very

A Short Introduction to the Soft-Collinear Effective Theory

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\[ |V_{xb}| \text{ and } |V_{tx}| \]
A workshop on semileptonic and radiative B decays
SLAC, December 2002
Effective field Theory of highly energetic particles that have a small invariant mass

- **E >> M:** Near the lightcone
  - \( p = (p^+, p^-, p^\perp) \sim Q(M^2/Q^2, 1, M/Q) \sim Q(\Box^2, 1, \Box) \)
  - \( \Box \ll 1 \), and \( p^2 \sim \Box^2 \)

- SCET has the right degrees of freedom for describing energetic particles interacting with soft “stuff”

Analogous to HQET: Effective Field Theory of Heavy and soft degrees of freedom--describes heavy particles interacting with soft “stuff”
If you only remember one thing…
Remember this picture:

Heavy: HQET

Light and energetic: SCET

SCET describes the light and energetic particles
SCET is QCD in a limit
Pion momentum: \( \mathbf{p} = (2.640 \text{ GeV}, 0, 0, -2.636 \text{ GeV}) \approx Q \mathbf{n} \quad \mathbf{n} = (1, 0, 0, -1) = (0, 2, 0) \) LC coordinates

Corrections are small \( \sim \frac{m}{Q_{\text{QCD}}} \), \( m \) relative to \( Q \)

Expansion in \( \frac{m_{\text{QCD}}}{Q} \) or \( \sqrt{\frac{m_{\text{QCD}}}{Q}} \)
Motivation

Systematic: power counting in small parameter

Understand Factorization in a universal way
- Key to separate hard contributions from soft & collinear
- Systematic corrections to factorization (power counting)

Symmetries
- Reduce the number form-factors
- In HQET where there is only the Isgur-Wise function

Sum infrared logarithms
- Sudakov logarithms
So…what’s it good for?

SCET couple to HQET can be used for any decays involving stationary heavy, and fast light particles:

\[
\begin{align*}
B & \to D, \\
B & \to K^0, B & \to K^0, B & \to K^+ e^-, \\
B & \to K^0, B & \to K^0, \\
B & \to X_u e, B & \to X_s e, & \to X^... \\
DIS, Drell Yan, \\
\end{align*}
\]
B $\rightarrow$ D $\Box$ factorization

Heavy: HQET

Light & Fast: SCET

$\frac{i}{2} m_b \ E_\Box f_\Box \ F^{B\Box D(0)} \int dx \ T(x,\Box) \ \Box(x,\Box)$

Soft $B\Box$ D form factor

Hard coefficient calculate in PT: $\Box_S(M_b)$

Light-cone pion wavefunction: nonperturbative

References:
- Bauer, Pirjol, Stewart: SCET (proved to all orders in $\Box$) Phys. Rev. Lett. 87: 201806, 2001
Semi-leptonic heavy-to-light

Selected history:

Brodsky et al. (1990)  
Hard part, $1/x^2$ singularity

Li & Yu (1996)  
k$_T$ factorization, Sudakov suppression

Bagan, Ball, Braun (1997)  
Light-cone sum rules

Charles et al. (1998)  
Symmetry relations: $\bar{Q}(E)$, $\bar{D}(q)$, $\bar{D}(q)$

$O(a_s)$ corrections, factorization proposal

Bauer et al. (2000)  
Collinear gluons, $C_1(\mathcal{P})$, soft factorization

Descotes, Sachradja (2001)  
More on Sudakov suppression

Bauer, Pirjol, Stewart (2002)  
Factorization in SCET

Pirjol & Stewart (2002)  
Details of factorization in SCET
Semi-leptonic heavy-to-light

\[ \text{e.g. } B \rightarrow \ell \bar{\nu} \text{ at large recoil} \]

**SCET**

**HQET**

**SCET factorization:** all orders in $\bar{\nu}_s$, leading order in $\bar{\nu}$:

\[
F(M^2) = \frac{1}{2} f_B f_M \int dz \int dx \int dr_+ T(z, M, \bar{\nu}_0) J(z, x, r_+, M, \bar{\nu}_0, \bar{\nu}) \bar{\nu}(x, \bar{\nu}) \bar{\nu}^+(r_+, \bar{\nu}) \\
+ C_k(M, \bar{\nu}) \bar{\nu}_k(q, \bar{\nu})
\]

- Factorizable piece
- Non-perturbative parameters:
  - decay constants, LC wave functions

**Non-factorizable piece**

- Non-perturbative form factors
- (restricted by symmetries in SCET)

\[ \text{Note both the pieces are same order in power counting!} \]

Bauer, Pleijol, Stewart: hep-ph/0211069
B ☢ ☢ l ☢: $Q^2$ range where SCET is valid

- $m_r = 770$ Mev
- Remember for SCET to be valid we need $Q \gg \bar{q}_{QCD}, m_r$

<table>
<thead>
<tr>
<th>$Q^2$ (GeV$^2$)</th>
<th>E (GeV)</th>
<th>P (GeV)</th>
<th>$m_r/E$ or $m_r/2E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.70</td>
<td>2.58</td>
<td>0.286 0.143</td>
</tr>
<tr>
<td>0.25</td>
<td>2.67</td>
<td>2.56</td>
<td>0.288 0.144</td>
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<tr>
<td>1</td>
<td>2.60</td>
<td>2.48</td>
<td>0.300 0.150</td>
</tr>
<tr>
<td>2.25</td>
<td>2.48</td>
<td>2.36</td>
<td>0.310 0.155</td>
</tr>
<tr>
<td>4</td>
<td>2.32</td>
<td>2.19</td>
<td>0.330 0.165</td>
</tr>
<tr>
<td>6.25</td>
<td>2.10</td>
<td>1.96</td>
<td>0.360 0.180</td>
</tr>
</tbody>
</table>

Too Big?!?!
Heavy-to-light factorization in SCET: Details

\[ F(Q^2) = \frac{1}{2} f_B f_M \int dz \int dx \int dr_+ T(z, Q, \bar{c}_0) J(z, x, r_+, Q, \bar{c}_0, \bar{c}) \bar{c}_-(x, \bar{c}) \bar{c}_-(r_+, \bar{c}) \]

- Decay constants
  - Calculable
- \( C_k(Q, \bar{c}) \bar{c}_k(Q, \bar{c}) \)
  - Calculable
- Soft form factor

- \( T(z, Q, \bar{c}_0) \) & \( C_k(Q, \bar{c}) \): Expansion in \( \bar{c}_s(Q) \)
- \( J(z, x, r_+, Q, \bar{c}_0, \bar{c}) \): Expansion in \( \bar{c}_s(Q) \)

\[ Q \sim \{m_b, E = m_b - q^2/(2m_b)\} \]
Factorization in $B \to D \pi (\pi)$


$$F(M) = f^{B \to D}(0) \int dx \, T^I(x) \, \frac{\alpha_s}{\pi} \left( x \right) + \int d\mu \, dx \, dy \, T^{II}(\mu, x, y) \, \frac{\alpha_s}{\pi} \left( x \right) \left( \frac{\alpha_s}{\pi} \left( y \right) \right)$$

• Was shown to hold to order $\alpha_s$

Perturbative QCD: Keum, Li, Sanda: hep-ph/0201103

$$F(M) = 0 + \int d\mu \, dx \, dy \, T^{II}(\mu, x, y) \, \frac{\alpha_s}{\pi} \left( x \right) \left( \frac{\alpha_s}{\pi} \left( y \right) \right)$$

• Sum Sudakov logarithms

No proof in SCET yet

• It is not a given that this will give the above formula
• Wait and see…
What’s to come?

- Proof of factorization in $B \rightarrow \rho \ell \nu$
  - Phenomenology

- Phenomenology in heavy-to-light semileptonic decays
  - Forward backward asymmetry
  - Extraction of form factors
  - Decay rates