Solving light-front QCD with ab-initio many-body basis function methods

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Light-Cone QCD and Nonperturbative Hadron Physics
(LC 2006)
Minneapolis, Minnesota
May 15-19 , 2006

I. Ab initio approach to quantum many-nucleon systems
II. Ingredients for light front QCD from constituent quark models, light front Φ^4_{1+1}, and transverse lattice QCD
III. The transverse basis function approach (2D HO, ADS/CFT)
IV. Conclusions and Outlook

Collaborators on light-front papers discussed here

Stan Brodsky, SLAC
Dipankar Chakrabarti, Florida State University
Avaroth Harindranath, Saha Institute of Nuclear Physics, Calcutta, India
Richard Lloyd, University of Arkansas
Lubo Martinovic, Institute of Physics Institute, Bratislava, Slovakia
Asmita Mukherjee, Indian Institute of Technology, Mumbai, India
Grigorii Pivovarov, Institute for Nuclear Research, Moscow, Russia
Peter Peroncik, John R. Spence, Iowa State University (ISU)
Guy de Teramond, University of Costa Rica

Constructing the non-perturbative theory bridge between
"Short distance physics" ⟺ "Long distance physics"

Asymptotically free current quarks Chiral symmetry High momentum transfer processes	⟺	Constituent quarks Broken Chiral symmetry Meson and Baryon Spectroscopy Moments, GPD's, etc.
Bare NN, NNN interactions fitting 2-body data Short range correlations & strong tensor correlations	⟺	Effective NN, NNN interactions describing low energy nuclear data Mean field, pairing, & quadrupole, etc., correlations
H(bare operators) Bare transition operators	⟺	Heff Effective charges, transition ops, etc.

BOLD CLAIM
We now have the tools to accomplish this program in nuclear many-body theory

Fundamental Challenges

- What is the Hamiltonian
- How to renormalize to a finite representation
- How to solve for non-perturbative observables

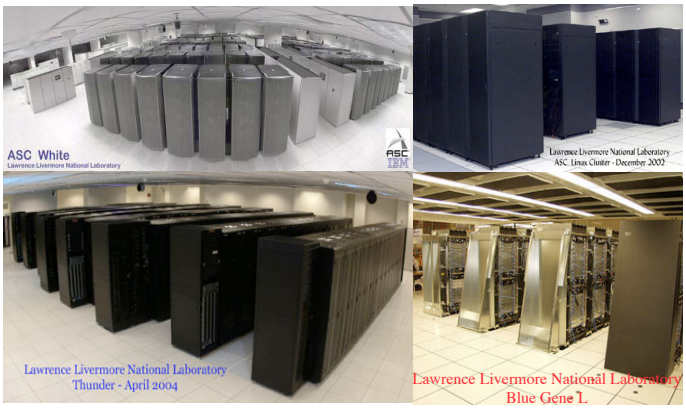
Focii of the Nuclear Many-Body and Light-Front QCD communities for several decades!

What's New?

- Progress in Hamiltonian renormalization programs
- Progress in developing/implementing light front representations - both longitudinal and transverse
- Progress in developing/evaluating non-perturbative observables with light front amplitudes
- Advances in large sparse matrix algorithms
- Advances in parallel architecture supercomputers

It is time to "Just Do It" - Stan Brodsky LC2004

Computational aspects of the ab-initio NCSM



System	Proc x Nodes	Memory/node (GB)	Performance (Gflop/s)
Thunder[5]	4x1024	8	22,938

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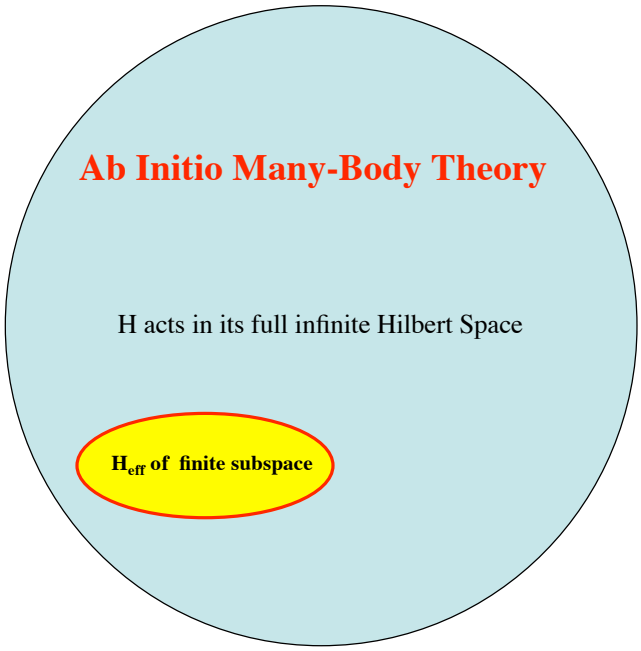


Effective Hamiltonian for A-Particles

Lee-Suzuki-Okamoto Method plus Cluster Decomposition

P. Navratil, J.P. Vary and B.R. Barrett,
Phys. Rev. Lett. 84, 5728(2000); Phys. Rev. C62, 054311(2000)
C. Viazminsky and J.P. Vary, J. Math. Phys. 42, 2055 (2001);
K. Suzuki and S.Y. Lee, Progr. Theor. Phys. 64, 2091(1980);
K. Suzuki, *ibid*, 68, 246(1982);
K. Suzuki and R. Okamoto, *ibid*, 70, 439(1983)

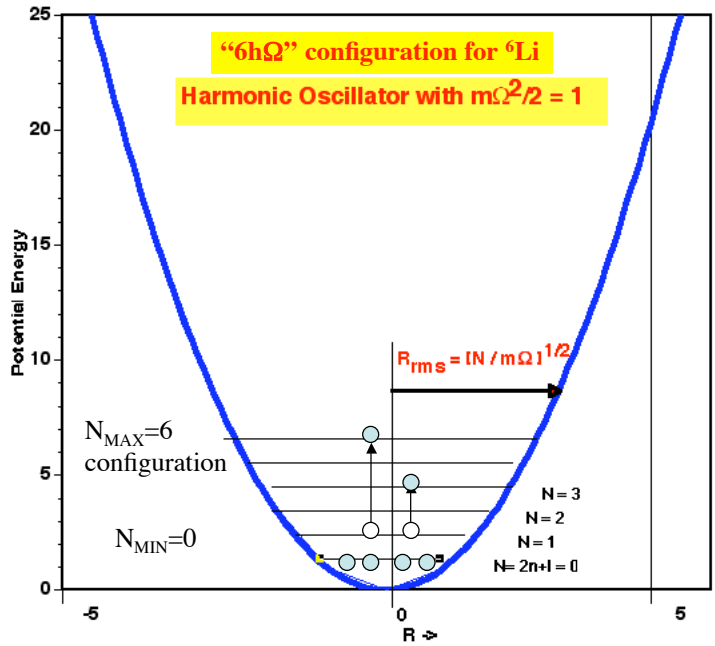
Preserves the symmetries of the full Hamiltonian:
Rotational, translational, parity, etc., invariance

$$H_{\mathcal{A}} = T_{rel} + V = \sum_{i<j}^{\mathcal{A}} [\frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} + V_{ij}] + V_{NNN}$$

Select a finite oscillator basis space (P-space) and evaluate an a- body cluster effective Hamiltonian:

$$\mathcal{H} = T_{rel} + \mathcal{V}^{(a)}$$

Guaranteed to provide exact answers as $a \rightarrow A$ or as $P \rightarrow 1$.



Guide to the methodology

Select a subsystem (cluster) of a < A Fermions.

Develop a unitary transformation for a finite space, the "P-space" that generates the exact low-lying spectra of that cluster subsytem. Since it is unitary, it preserves all symmetries.

Construct the A-Fermion Hamiltonian from this a-Fermion cluster Hamiltonian and solve for the A-Fermion spectra by diagonalization.

Guaranteed to provide the full spectra as either a --> A or as P --> 1

Key equations to solve at the a-body cluster level

Solve a cluster eigenvalue problem in a very large but finite basis and retain all the symmetries of the bare Hamiltonian

$$P_a = \sum_{P \in \mathcal{P}} |\alpha_P\rangle\langle\alpha_P|$$

$$Q_a = \sum_{Q \in \mathcal{Q}} |\alpha_Q\rangle\langle\alpha_Q|$$

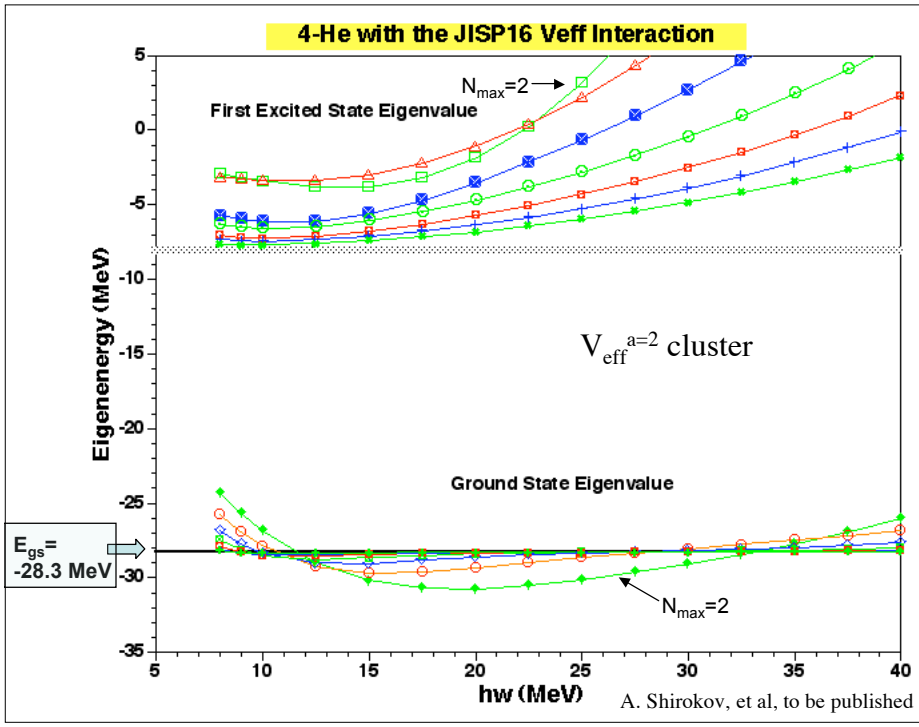
$$P_a + Q_a \approx 1_a$$

$$H_a^{\Omega}|k\rangle = E_k|k\rangle$$

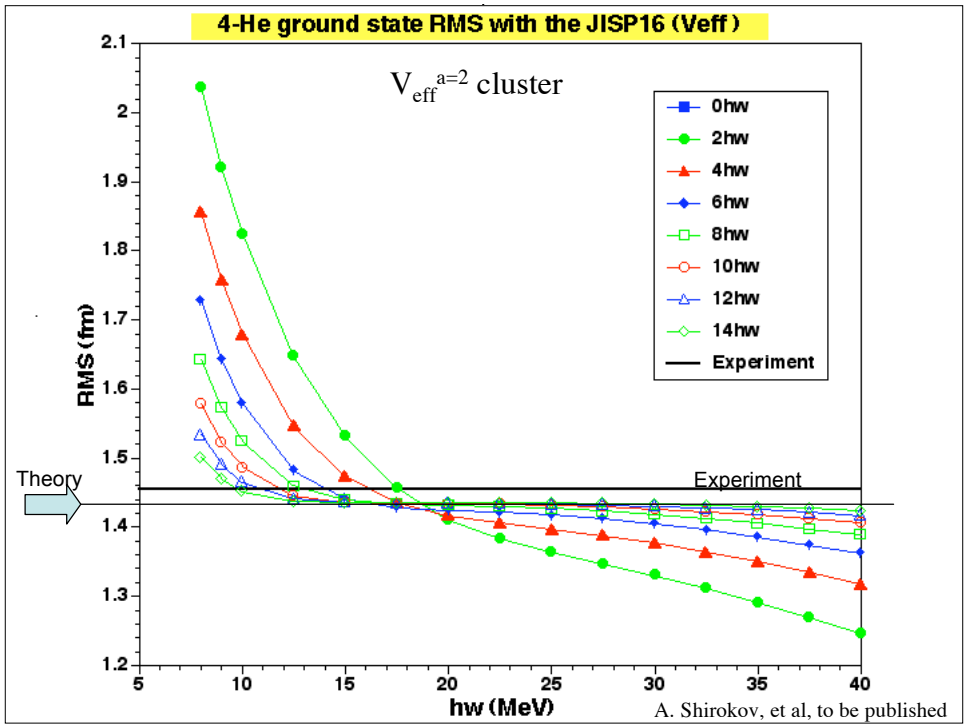
$$\langle\alpha_Q|\omega|\alpha_P\rangle = \sum_{k \in K} \langle\alpha_Q|k\rangle\langle\hat{k}|\alpha_P\rangle$$

$$where: \langle\hat{k}|\alpha_P\rangle = Inverse\{\langle k|\alpha_P\rangle\}$$

$$\mathcal{H}^{(a)} = (P_a + \omega^T\omega)^{-1/2}(P_a + P_a\omega^T Q_a)H_a^{\Omega}(Q_a\omega P_a + P_a)(P_a + \omega^T\omega)^{-1/2}$$



A. Shirokov, et al, to be published



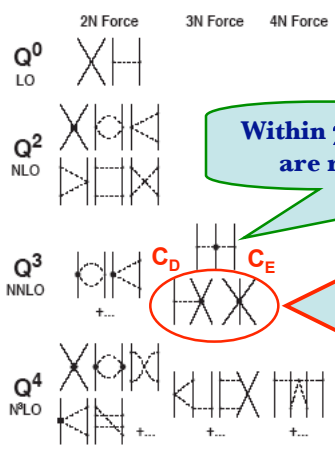
Effective Nucleon Interaction

(Chiral Perturbation Theory)

Chiral perturbation theory (χPT) allows for controlled power series expansion

Expansion parameter : $\left(\frac{Q}{\Lambda_\chi}\right)^v$, Q – momentum transfer,

$\Lambda_\chi \approx 1\ GeV$, χ - symmetry breaking scale



Within χPT 2π-NNN Low Energy Constants (LEC) are related to the NN-interaction LECs $\{c_i\}$.

Terms suggested within the Chiral Perturbation Theory

Regularization is essential, which is obvious within the Harmonic Oscillator wave function basis.

R. Machleidt, D. R. Entem, nucl-th/0503025

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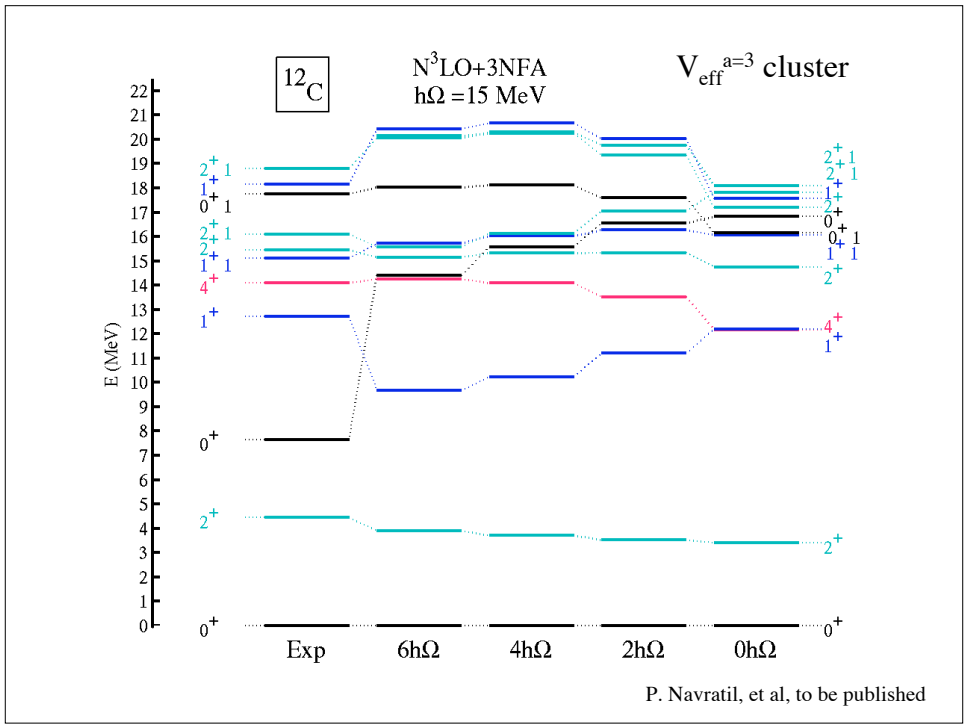
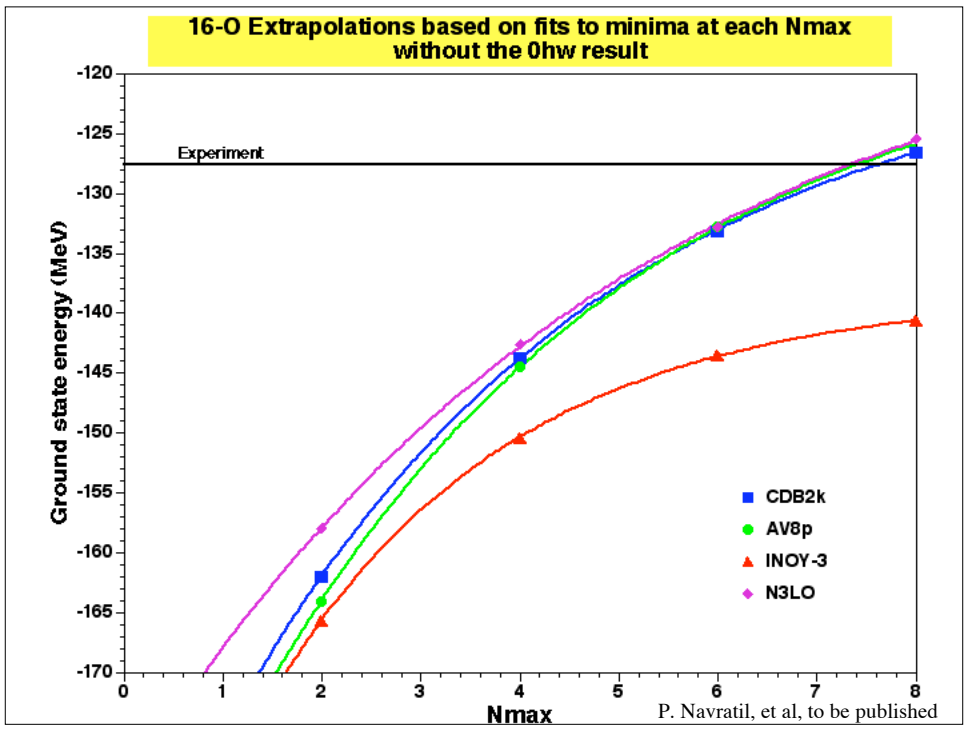
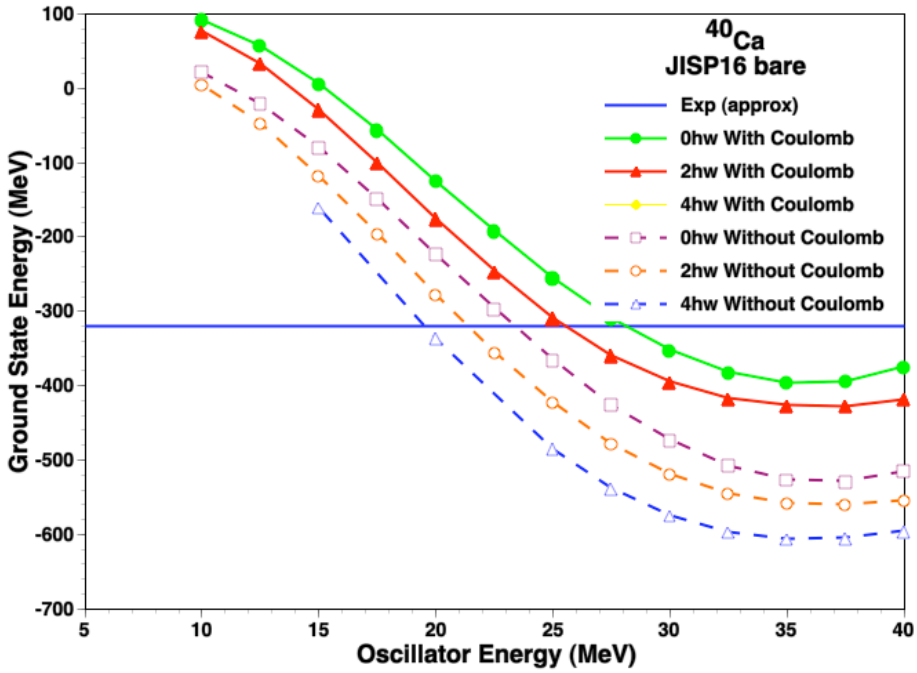

12C
N3LO+3NFA
hΩ =15 MeV
Veff a=3 cluster
E (MeV)
Exp
6hΩ
4hΩ
2hΩ
0hΩ
P. Navratil, et al, to be published


16-O Extrapolations based on fits to minima at each Nmax
without the 0hw result
Experiment
Ground state energy (MeV)
Nmax
CDB2k
AV8p
INOY-3
N3LO
P. Navratil, et al, to be published



A. Shirokov, et al, to be published

Constituent Quark Models of Exotic Mesons
R. Lloyd, PhD Thesis, ISU 2003
R. Lloyd and J. P. Vary, Phys. Rev. D 70: 014009 (2004)

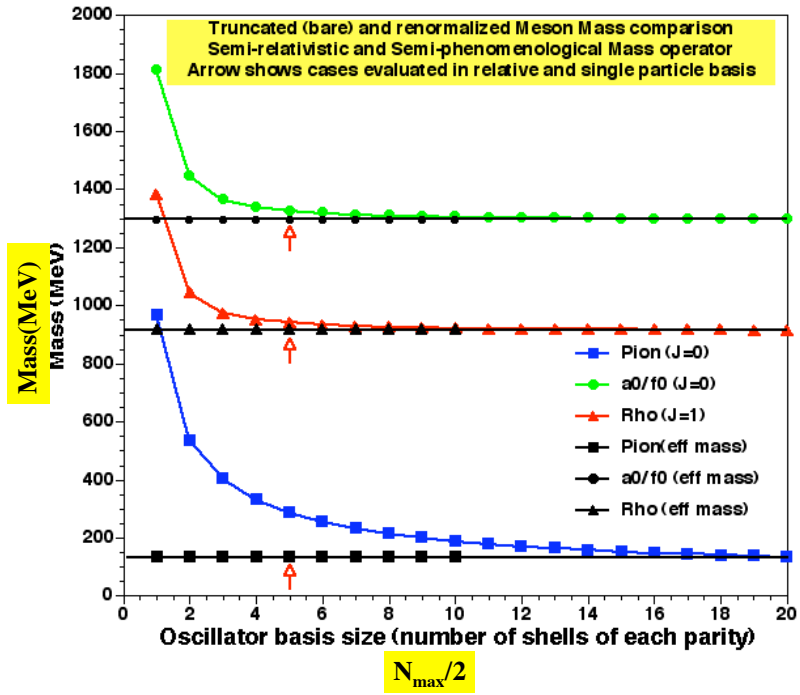
H = T + V(OGE) + V(confinement)

Solve in HO basis as a bare H problem & study dependence on cutoff

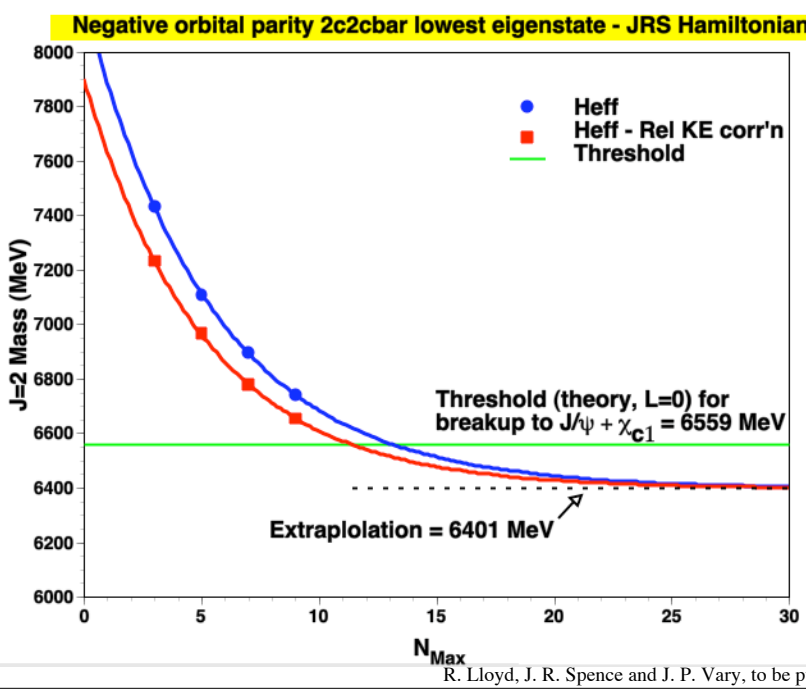
Symmetries:
Exact treatment of color degree of freedom <-- Major new accomplishment
Translational invariance preserved
Angular momentum and parity preserved

Next generation - Lee-Suzuki form (preliminary results!):
More realistic H_{eff} in equal time quantization
fit quark masses and couplings to wider range of mesons and baryons

Beyond that generation:
H_{eff} derived from QCD using light-front quantization



R. Lloyd, J. R. Spence and J. P. Vary, to be published



R. Lloyd, J. R. Spence and J. P. Vary, to be published

Φ^4 in 1+1 Dimensions

Burning issues

Demonstrate degeneracy - Spontaneous Symmetry Breaking
Topological features - soliton mass and profile (Kink, Kink-Antikink)
Quantum modes of kink excitation
Phase transition - critical coupling, critical exponent and
the physics of symmetry restoration
Role and proper treatment of the zero mode constraint
Chang's Duality

Φ^4 in 1+1 Dimensions
DLCQ with Coherent State Analysis

A. Derive the Hamiltonian and quantize it on the light front, investigate coherent state treatment of vacuum
A. Harindranath and J.P. Vary, Phys Rev D**36**, 1141(1987)

B. Obtain vacuum energy as well as the mass and profile functions of soliton-like solutions in the symmetry-broken phase:

PBC: SSB observed, Kink + Antinkink ~ coherent state!
Chakrabarti, Harindranath, Martinovic, Pivovarov and Vary, Phys. Letts. B **617**, 92 (2005); hep-th/0310290.

APBC: SSB observed, Kink ~ coherent state!
Chakrabarti, Harindranath, Martinovic and Vary, Phys. Letts. B**582**, 196 (2004); hep-th/0309263

C. Demonstrate onset of Kink Condensation:
Chakrabarti, Harindranath and Vary, Phys. Rev. D**71**, 125012(2005); hep-th/0504094

Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi + \mu^2 \phi^2) - \frac{\lambda}{4!}\phi^4$$

Hamiltonian quantized in light front coordinates with, for example, anti - periodic boundary conditions:

$$H = H_0 + H_1 + H_2$$

$$H_0 = -\mu^2 \sum_{n=1/2}^{K} \frac{1}{n} a_n^+ a_n$$

$$H_1 = \frac{\lambda}{4\pi} \sum_{k \le l, m \le n}^{K} \frac{1}{N_{kl}^2} \frac{1}{N_{mn}^2} \frac{a_k^+ a_l^+ a_m a_n}{\sqrt{klmn}} \delta_{m+n,k+l}$$

$$H_2 = \frac{\lambda}{4\pi} \sum_{k,l \le m \le n}^{K} \frac{1}{N_{lmn}^2} [\frac{a_k^+ a_l a_m a_n + a_n^+ a_m^+ a_l^+ a_k}{\sqrt{klmn}}] \delta_{k,m+n+l}$$

$$N_{kl}^2 = 1,\ k \neq l \qquad N_{lmn}^2 = 1, l \neq m \neq n$$

$$= 2!, k = l \qquad = 2!, l = m \neq n, l \neq m = n$$

$$= 3!, l = m = n$$

where all indecies are half odd integers.

Set up a normalized and symmetrized set of basis states, where, with n_k representing the number of bosons with light front momentum k:

$$P^+ = \text{K / L conserved} \Rightarrow \sum_k n_k k = K$$

$$P^- = HL \ \ \& \ H|\psi_i\rangle = E_i|\psi_i\rangle$$

Fully covariant mass - squared spectra emerges

$$P^+ P^- = M_i^2 = KE_i$$

Must extrapolate to the continuum limit, K - > ∞

At finite K, we compare with a constrained variational treatment based on the coherent state ansatz of
J.S. Rozowski and C.B. Thorn, Phys. Rev. Lett. 85, 1614 (2000).

DLCQ matrix dimension in even particle sector (APBC)

K	Dimension
16	336
32	14219
40	67243
45	165498
50	389253
55	880962
60	1928175

Light front momentum probability density in DLCQ:

$$\chi(n) = \langle K | a_n^+ a_n | K \rangle$$

Normalized:

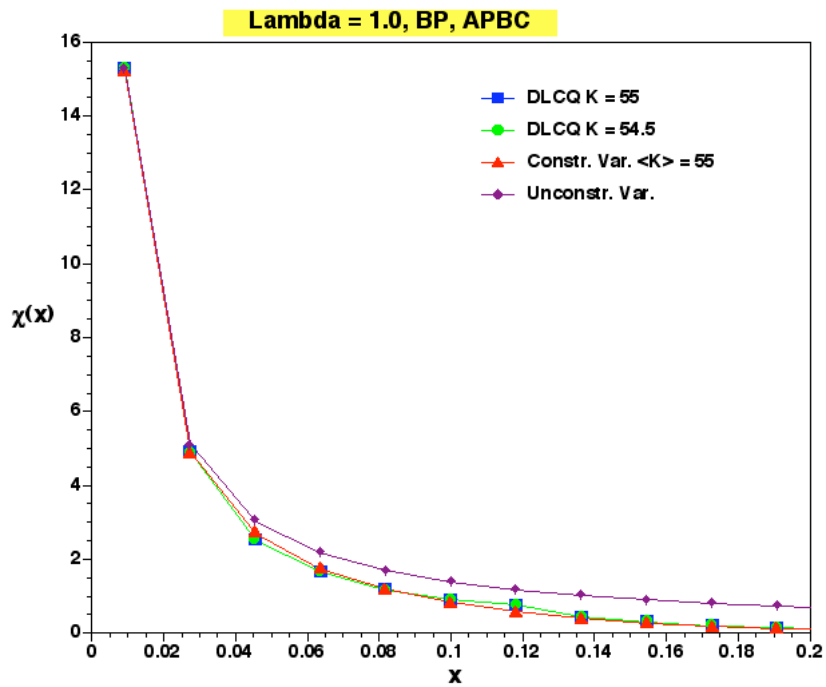
$$\sum_n n\chi(n) = K$$

Light cone momentum fraction:

$$x = n / K$$

"Bjorken x"

Compares favorably with results from a constrained variational treatment based on the coherent state



Extract the Vacuum energy density and Kink Mass

$$M_0^2 = E_{0v}K + M_{kink}^2$$
$$\Rightarrow E_0 = E_{0v} + M_{kink}^2/K$$

Note :

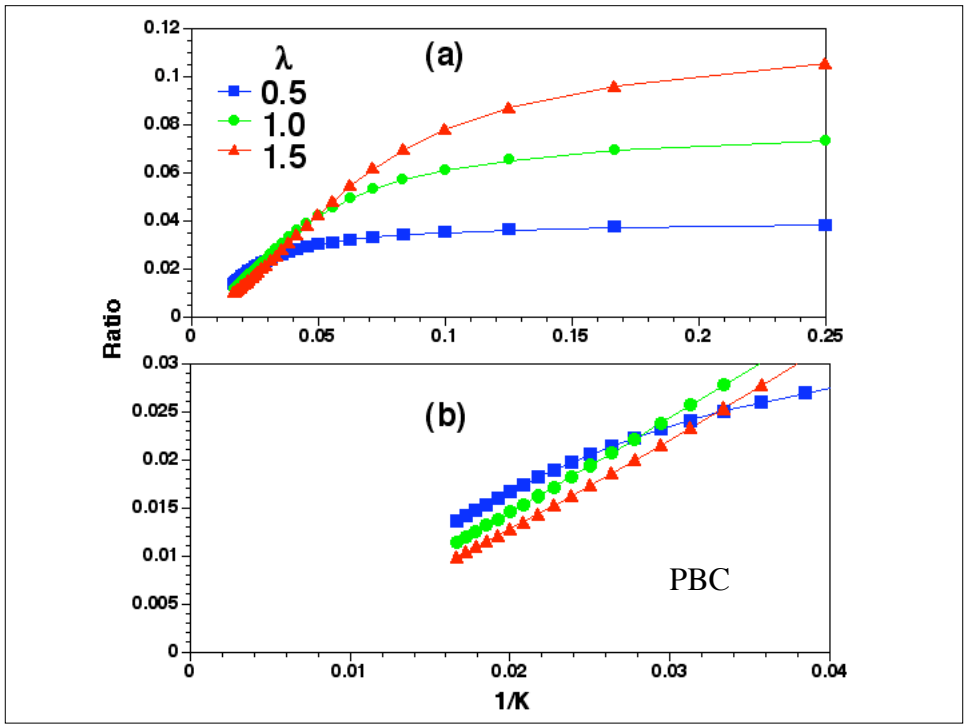
$\phi \rightarrow -\phi$ symmetry

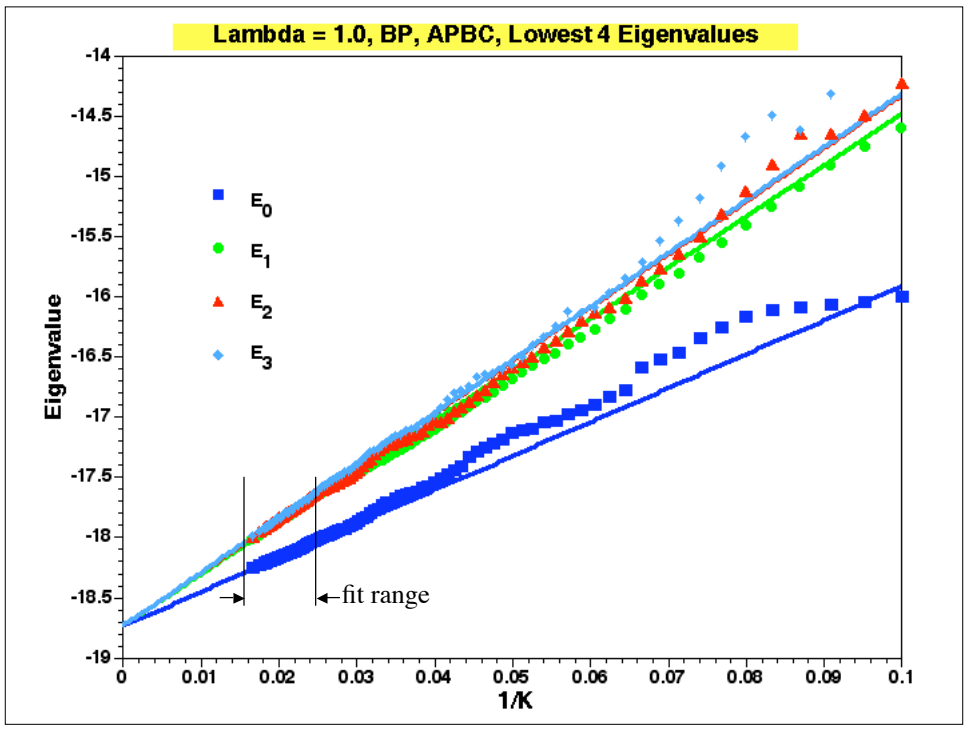
$\Rightarrow$ Decoupling of even and odd boson sectors

$\Rightarrow$ Exact degeneracy only in the K $\rightarrow \infty$ limit

All results to date are in the broken phase: $\mu^2 = -1$

Define a Ratio = $[M^2_{even} - M^2_{odd}]/$ [Vac Energy Dens]^2


λ
0.5
1.0
1.5
(a)
(b)
Ratio
PBC
1/K


Lambda = 1.0, BP, APBC, Lowest 4 Eigenvalues
E0
E1
E2
E3
Eigenvalue
fit range
1/K

Vacuum Energy

λ	Classical	DLCQ-APBC	DLCQ-PBC zero modes?
0.5	-37.70	-37.81(7)	-37.90(4)
1.0	-18.85	-18.71(5)	-18.97(2)
1.25	-15.08	-14.91(5)	15.19(5)

Soliton Mass

λ	Classical	Semi-Classical	DLCQ-APBC	DLCQ-PBC zero modes?
0.5	11.31	10.84	11.6(2)	11.26(4)
1.00	5.657	5.186	5.22(8)	5.563(7)
1.25	4.526	4.054	4.07(6)	4.43(4)

Fourier Transform of the Soliton (Kink) Form Factor
Ref: Goldstone and Jackiw

$$\phi_c(x^- - a) = \int_{-\infty}^{\infty} dq^+ \exp\left\{-\frac{i}{2} q^+ a\right\} \langle K + q^+ | \Phi(x^-) | K \rangle$$

Evaluated within DLCQ choosing $a = 0$:

$$= \sum_{l=\frac{1}{2},\frac{3}{2},\frac{5}{2},\ldots}^{\infty} \frac{1}{\sqrt{4\pi l}} \langle K \pm l | \left\{ a_l e^{-l\pi \frac{x^-}{L}} + a_l^+ e^{l\pi \frac{x^-}{L}} \right\} | K \rangle$$

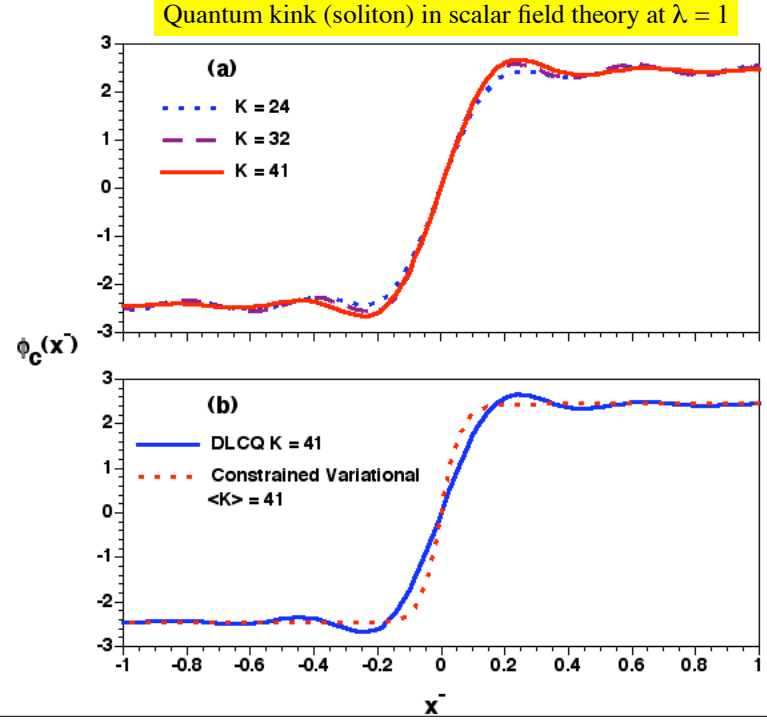
Note: Issue of the relative phases - fix arbitrary phases via guidance of the the coherent state analysis:

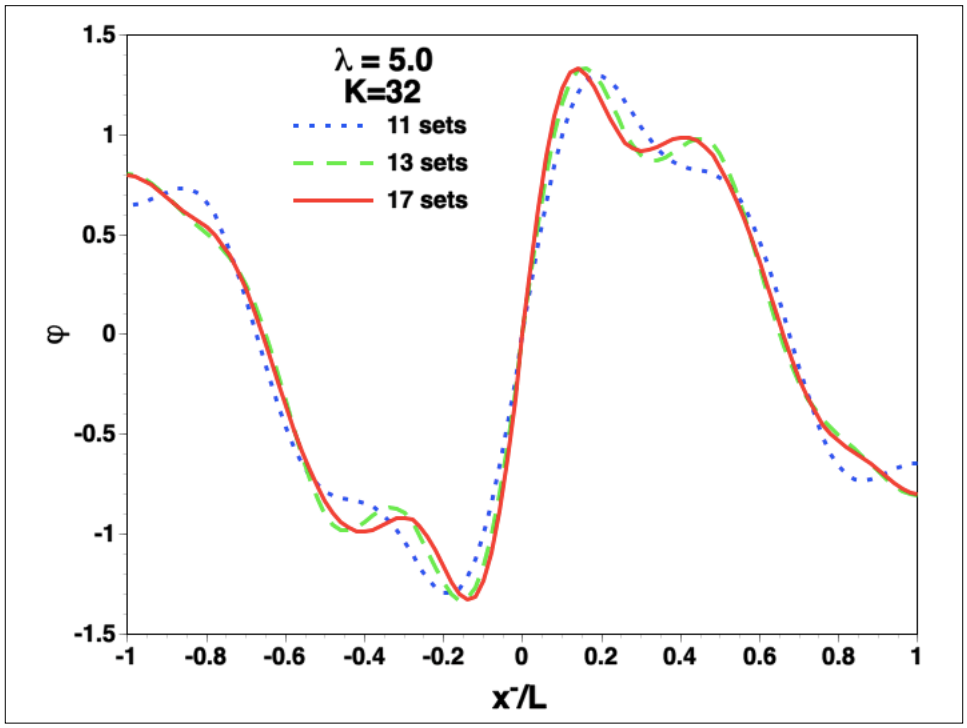
$$\mathbf{Sign}\left\{\langle K + l | a_l^+ | K \rangle\right\} = +$$

$$\mathbf{Sign}\left\{\langle K - l | a_l | K \rangle\right\} = -$$

Cancellation of imaginary terms and vacuum expectation value, $\langle \phi \rangle$, are non-trivial tests of resulting kink structure.

Quantum kink (soliton) in scalar field theory at λ = 1





What is the nature of this phase transition?

We observe dramatic changes in:

(1) Mass spectroscopy
(2) Form factor -> Kink condensation signal
(3) Parton distributions

Non-trivial achievements in the Hamiltonian framework

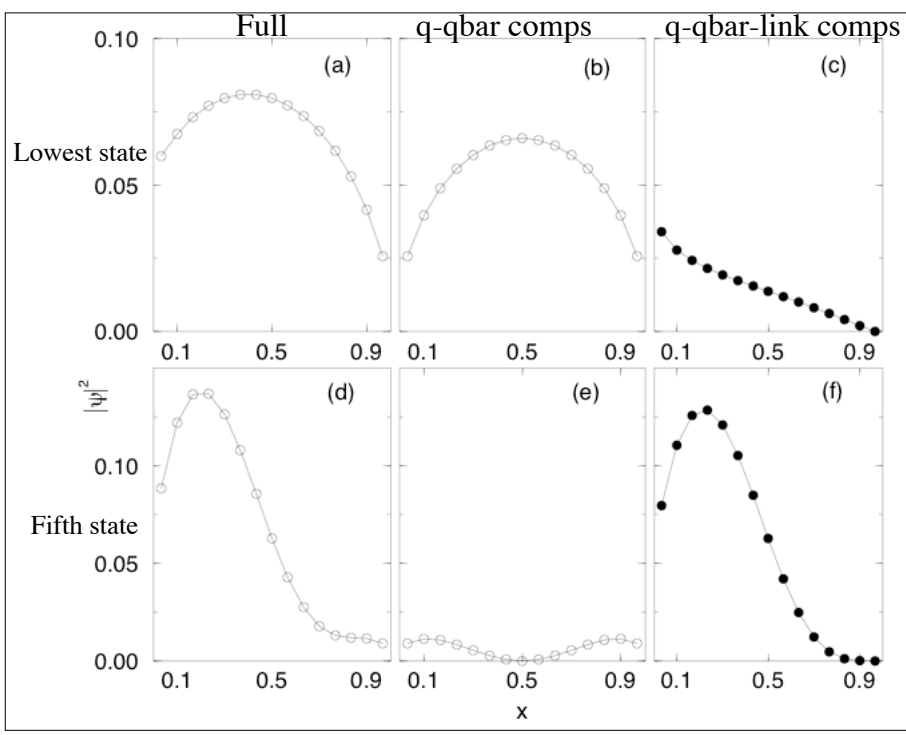
QCD applications in the $q\bar{q}$ + $q\bar{q}$ -link approximation for mesons

DLCQ for longitudinal modes and a transverse momentum lattice

- Adopt QCD Hamiltonian of Bardeen, Pearson and Rabinovici
- Restrict the P-space to q-qbar and q-qbar-link configurations
- Does not follow the effective operator approach (yet)
- Introduce regulators as needed to obtain cutoff-independent spectra

S. Dalley and B. van de Sande,
Phys. Rev. D67, 114507 (2003); hep-ph/0212086

D. Chakrabarti, A. Harindranath and J.P. Vary,
Phys. Rev. D69, 034502 (2004); hep-ph/0309317



Codes and their Applications

- MFDn V10-b3 NN+NNN interactions in the Ab-initio No-Core Shell Model (NCSM)
- MFDq V9-b5 multi-Q + multi-Q systems in equal-time 3D harmonic oscillator basis with color and multiple flavors
- MFBD V9-b5 scalar field theory in 1+1 dimensions
- MFDMC V9-b5 transverse lattice QCD (QQ + QQlink)
- MFBDQ V10-b3 QCD in light-cone quantization for mesons & baryons under development (details follow)

Core technology - Lanczos algorithm on parallel processors

Many-Fermion Dynamics (MFD)
General Purpose F90/MPI Code

- Accurate - verified using 3 independent codes
- Functional - Spectra and other observables
- Flexible - Alternative many-Fermion system apps
- Parallel - Multi-processor enabled
- Scalable - Behavior with increase in processors
- Balanced - all processors equally loaded
- Robust/stable - accurate in single precision
- User-friendly - Documented, several groups use it
- Portable - SGI-Origin, DEC workstation clusters, IBM
- Restartable - 10 run/restart modes - key to large scale apps
- Efficient - both in speed, disk space and RAM
- Productive - employed in multiple investigations

Overview of MFBDQ Development

status report - May 19, 2006

Selected/implemented/tested

- Degrees of Freedom (identical particle statistics, coordinates, spin, flavor, color)
- Basis space(s)
- Symmetries
- Lanczos algorithm

Under development/implementation/testing

- Hamiltonian vertex matrix elements of Light-Front QCD

Selection of degrees of freedom and the basis functions for solving baryons

Quarks and Gluons

Multiple Flavors

SU(3) Color

Spin and Helicity

Longitudinal modes of DLCQ

Transverse modes of

2D HO or AdS/CFT holography

Initial Application to Baryons

- Longitudinal DLCQ basis
- Transverse 2D HO or AdS/CFT basis
- Two Flavors (u,d) and limited to one gluon
- SU(3) color
- Simplified H_{QCD} -> progressing to greater realism

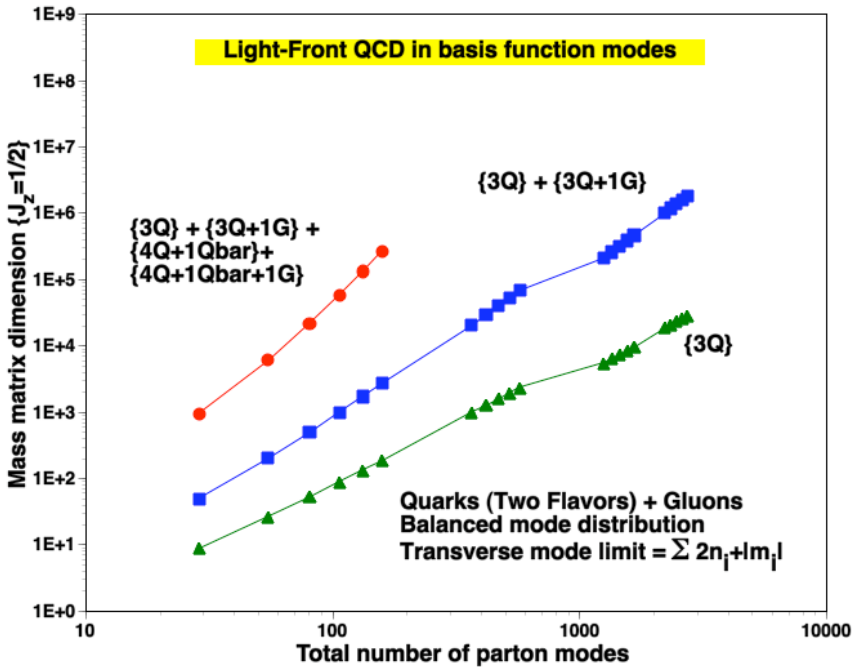
Symmetries and Constraints

Explicit but flexible symmetries/constraints
Flexible => all but first are input parameters

- Identical particle statistics of Fermions and Bosons
- Total baryon number = 1
- Total charge = 1
- Total SU(3) color projection = 0
- Total angular momentum projection $J_z = 1/2$
- Total number of q-qbar pairs le 1
- Total number of gluons le 1

Symmetries/constraints via Lagrange method

- Total transverse momentum eliminated by exact factorization of the light-front wavefunction
- Total color-singlet states isolated



Conclusions

- Similarity of "two-scale" problems in Nuclear and Particle Physics
- Ab-initio theory is a convergent exact method for solving many-particle Hamiltonians
- Method has been demonstrated as exact in the Nuclear Physics applications
- Quasi-exact results for 1+1 scalar field theory obtained
- Non-perturbative vacuum expectation value (order parameter)
- Kink mass & profile obtained
- Critical properties (coupling, exponent) emerging
- Evidence of Kink condensation obtained
- Sensitivity to boundary conditions needs further study
- Role of zero modes yet to be fully clarified
- Initial applications to constituent quark models (equal time) - novel predictions
- Initial applications to QCD (3 + 1) on transverse lattice proved encouraging
- Project underway to implement QCD (3 + 1) for baryons with Light Front Basis Functions
- Advent of parallel computing has made new physics domains accessible:
  algorithm improvements have achieved fully scalable and load-balanced codes.

"…I will sum up by saying that light-front QCD is not for the faint of heart, but for a few good candidates it is a chance to be a leader in a much smaller community of researchers than one faces in the other areas of high-energy physics, with, I believe, unusual promise for interesting and unexpected results."

K.G. Wilson, "The Origins of Lattice Gauge Theory," Nuclear Physics B, Suppl., **140** (2005) p3

Community Effort?
"Light Front QCD Collaboration"
Facilitates entry of new researchers - eg grad students

Should we emulate the lattice QCD community and organize a light-front community to co-develop and share via web:

- Computer codes
- Light-front amplitudes (data files)
- Manuals/tutorials/codes for their use
- Computer cycles (a "light-front grid")