. + \frac{\alpha_s}{\pi} \left[ \frac{7}{2} - \frac{2\pi^2}{9} - \frac{10}{9} \log \left( 1 - \frac{x_B}{u_B} \right) \right] \right\}$$

$$\simeq w_{\text{slope}}(u_B - x_B^c)$$
What have we done?

Formula for $V_{ub}$ by relating $b \rightarrow u \ell \bar{v}$ and $b \rightarrow s \gamma$ rates

Leading order structure function $f(k_\perp)$ cancels

Accurate to $O[\alpha_s(1 - x), (1 - x)^3, \Lambda/m_b]$

“Simple” convolution of $b \rightarrow s \gamma$ rate with linear function

What about the breakdown of the perturbative series?

Large $\log(1 - x_B^c)$ corrections as the cut approaches 1
Stacking the logs

The logs can be summed, net effect is change in $W(u_B)$

$$W(u_B) = u_B^2 \int_{x_B}^{u_B} dx_B \ K \left[ x_B; \frac{4}{3 \pi \beta_0} \log \left( 1 - \alpha_s \beta_0 \ell_{x/u} \right) \right]$$

where $\ell_{x/u} = - \log(- \log(x_B/u_B))$

$W(u_B)$ is still approximately linear, just with a different slope

Correctly includes leading and next-to-leading logs

Shifts central value, but errors still dominated by $\Lambda/m_b$
What about those errors?

- Theoretical uncertainty of $\mathcal{O}(\alpha_s^2, \alpha_s(1 - x_B^C), \Lambda/m_b)$
- Also have parton-hadron duality errors – difficult to quantify
- How many resonances? Only about 10% of rate
- Should also do other extractions

Quantifiable uncertainty dominated by $\Lambda/m_b$

- Subleading in $\Lambda/m_b$ can have large effect in endpoint region
- Known and unknown contributions enter with new structure functions

Best solution – lower cut as much as possible
Other inclusive extractions

- Can do similar analysis for hadronic invariant mass
  - Combine rates to remove leading structure function
  - Remove large corrections by summing logs
  - Hadronic mass spectrum captures $\sim 80\%$ of rate
    $\Rightarrow$ smaller duality errors?

- Dilepton invariant mass
  - Captures $\sim 20\%$ of rate
  - Do not need to worry about leading structure function
  - Question as to how low can push cut
    Errors grow quickly with $q^2$

- Mixed cut ($q^2$ and $m_X$)
  - Best of both worlds?
  - Some model dependence
Conclusions

- $V_{ub}$ can and should be measured in a variety of ways
- Different extractions have advantages and disadvantages
- Theoretical uncertainties on the order of 10% possible
- Best way to reduce error is to lower cuts (or wait for lattice)
- Must wait to see convergence of extractions