Rough Layout and Rate Estimate for Beam Test in SLAC ESA

of Synchrotron Radiation Detector Prototypes for LC Beam Monitor

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Introduction
This note gives a rough experiment layout and counting rate estimate for the beam test of a proposed synchrotron light beam monitor for LC spectrometry. The idea for this detector is documented in Ref [1]. The proposal for a beam test SLAC Proposal T-475(June 2004) can be found at http://www.slac.stanford.edu/xorg/lcd/ipbi/documentation.html.

The method proposed for the LC detectors is similar to that employed by the WISRD spectrometer[2] at the SLAC Linear Collider/SLD interaction region. The idea is to generate two stripes of synchrotron light, one up-beam and one down-beam of a well know bend magnet somewhere in the extraction line system of the LC. Measurements of the positions and shapes of the two synchrotron stripes, together with precise knowledge of the geometry and the field in the bend magnet, can be used to determine many properties of the spent beam from the Linear Collider. A main goal is to determine the beam energy from the distance between the stripes. Other interesting features would be to extract information about the spectrum of the spent beam, and therefore information about the e+e- collision, from the shapes of the synchrotron stripes. The purpose of the proposed test is to begin testing candidate detector technologies in a synchrotron beam and learn how to design a detector for the LC. This note gives a very crude estimate of the expected counting rate in the proposed detectors.

Experiment Layout
A sketch of the main features of the basic layout is shown in Fig 1. The synchrotron light will be generated in a wiggler magnet aligned at 0.005 rad to the main beam line. The incident beam will be first deflected horizontally in a bend magnet B1, passed through the vertical wiggler, and then after the wiggler it will be deflected back in B2 offset and parallel to the original direction so it can exit the ESA hall into Beam Dump East. The synchrotron stripe from the wiggler will proceed forward and be extracted from the vacuum system downstream at some point conveniently far enough away from the electron beam to allow space for the exit window and the detectors. This arrangement would test one leg of the system proposed for the LC, and is mainly aimed at understanding the performance of the detectors. A second leg with a calibrated bend magnet between would be needed to measure the beam energy.

Some considerations in choosing the geometry of the layout are:

- The space required by the setup is partly constrained by available room in the SLAC
Figure 1: Top figure: schematic layout (not to scale) of the proposed synchrotron stripe detector test in SLAC ESA. Bottom figure: looking down beam at a projection in $x$ vrs $y$ of the synchrotron stripes from the wiggler (vertical red blob) and the bend magnets (horizontal green blob) and the exit beam pipe at the location of the fiber detectors for the case of $L2 = 680$ inches in the top figure.
ESA, and needs to be kept to a reasonably short length. The length required for the
bends and wiggler is partly determined by the length of the bends, and by the space
needed for coils, flanges and access. The length 120 inches shown in Fig 1 is estimated
for the 10D45 magnets, and the 10D90s would be a bit longer. This length, together with
the bend angle, determines the exit beam offset and thus the radius required for the exit
beam pipe.

• The bend angle in the first bend is constrained to small values by the need to permit
both the bent beam and the straight-ahead beam through the 1.5 inch wide pole gap of
the wiggler (minus vacuum chamber walls). Allowing clear passage for the straight-ahead
beam facilitates operations with the bend magnets off.

• The bend angle is constrained on the small side by the need to extract the synchrotron
beam from the beam pipe in a reasonably short distance down-beam from the wiggler.
The exit beam pipe is set at 2 inch radius to limit background from upstream scattering of
beam halo. The synchrotron stripe must clear the beam pipe with some room to spare to
permit installation of the fiber detectors. The pipe radius and the synchrotron clearance,
together with the 5 mr bend angle determines the distance from B2 to the detectors.
Three cases are shown in a table in Fig 1. For the rate estimates the middle value for
L2 = 680 inches corresponding to clearance of 2 inches is used. This choice affects the
fraction of bend spray that could intercept the detectors.

More study with simulations of signal and background rates will be needed to choose the
final values for the bend angle, exit pipe size, synchrotron clearance and other parameters,
but they will be similar to the ones shown in Fig 1.

The proposed detector will be made of various sizes of quartz fibers arranged in various
orientations to sample the stripe from the wiggler and connected to a multi-anode PMT. The
detector works by registering light in the PMT from Čerenkov light made in the quartz from
recoil electrons (and a few positrons) generated in the quartz by the synchrotron photons.
The purpose of this note is to estimate the counting rate in such a detector using rough
numbers for the synchrotron flux produced in the wiggler and crude estimates of the response
of the quartz/PMT system to those photons. For this estimate we assume the fibers are 0.6
mm thick oriented vertically side-by-side with the center of the bunch centered on the wiggler
stripe.

An important difference between this test and the proposed LC arrangement is that the
beam energy in the test will be of the order of 28 GeV, much lower than the many hundreds
of GeV at the LC. This results in synchrotron light spectra which are lower in energy than
those at the LC, and a main point of this estimate is to show that such a low energy beam
is still useful for a test.

Another point to be studied is how to optimize the geometry of the test. One question is
which of the available bend magnets would be best. The current best choices are either 10D45
or 10D90 magnets from the original SPEAR ring at SLAC. They have gaps 10 inches wide
and 2.5 inches tall, and are 45 and 90 inches long respectively. Longer magnets extend the
size of the total layout, and being heavier, are somewhat more costly to mount. However we
shall see that longer magnets give much less background synchrotron light in the detectors,
and are probably preferred.
The wiggler magnet to be used is also from the early history of SPEAR and is documented in Ref [3]. It is oriented to bend electrons vertically giving a vertical synchrotron stripe at the detectors.

Parameters of Čerenkov Light in Si and Synch Light in the Bend Magnets

Electrons at Čerenkov threshold in quartz have:

\[
\beta = 1/n = 0.6859 \quad (1)
\]
\[
\text{total energy} = 0.7022 \text{ MeV} \quad (2)
\]
\[
\text{kinetic energy} = 0.2269 \text{ MeV} \quad (3)
\]

Calculate the critical energy \( E_c \) in the wiggler from \( E_c = 0.0665 \text{ B(kG)} \text{ E}^2 \text{ (GeV)} \). Using 17 kG max pole field and beam energy \( E_0 = 28 \text{ GeV} \) gives \( E_c = 0.886 \text{ MeV} \) for the wiggler. In the actual wiggler field some of the path length is at lower than the maximum field, so part of the photon spectrum will have lower critical energies.

Consider two possible bend magnets, 10D45 (length = 45 inch) and 10D90 (length = 90 inch) and bend angle of 0.005 rad. This gives bend total BL = 4.66 (kG-m). Field and critical energy in the bends is

\[
\begin{align*}
10D45 & = 4.087 \text{ kG} & E_{c45} & = 0.213 \text{ MeV} \\
10D90 & = 2.044 \text{ kG} & E_{c90} & = 0.106 \text{ MeV}
\end{align*}
\]  

(4)  

To estimate synchrotron photon rates we use some cool notes from Hobey DeStaebler from his work on backgrounds in PEPII and BaBar. These were published in the Appendix A of Ref [4], included below. Hobey gives lots of useful formulas and some tables. The most important is a formula for the average number of photons emitted along a circular arc in a bend magnet

\[
< n >= 0.6181 \text{ B(kG)L(m) photons/electron} \quad (6)
\]

and Table A.1 showing the fraction of the number of photons and the fraction of the energy carried by photons above \( E/E_c \) for various \( E \).

The main parameters for the wiggler are: BL = 0.831 kG-m for one half pole at 17 kG full field (practical maximum operating value scaled from 20 kG field given in SLAC PUB 2289). The wiggler has 6 full poles. The synchrotron flux from the wiggler makes a vertical stripe at the fiber detector with total flux of

\[
< n > = 0.6181(2)(0.831)(6) = 6.19 \text{ photons/electron} \quad (7)
\]  

(8)

The horizontal stripe of synchrotron photons from the bend magnets acts as a background in the fiber detectors. If the fibers intercept the photons from the bend, they generate uninteresting rate that confuses the measurement of the position and shape of the vertical wiggler stripe. At the LC the fiber detector is planned to be located above or below the bend stripe to avoid that background. Here in the test beam the photons from the bend stripe
Figure 2: Notation used in Appendix A for synchrotron photons from a segment of a circular arc bend. Formulas for photon flux are integrated over the angular distribution perpendicular to the bend plane.

\[ d\phi = \frac{ds}{\rho} \]

Figure 3: Distributions of synchrotron photons vs energy \((k\text{ is the photon energy, } k_c\text{ is the critical energy})\). Top fig: number distribution of photons; bottom fig: energy distribution carried by the photons. From Hobey DeStaebler, priv comm. Values plotted are from tables by R. A. Mack, Ref. 55 in Appendix A.
are sufficiently low in energy that they do not present a serious background because they make recoil electrons in the quartz mostly below Čerenkov threshold. As we show below, the 10D90 magnets are better in this respect, because the bend field for a given bend angle is lower than for the 10D45 magnets, and the synchrotron critical energy is lower. The number of photons in the full bend stripe is same for 10D45 and 10D90 (same BL)

\[< n > = (0.618)(4.66kG - m)(2 \text{ bends}) \]
\[= 5.76 \text{ photons/electron} \]  

**Čerenkov Signal in Quartz from Recoil Comptons above Cherenkov Threshold**

Using Compton formulas for max recoil energy \( T_{\text{max}} \) and looking at pictures of distributions for Compton scattering in text books (see Fig 5 and 6), we estimate that photons above 0.4 MeV can make recoil electrons with kinetic energy above the 0.2 MeV Čerenkov threshold in quartz. For photons in the range 0.8 to 1.0 MeV (the most useful ones we have) the fraction of recoil Compton electrons above 0.2 MeV is 0.8 to 0.9. We use 0.8 in the following estimates. Using Table A.1 from Appendix A., we get the fraction of synchrotron photons above 0.4 MeV for each source (wiggler, 10D45, 10D90)

For the wiggler we assume all the photons made impinge on the detector. This corresponds to putting the fibers to cover the full stripe from the wiggler. In this position they would also intercept a small portion of the tip of the synchrotron stripe from the bends. We can do this if the synchrotron photons from the bends are low enough energy so as to not produce much signal in the fibers, which is probably the case (see below). If the fibers have to be pulled up or down to stay out of the plane of the bend stripe, then the rates from the wiggler will be reduced accordingly.

For the bends only a fraction of the stripe hits the detector. For the case of two bends, the bend stripe is only on one side of the incident beam. We estimate the fraction of the bend stripe intercepted by the detectors to be:

\[0.08 = (0.8 \text{ cm detector width in the stripe}) / (10 \text{ cm stripe width})\]

The interaction of synchrotron photons in the fibers is dominated by Compton scattering, with a small amount of pair production for the higher energies (See Fig. 4). The attenuation coefficient (Particle Data Book) for 0.5 to 2 MeV photons in Si is approximately \( t_{\text{atten}} = 20 \) (g/cm\(^2\)). The quartz fiber thickness is \( t = 0.6 \) (mm) and density is \( \rho = 2.33 \) (g/cm\(^3\)), so \( t\rho = 0.14 \) (g/cm\(^2\)). The transmission of photons through the fibers is then \( e^{-(\frac{t\rho}{t_{\text{atten}}})} = 0.99302 \). The fraction of synchrotron photons that interact in the fibers is 0.007.

Note: the recoil Compton electrons are produced mostly at a range of fairly wide angles to the incident photons (see Fig. 5), and they have low kinetic energy and fairly short ranges (0.1 to 0.8 g/cm\(^2\)). They will straggle in the fibers producing Čerenkov light as long as they remain above 0.2 MeV. Any material in front of the fibers, such as the vacuum exit window or any preradiator, will attenuate the low energy photons, mostly by photo electric effect, but will not significantly enhance the number of recoil electrons through the fibers because the cross section is dominated by Compton scattering that produces recoil electrons at wide angles and is mostly below the pair threshold.

6
Figure 4: Photon cross sections in carbon, similar to Si (Particle Data Book). Also indicated is a sketch of the approximate distribution of the synchrotron photons from the wiggler with critical energy $E_c = 0.886 \text{ MeV}$. The interaction cross section is dominated by Compton scattering ($\sigma_{\text{incoherent}}$).
Figure 5: Angular distribution of Compton scattered photons, from "Nuclei and Particles", p.55, E. Segre, Benjamin (1965). In our useful photon range 0.5 to about 2 MeV ($\alpha \simeq 1$ to 4) the cross section is dominated by photons scattered at forward angles with recoil electrons at wide angles.

Figure 6: Energy distribution of Compton recoil electrons, from "Techniques for Nuclear and Particle Physics Experiments", p.57, W. R. Leo, Springer-Verlag (1994). For photon energies above 0.8 MeV about 0.8 to 0.9 of the recoil electrons are above 0.2 MeV Čerenkov threshold in quartz.
The final number needed to calculate counting rates is the efficiency for a single recoil electron in a fiber to make a signal in the PMT. This is impossible to calculate by hand as it involves all the Compton and Čerenkov angles, absorption and propagation of light in the fibers, PMT efficiency etc. For the electron detection efficiency we use the number obtained in preliminary work for a proposal for LC detector development Ref [5]. Using a cosmic ray telescope of scintillating fibers on each side of a set of quartz fibers, the efficiency for observing a photon in the PMT from the quartz per cosmic ray was found to be 0.5%. One photon in the PMT is not much, and cosmics are minimum ionizing particles at many MeV that make Čerenkov light over their full path in the fibers, so this may be an over estimate of the useful efficiency of the relatively low energy recoil Compton electrons.

Summarizing, the factors for rates in the fiber detector from the wiggler are:

- beam energy $E_0 = 28$ GeV and intensity $I = 10^{11}$ e/pulse
- synchrotron flux from wiggler – 6.19 photons/electron
- fraction above 0.4 MeV that can make Čerenkov light (from Hobey’s Table A.1) = 0.18
- fraction that make Compton interactions in fibers = 0.007
- fraction of Compton interactions that make recoil above 0.2 MeV = 0.8
- efficiency for one Compton electron to make a signal in the PMT = 0.005

These factors yield for the total detected counts in all fibers from the wiggler

$$N_{\text{detected}} = (6.19 \times 10^{11})(0.18)(0.8)(0.007)(0.005)$$

$$= 3.11 \times 10^6 \text{ electrons/pulse}$$

Similary, we can estimate rates in the fibers from the bend stripe.

- synchrotron flux 5.76 photons/electron from two bends
- fraction above 0.4 MeV (from Hobey’s Table A.1) = 0.02 (10D45) = 0.002 (10D90) The lower bend field and lower critical energy in the 10D90 reduces the rate of high energy photons by a factor of roughly 10 compared to the 10D45. This is good.
- fraction of Compton interactions that make recoil above 0.2 MeV = 0.8
- fraction of bend stripe hitting the detector = 0.08
- fraction incident that make Compton in fibers = 0.007
- efficiency for one Compton electron to make signal in PMT = 0.005

The total detected counts from the bends is then:

$$2.58 \times 10^4 \text{ electrons/pulse(10D45)}$$

$$2.58 \times 10^3 \text{ electrons/pulse(10D90)}$$

This yields ratios of signal/background: 120 (10D45) and 1200 (10D90)

There are lots of crude approximations here, but its a start.
APPENDIX A

FORMULAS FOR SYNCHROTRON RADIATION

11.1. Bend Magnets

These formulas pertain to electrons in circular motion.\(^5,3,5,4\)

The average number of photons radiated in path length \(ds\) is

\[
\frac{dn}{ds} = \frac{5}{2\sqrt{3}} \frac{\alpha \gamma}{\rho},
\]

\(n = \langle n \rangle = 20.6 \ E(\text{GeV}) \phi \ \text{(rad)}
\]

\(= 0.618 \ B \ I(\text{kG} \cdot \text{m})\). \hspace{1cm} (A.1a)

The spectrum is a universal function of the characteristic energy

\[
k_c = \frac{3}{2} \gamma^3 \frac{\hbar c}{\rho} = \frac{3}{2} \frac{\gamma^3}{\alpha} \frac{r_e}{\rho} mc^2,
\]

\(= 2.22(\text{keV}) \left( \frac{E}{10 \ \text{GeV}} \right) \left( \frac{1 \ \text{km}}{\rho} \right), \hspace{1cm} (A.2a)\)

\(= 6.66(\text{keV}) \left( \frac{E}{10 \ \text{GeV}} \right)^2 \left( \frac{B}{1 \ \text{kG}} \right). \hspace{1cm} (A.2b)\)

The normalized photon energy is

\[
v = \frac{k}{k_c}.
\]

(A.3)

The normalized number distribution of the photons is

\[
\frac{1}{n} \frac{dn}{dv} = \frac{3}{5\pi} \int_{v_0}^{\infty} K_{8/3}(y) \, dy,
\]

\(\approx 0.4105v^{-2/3}(1 - 0.8438v^{2/3} + 0.0v^{4/3} + \ldots) \ v << 1, \)

\(\approx \frac{3}{5\sqrt{2}} \frac{\epsilon^{-\nu}}{v_0} \left(1 + \frac{55}{6} \frac{1}{\epsilon} - \frac{0.9791}{\epsilon^2} + \ldots\right) \ \ v >> 1. \)

Half of the energy is carried above \(v = 1\), by only 8.7% of the photons. Half of the photons are above \(v = 0.078\).
For convenience, Table A.1 lists some values for the spectrum (A.4) and its integrals. \( f_N \) is the fraction of the number of photons above \( v \), and \( f_E \) is the fraction of the energy carried by photons above \( v \).

<table>
<thead>
<tr>
<th>( v )</th>
<th>( dn/n \ dv )</th>
<th>( f_N )</th>
<th>( f_E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>8.50</td>
<td>0.7381</td>
<td>0.9579</td>
</tr>
<tr>
<td>0.03</td>
<td>3.91</td>
<td>0.6277</td>
<td>0.9912</td>
</tr>
<tr>
<td>0.1</td>
<td>1.562</td>
<td>0.4628</td>
<td>0.9502</td>
</tr>
<tr>
<td>0.2</td>
<td>0.863</td>
<td>0.3483</td>
<td>0.9052</td>
</tr>
<tr>
<td>0.3</td>
<td>0.584</td>
<td>0.2775</td>
<td>0.8485</td>
</tr>
<tr>
<td>0.5</td>
<td>0.333</td>
<td>0.1896</td>
<td>0.7369</td>
</tr>
<tr>
<td>1.</td>
<td>0.1244</td>
<td>0.08677</td>
<td>0.5000</td>
</tr>
<tr>
<td>2.</td>
<td>0.0288</td>
<td>0.02326</td>
<td>0.2150</td>
</tr>
<tr>
<td>3.</td>
<td>0.00818</td>
<td>0.000703</td>
<td>0.0886</td>
</tr>
<tr>
<td>5.</td>
<td>0.00081</td>
<td>0.000737</td>
<td>0.0142</td>
</tr>
</tbody>
</table>

Table A.1. Photon energy spectrum \( v \) for electrons in circular motion, normalized to the critical energy. \( f_N \) is the number of photons above \( v \), and \( f_E \) is the fraction of energy carried by photons above \( v \).

The energy loss per electron has average value

\[
\langle U \rangle = \langle n \rangle \langle k \rangle \quad \langle A.5 \rangle
\]

\[
= 1.267 \text{ (keV)} \left( \frac{E}{10 \text{ GeV}} \right)^2 \left( \frac{B}{1 \text{ kG}} \right)^2 L(m) \quad \langle A.5a \rangle
\]

\[
= 140.8 \text{ (keV)} \left( \frac{E}{10 \text{ GeV}} \right)^4 \left( \frac{1 \text{ km}}{\rho} \right) \phi(\text{rad}) \langle A.5b \rangle
\]

and variance

\[
\text{var}(U) = \langle (U - \langle U \rangle)^2 \rangle = \langle n \rangle \langle k^2 \rangle \quad \langle A.6 \rangle
\]

\[
= 11.17(\text{keV}^2) \left( \frac{E}{10 \text{ GeV}} \right)^4 \left( \frac{B}{1 \text{ kG}} \right)^3 L(m) \langle A.6a \rangle
\]

\[
= 414 (\text{keV}^2) \left( \frac{E}{10 \text{ GeV}} \right)^7 \left( \frac{1 \text{ km}}{\rho} \right)^2 \phi(\text{rad}) \langle A.6b \rangle
\]
where \( \langle u \rangle = 0.3079 \) and \( \langle u^2 \rangle = 0.4074 \) have been used.\(^{53}\) Equation (A.6) arises because \( \text{var}(U) \) depends on fluctuations in \( n \) as well as in \( k \); a Poisson distribution for \( n \) has been used. The angular distribution of the radiated energy integrated over all \( k \) is

\[
\frac{1}{U} \frac{dU}{dw} = \frac{21}{32} \frac{1 + (12/7)w^2}{(1 + w^2)^{7/2}}, \quad -\infty \leq w \leq \infty, \quad (A.7)
\]

\[w = \gamma \psi, \quad (A.7a)\]

where \( \psi \) is the angle perpendicular to the bend plane. For some cases of heating by bend SR, it is useful to have an approximate expression for the full double differential distribution. For fixed \( \nu \), approximate the \( w \) dependence with a Gaussian. Then

\[
\frac{1}{U} \frac{dU}{dv dw} \approx \frac{1}{U} \frac{dU}{dv} \frac{1}{\sqrt{2\pi} \sigma_w} \exp \left\{ \frac{-w^2}{2\sigma_w^2} \right\}, \quad (A.8)
\]

\[\sigma_w \approx \left( \frac{0.353}{\nu} \right)^{0.44}, \quad (A.9)\]

This approximation is reasonable at the 10–20% level for \( 0.1 < \nu < 3 \). Outside this range, Eq. (A.9) overestimates the effective \( \sigma \).

11.2. Quadrupoles

A photon spectrum integrated over a quadrupole field may be derived from (A.4).\(^{54}\) This spectrum is not very useful for background calculations because only the spectrum hitting the mask is interesting, not the spectrum going down the beampipe. The moments of a quad spectrum may be interesting for power reasons. For a Gaussian beam, the field scale is \( B_0 \), the quad field at 1 \( \sigma \) of the beam. The number of radiated photons and the radiated energy in units of the values for a bend magnet with \( B_0 \) are, with \( b \) the beam centroid offset in \( \sigma \) units:

12
11.3. Random Sampling from the Synchrotron Radiation Distribution

The nicest routine I know for random sampling from the spectrum Eq. (A.4) is due to Yokoya.\textsuperscript{57} RN is a random number uniform between 0 and 1 representing the integral number distribution between \(v\) and infinity. The returned values of \(v\) are within 0.05\% of Mack's values,\textsuperscript{1} at least for 0.001 < \(v\) < 12.

\begin{verbatim}
DATA YA1/0.5552 /, YAS/0.3063 /, YAS/0.1418 /, YAS/0.4184 /,
% YBO/0.01192 /, YB1/0.2065 /, YB2/-0.3281 /,
% YCO/0.003514 /, YC1/0.1927 /, YC2/0.6877 /,
% YDD/148.3 /, YD1/675.0 /, YD0/-692.2 /, YD1/-725.5 /

IF(RN.GT.342)THEN
P1=1.0-RN
P2=P1*P1
V=((YA4+YA2+YA1)*P2+YA2)*P2+YA1)*P2*P1
ELSEIF(RN.GT.0.0297)THEN
V=((YB2+RN*YB1)*RN+YB0)/((RN*YC2)*RN+YC1)*RN*YCO
ELSE
T=LOG(RN)
V=T1+(YD1*T1+YD0)/((T1+YB1)*T1+YD0)
\end{verbatim}
11.4. Short-Bend Radiation

A magnet in which the bend angle is less than $1/\gamma$ is called a short magnet; the spectrum does not follow Eq. (A.4) and depends on the $z$ variation of $B$. The amount of energy is roughly the same as given in Section A.2 above, but the scale or characteristic energy is greater,\textsuperscript{54} where with $k_{c-long}$ given by Eq. (A.2),

$$k_{c-short} \approx k_{c-long} \frac{\frac{2}{\phi}}{\phi}.$$ \hspace{1cm} (A.10)

11.5. General SR Spectrum

The spectrum in Section A.1 is for $k_c \ll E$. The general case is,\textsuperscript{58,59,60}

$$\frac{1}{n} \frac{dn}{dy} = \frac{3}{5\pi} \frac{1}{(1 + \xi y)^2} \times \left[ \int_{y}^{\infty} K_{5/2}(x) \, dx + \frac{\xi^2 y^2}{1 + \xi y} K_{2/2}(y) \right],$$ \hspace{1cm} (A.11)

$$y = \frac{k_c}{1 - \frac{k_c}{E}} = \frac{\nu}{1 - u} = \nu(1 + \xi y),$$ \hspace{1cm} (A.12)

$$\xi = \frac{k_c}{E} = \frac{3}{2} \Upsilon,$$ \hspace{1cm} (A.13)

where $n$ is given by (A.1), and $\nu$ only enters in the combination $y$. For $\xi = 0$, (A.11) reduces to (A.4). $\Upsilon$ is frequently used as a measure of $k_c/E$ rather than $\xi$.\textsuperscript{59,60} For Monte Carlo sampling of (A.11) see Reference 57.
APPENDIX D

NOTATION

\(a\)  Radius
\(b\)  Radius
\(c\)  Velocity of light
\(f_b\) Bunch crossing frequency, \(f_b = c/s_b\)
\(k\)  HOM loss parameter, usually \(pV/C\)
\(k\)  Photon energy, usually keV
\(k'\) Scattered photon energy
\(k_e\) Characteristic energy of synchrotron radiation
\(m\)  Electron mass (energy or momentum, i.e., missing factors of c)
\(n\)  Number of radiated SR photons
\(p\)  Momentum
\(q\)  Bunch charge
\(q^2\) (Momentum transfer)²
\(r_c\) Classical radius of electron, \(2.82 \times 10^{-13}\) cm
\(s\)  Path length along orbit
\(s_b\) Bunch spacing
\(u\)  Energy loss normalized to \(E\), e.g., \(k/E\)
\(v\)  Normalized photon energy, \(k/k_e\)
\(w\)  Angle normalized to \(1/\gamma\), \(w = \gamma \times \text{angle}\)
\(z\)  Distance along beam axis
\(A\)  Atomic weight
\(B\)  Magnetic field, usually kG
\(E\)  Beam energy
\(I\)  Beam current
\(L\)  Magnet length, usually m
\(SR\)  Synchrotron radiation
\(T\)  Kinetic energy
\(U\)  Energy radiated
\(Z\)  Atomic number
\(Z_a\)  377 ohms
\(\alpha\)  1/137
\(\beta\)  \(E/m\)
\(\delta\)  Skin depth, usually \(\mu\)m
\(\mu\)  RF magnetic permeability relative to vacuum
\(\phi\)  Bend angle
\(\sigma\)  Cross section
\(\sigma_s\)  Rms bunch length, Gaussian parameter
\(\sigma(Z)\)  DC electrical conductivity of material of atomic number \(Z\)
\(\theta\)  Scattering angle
\(\rho\)  Radius of curvature
REFERENCES

1. A. R. Clark (LBL) wrote QSRAD in the early 1970s for use with PEP. Many versions exist now. There is no write-up, as far as I know.


3. B. Rossi, High-Energy Particles, Prentice-Hall, 1952. These formulas may be found in many other places.


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232 Interaction Region Considerations


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Interaction Region Considerations


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