Beyond the Ladder

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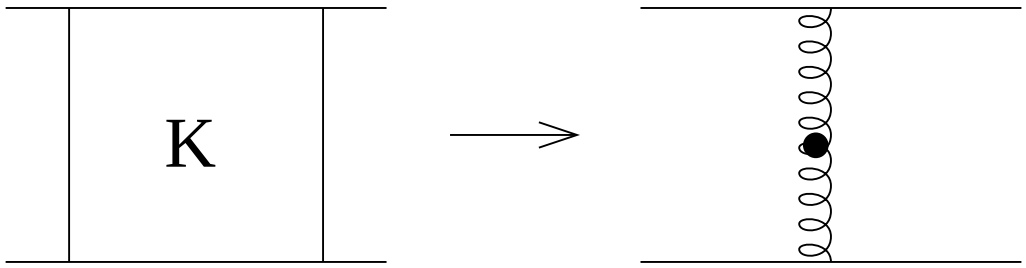
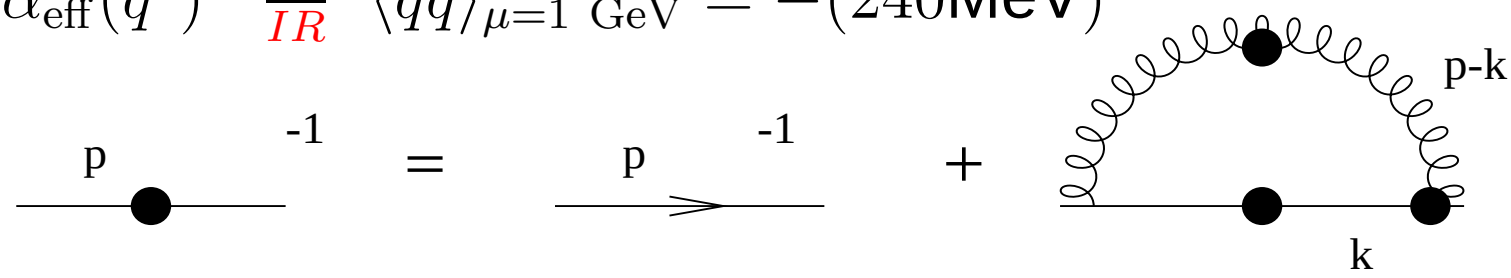
Background/Aims

- Fully covariant Bethe-Salpeter description of bound states
- Rarely does one go beyond ladder approximation
- How good or bad is the ladder approximation?
- Need to study the ladder truncation of a more complete solution
- Employ a simple model that can be solved to high order and give some insight
- Realistic enough to have DCSB, manifest covariance, quark confinement and good meson masses
- Utilize Munczek-Nemirovsky kernel (integral eqns $\Rightarrow$ algebraic eqns)

NOT this Ladder-Rainbow Model



- $K_{\rm BSE} \to -\gamma_\mu \frac{\lambda^a}{2} \; 4\pi\alpha_{\rm eff}(q^2) \; D^{\rm free}_{\mu\nu}(q) \; \gamma_\nu \frac{\lambda^a}{2}$
- $\alpha_{\rm eff}(q^2) \underset{UV}{\to} \alpha_s^{1-\rm loop}(q^2)$
- $\alpha_{\rm eff}(q^2) \underset{IR}{\to} \langle \bar{q}q \rangle_{\mu=1\ \rm GeV} = -(240\text{MeV})^3$

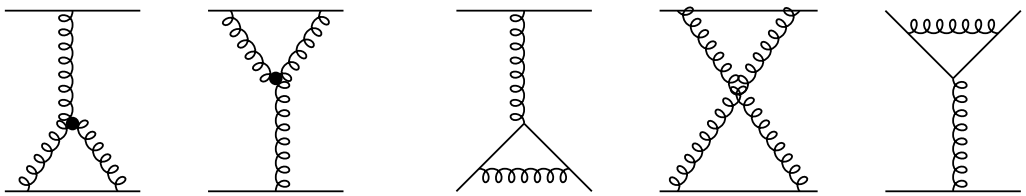


- Above maps vertex dressing in with effective 2-point fn

From Gluon vertex to BSE Kernel

- $K_{\rm BSE}(x',y';x,y) = -\frac{\delta}{\delta S(x,y)}\Sigma(x',y')$
- Vertex $\Gamma_\mu(p,q) = \sum$ diagrams $\Rightarrow K_{\rm BSE} = \sum$ diagrams
- If Σ contains:

- $K_{\rm BSE}$ contains:



- Axial vector and vector WTIs (color singlet) preserved, if all q props are self-consistently dressed (Munczek 95); Goldstone Thm preserved
- Independent of model parameters. Model does not fight chiral symmetry, use light vector mesons to fix parameters

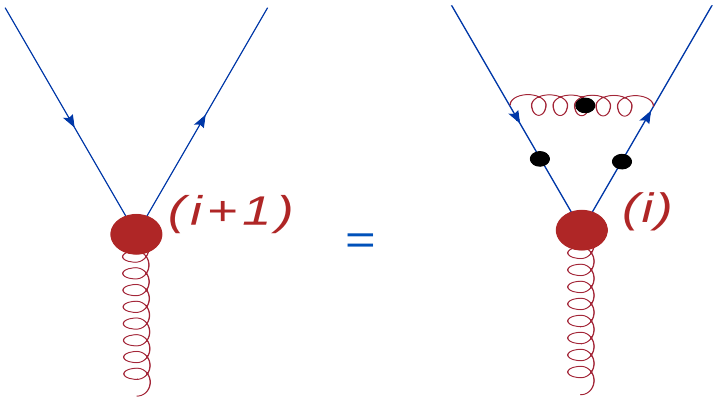
Algebraic model

- Effective gluon line (Munczek-Nemirovsky 95)
  $g^2 D_{\mu\nu}(k) \to \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)(2\pi)^4 \mathcal{G}^2 \delta^4(k)$
- One IR parameter $\mathcal{G} \sim 0.5$ GeV (m_ρ)
- Interactions conserve quark momentum $\Rightarrow$ DSE and BSE are algebraic
- Eqns are UV finite, all $Z_i = 1$
- UV is mis-represented, we focus upon IR-dominated physics (meson masses)

Algebraic Vertex: An earlier attempt

- Note: a ladder sum for gluon vertex is an algebraic recursive relation

Implemented: Bender, Detmold, Roberts, Thomas, PRC, nucl-th/0202082

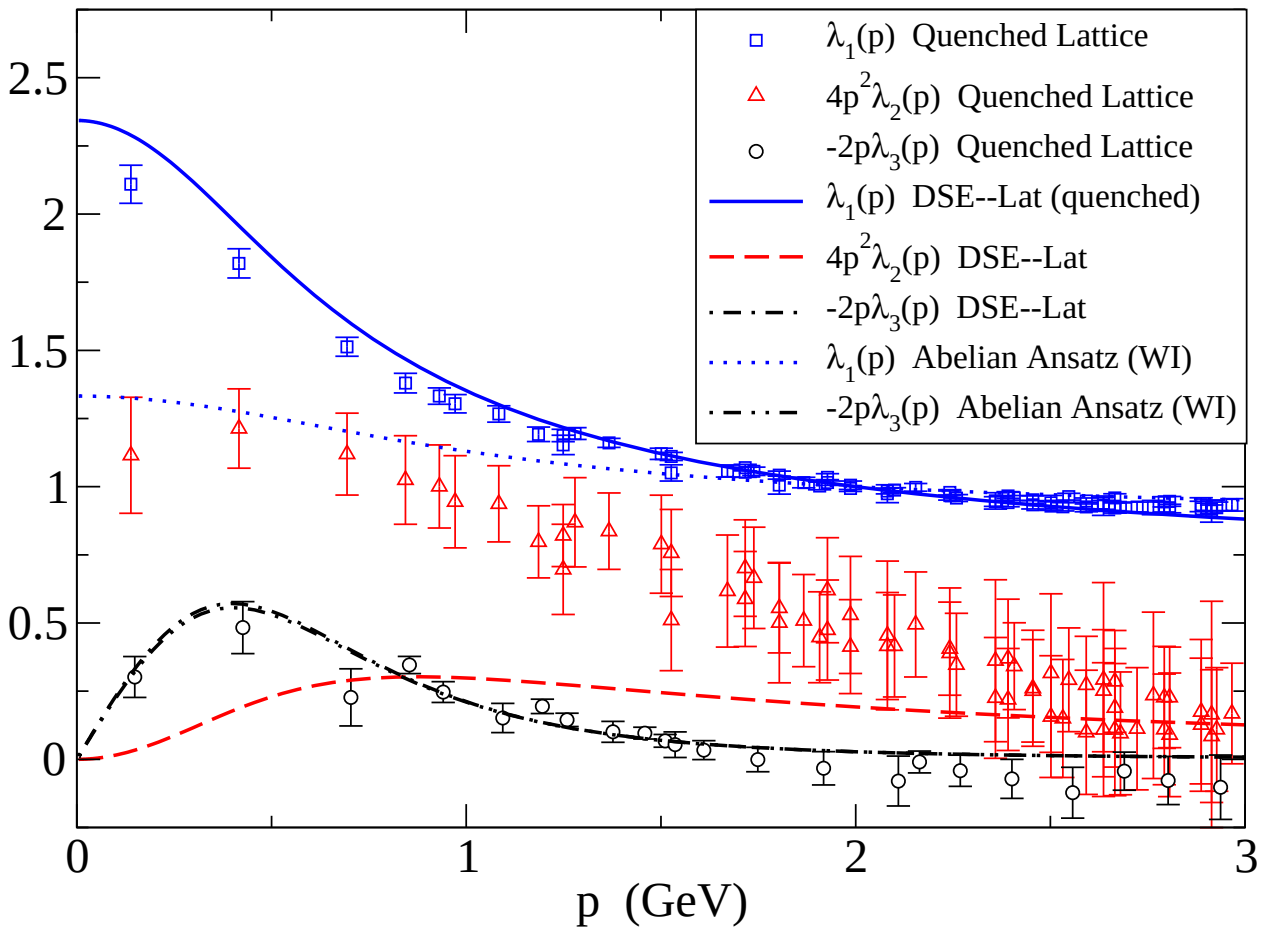


- A rare BSE study of mesons (and diquarks) with ∞ diagrams in $K_{\rm BSE}$
- Is ladder-summed gluon vertex realistic ?
- Lattice vertex data and 1-loop QCD analysis $\Rightarrow$ No

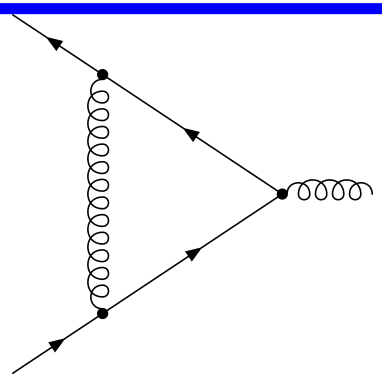
DSE Model for $k = 0$ gluon-quark vertex

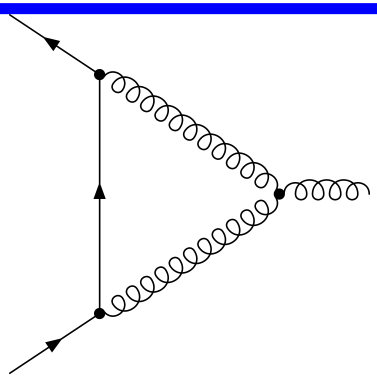
$$\Gamma_\nu(p;0) = \gamma_\nu\, \lambda_1(p) - 4p_\nu\, \not{p}\lambda_2(p) - 2ip_\nu\lambda_3(p)$$

WI: $\lambda_1 \sim A, \quad \lambda_2 \sim -A'/2, \quad \lambda_3 \sim B'$



Dressed gluon-quark vertex: 1-loop pQCD





$$(C_{\rm F} - \frac{C_{\rm A}}{2}) = -\frac{1}{2N_c} \qquad \frac{C_{\rm A}}{2} = \frac{N_c}{2}$$

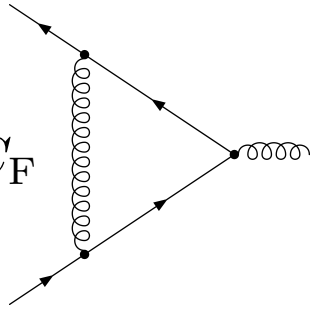
(weak repulsive) (v. strong attractive)

- Satisfies Slavnov-Taylor Id to $\mathcal{O}(g^3)$

  $ik_\nu \Gamma_\nu = G(k^2)[(1+B)\, S^{-1}(p_+) - S^{-1}(p_-)\,(1+\tilde{B})]$

- Color singlet vector channel: $C_{\rm F} = (N_c^2 - 1)/2N_c$ (strong attraction)

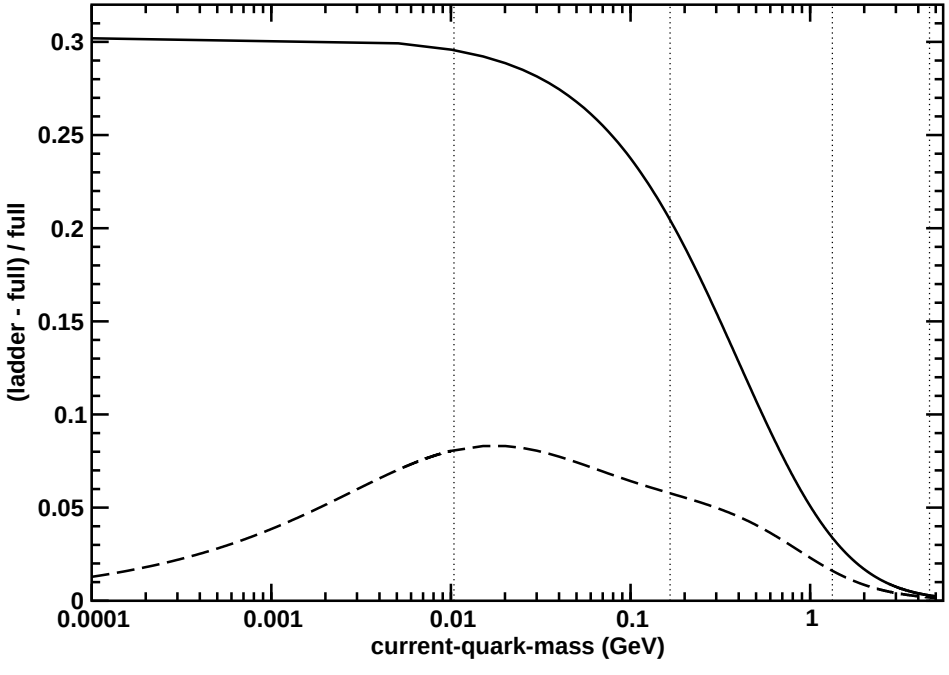
- Both in effective model: $\mathcal{C}\, C_{\rm F}$



$-1/8 < \mathcal{C} < 1$

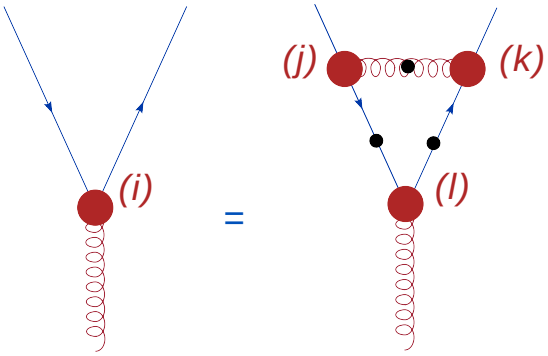
Algebraic Vertex and BSE—incl eff 3-gluon coupling

- Enters quark-gluon vertex and $K_{\rm BSE}$, preserves chiral symmetry
- Implemented in DSE$_q$ and meson BSE via (algebraic) MN model
- nucl-th/0403012, Bhagwat, Höll, Krassnigg, Roberts, PCT
- cf Ladder-rainbow: 30% reduction in $M_{\rm V}$ minor change in $M_{\rm PS}$

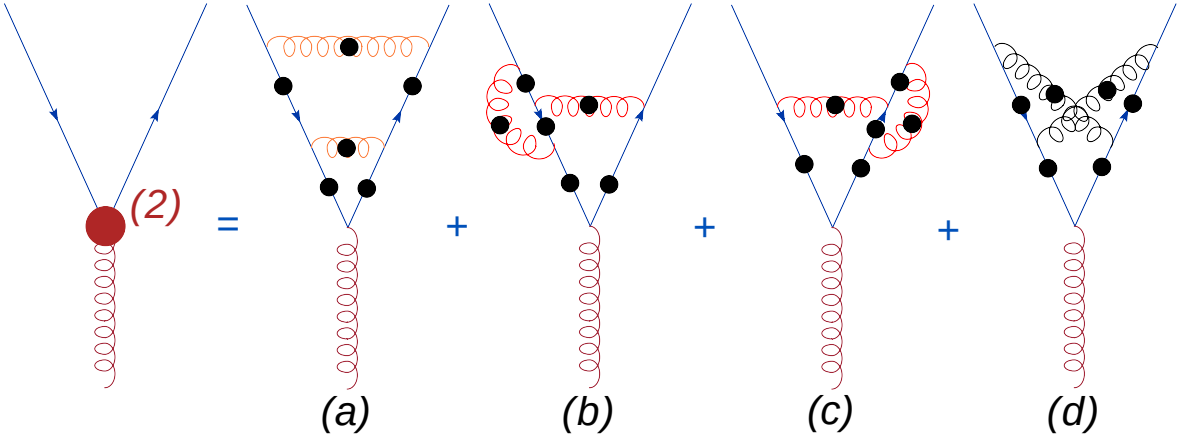


Include eff 3-gluon coupling plus self-cons vertices

- Previously: $n+1$ vertex diagrams with n gluon lines
- Now all vertices self-consistent: $1+n(n+1)(n+2)/6$ vertex diagrams having n gluon lines
- H. Matevosyan, A.W.Thomas, PCT 2006



- For example



Algebraic Eqns

- Vertex $\Gamma_\mu(p) = \gamma_\mu - \mathcal{C}\mathcal{G}^2\Gamma_\sigma(p)S(p)\Gamma_\mu(p)S(p)\Gamma_\sigma(p)$
- Vertex form
  $\Gamma_\mu(p) = \alpha_1(p^2)\gamma_\mu + \alpha_2(p^2)\gamma\cdot\ p_\mu - \alpha_3(p^2)ip_\mu + \alpha_4(p^2)i\gamma_\mu\ \gamma\cdot p$
- $\Gamma_\mu(p) = \sum_n \Gamma^n_\mu(p), \quad n =$ order of dressing
- Quark propagator $S^{-1}(p) = i\gamma\cdot p + m + \mathcal{G}^2\gamma_\mu S(p)\Gamma_\mu(p)$
- BSE $\Gamma_M(P) = -\mathcal{G}^2\,\gamma_\mu S(\frac{P}{2})\left\{\Gamma_M(P)\,S(-\frac{P}{2})\,\Gamma_\mu(-\frac{P}{2}) + \Lambda_{M\mu}(0;P)\right\}$
- With full model solution
  - Fit $\mathcal{C}$ to lattice q propagator $A(0), B(0)$ and lattice vertex ampls $\alpha_i(0)$
  - Fit $\mathcal{G}, m$ to m_ρ, m_π
- Examine truncations: ladder-rainbow $(n=0)$, 1-loop $(n=1)$, etc

Vertex dressing effect on $m_\pi, \quad m_\rho$

Vertex Dressing	m_π	m_ρ	Δm_ρ	$\frac{\Delta m_\rho}{m_\rho}$	$\frac{\Delta m_\rho}{m_\rho}$ prev
$n = 0$ (LR)	0.140	0.850	+0.074	+0.095	+0.295
$n = 1$ (1-loop)	0.135	0.759	-0.017	-0.022	—
$n = 2$	0.135	0.781	+0.005	+0.006	+0.096
$n = 3$	0.135	0.772	-0.004	-0.005	N/A
$n = 4$	0.135	0.778	+0.002	+0.003	N/A
$n = \infty$ (full model)	0.135	0.776	0.0	0.0	0.0

Accuracy of ladder-rainbow vs quark mass

	ladder-rainbow	full model	LR % error	
	$n = 0$	$n = \infty$	this model	prev
$m_{u,d} = 0.011$				
m_ρ	0.850	0.776	9.5%	30%
$\mathcal{BE}_\rho$	0.346	0.311	11%	
$m_s = 0.165$				
m_ϕ	1.08	1.02	6.0%	21%
$\mathcal{BE}_\phi$	0.350	0.320	9.0%	
$m_c = 1.35$				
$m_{J/\psi}$	3.11	3.09	0.3%	3.5%
$\mathcal{BE}_{J/\psi}$	0.260	0.260	0%	
$m_b = 4.64$				
m_Υ	9.46	9.46	0%	0%
$\mathcal{BE}_\Upsilon$	0.100	0.100	0%	

Summary: vector and pseudoscalars

- From full model solutions:

$m_{u,d} = 0.011$	$m_s = 0.165$	$m_c = 1.35$	$m_b = 4.64$
$m_\rho = 0.776$	$m_\phi = 1.02$	$m_{J/\psi} = 3.09$	$m_{\Upsilon(1S)} = 9.46$
$\mathcal{BE}_\rho = 0.311$	$\mathcal{BE}_\phi = 0.320$	$\mathcal{BE}_{J/\psi} = 0.260$	$\mathcal{BE}_\Upsilon = 0.100$
$m_\pi = 0.135$	$m_{0^-_{s\bar{s}}} = 0.61$	$m_{\eta_c} = 2.97$	$m_{\eta_b} = 9.43$
$\mathcal{BE}_\pi = 0.953$	$\mathcal{BE}_{0^-} = 0.727$	$\mathcal{BE}_{\eta_c} = 0.380$	$\mathcal{BE}_{\eta_b} = 0.130$

Summary

- Vertex dressing dominated by 3-gluon coupling
- Here we include self-consistent dressing on all available vertices in an algebraic model
- Observe cancellation among different orders of dressing
- Chiral symmetry-preserving $K_{\rm BSE}$ constructed
- Compared to full solution, ladder-rainbow truncation gives 10% error for m_ρ decreasing to $<$ 1% for J/ψ and Υ
- Limitations: no scalar or axial vector solutions, omitted non-planar vertex diagrams and 4-gluon coupling
- Still a rare BSE solution with ∞ diagrams in $K_{\rm BSE}$
- Can one produce $K_{\rm BSE}$ directly from a phenomenological gluon-quark vertex?