Solving light-front QCD with ab-initio many-body basis function methods

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I. Ab initio approach to quantum many-nucleon systems
II. Ingredients for light front QCD from constituent quark models, light front $\Phi^4_{1+1}$, and transverse lattice QCD
III. The transverse basis function approach (2D HO, ADS/CFT)
IV. Conclusions and Outlook

Collaborators on light-front papers discussed here

Stan Brodsky, SLAC
Dipankar Chakrabarti, Florida State University
Avaroth Harindranath, Saha Institute of Nuclear Physics, Calcutta, India
Richard Lloyd, University of Arkansas
Lubo Martinovic, Institute of Physics Institute, Bratislava, Slovakia
Asmita Mukherjee, Indian Institute of Technology, Mumbai, India
Grigori Pivovarov, Institute for Nuclear Research, Moscow, Russia
Peter Peroncik, John R. Spence, Iowa State University (ISU)
Guy de Teramond, University of Costa Rica
Constructing the non-perturbative theory bridge between “Short distance physics” and “Long distance physics”

- Asymptotically free current quarks
- Chiral symmetry
- High momentum transfer processes

- Constituent quarks
- Broken Chiral symmetry
- Meson and Baryon Spectroscopy
- Moments, GPD’s, etc.

- Bare NN, NNN interactions
- fitting 2-body data
- Short range correlations &
- strong tensor correlations

- Effective NN, NNN interactions
- describing low energy nuclear data
- Mean field, pairing, &
- quadrupole, etc., correlations

- \( H(\text{bare operators}) \)
- \( \text{Bare transition operators} \)

- \( \text{Heff} \)
- \( \text{Effective charges, transition ops, etc.} \)

**BOLD CLAIM**

*We now have the tools to accomplish this program in nuclear many-body theory*

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**Fundamental Challenges**

- What is the Hamiltonian
- How to renormalize to a finite representation
- How to solve for non-perturbative observables

*Focii of the Nuclear Many-Body and Light-Front QCD communities for several decades!*
What’s New?

- Progress in Hamiltonian renormalization programs
- Progress in developing/implementing light front representations - both longitudinal and transverse
- Progress in developing/evaluating non-perturbative observables with light front amplitudes
- Advances in large sparse matrix algorithms
- Advances in parallel architecture supercomputers

It is time to “Just Do It” - Stan Brodsky LC2004

Computational aspects of the ab-initio NCSM

<table>
<thead>
<tr>
<th>System</th>
<th>Proc x Nodes</th>
<th>Memory/node (GB)</th>
<th>Performance (Gflop/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thunder⁵</td>
<td>4x1024</td>
<td>8</td>
<td>22,938</td>
</tr>
</tbody>
</table>

IOWA STATE UNIVERSITY
Ab Initio Many-Body Theory

$H$ acts in its full infinite Hilbert Space

$H_{\text{eff}}$ of finite subspace

Effective Hamiltonian for $A$-Particles
Lee-Suzuki-Okamoto Method plus Cluster Decomposition

C. Viazminsky and J.P. Vary, J. Math. Phys. 42, 2055 (2001);
K. Suzuki and S.Y. Lee, Progr. Theor. Phys. 64, 2091 (1980);
K. Suzuki, *ibid*, 68, 246 (1982);

Preserves the symmetries of the full Hamiltonian:
Rotational, translational, parity, etc., invariance

$$H_A = T_{\text{rel}} + V = \frac{1}{2m_A} \sum_{i,j} \left( \vec{p}_i - \vec{p}_j \right)^2 + V_{ij} + V_{\text{NNN}}$$

Select a finite oscillator basis space ($P$-space) and evaluate an $a$-body cluster effective Hamiltonian:

$$\mathcal{H} = T_{\text{rel}} + V^{(a)}$$

Guaranteed to provide exact answers as $a \rightarrow A$ or as $P \rightarrow 1$. 
Select a subsystem (cluster) of a < A Fermions.

Develop a unitary transformation for a finite space, the “P-space” that generates the exact low-lying spectra of that cluster subsystem. Since it is unitary, it preserves all symmetries.

Construct the A-Fermion Hamiltonian from this a-Fermion cluster Hamiltonian and solve for the A-Fermion spectra by diagonalization.

Guaranteed to provide the full spectra as either a --> A or as P --> I.
Key equations to solve at the a-body cluster level

Solve a cluster eigenvalue problem in a very large but finite basis and retain all the symmetries of the bare Hamiltonian

\[ P_a = \sum_{\alpha} |\alpha\rangle \langle \alpha| \]

\[ Q_a = \sum_{\beta} |\beta\rangle \langle \beta| \]

\[ P_a + Q_a = I_a \]

\[ H_a^{\alpha} |k\rangle = E_k |k\rangle \]

\[ \langle \alpha_q |\omega| \alpha_p \rangle = \sum_{k \in \mathbb{K}} \langle \alpha_q |k\rangle \langle k| \alpha_p \rangle \]

where \( \langle k| \alpha_p \rangle = \text{Inverse} \{ \{k| \alpha_p \} \} \)

\[ H^{(a)} = (P_a + \omega \omega^T)^{-1/2} (P_a + P_a \omega \omega^T Q_a) H_a^{\alpha} (Q_a \omega P_a + P_a)(P_a + \omega \omega^T)^{-1/2} \]

4-He with the JISP16 Veff Interaction

- First Excited State Eigenvalue
- Ground State Eigenvalue

\( V_{\text{eff}}^{a=2} \) cluster

- \( N_{\text{max}} = 2 \)

\( E_a = -28.3 \text{ MeV} \)

A. Shirokov, et al, to be published
Chiral perturbation theory (χPT) allows for controlled power series expansion

Expansion parameter: \( \frac{Q}{\Lambda_{\chi}} \)

\( \Lambda_{\chi} \approx 1 \text{ GeV} \); \( \chi \) - symmetry breaking scale

Within χPT 2π-NNN Low Energy Constants (LEC) are related to the NN-interaction LECs \( \{c_i\} \).

Terms suggested within the Chiral Perturbation Theory

Regularization is essential, which is obvious within the Harmonic Oscillator wave function basis.
Constituent Quark Models of Exotic Mesons


\[ H = T + V(\text{OGE}) + V(\text{confinement}) \]

Solve in HO basis as a bare H problem & study dependence on cutoff

**Symmetries:**
- Exact treatment of color degree of freedom \(<-- \text{Major new accomplishment}\)
- Translational invariance preserved
- Angular momentum and parity preserved

**Next generation - Lee-Suzuki form (preliminary results!):**
- More realistic \( H_{\text{eff}} \) in equal time quantization
- Fit quark masses and couplings to wider range of mesons and baryons

**Beyond that generation:**
- \( H_{\text{eff}} \) derived from QCD using light-front quantization
Truncated (bare) and renormalized meson mass comparison
Semi-relativistic and Semi-phenomenological mass operator
Arrow shows cases evaluated in relative and single particle basis

R. Lloyd, J. R. Spence and J. P. Vary, to be published

Negative orbital parity 2c2cbar lowest eigenstate - JRS Hamiltonian

Threshold (theory, L=0) for breakup to J/ψ + χc1 = 6559 MeV
Extrapolation = 6401 MeV

R. Lloyd, J. R. Spence and J. P. Vary, to be published
Burning issues

Demonstrate degeneracy - Spontaneous Symmetry Breaking
Topological features - soliton mass and profile (Kink, Kink-Antikink)
Quantum modes of kink excitation
Phase transition - critical coupling, critical exponent and the physics of symmetry restoration
Role and proper treatment of the zero mode constraint
Chang’s Duality

Φ⁴ in 1+1 Dimensions

DLCQ with Coherent State Analysis

A. Derive the Hamiltonian and quantize it on the light front, investigate coherent state treatment of vacuum

B. Obtain vacuum energy as well as the mass and profile functions of soliton-like solutions in the symmetry-broken phase:
   PBC: SSB observed, Kink + Antinkink ~ coherent state!
   APBC: SSB observed, Kink ~ coherent state!
       Chakrabarti, Harindranath, Martinovic and Vary, Phys. Letts. B582, 196 (2004); hep-th/0309263

C. Demonstrate onset of Kink Condensation:
   Chakrabarti, Harindranath and Vary, Phys. Rev. D71, 125012(2005); hep-th/0504094
Lagrangian density
\[ \mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi \partial^{\mu} \phi + \mu^2 \phi^2) - \frac{\lambda}{4!} \phi^4 \]

Hamiltonian quantized in light front coordinates with, for example, anti-periodic boundary conditions:
\[ H = H_0 + H_1 + H_2 \]

\[ H_0 = -\mu^2 \sum_{n=0}^{K} \frac{1}{n!} a_n^* a_n \]

\[ H_1 = \frac{\lambda}{4\pi} \sum_{k,l,m,n=0}^{K} \frac{1}{N_{kl}^2} \frac{1}{N_{mn}^2} \frac{1}{\sqrt{klmn}} \delta_{m+n,k+l} \]

\[ H_2 = \frac{\lambda}{4\pi} \sum_{k,l,m,n=0}^{K} \frac{1}{N_{limn}^2} \left[ \frac{a_n^* a_m^* a_l^* a_k^*}{\sqrt{klmn}} \right] \delta_{k,m+n+l} \]

where all indices are half odd integers.

Set up a normalized and symmetrized set of basis states, where, with \( n_k \) representing the number of bosons with light front momentum \( k \):

\[ P^+ = \frac{K}{L} \text{ conserved} \Rightarrow \sum_k n_k k = K \]

\[ P^- = HL \& H|\psi_i> = E_i|\psi_i> \]

Fully covariant mass-squared spectra emerges
\[ P^+ P^- = M_i^2 = KE_i \]

Must extrapolate to the continuum limit, \( K \rightarrow \infty \)

DLCQ matrix dimension in even particle sector (APBC)

<table>
<thead>
<tr>
<th>K</th>
<th>Dimension</th>
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</thead>
<tbody>
<tr>
<td>16</td>
<td>336</td>
</tr>
<tr>
<td>32</td>
<td>14219</td>
</tr>
<tr>
<td>40</td>
<td>67243</td>
</tr>
<tr>
<td>45</td>
<td>165498</td>
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<tr>
<td>50</td>
<td>389253</td>
</tr>
<tr>
<td>55</td>
<td>880962</td>
</tr>
<tr>
<td>60</td>
<td>1928175</td>
</tr>
</tbody>
</table>

Light front momentum probability density in DLCQ:

\[ \chi(n) = \langle K | a^*_{-n} a_n | K \rangle \]

Normalized:

\[ \sum_n n \chi(n) = K \]

Light cone momentum fraction:

\[ x = n / K \]

“Bjorken x”

Compares favorably with results from a constrained variational treatment based on the coherent state
Extract the Vacuum energy density and Kink Mass

\[ M_0^2 = E_0 K + M_{\text{kink}}^2 \]
\[ \Rightarrow E_0 = E_{00} + M_{\text{kink}}^2 / K \]

Note:
- $\phi \rightarrow -\phi$ symmetry
- Decoupling of even and odd boson sectors
- Exact degeneracy only in the $K \rightarrow \infty$ limit

All results to date are in the broken phase: \( \mu^2 = -1 \)

Define a Ratio = \( [M_{\text{even}}^2 - M_{\text{odd}}^2] / [\text{Vac Energy Dens}]^2 \)
### Vacuum Energy

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Classical</th>
<th>DLCQ-APBC</th>
<th>DLCQ-PBC zero modes?</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-37.70</td>
<td>-37.81(7)</td>
<td>-37.90(4)</td>
</tr>
<tr>
<td>1.0</td>
<td>-18.85</td>
<td>-18.71(5)</td>
<td>-18.97(2)</td>
</tr>
<tr>
<td>1.25</td>
<td>-15.08</td>
<td>-14.91(5)</td>
<td>15.19(5)</td>
</tr>
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</table>

### Soliton Mass

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Classical</th>
<th>Semi-Classical</th>
<th>DLCQ-APBC</th>
<th>DLCQ-PBC zero modes?</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>11.31</td>
<td>10.84</td>
<td>11.6(2)</td>
<td>11.26(4)</td>
</tr>
<tr>
<td>1.0</td>
<td>5.657</td>
<td>5.186</td>
<td>5.22(8)</td>
<td>5.563(7)</td>
</tr>
<tr>
<td>1.25</td>
<td>4.526</td>
<td>4.054</td>
<td>4.07(6)</td>
<td>4.43(4)</td>
</tr>
</tbody>
</table>
Fourier Transform of the Soliton (Kink) Form Factor
Ref: Goldstone and Jackiw

\[ \phi_e(x^- - a) = \int_{-\infty}^{\infty} dq^+ \exp\left(-\frac{i}{2} q^+ a\right) \langle K + q^+ | \phi(x^-) | K \rangle \]

Evaluated within DLCQ choosing \( a = 0 \):

\[ = \sum_{l \in \mathbb{Z}} \frac{1}{\sqrt{4\pi l}} \langle K \pm l | \left\{ a_L e^{-l i x^- \tau} + a_R e^{l i x^- \tau} \right\} | K \rangle \]

Note: Issue of the relative phases - fix arbitrary phases via guidance of the coherent state analysis:

\[ \text{Sign} \left\{ \langle K + l | \mu_l^* | K \rangle \right\} = + \]
\[ \text{Sign} \left\{ \langle K - l | \mu_l^* | K \rangle \right\} = - \]

Cancellation of imaginary terms and vacuum expectation value, \( \langle \phi \rangle \), are non-trivial tests of resulting kink structure.
What is the nature of this phase transition?

We observe dramatic changes in:

1. Mass spectroscopy
2. Form factor $\rightarrow$ Kink condensation signal
3. Parton distributions

Non-trivial achievements in the Hamiltonian framework
QCD applications in the $q\bar{q} + q\bar{q}$-link approximation for mesons

DLCQ for longitudinal modes and a transverse momentum lattice

- Adopt QCD Hamiltonian of Bardeen, Pearson and Rabinovici
- Restrict the P-space to q-qbar and q-qbar-link configurations
- Does not follow the effective operator approach (yet)
- Introduce regulators as needed to obtain cutoff-independent spectra

S. Dalley and B. van de Sande,

D. Chakrabarti, A. Harindranath and J.P. Vary,
Codes and their Applications

- **MFDn V10-b3**  
  NN+NNN interactions in the Ab-initio No-Core Shell Model (NCSM)

- **MFDq V9-b5**  
  multi-Q + multi-Q systems in equal-time 3D harmonic oscillator basis with color and multiple flavors

- **MFBD V9-b5**  
  scalar field theory in 1+1 dimensions

- **MFDMC V9-b5**  
  transverse lattice QCD (QQ + QQlink)

- **MFBDQ V10-b3**  
  QCD in light-cone quantization for mesons & baryons under development (details follow)

Core technology - Lanczos algorithm on parallel processors

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Many-Fermion Dynamics (MFD)  
General Purpose F90/MPI Code

- Accurate - verified using 3 independent codes
- Functional - Spectra and other observables
- Flexible - Alternative many-Fermion system apps
- Parallel - Multi-processor enabled
- Scalable - Behavior with increase in processors
- Balanced - all processors equally loaded
- Robust/stable - accurate in single precision
- User-friendly - Documented, several groups use it
- Portable - SGI-Origin, DEC workstation clusters, IBM
- Restartable - 10 run/restart modes - key to large scale apps
- Efficient - both in speed, disk space and RAM
- Productive - employed in multiple investigations
Overview of MFBDQ Development
status report - May 19, 2006

Selected/implemented/tested
➢ Degrees of Freedom (identical particle statistics, coordinates, spin, flavor, color)
➢ Basis space(s)
➢ Symmetries
➢ Lanczos algorithm

Under development/implementation/testing
➢ Hamiltonian vertex matrix elements of Light-Front QCD

Selection of degrees of freedom
and the basis functions for solving baryons

Quarks and Gluons
Multiple Flavors
SU(3) Color
Spin and Helicity
Longitudinal modes of DLCQ
Transverse modes of
2D HO or AdS/CFT holography
Initial Application to Baryons

- Longitudinal DLCQ basis
- Transverse 2D HO or AdS/CFT basis
- Two Flavors (u,d) and limited to one gluon
- SU(3) color
- Simplified H_{QCD} -> progressing to greater realism

Symmetries and Constraints

Explicit but flexible symmetries/constraints
Flexible => all but first are input parameters

- Identical particle statistics of Fermions and Bosons
- Total baryon number = 1
- Total charge = 1
- Total SU(3) color projection = 0
- Total angular momentum projection J_z = 1/2
- Total number of q-qbar pairs le 1
- Total number of gluons le 1

Symmetries/constraints via Lagrange method

- Total transverse momentum eliminated by exact factorization of the light-front wavefunction
- Total color-singlet states isolated
Conclusions

- Similarity of “two-scale” problems in Nuclear and Particle Physics
- Ab-initio theory is a convergent exact method for solving many-particle Hamiltonians
- Method has been demonstrated as exact in the Nuclear Physics applications
- Quasi-exact results for 1+1 scalar field theory obtained
- Non-perturbative vacuum expectation value (order parameter)
- Kink mass & profile obtained
- Critical properties (coupling, exponent) emerging
- Evidence of Kink condensation obtained
- Sensitivity to boundary conditions needs further study
- Role of zero modes yet to be fully clarified
- Initial applications to constituent quark models (equal time) - novel predictions
- Initial applications to QCD (3 + 1) on transverse lattice proved encouraging
- Project underway to implement QCD (3 + 1) for baryons with Light Front Basis Functions
- Advent of parallel computing has made new physics domains accessible: algorithm improvements have achieved fully scalable and load-balanced codes.
“…I will sum up by saying that light-front QCD is not for the faint of heart, but for a few good candidates it is a chance to be a leader in a much smaller community of researchers than one faces in the other areas of high-energy physics, with, I believe, unusual promise for interesting and unexpected results.”


Community Effort?
“Light Front QCD Collaboration”
Facilitates entry of new researchers - eg grad students

Should we emulate the lattice QCD community and organize a light-front community to co-develop and share via web:

- Computer codes
- Light-front amplitudes (data files)
- Manuals/tutorials/codes for their use
- Computer cycles (a “light-front grid”)
