Holographic decays of Stringy mesons

with
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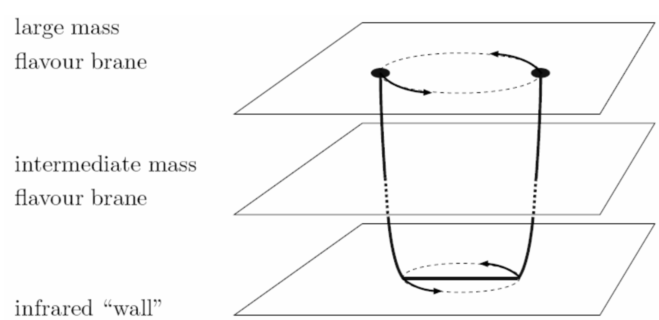
•In spite of the fact that we still lack the string theory of QCD ,quite remarkably many properties of hadron physics do get reproduced in confining SUGRA models.

•However, so far most these properties concern spectra of states

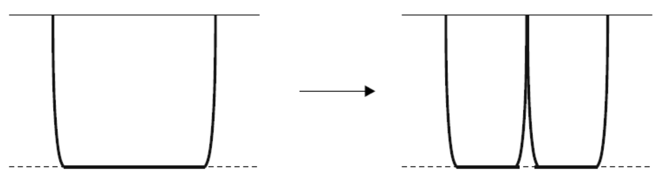
•A first attempt to compute decay rate of low spin mesons was done by Sakai Sugimoto in the context of gravity/gauge duality

•High spin mesons cannot be described by SUGRA modes but only as string configurations.



•The purpose of this work is to compute the decay width of the stringy meson into two stringy mesons.

•Such a process takes the form



•We need to compute the probability that the string will reach a flavor brane times the probability that the string splits when it is on the brane

•We then compare the holographic results to the phenomenological CNN and Lund models which are based on massive quark anti-quark pair connected by a flux tube

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Outline

•The laboratory –Witten’s model of near extremal D4 branes

•Flavor probe branes in confining backgrounds

•Stringy mesons- an exercise in classical string theory

•String with massive endpoints corrections to the Regge trajectories

•The “Old description” of a decay of a meson-The CNN model (Casher, Neuberger, Nussinov) and the Lund model

•Corrections due to masses and the centrifugal barrier

•Holographic decay- qualitative picture

•The split of an open sting in flat space time- an exercise in string loop

•The decay width- the wave function of the fluctuating string

•Flat space-time approximation, corrections due to curvature

•String bit model

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•Comparison of the calculated width and experiments summary

The laboratory – Witten's model

- Our laboratory is the confining background of the near extremal D4 branes in the limit of large temperature. This is believed to be in the same universality class as the low energy effective action of pure YM theory in 4D.

- The background is given by

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left[\eta_{\mu\nu}\mathrm{d}X^\mu \mathrm{d}X^\nu + f(U)\mathrm{d}\theta^2\right] + \left(\frac{R}{U}\right)^{3/2}\left[\frac{\mathrm{d}U^2}{f(U)} + U^2 \mathrm{d}\Omega_4\right]$$

$$e^\phi = g_s\left(\frac{U}{R}\right)^{3/4}, \qquad F_4 = \frac{2\pi N_c}{V_4}\epsilon_4, \qquad f(U) = 1 - \left(\frac{U_\Lambda}{U}\right)^3.$$

- It is an ancient wisdom that it admits an <u>area law Wilson loop</u> and a mass gap in the glue-ball spectrum.

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- The size of the thermal circle $L_\Lambda = \frac{4}{3}\pi\left(\frac{R^3}{U_\Lambda}\right)^{1/2}$ determines the scale of the system $M_\Lambda = 2\pi/L_\Lambda$

- 't Hooft parameter is $g^2_{\rm YM}N = 2\,M_\Lambda\,R^3/\alpha'$ where $R^3 = \pi g_s N_c l_s^3$

- The effective string tension is

$$T_{\rm eff} = \frac{1}{2\pi\alpha'}\sqrt{g_{00}g_{xx}}\,|_{\rm wall} = \frac{1}{2\pi\alpha'}\left(\frac{U_\Lambda}{R}\right)^{3/2} = \frac{2}{27\,\pi}M^2_\Lambda\,(g^2_{\rm YM}N)$$

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Flavor probe branes in confining backgrounds

- One way to incorporate flavor into the game is to introduce flavor barnes. If the number of flavor branes Nf <<Nc then the branes can be treated as probes whose dynamics is governed by a DBI +CS action.
- The open strings between the original Nc branes and the flavor branes play the role of quarks in the fundamental representation..
- This was proposed by Karch and Katz in the context of the AdS5 xS5
- The first time it was applied in a confining background was in the context of the KS model Sakai Sonnenschein with D7 branes .
- Myers et al introduced D6 branes into Witten's model.
- A model with *$U_L(N_f)$ x $U_R(N_f)$* flavor chiral symmetry was proposed by Sakai and Sugimoto using D8 anti- D8 branes.
- Recently an analogous non-critical model based on D4 anti- D4 branes was also analyzed. (Casero,Paredes, Sonnenschein)

•Here in this work we use the D6 brane model but the analysis can be adopted also to the other models.

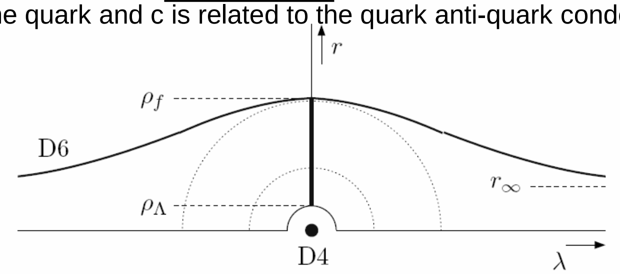
$$\mathrm{d}\tilde{s}^2 = K(\rho)\Big[\mathrm{d}\rho^2 + \rho^2\mathrm{d}\Omega_4^2\Big] = K(\rho)\Big[\mathrm{d}\lambda^2 + \lambda^2\mathrm{d}\Omega_2 + \mathrm{d}r^2 + r^2\mathrm{d}\phi^2\Big]$$

$$U(\rho)^{3/2} \equiv \rho^{3/2} + \frac{U_\Lambda^2}{4\rho^{3/2}}\,, \qquad K(\rho) = R^{3/2}U^{1/2}\rho^{-2}\,, \qquad \rho^2 = \lambda^2 + r^2$$

•The plane (r,ϕ) is perpendicular to the D6 brane.

•We solve the equation of motion of the brane

•Asymptotically $\boxed{r = r_\infty + \frac{c}{\lambda}}$ r_∞ is related to the QCD mass of the quark and c is related to the quark anti-quark condensate



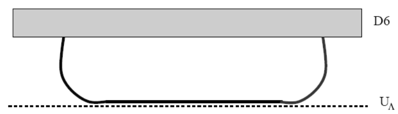
Stringy mesons- An exercise in classical string theory

- The laboratory is the NED4 model with D6 flavor probe brane
- The 4d metric is parameterized

$$dX^\mu dX_\mu = -(dX^0)^2 + dR^2 + R^2 d\theta^2 + (dX^3)^2.$$

- We look for solutions of the classical equations of the form of spinning open string with endpoints on the probe brane

$$\boxed{X^0 = e\tau, \quad \theta = e\omega\tau, \quad R = R(\sigma), \quad r = r(\sigma), \quad \lambda = \lambda(\sigma),}$$



- The boundary conditions:
  Dirichlet D6 $\quad \boxed{r(\sigma) = r(\lambda(\sigma))\Big|_{\sigma=-\pi/2,\pi/2}.}$

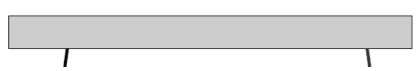
  Neuman D6 $\quad \boxed{\left(\partial_\sigma X^0 = \partial_\sigma \theta = \partial_\sigma R = \partial_\sigma \lambda\right)\Big|_{\sigma=0,\pi} = 0}$

- $\lambda = 0.$ is a solution of the equation of motion
- Now the Nambu Goto action reads

$$\boxed{S = -T_s \int d\sigma d\tau \sqrt{(U/R_{D4})^{3/2}(\partial_\tau X^{02} - R^2 \partial_\tau \theta^2)((U/R_{D4})^{3/2}\partial_\sigma R^2 + K\partial_\sigma \rho^2)},}$$

- The string ends transversely to the D6 brane

$$d\rho/dR = d\rho/d\sigma \cdot d\sigma/dR = \infty$$



The NG equation of motion

$$\frac{d}{dR}\left[\frac{U\mathcal{E}\dot\rho}{\rho^2\sqrt{UR_{D4}^{-3}+\frac{\dot\rho^2}{\rho^2}}}\right] = \frac{dU}{d\rho}\frac{\mathcal{E}}{\sqrt{UR_{D4}^{-3}+\frac{\dot\rho^2}{\rho^2}}}\left(\frac{3}{2}UR_{D4}^{-3}+\frac{\dot\rho^2}{\rho^2}\right)-\frac{\dot\rho^2 U\mathcal{E}}{\rho^3\sqrt{UR_{D4}^{-3}+\frac{\dot\rho^2}{\rho^2}}},$$

- The Noether charges associated with the shift of X^0 and θ.

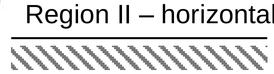
$$E = T_s\int dR\frac{e\sqrt{(U/R_{D4})^{3/2}+K\dot\rho^2}}{\mathcal{E}}(U/R_{D4})^{3/4},$$

$$J = T_s\int dR\frac{e\omega R^2\sqrt{(U/R_{D4})^{3/2}+K\dot\rho^2}}{\mathcal{E}}(U/R_{D4})^{3/4}$$

- There are two solutions

Region I- vertical $\dot\rho\to\infty$

$\dot\rho\to 0$ Region II – horizontal



wall

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•Sewing together the vertical and horizontal solutions requires that

$$\int d\tau\delta R\sqrt{1-\omega^2R_0^2}(U_\Lambda/R_{D4})^{3/2}-\frac{\omega^2R_0}{\sqrt{1-\omega^2R_0^2}}\int d\tau\int_{\rho_\Lambda}^{\rho_f}d\rho\delta R\frac{U(\rho)}{\rho}.$$

Namely:

$$1-\omega^2R_0^2=\omega^2R_0\frac{1}{(U_\Lambda/R_{D4})^{3/2}}\int_{\rho_\Lambda}^{\rho_f}d\rho\frac{U(\rho)}{\rho}.$$

Now the mass of the ``quark" is defined by

$$m_q = T_s\int_{\rho_f}^{\rho_\Lambda}d\rho\sqrt{g_{00}g_{\rho\rho}} = T_s\int_{\rho_f}^{\rho_\Lambda}d\rho\frac{U}{\rho}.$$

and hence we the find the "classical relation"

$$T=\hat m_q w^2R_0 \qquad \hat m_q=\frac{m}{\sqrt{1-w^2R_0^2}}$$

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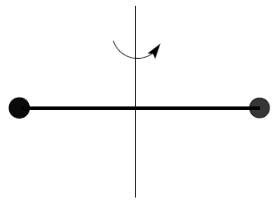
•Indeed the same result is derived from a toy model of a string with massive particles at its ends.



•The NG action of an open string in flat space-time combined with the action of two relativistic particles

$$
\begin{aligned}
S &= -T\int d\tau \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\sigma \sqrt{-[(\dot{t})^2 - \dot{R}^2 - R^2\dot{\theta}^2][(t')^2 - R'^2 - R^2\theta'^2] + [-\dot{t}t' + \dot{R}R' + R^2\dot{\theta}\theta']^2} \\
&- m\int d\tau \sqrt{-[(\dot{t})^2 - \dot{R}^2 - R^2\dot{\theta}^2]}\Big|_{\sigma=-\frac{\pi}{2}} - m\int d\tau \sqrt{-[(\dot{t})^2 - \dot{R}^2 - R^2\dot{\theta}^2]}\Big|_{\sigma=\frac{\pi}{2}},
\end{aligned} \tag{3.1}
$$

Yields exactly the same result if we take $m=m_q$

•The energy and angular momentum from the vertical parts are

$$
E_I = \frac{2m_q}{\sqrt{1-\omega^2 R_0^2}}, \qquad J_I = \frac{2m_q \omega {R_0}^2}{\sqrt{1-\omega^2 R_0^2}}
$$

•The horizontal string part contributes

$$
\begin{aligned}
E_{II} &= T_s \int_{-R_0}^{R_0} dR \frac{e\sqrt{U^{3/2} + K\dot{\rho}^2}}{\mathcal{E}} (U/R_{D4})^{3/4} \\
&= T_s (U_\Lambda/R_{D4})^{3/2} \frac{2}{\omega} \arcsin(\omega R_0) \\
J_{II} &= T_s \int_{-R_0}^{R_0} dR\, e\omega R^2 \frac{e\sqrt{U^{3/2} + K\dot{\rho}^2}}{\mathcal{E}} U^{3/4} \\
&= T_s (U_\Lambda/R_{D4})^{3/2} \frac{1}{\omega^2} (\arcsin(\omega R_0) - \omega R_0 \sqrt{1-\omega^2 {R_0}^2})
\end{aligned}
$$

so that altogether with x = wR we get

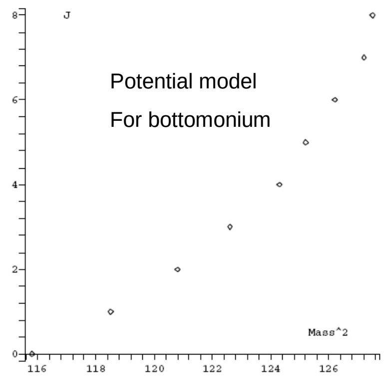
$$
\begin{aligned}
E &= \frac{2T_g}{\omega}\left(\arcsin x + \frac{1}{x}\sqrt{1-x^2}\right) \\
J &= \frac{T_g}{\omega^2}\left(\arcsin x + \frac{3}{2}x\sqrt{1-x^2}\right)
\end{aligned}
$$

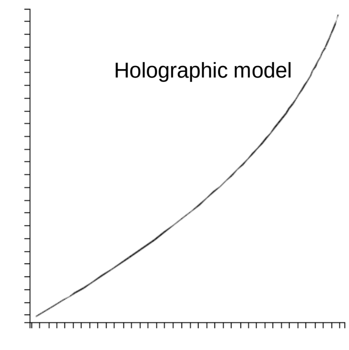
•For light quarks with x~1 we get the following correction to the Regge trajectory

$$J = \frac{1}{\pi T_g} E^2 \left(1 + \frac{\sqrt{2}}{\sqrt{\pi}} \left(\frac{m_q}{E} \right)^{1/2} - \frac{\pi - 1}{\pi} \frac{m_q}{E} + \dots \right)$$

For heavy quarks the trajectory looks like

$$J = \frac{2m_q^{1/2}}{T_g}(E-2m_q)^{3/2} - \frac{11}{12}\frac{1}{m_q^{1/2}T_g}(E-2m_q)^{5/2} + \frac{163}{144}\frac{1}{T_g m_q^{3/2}}(E-2m_q)^{7/2}$$





hologra

•It is straightforward to generalize the discussion to the case that the string ends on two different probe branes, namely two different masses.

•In general there are several stacks of probe branes characterized by their distance from the wall ρ_{f_i}

•For convenience we group the probe branes into three classes

$$\begin{aligned} m_l &\approx T_{\text{eff}}(\rho_{f_l} - \rho_\Lambda) \ll \Lambda_{\text{QCD}}\,, \\ m_m &\approx T_{\text{eff}}(\rho_{f_l} - \rho_\Lambda) \sim \Lambda_{\text{QCD}}\,, \\ m_h &\approx T_{\text{eff}}(\rho_{f_l} - \rho_\Lambda) \gg \Lambda_{\text{QCD}}\,. \end{aligned}$$

•Accordingly there are six types of mesons

$$(l,l)\,,\ (l,m)\,,\ (l,h)\,,\ (m,m)\,,\ (m,h)\,,\ (h,h)\,.$$

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The "Old description" of a decay of a meson-
The CNN model and the Lund model

•In this model the meson is built from a quark/anti-quark pair with a color electric flux tube between them

•When a new pair is created along the flux tube it will be pulled apart and tear the original tube into two tubes.

•A use is made of Schwinger's calculation of the probability of creating a pair in a constant electric field. The decay probability per unit time and volume is given by

$$P = \frac{g^2\mathcal{E}^2}{16\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{2\pi m_q^2 n}{g\mathcal{E}}\right) = \frac{T_{\text{eff}}^2}{\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{\pi m_q^2 n}{2T_{\text{eff}}}\right) .$$

•The probability of the decay of the meson $1 - e^{-V_4(t_M)P}$

•Finally the width is

$$\left(\frac{\Gamma}{M}\right)_{\text{rot}} = \frac{2r_t^2}{T_{\text{eff}}}P = (0.6 - 8.5)\times 10^{-2}$$

Corrections due to masses and centrifugal barrier

•The massive particles at the end of the flux tube change the relation between the length and the mass

$$\frac{L}{M} = \frac{2}{\pi T_{\text{eff}}} - \frac{m_1+m_2}{2T_{\text{eff}}M} + \mathcal{O}\left(\frac{m_i^2}{M^2}\right)$$

•A WKB approximation without a barrier reproduces the CNN result. An improvement can be achieved by incorporating centifugal barrier

$$P \sim \exp\left(-2\int_0^{r_c}\mathrm{d}r\sqrt{(E-V(r))^2 - m_q^2 - \frac{l(l+1)}{r^2}}\right)$$

•The probability for a breaking of the tube is modified to give

$$P \sim \exp\left[-\frac{\pi m_q^2}{2T_{\text{eff}}}\frac{\left(1+\frac{w}{6m(1-w^2R_q^2)}\right)^2}{\left(1-\frac{w^3R_q}{2T_{\text{eff}}(1-w^2R_q^2)^2}\right)^{3/2}}\right] ,$$

Holographic decay- qualitative picture

- Quantum mechanically the stringy meson is unstable.
- Fluctuations of endpoints splitting of the string
- The string has to split in such a way that the new endpoints are on a flavor brane.
- The decay probability= (to split at a given point) X (that the split point is on a flavor brane)

•The probability to split of an open string in flat space time was computed by Dai and Polchinski and by Turok et al.

$$\boxed{\Gamma = \frac{1}{\pi^{23}\sqrt{2^{45}}}M}$$

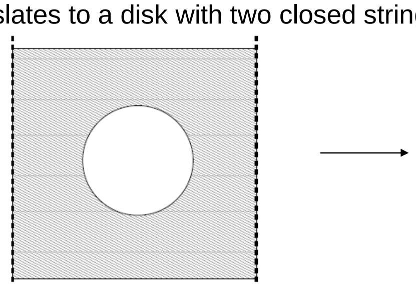
The split of an open sting in flat space time

An exercise in one loop string calculation

•Intuitively the string can split at any point and hence we expect width~ L

•The idea is to use the optical theorem and compute the total rate by computing the imaginary part of the self energy diagram

•Consider a string streched around a long compact spatial direction. A winding state splits and joins. In terms of vertex operators it translates to a disk with two closed string vetex operators



•The corresponding amplitude takes the form

$$i\mathcal{A} = \frac{iTN}{g^2} L \left[\frac{\kappa}{2\pi\sqrt{L}}\right]^2 \int_{|z|<1} \mathrm{d}^2 z \,\langle : e^{ip_0 X(0)} :: e^{-ip_0 X(z)} : \rangle$$

where k is the gravitational coupling, g the coefficient of the open string tachyon, the factor L comes from the zero mode.

•Using the ope's we get

$$\langle : e^{ip_0 X(0)} :: e^{-ip_0 X(z)} : \rangle = |z\bar{z}|^{-2}(1 - z\bar{z})^{-\gamma}$$

where $\gamma = \frac{L^2 T}{2\pi} - 2.$

•Performing the integral, taking the imaginary part

$$\Gamma = -\operatorname{Im}\delta m = -\frac{1}{2m}\operatorname{Im}\mathcal{A} = \frac{TN\kappa^2}{4g^2 E}\gamma \to \frac{TN\kappa^2}{8\pi g^2} L = \frac{g^2 T^{13}}{2^{26}\pi^{12}} NL$$

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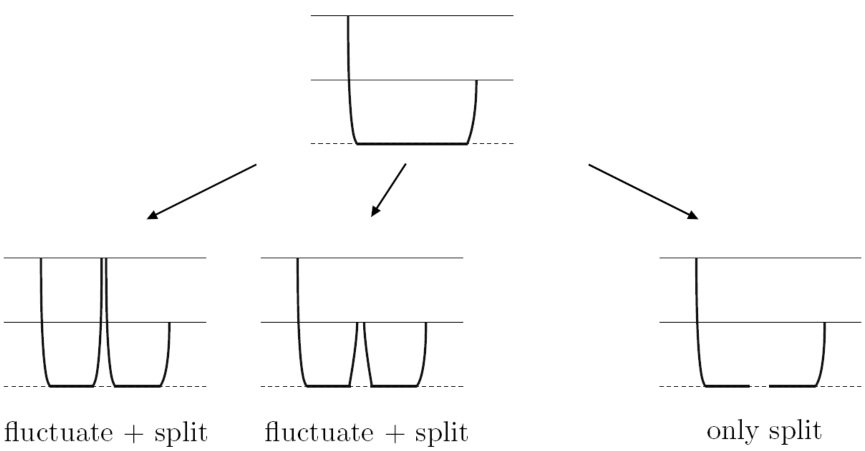
Splits of a (h,m) meson



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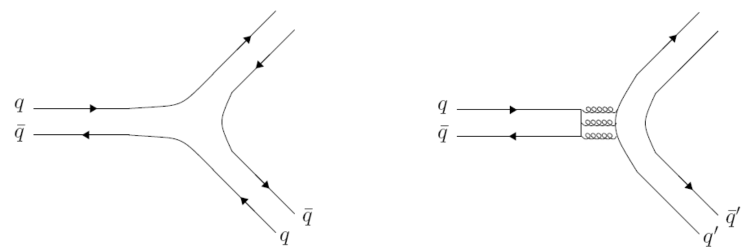


Figure 5: The Zweig rule illustrated. The dominant decay channel for mesons is the process on the left, in which the original quarks are part of the mesons in the outgoing state. The process on the right, in which the quark and anti-quark which constitute the initial meson annihilate, is suppressed.

Meson decay width

- We now compute the probability that due to quantum fluctuations, the horizontal part of the string reaches the probe brane
- We express the spectrum of fluctuations in terms of normal modes, and write its wave function $\Psi_n[\mathcal{N}_n]$.The total wave function is

$$\boxed{\Psi\left[\{\mathcal{N}_n\}\right] = \prod_n \Psi_n\left[\mathcal{N}_n(X^M)\right]}$$

- The probability is formally given by

$$\boxed{\mathcal{P}_{\text{fluct}} = \int'_{\{\mathcal{N}_n\}} \left|\, \Psi\left[\{\mathcal{N}_n\}\right] \right|^2}$$

Where the integral is taken only over configurations which obey

$$\boxed{\max\left(U(\sigma)\right) \geq U_B}$$

$U(\sigma)$ is a linear combination of all the modes.

Flat space time approximation

•We assume the space around the wall is flat and defiene the coordinates

$$\eta^2 = \frac{U - U_\Lambda}{U_\Lambda} . \qquad \eta = \sqrt{\frac{3}{4}} U_\Lambda^{1/2} R^{-3/2} \, z$$

•The metric then reduces to

$$\mathrm{d}s^2 \sim \left(\frac{U_\Lambda}{R}\right)^{3/2} \left(\eta_{\mu\nu} \mathrm{d}X^\mu \mathrm{d}X^\nu + \mathrm{d}z^2\right) + (R^3 U_\Lambda)^{1/2} \mathrm{d}\Omega_4^2$$

•The fluctuations both for light and heavy mesons have Dirichlet b.c, hence

$$z(\sigma, \tau) = \sum_{n>0} z_n \cos(n\sigma)$$

•The action for the fluctuations in the z direction is

$$S_{\text{fluct}} = \frac{L}{2\alpha'_{\text{eff}}} \int \mathrm{d}T \left[\sum_{n>0} \left(-(\partial_T z_n)^2 + \frac{n^2}{L^2} z_n^2 + \ldots\right)\right]$$

• This system is equivalent to infinite number of uncoupled harmonic oscillators with frequencies n/L and mass L/α'_{eff}

•The total wave function is

$$\Psi(\{z_n\}, \{x_n\}) = \Psi_{\text{long}}(\{x_n\}) \times \Psi_{\text{sphere}}(\{y_n\}) \times \Psi_\theta(\{\theta_n\}) \times \Psi_{\text{trans}}(\{z_n\})$$

However only the fluctuations along z are relevant

$$\Psi[\{z_n\}] = \prod_{n=1}^{\infty} \Psi_0(z_n)$$

•The wave function for the individual modes is

$$\Psi_0(z_n) = \left(\frac{n}{\pi \alpha'_{\text{eff}}}\right)^{1/4} \exp\left(-\frac{n}{2\alpha'_{\text{eff}}} z_n^2\right) .$$

•The probability that the string does not touch the probe brane is given by integrating all configurations such that

$$\sum_{n>0} |z_n| \leq z_B$$

•To simplify the calculations we find a lower bound by integrating over modes such that for each $z_n < z_B$

$$\mathcal{P}_{\text{fluct}}^{\min} = 1 - \lim_{N \to \infty} \int_0^{z_B} \mathrm{d}z_1 \int_0^{z_B} \mathrm{d}z_2 \cdots \int_0^{z_B} \mathrm{d}z_N \, |\Psi(\{z_n\})|^2$$

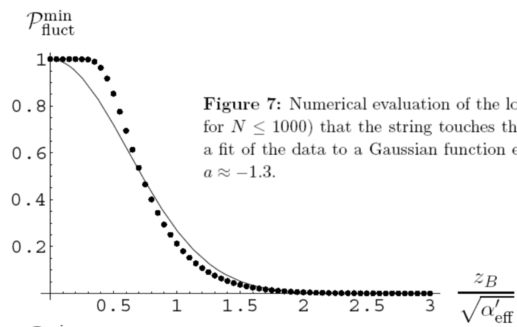


Figure 7: Numerical evaluation of the lower bound on the probability $\mathcal{P}_{\min}$ (i.e. (4.27), for $N \leq 1000$) that the string touches the flavour brane (black dots). Also displayed is a fit of the data to a Gaussian function $\exp(-az^2/\alpha'_{\text{eff}})$ (blue curve). The best fit gives $a \approx -1.3$.

$$\boxed{\mathcal{P}^{\min}_{\text{fluct}} \approx \exp\left(-1.3\frac{z_B^2}{\alpha'_{\text{eff}}}\right)}$$

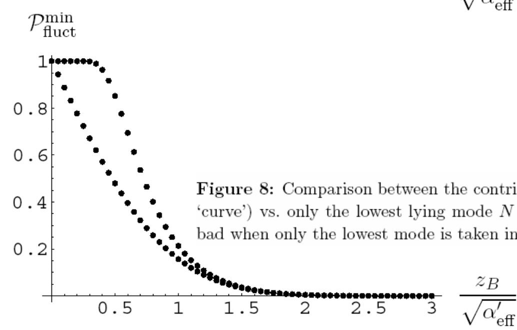


Figure 8: Comparison between the contribution to $\mathcal{P}^{\min}_{\text{fluct}}$ of all modes $N \leq 1000$ (upper 'curve') vs. only the lowest lying mode $N = 1$ (lower 'curve'). The fit to a Gaussian is bad when only the lowest mode is taken into account.

holografic decay

Curved space-time approximation

•Let us study the effects of the curvature on the width. We use now the full metric around the wall to find the following fluctuations action

$$S = \frac{1}{2\pi\alpha'}\int \mathrm{d}\tau\mathrm{d}\sigma\,\left[\frac{4}{3}(R^3U_\Lambda)^{1/2}(\dot{\eta}^2 - \eta'^2) - 3\,L^2\,\cos^2(\sigma)\left(\frac{U_\Lambda}{R}\right)^{3/2}\eta^2\right]$$

•We find that the equation of motion is a Mathieu equation

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} + \frac{\mathrm{d}^2}{\mathrm{d}\sigma^2} - \frac{9}{8}\frac{L^2\,U_\Lambda}{R^3}\,\big(1+\cos(2\sigma)\big)\right]\eta(\tau,\sigma) = 0$$

with the boundary conditions $\eta(\tau, -\frac{\pi}{2}) = \eta(\tau, \frac{\pi}{2}) = 0$

•Define $\eta(\tau,\sigma) = e^{i\omega\tau}\,f(\sigma)$ the solution that obeys the left boundary condition is

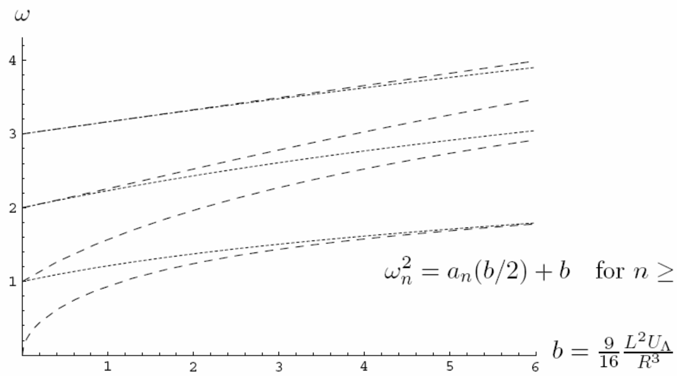
$$f(\sigma) = C\left(\omega^2 - b, \frac{b}{2}, -\frac{\pi}{2}\right)S\left(\omega^2 - b, \frac{b}{2}, \sigma\right) - S\left(\omega^2 - b, \frac{b}{2}, -\frac{\pi}{2}\right)C\left(\omega^2 - b, \frac{b}{2}, \sigma\right)$$

holografic decay

•The parameter b $b \equiv \frac{9}{16}\frac{L^2 U_\Lambda}{R^3} = \left(\frac{L}{L_\Lambda}\right)^2 = \frac{9}{16}\frac{\alpha'}{\mathcal{R}^2}\left(\frac{L^2}{\alpha'_{\text{eff}}}\right) = \frac{27}{4}\pi^2\frac{J}{g^2_{\text{YM}}N}$

•At vanishing b we are back in flat space.

•We need to tune ω^2 such that the right boundary condition is satisfied



•We now use the equation of motion to eliminate η'' and the known frequencies and derive a system of harmonic oscillators

$$S = \frac{L}{2\alpha'}\frac{4}{3}(R^3 U_\Lambda)^{1/2}\int \mathrm{d}T \sum_n \left[\left(\frac{\mathrm{d}\eta_n}{\mathrm{d}T}\right)^2 - \frac{\omega_n^2}{L^2}\eta_n^2\right]$$

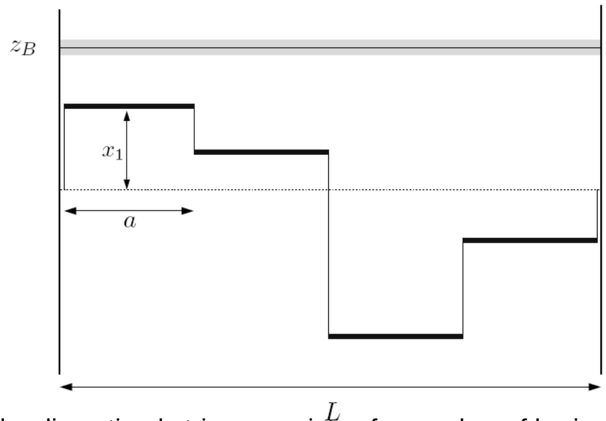
•The wave function of the ground state behaves like

$$\Psi[\eta_n] \sim \exp\left[-\frac{2}{3\alpha'}(R^3 U_\Lambda)^{1/2}\big(a_n(b/2)+b\big)^{1/2}\,\eta_n^2\right] .$$

•For leading n $\omega.$ ~ n^2 namely the flat space result and no J dependence of the exponential factor. However for larger n the curvature tends to suppress the decay for higher spin mesons. On the other hand recall that the finite mass effects tend to enhance the decay for larger J. There are thus two competing effects.

String bit approximation

•Using a string bit model the integration over the right subset of configurations becomes more managable.



•The discretized string consists of a number of horizontal rigid rods connected by vertical springs.

•The mass of each bead is M, the length is L=(N+1)a and the action is

$$\boxed{S = \frac{1}{2}\int \mathrm{d}t \left(\sum_{n=1}^{N} M\,\dot{x}_n^2 - \frac{T_{\rm eff}}{a} \sum_{n=1}^{N+1} \left(x_n - x_{n-1}\right)^2 \right)}$$

•The normal modes and their frequencies are

$$y_m = \frac{1}{N+1} \sum_{n=1}^{N} \sin\left(\frac{mn\pi}{N+1}\right) x_n\,, \qquad \omega_m^2 = \frac{4\,T_{\rm eff}\,N(N+1)}{M_{\rm tot}\,L}\,\sin^2\left(\frac{m\pi}{2(N+1)}\right)$$

• In the relativistic limit and large N $\qquad \omega_m^2 = m^2\pi^2/L^2$

•The action now is of N decoupled normal modes

$$S = (N+1)M \int \mathrm{d}t \sum_{m=1}^{N} \left(\dot{y}_m^2 - \omega_m^2 y_m^2\right)$$

•The wave function is a product of the wave functions of the normal modes

$$\boxed{\Psi\left(\{y_1, y_2, \ldots\}\right) = \prod_{m=1}^{N} \left(\frac{2(N+1)M\omega_m}{\pi}\right)^{1/4} \exp\left(-(N+1)M\omega_m\, y_m^2\right)}$$

•Note that the width of the Gaussian depends on T_{eff} and not on L

$$\boxed{\lim_{N\to\infty}(N+1)M\omega_m = \lim_{N\to\infty}(N+1)\frac{T_{\text{eff}}L}{N}\frac{m\pi}{L} = T_{\text{eff}}\pi m}$$

•The integration interval is when the bead is "at the brane" defined by

$$I_{\text{brane}} : \big[-z_B - \Delta,\, -z_B\big] \,\cup\, \big[z_B,\, z_B + \Delta\big]\,,$$
$$I_{\text{space}} : \big\langle -\infty,\, -z_B - \Delta\big] \,\cup\, \big[-z_B,\, z_B\big] \,\cup\, \big[z_B + \Delta,\, \infty\big\rangle$$

•By computing the decay width for various values of N and extrapolating to large N we find that the decay rate is approximated by

$$\boxed{\Gamma_{\text{beads}} = \text{const.} \cdot \exp\left(-1.0\,\frac{z_B^2}{\alpha'_{\text{eff}}}\right)\,\Gamma_{\text{open}}}$$

holografic decay

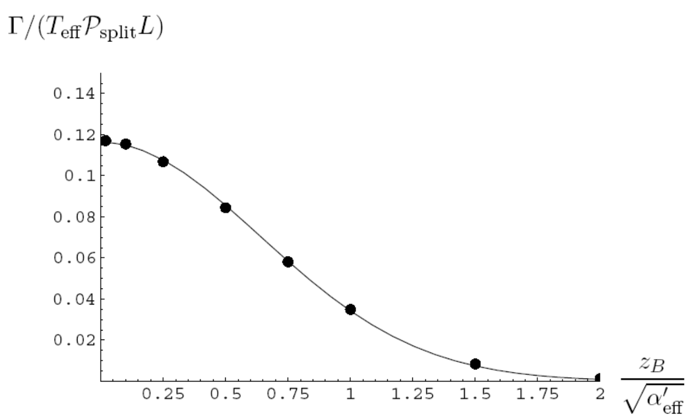


Figure 12: Total decay width (divided by $T_{\text{eff}}\mathcal{P}_{\text{split}}L$) for a six-bead system, with a brane width of $0.1\sqrt{\alpha'_{\text{eff}}}$, as a function of the distance z_B of the IR "wall" to the flavour brane. Superposed is a best-fit Gaussian, with parameters $\Gamma = 0.12\,\exp(-1.22\,z_B^2/\alpha'_{\text{eff}})$. The string is allowed to split if a bead is at the brane, but the other beads are allowed to be anywhere (both below and above the brane).

holografic decay

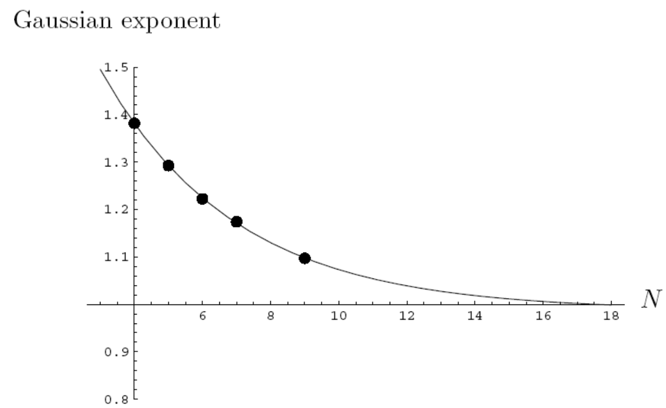


Figure 13: The behaviour of the scale of the Gaussian shape of the decay width as a function of the number of beads N. A best-fit exponential curve has been superposed, which suggest that the scale approaches one in the large-N limit.

Summary and comparison with experiment

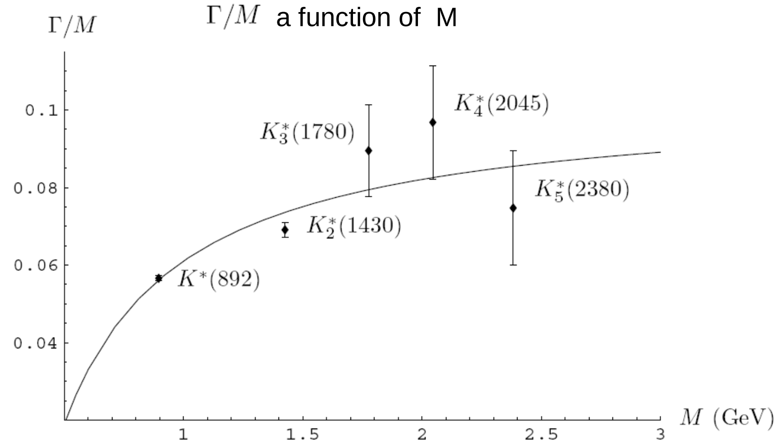
- The main idea in this work is the relation between the decay rate of a mesonic string and the fluctuations of the horizontal part of the U-shaped string.
- The probability for the string to break was determined as the probability of an open sting in flat space time to break multiplied by the probability of the two new endpoint to reconnect with the probe.
- The decay width of high spin mesons exhibits

(a) Linear dependence on the string length
(b) Exponential suppression with the mass of the product quarks
(c) Flavor conservation
(d) 1/N dependence in large N
(e) The Zweig rule.

- The result is in a very good agreement with Lund model
- However the precise exponent is different .

•The basic CNN model predicts $\Gamma/M = \mathrm{const.}$ In fact Γ/L is *constant* and hence incorporating the corrections due to the massive endpoints we find the following blue curve which fits the data points of the K* mesons

Γ/M a function of M



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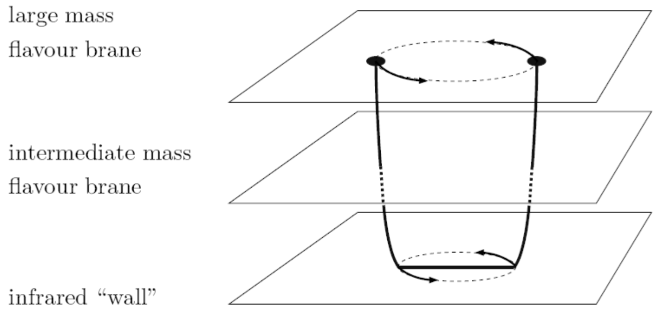
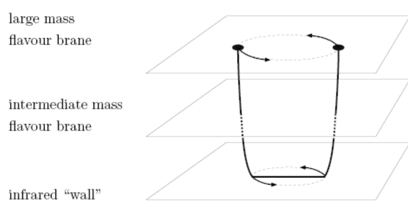


Figure 1: A high-spin meson composed of heavy quarks, represented in the dual string picture as an open string ending on a flavour brane far away from the infrared "wall". To good approximation, the string consists of two vertical segments, called "region I" and one horizontal segment, called "region II".

holografic decay



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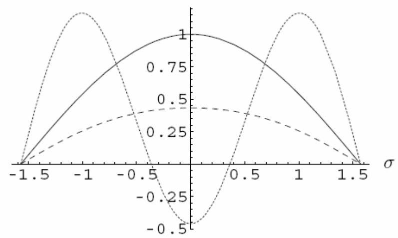
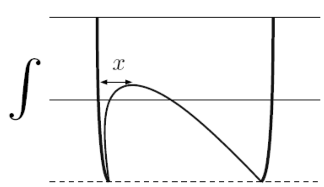


Figure 10: The first excited modes (i.e. corresponding to the frequencies for $n = 1$ in equation (4.36)), with arbitrary normalisation. Dashing is as in figure (9). Also depicted is the mode which survives for $b = 0$, i.e. in the absence of curvature (solid curve).



$K\left[\{\mathcal{N}_n\}\right] \mathrm{d}x \approx \; 2L \; \times \kappa_{\max}^{-1}$

Figure 6: The approximation used to separate the L-dependent factor in the decay width from the dimensionless remainder. The integral over all configurations which touch the string at two points and have a maximum at $U = U_B$ is, after taking into account the dimensionful measure factor $K\left[\{\mathcal{N}_n\}\right]$, approximately equal to L times the volume of this subspace of configuration space.

$$\Gamma = T_{\text{eff}}\, \mathcal{P}_{\text{split}} \int_{\{\mathcal{N}_n\}}' \left| \Psi\left[\{\mathcal{N}_n\}\right] \right|^2 K\left[\{\mathcal{N}_n\}\right] \; < \; T_{\text{eff}}\, \mathcal{P}_{\text{split}}\, L\, \kappa_{\max} \int_{\{\mathcal{N}_n\}}' \left| \Psi\left[\{\mathcal{N}_n\}\right] \right|^2,$$

holografic decay

We extract the equations of motion from the variation of the action

$$
\begin{aligned}
0 &= \delta S \\
&= \int d\sigma d\tau\ \delta R\Big[-\partial_\sigma\Big(\frac{(U/R_{D4})^{3/2}\partial_\sigma R\sqrt{(\partial_\tau X^{02}-R^2\partial_\tau\theta^2)(U/R_{D4})^{3/2}}}{\sqrt{(U/R_{D4})^{3/2}\partial_\sigma R^2+K\partial_\sigma\rho^2}}\Big) \\
&- \frac{R\partial_\tau\theta^2\sqrt{(U/R_{D4})^{3/2}((U/R_{D4})^{3/2}\partial_\sigma R^2+K\partial_\sigma\rho^2)}}{\sqrt{\partial_\tau X^{02}+R^2\partial_\tau\theta^2}}\Big] \\
&+ \int d\sigma d\tau\ \delta\rho\Big[-\partial_\sigma\Big(\frac{K\partial_\sigma\rho\sqrt{(\partial_\tau X^{02}-R^2\partial_\tau\theta^2)(U/R_{D4})^{3/2}}}{\sqrt{(U/R_{D4})^{3/2}\partial_\sigma R^2+K\partial_\sigma\rho^2}}\Big) \\
&+ \frac{\sqrt{\partial_\tau X^{02}-R^2\partial_\tau\theta^2}(3\frac{dU}{d\rho}R_{D4}^{-1}((U/R_{D4})^2\partial_\sigma R^2+1/2(U/R_{D4})^{1/2}K\partial_\sigma\rho^2)+(U/R_{D4})^{3/2}\frac{dK}{d\rho}\partial_\sigma\rho^2)}{2\sqrt{(U/R_{D4})^{3/2}((U/R_{D4})^{3/2}\partial_\sigma R^2+K\partial_\sigma\rho^2)}}\Big] \\
&- \int d\sigma d\tau\ \delta X^0\partial_\tau\Big(\frac{\partial_\tau X^0\sqrt{(U/R_{D4})^{3/2}((U/R_{D4})^{3/2}\partial_\sigma R^2+K\partial_\sigma\rho^2)}}{\sqrt{\partial_\tau X^{02}-R^2\partial_\tau\theta^2}}\Big) \\
&+ \int d\sigma d\tau\ \delta\theta\partial_\tau\Big(\frac{R^2\partial_\tau\theta\sqrt{(/R_{D4})U^{3/2}((U/R_{D4})^{3/2}\partial_\sigma R^2+K\partial_\sigma\rho^2)}}{\sqrt{\partial_\tau X^{02}-R^2\partial_\tau\theta^2}}\Big) \\
&+ \int d\tau\delta R\frac{(U/R_{D4})^{3/2}\partial_\sigma R\sqrt{(\partial_\tau X^{02}-R^2\partial_\tau\theta^2)(U/R_{D4})^{3/2}}}{\sqrt{(U/R_{D4})^{3/2}\partial_\sigma R^2+K\partial_\sigma\rho^2}}\Big|_{\sigma=-\pi/2}^{\sigma=\pi/2} \\
&+ \int d\tau\delta\rho\frac{K\partial_\sigma\rho\sqrt{(\partial_\tau X^{02}-R^2\partial_\tau\theta^2)(U/R_{D4})^{3/2}}}{\sqrt{(U/R_{D4})^{3/2}\partial_\sigma R^2+K\partial_\sigma\rho^2}}\Big|_{\sigma=-\pi/2}^{\sigma=\pi/2}.
\end{aligned}
$$

holografic decay

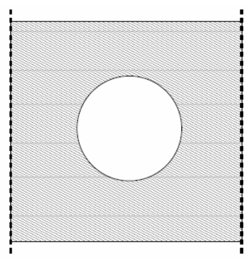
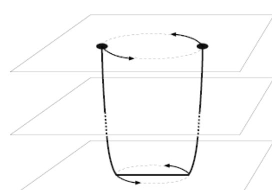


Figure 16: The setup for the computation of the open string decay width as used by Dai and Polchinski [7].

holografic decay



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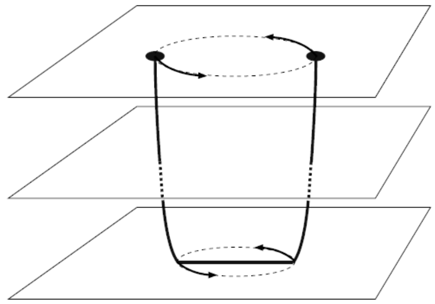
holografic decay



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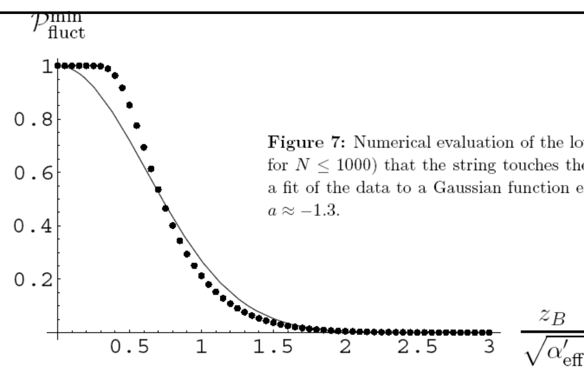


Figure 7: Numerical evaluation of the lower bound on the probability $\mathcal{P}_{\min}$ (i.e. (4.27), for $N \leq 1000$) that the string touches the flavour brane (black dots). Also displayed is a fit of the data to a Gaussian function $\exp(-az^2/\alpha'_{\text{eff}})$ (blue curve). The best fit gives $a \approx -1.3$.

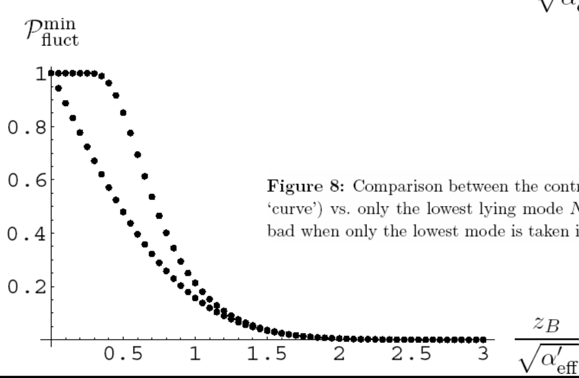
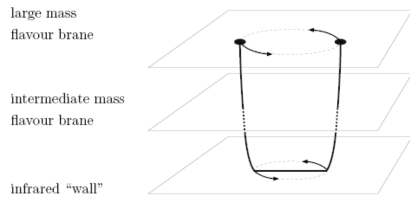


Figure 8: Comparison between the contribution to $\mathcal{P}^{\min}_{\text{fluct}}$ of all modes $N \leq 1000$ (upper ‘curve’) vs. only the lowest lying mode $N = 1$ (lower ‘curve’). The fit to a Gaussian is bad when only the lowest mode is taken into account.



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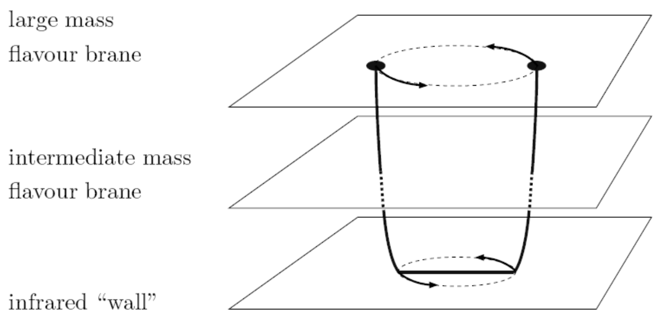

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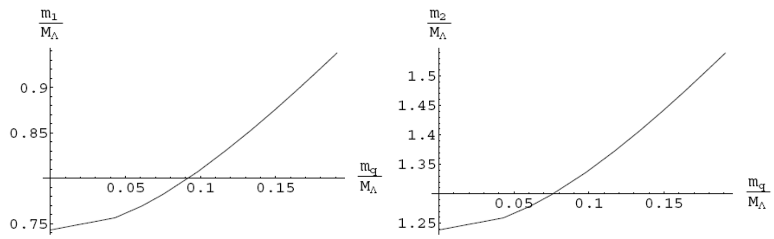

Figure 1: Plots of the masses of the two lightest vector mesons varying the constant u_0. $a = 0$ has been taken.

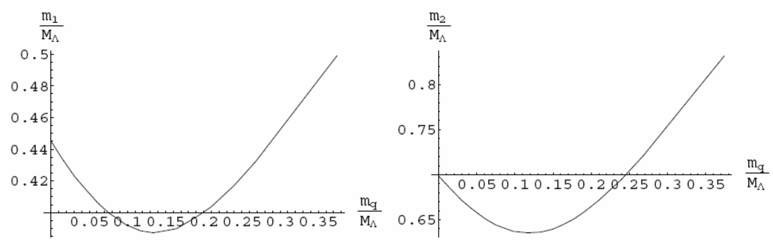

Figure 3: Plots of the masses of the two lightest vector mesons computed with $\tilde{a} = 1$.

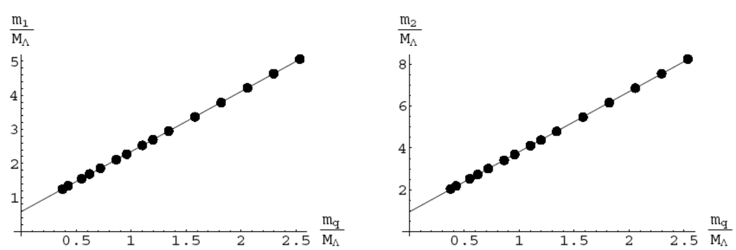

Figure 6: Asymptotically, the meson masses grow linearly in the parameter m_q. Here we present the plot for the two lightest vector mesons.

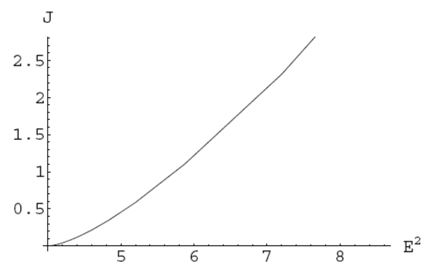

Figure 5: The plot shows an extrapolation to the small angular momentum region of formulae (B.22) and (B.23). Our approximate analysis shows that in this region the (E^2, J) trajectory deviates from the linear Regge behavior. Moreover, it is evident that there is a non-zero negative intercept.