Unifying the Mechanisms for SSAs in Hard Process

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Outline

- Introduction
- Naïve parton model fails for large SSAs
- Two mechanisms: Sivers and twist-3
- Unifying these two
- Summary
What is Single Spin Asymmetry?

- Scattering a transverse spin polarized proton on an unpolarized target (another hadron or a photon)

- Physically, the cross section contains a term

\[ d\sigma \propto \vec{S} \cdot (\vec{p} \times \vec{K}_\perp) \]
Sample Exp. Data

- A. Bravar et al., E704, PRL77, 2626 (1996)

\[ \bar{p} + p \rightarrow \pi + X \]

**FIG. 3.** $A_N$ data as a function of $x_F$ for $\pi^-$ and $\pi^+$ for $p_T \geq 0.5$ GeV/c. $A_N$ data for $\pi^0$ in a similar $p_T$ range are also shown [5]. The first $\pi^-$ and $\pi^+$ data points are offset by $-0.01$ and $+0.01$ $x_F$ units, respectively.
Examples: SSAs at RHIC

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STAR

Central rapidity!!

PHENIX

BRAHMS preliminary
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5
In Deep Inelastic Scattering, HERMES, A. Airapetian et al.,
hep-ex/0408013, hep-ex/0507013

\[ \langle Q^2 \rangle = 2.41 \text{GeV}^2, \langle P_\perp \rangle = 0.41 \text{GeV} \]

\[ 0.023 < x < 0.4, \quad 0.2 < z < 0.7 \]
Why Does SSA Exist?

- Single Spin Asymmetry requires
  - Helicity flip: one must have a reaction mechanism for the hadron to change its helicity (in a cut diagram)
  - Final State Interactions (FSI): to generate a phase difference between two amplitudes
    The phase difference is needed because the structure $S \cdot (p \times k)$ violate the naïve time-reversal invariance
Naïve Parton Model Fails to Explain Large SSAs

- If the underlying scattering mechanism is hard, the naïve parton model generates a very small SSA: (G. Kane et al, PRL41, 1978)
  - It is in general suppressed by $\alpha_s m_q/Q$
- We have to go beyond the naïve parton model to understand the large SSAs observed in hadronic reactions
Two Mechanisms in QCD

- Transverse Momentum Dependent (TMD) Parton Distributions and Fragmentations
  - Sivers function, Sivers 90
  - Collins function, Collins 93
  - Gauge invariant definition of the TMDs: Brodsky, Hwang, Schmidt 02; Collins 02; Belitsky, Ji, Yuan 02; Boer, Mulders, Pijman, 03
  - The QCD factorization: Ji, Ma, Yuan, 04; Collins, Metz, 04

- Twist-three Correlations (collinear factorization)
  - Efremov-Teryaev, 82, 84
  - Qiu-Sterman, 91, 98
How Do They Contribute?

- **TMD**: the quark orbital angular momentum leads to hadron helicity flip
- The factorizable final state interactions --- the gauge link provides the phase

- **Twist-three**: the gluon carries spin, flipping hadron helicity
- The phase comes from the poles in the hard scattering amplitudes
A General Diagram in Twist-3

Collinear Factorization:

$$d\sigma \propto \epsilon^\alpha S_{12} p_{h12} \int \frac{dx}{x} \frac{dz}{z} \hat{q}(z) T_F(x, x - xg) \times \cdots$$

Qiu, Sterman, 91
The TMD Factorization

\[ F_{UT}(x_B, z_h, P_{h\perp}, Q^2) = \sum_{q=u,d,s,...} e_q^2 \int d^2 \vec{k}_\perp d^2 \vec{p}_\perp d^2 \vec{\ell}_\perp \frac{\vec{k}_\perp \cdot \vec{P}_{h\perp}}{|P_{h\perp}|} \]

\[ \times q_T(x_B, k_\perp, \mu^2, x_B \zeta, \rho) \hat{q}_h(z_h, p_\perp, \mu^2, \bar{\zeta}/z_h, \rho) S(\vec{\lambda}_\perp, \mu^2, \rho) \]

\[ \times H(Q^2, \mu^2, \rho) \delta^2(z_h \vec{k}_\perp + \vec{p}_\perp + \vec{\lambda}_\perp - \vec{P}_{h\perp}) \]
Previous Attempts to Connect these Two

- Relations at the matrix elements level
  \[ T_F(x,x) = \int d^2k \ |k|^2 q_T(x,k) \]

- Qiu-Sterman
- Sivers

- Boer, Mulders, Pijman 03

- Attempts to connect these two in some processes, no definite conclusion:
  - Ma, Wang, 03
  - Bacchetta, 05
Unifying the Two Mechanisms (P⊥ dependence of SSAs)

- At low P⊥, the non-perturbative TMD Sivers function will be responsible for its SSA
- When P⊥ ∼ Q, purely twist-3 contributions
- For intermediate P⊥, Λ_{QCD} ≪ P⊥ ≪ Q, we should see the transition between these two
- An important issue, at P⊥ ≪ Q, these two should merge, showing consistence of the theory

Factorization guidelines

Reduced diagrams for different regions of the gluon momentum:
along P direction, P', and soft

Collins-Soper 81
Detailed Calculations: Soft and Hard Poles

- Soft: \( x_g = 0 \)
- Hard: \( x_g \neq 0 \)

Soft Pole: \[ \frac{1}{-x_g + i\epsilon} \]

Hard Pole: \[ \frac{1}{-x_g - x_B + x + i\epsilon} \]
Cross sections

- Summing all diagrams,

\[
\frac{d\Delta \sigma}{dx_B dz d^2 P_{h\perp}} = \epsilon^{\alpha\beta} S_{\perp}^\alpha P_{h\perp}^\beta \int \frac{dx}{x} \frac{dz}{z} \tilde{q}(z) \delta (\tilde{q}_\perp^2 - \cdots) \left\{ \frac{1}{2N_c} \left[ x \frac{\partial}{\partial x} T_F(x, x) \right] D^s \right. \\
+ \frac{1}{2N_c} T_F(x, x) N^s \\
+ T_F(x, x_B) N^h \right\}
\]

- For $P_{\perp} \sim Q$, many other terms contribute, see Eguchi, Koike, Tanaka, hep-ph/0604003
\( P_{h\perp} \ll Q \) limit

- Keeping the leading order of \( q_{\perp}/Q \),

\[
\left. \frac{d\Delta \sigma}{dx_Bdzd^2P_{h\perp}} \right|_{asspt.} = \epsilon^{\alpha\beta} S^{\alpha}_{\perp} P^{\beta}_{h\perp} \frac{1}{(P_{h\perp}^2)^2} \int \left\{ \delta(\tilde{\xi} - 1) A + \delta(\xi - 1) B \right\}
\]

\[
A = \frac{1}{2N_c} \left[ x \frac{\partial}{\partial x} T_F(x, x)(1 + \xi^2) + T_F(x, x_B) \ldots \right]
\]

\[
B = C_F T_F(x, x) \left[ \frac{1 + \tilde{\xi}^2}{(1 - \tilde{\xi})_+} + 2\delta(\tilde{\xi} - 1) \ln \frac{Q^2}{\tilde{q}_{\perp}^2} \right],
\]

- Hard-pole contribution plays very important role to get the above result, and this result should be reproduced by the Sivers function at the same kinematical limit, from the TMD factorization.
TMD Factorization

- When $q_\perp \ll Q$, a TMD factorization holds,

$$
\frac{d^3 \Delta \sigma(S_\perp)}{dx_Bdz_hd^2P_{h\perp}} = \frac{\epsilon^{\alpha\beta} S_{\perp\alpha} P_{h\perp\beta}}{M_P} \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\lambda}_\perp H(Q)
\times \frac{\vec{k}_\perp \cdot \vec{P}_{h\perp}}{P_{h\perp}^2} \delta(2)(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\lambda}_\perp - \vec{q}_\perp)
\times q_T(x_B, k_\perp, \zeta) \bar{q}(z_h, p_\perp, \zeta) (S(\lambda_\perp))^{-1}
$$

- When $q_\perp \gg \Lambda_{QCD}$, the $P_t$ dependence of all distributions and soft factor can be calculated from pQCD, by radiating a hard gluon
Fragmentation function at $p_\perp \gg \Lambda_{\text{QCD}}$

\[ \tilde{q}(z_h, p_\perp) = \frac{\alpha_s}{2\pi^2 p_\perp^2} C_F \int \frac{dz}{z} \tilde{q}(z) \times \left[ \frac{1 + \tilde{\xi}^2}{(1 - \tilde{\xi})^+} + \delta(\tilde{\xi} - 1) \left( \ln \frac{\tilde{\xi}^2}{p_\perp^2} - 1 \right) \right] \]

See, e.g., Ji, Ma, Yuan, 04
\[ S(\lambda_{\perp}) = \frac{\alpha_s}{2\pi^2} \frac{1}{\lambda_{\perp}^2} C_F \left( \ln \rho^2 - 2 \right) \]
Sivers Function from twist-3: soft poles

(a) 

(b) 

(c) 

(d)
Hard Poles for Sivers Function

(a)  

(b)  

(c)  

(d)  

(e)  

(f)
Sivers Function at Large $k_\perp$

\[ q_T(x, k_\perp) = -\frac{\alpha_s}{4\pi^2} \frac{2M_p}{(k_\perp^2)^2} \int \frac{dx}{x} \left\{ A + C_F T_F(x) \right\} \times \delta(\xi - 1) \left( \ln \frac{\zeta^2 / \vec{k}_\perp^2}{-1} \right) \]

- $1/k_\perp^4$ follows a power counting
- Drell-Yan Sivers function has opposite sign
  (Brodsky, et al., 02; Collins, 02; Belitsky, et al., 02)
- Plugging this into the factorization formula, we indeed reproduce the polarized cross section calculated from twist-3 correlation
Concluding Remarks

- Two mechanisms generating the large SSAs are unified for a physical process and observable
- This unification will impose a rigorous constraint for phenomenological studies of SSAs
- Extension to other processes, and other observables are also interested to follow
- $P_t$ scan of SSAs will show the transition from perturbative region to nonperturbative region
Final Results

- $P_\perp$ dependence

$$\frac{d\Delta \sigma}{d^2 q_\perp dy} = \int q_T(z_1, k_\perp)\bar{q}(z_2, k_\perp) + \left( \frac{d\Delta \sigma^{QS}}{d^2 q_\perp dy} - \frac{d\Delta \sigma^{QS}}{d^2 q_\perp dy} \bigg|_{aspt.} \right)$$

Sivers function at low $P_\perp$

Qiu-Sterman Twist-three

- Which is valid for all $P_\perp$ range
Transition from Perturbative region to Nonperturbative region?

- Compare different region of $P_{\perp}$

![Graph showing transition from perturbative to nonperturbative region with data points and arrows indicating the transition regions.](image-url)