Beyond the Ladder

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Background/Aims

- Fully covariant Bethe-Salpeter description of bound states
- Rarely does one go beyond ladder approximation
- How good or bad is the ladder approximation?
- Need to study the ladder truncation of a more complete solution
- Employ a simple model that can be solved to high order and give some insight
- Realistic enough to have DCSB, manifest covariance, quark confinement and good meson masses
- Utilize Munczek-Nemirovsky kernel (integral eqns $\Rightarrow$ algebraic eqns)
$K_{\text{BSE}} \rightarrow -\gamma_\mu \frac{\lambda^a}{2} \ 4\pi \alpha_{\text{eff}}(q^2) \ D^\text{free}_{\mu\nu}(q) \ \gamma_\nu \frac{\lambda^a}{2}$

$\alpha_{\text{eff}}(q^2) \xrightarrow{\text{UV}} \alpha_s^{1-\text{loop}}(q^2)$

$\alpha_{\text{eff}}(q^2) \xrightarrow{\text{IR}} \langle \bar{q}q \rangle_{\mu=1} \text{GeV} = -(240\text{MeV})^3$

Above maps vertex dressing in with effective 2-point fn
From Gluon vertex to BSE Kernel

- \( K_{\text{BSE}}(x', y'; x, y) = -\frac{\delta}{\delta S(x, y)} \Sigma(x', y') \)

- Vertex \( \Gamma_\mu(p, q) = \sum \text{diagrams} \Rightarrow K_{\text{BSE}} = \sum \text{diagrams} \)

- If \( \Sigma \) contains:

- \( K_{\text{BSE}} \) contains:

- Axial vector and vector WTIs (color singlet) preserved, if all q props are self-consistently dressed (Munczek 95); Goldstone Thm preserved

- Independent of model parameters. Model does not fight chiral symmetry, use light vector mesons to fix parameters
Algebraic model

- Effective gluon line (Munczek-Nemirovsky 95)
  \[ g^2 D_{\mu\nu}(k) \rightarrow \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) (2\pi)^4 G^2 \delta^4(k) \]

- One IR parameter \( G \sim 0.5 \text{ GeV} \ (m_\rho) \)

- Interactions conserve quark momentum \( \Rightarrow \) DSE and BSE are algebraic

- Eqns are UV finite, all \( Z_i = 1 \)

- UV is mis-represented, we focus upon IR-dominated physics (meson masses)
**Algebraic Vertex: An earlier attempt**

- Note: a ladder sum for gluon vertex is an algebraic recursive relation
  \[(i+1) = (i)\]

  Implemented: Bender, Detmold, Roberts, Thomas, PRC, nucl-th/0202082

- A rare BSE study of mesons (and diquarks) with \(\infty\) diagrams in \(K_{BSE}\)

- Is ladder-summed gluon vertex realistic?
  Yes

- Lattice vertex data and 1-loop QCD analysis \(\Rightarrow No\)
DSE Model for $k = 0$ gluon-quark vertex

\[
\Gamma_\nu(p; 0) = \gamma_\nu \lambda_1(p) - 4p_\nu \not{p} \lambda_2(p) - 2ip_\nu \lambda_3(p)
\]

WI: $\lambda_1 \sim A$, $\lambda_2 \sim -A'/2$, $\lambda_3 \sim B'$
Dressed gluon-quark vertex: 1-loop pQCD

\[ (C_F - \frac{C_A}{2}) = - \frac{1}{2N_c} \]
(weak repulsive)

\[ \frac{C_A}{2} = \frac{N_c}{2} \]
(v. strong attractive)

Satisfies Slavnov-Taylor Id to \( O(g^3) \)

\[ ik \nu \Gamma_\nu = G(k^2) \left[ (1 + B) S^{-1}(p_+) - S^{-1}(p_-) (1 + \tilde{B}) \right] \]

Color singlet vector channel: \( C_F = (N_c^2 - 1)/2N_c \) (strong attraction)

Both in effective model: \( C \sim C_F \quad -1/8 < C < 1 \)
Algebraic Vertex and BSE—incl eff 3-gluon coupling

- Enters quark-gluon vertex and $K_{\text{BSE}}$, preserves chiral symmetry
- Implemented in $\text{DSE}_q$ and meson BSE via (algebraic) MN model
- nucl-th/0403012, Bhagwat, Höll, Krassnigg, Roberts, PCT
- cf Ladder-rainbow: 30% reduction in $M_V$
  minor change in $M_{PS}$
Include eff 3-gluon coupling plus self-cons verts

- Previously: \( n + 1 \) vertex diagrams with \( n \) gluon lines
- Now all vertices self-consistent: \( 1 + n(n + 1)(n + 2)/6 \) vertex diagrams having \( n \) gluon lines

H. Matevosyan, A.W. Thomas, PCT 2006

For example
**Algebraic Eqns**

- **Vertex**
  \[
  \Gamma_{\mu}(p) = \gamma_{\mu} - CG^2 \Gamma_{\sigma}(p) S(p) \Gamma_{\mu}(p) S(p) \Gamma_{\sigma}(p)
  \]

- **Vertex form**
  \[
  \Gamma_{\mu}(p) = \alpha_1(p^2) \gamma_{\mu} + \alpha_2(p^2) \gamma \cdot p_{\mu} - \alpha_3(p^2) i p_{\mu} + \alpha_4(p^2) i \gamma_{\mu} \gamma \cdot p
  \]

- **Quark propagator**
  \[
  S^{-1}(p) = i \gamma \cdot p + m + G^2 \gamma_{\mu} S(p) \Gamma_{\mu}(p)
  \]

- **BSE**
  \[
  \Gamma_{M}(P) = -G^2 \gamma_{\mu} S(P) \left\{ \Gamma_{M}(P) S\left(-\frac{P}{2}\right) \Gamma_{\mu}\left(-\frac{P}{2}\right) + \Lambda_{M\mu}(0; P) \right\}
  \]

**With full model solution**

- Fit \( C \) to lattice q propagator \( A(0), B(0) \) and lattice vertex ampls \( \alpha_i(0) \)

- Fit \( \mathcal{G}, m \) to \( m_\rho, m_\pi \)

- Examine truncations: ladder-rainbow \( (n = 0) \), 1-loop \( (n = 1) \), etc
**Vertex dressing effect on** $m_\pi$, $m_\rho$

<table>
<thead>
<tr>
<th>Vertex Dressing</th>
<th>$m_\pi$</th>
<th>$m_\rho$</th>
<th>$\Delta m_\rho$</th>
<th>$\frac{\Delta m_\rho}{m_\rho}$</th>
<th>$\frac{\Delta m_\rho}{m_\rho}$ prev</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 0$ (LR)</td>
<td>0.140</td>
<td>0.850</td>
<td>+0.074</td>
<td>+0.095</td>
<td>+0.295</td>
</tr>
<tr>
<td>$n = 1$ (1-loop)</td>
<td>0.135</td>
<td>0.759</td>
<td>-0.017</td>
<td>-0.022</td>
<td>——</td>
</tr>
<tr>
<td>$n = 2$</td>
<td>0.135</td>
<td>0.781</td>
<td>+0.005</td>
<td>+0.006</td>
<td>+0.096</td>
</tr>
<tr>
<td>$n = 3$</td>
<td>0.135</td>
<td>0.772</td>
<td>-0.004</td>
<td>-0.005</td>
<td>N/A</td>
</tr>
<tr>
<td>$n = 4$</td>
<td>0.135</td>
<td>0.778</td>
<td>+0.002</td>
<td>+0.003</td>
<td>N/A</td>
</tr>
<tr>
<td>$n = \infty$ (full model)</td>
<td>0.135</td>
<td>0.776</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
## Accuracy of ladder-rainbow vs quark mass

<table>
<thead>
<tr>
<th></th>
<th>ladder-rainbow</th>
<th>full model</th>
<th>LR % error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n = 0$</td>
<td>$n = \infty$</td>
<td>this model</td>
</tr>
<tr>
<td>$m_{u,d} = 0.011$</td>
<td>0.850</td>
<td>0.776</td>
<td>9.5%</td>
</tr>
<tr>
<td>$m_\rho$</td>
<td>0.346</td>
<td>0.311</td>
<td>11%</td>
</tr>
<tr>
<td>$m_\phi$</td>
<td>1.08</td>
<td>1.02</td>
<td>6.0%</td>
</tr>
<tr>
<td>$m_c = 1.35$</td>
<td>3.11</td>
<td>3.09</td>
<td>0.3%</td>
</tr>
<tr>
<td>$m_{J/\psi}$</td>
<td>0.260</td>
<td>0.260</td>
<td>0%</td>
</tr>
<tr>
<td>$m_b = 4.64$</td>
<td>9.46</td>
<td>9.46</td>
<td>0%</td>
</tr>
<tr>
<td>$m_{\Upsilon}$</td>
<td>0.100</td>
<td>0.100</td>
<td>0%</td>
</tr>
</tbody>
</table>
### Summary: vector and pseudoscalars

From full model solutions:

<p>| | | | | |</p>
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<tr>
<td>$m_{u,d}$</td>
<td>0.011</td>
<td>$m_s$</td>
<td>0.165</td>
<td>$m_c$</td>
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<tr>
<td>$m_\rho$</td>
<td>0.776</td>
<td>$m_\phi$</td>
<td>1.02</td>
<td>$m_{J/\psi}$</td>
</tr>
<tr>
<td>$\mathcal{B}\varepsilon_\rho$</td>
<td>0.311</td>
<td>$\mathcal{B}\varepsilon_\phi$</td>
<td>0.320</td>
<td>$\mathcal{B}\varepsilon_{J/\psi}$</td>
</tr>
<tr>
<td>$m_\pi$</td>
<td>0.135</td>
<td>$m_{0_{s\bar{s}}}$</td>
<td>0.61</td>
<td>$m_{\eta_c}$</td>
</tr>
<tr>
<td>$\mathcal{B}\varepsilon_\pi$</td>
<td>0.953</td>
<td>$\mathcal{B}\varepsilon_{0^{-}}$</td>
<td>0.727</td>
<td>$\mathcal{B}\varepsilon_{\eta_c}$</td>
</tr>
</tbody>
</table>
Summary

- Vertex dressing dominated by 3-gluon coupling
- Here we include self-consistent dressing on all available vertices in an algebraic model
- Observe cancellation among different orders of dressing
- Chiral symmetry-preserving $K_{\text{BSE}}$ constructed
- Compared to full solution, ladder-rainbow truncation gives 10% error for $m_\rho$ decreasing to $< 1\%$ for $J/\psi$ and $\Upsilon$
- Limitations: no scalar or axial vector solutions, omitted non-planar vertex diagrams and 4-gluon coupling
- Still a rare BSE solution with $\infty$ diagrams in $K_{\text{BSE}}$
- Can one produce $K_{\text{BSE}}$ directly from a phenomenological gluon-quark vertex?