Highly Excited Mesons in QCD

A continuation/refinement of my Trento talk

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The problem is hard as it is essentially Minkowskian; Euclidean explorations (e.g. lattices) are of little help!

Analytic ideas are badly needed!

Use inspiration from strings and quasiclassics
Quasiclassics and level spacing

\[ \int p \, dx \sim E \ell \sim E^2 / \sigma \sim n \]

Equidistant spectrum:

\[ M_n^2 \sim \sigma n \]

To make the problem well-defined we have to go to \( N \to \infty \); otherwise high excitations continuum

excitation number

correction? (logarithmic in \( 't \) Hooft model)
Basic idea of high-spin mesons in duals of confining gauge theories (left). Open string corresponding to meson starts on a flavor brane, stretches to IR wall and then reaches up again to flavor brane. A decay process (right) requires that the string fluctuates, touches the flavor brane and then reconnects to it.

\[ \ell \times p \sim L; \quad \ell \times \sigma \sim p \]

\[ M_{\ell}^2 \sim p^2 \sim \sigma L \]

Sonnenschein et al. /2005

AdS/CFT
End-point of string are fermions (quarks)

- Sonnenschein et al./2005: Dynamics independent of spin orientation!
- In relativistic dynamics, usually $S$ and $L$ are bad, $J$ is good;
- Remarkably, this is NOT the case in string picture!

$$S_1 + S_2 \rightarrow S = 0 \text{ or } 1$$

Consider (a) $S=0$ and $J=L_0$; (b) $S=1$, $L=L_0-1$, $J=L+S = L_0$

Both mesons have same $J$, but opposite parities:

$$M^2 = \sigma \cdot L_0 = \sigma \cdot J$$ in one case and $$M^2 = \sigma(L_0 - 1) = \sigma(J - 1)$$ in the other

Hence, $$\Delta M_J; \pm = \frac{\sigma}{M_J} \sim \frac{1}{\sqrt{J}}$$
What about high radial excitations?

- J fixed, N → ∞, n >> 1

\[ \Gamma_J \sim \frac{\sigma \cdot \ell}{N} \sim \frac{M_J}{N} \left( 1 + J^{-1/2}(???) \right) \]

CNN – Casher-Neuberger-Nussinov, 1979
Proceed to radial excitations (say, $J=0$ or 1)

**Quasiclassically**

$$\Gamma_n = (B/N) M_n, \quad M_n \sim n^{1/2}$$

$B \sim 0.5$ phenomenologically & from 2D model (t Hooft model); Hence,

$$\frac{\Gamma_n}{M_n} \sim \frac{B}{N}$$

*The limits $N \to \infty$ and $n \to \infty$ do NOT commute!*

_Ads/CFT’ corrections ???_
Question: (say)

\[ \delta M_n = M_n V^{-M_n} A \] at \( n \gg 1 \), where \( n \) is the (radial) excitation number?

QCD SR, Trento
2005

Consider:

\[ \Pi (Q^2) = \langle T\{J(x), J(0)\} \rangle_q \]

Moreover, for the parity pair, say,

\[ \delta \Pi (Q^2) = \Pi_S (Q^2) - \Pi_P (Q^2) \]

we have
\[ \delta \Pi (Q^2) = \sum_n \left\{ f_{nS} \left( Q^2 + M_{nS}^2 \right)^{-1} - f_{nP} \left( Q^2 + M_{nP}^2 \right) \right\} \]

at large Euclidean \( Q^2 \) we get

\[ \sim \frac{1}{N} \langle \bar{\Psi} \Psi \rangle^2 \frac{1}{Q^4} \]

\[ \leftarrow \text{mod logs} \]

sum over resonances
To begin with, assume that $f_{nP}$ and $f_{nS}$ are degenerate.

These constants can be normalized from

$$\Pi (Q^2) \sim N Q^2 \log Q^2$$

$$f_n \sim N \Lambda^2 M_n^2 \sim N \Lambda^4 n$$

The minimal solution:

$$M_n^2 \delta M_n^2 = \text{const.}$$

or $| \delta M_n | \sim \Lambda/n^{3/2}$ or faster

Not a theorem! Assumes absence of high energy $\leftrightarrow$ low energy conspiracy (pretty evident; Cata et al., 06)
High energy ↔ low energy conspiracy: what it is?

Gives $1/Q^2$ which can be canceled by a dedicated low-lying resonance ...

Absence of high energy ↔ low energy conspiracy natural in QCD!
Peeters, Sonnenschein, Zamaklar AdS/CFT’??? No answer yet! 🧐

😊 Meanwhile, a simple heuristic argument:

Casher, 1979
quark mass operator

M. Shifman, 2006
\[ M_n(+) - M_n(-) = \Lambda (\Lambda / p_{\text{max}})^D \]

\[ \delta M_n^2 = \Lambda^2 \left( \Lambda^2 / M_n^2 \right)^{(D-1)/2} \]

If \( D=3 \), \( M_n^2 \delta M_n^2 = \text{const.} \)

Fixes \( \pi \) constant thru a generaliz. of GT:

\[ \langle \bar{B}_+ | A_\mu | B_- \rangle = J^\nu (B_- B_+) \cdot \left( g_{\mu\nu} - q_\mu q_\nu / q^2 \right) \]

\[ g_{\pi^+ \pi^-} f_{\pi} \sim M_- - M_+ \]

OK for 't Hooft model (D=1) too!
Observe that e.g. the vector and axial-vector mesons can be of two types:

\[ \bar{\chi}_\alpha \chi_\alpha \pm \bar{\eta}_\alpha \eta_\alpha, \quad \chi\{\alpha \eta_\beta\} \pm \text{h.c.} \]

Lorentz (1/2,1/2) \hspace{2cm} Lorentz (1,0)+(0,1)

In terms of the "left" and "right" isospins:

\[ (1/2,0) \times (1/2,0) \rightarrow (1,0) + (0,0) \quad (1/2,0) \times (0,1/2) \rightarrow (2 \text{ quadruplets}) \]

I triplet + I singlet, vector isovector, axial-vector isovector + 2 isosinglets

I triplet + I singlet, vector isoscalar + axial-vector isovector + vector isovector, axial-vector isoscalar

😊 8+8 ..... If spin independence of string (above) is correct, all split by \( \delta M_n \sim 1/n^{3/2} \) ★★★★★
Pre-conclusions:

Give us ....  1973
Give us ....  1979
Give us ....  1985
please...  1991
We beg ....  2001
for a WEAK parameter

Take What's 
SUSY available !!!

Strings???
CONCLUSIONS:

- For high excitations use quasiclassics/string ideas
- Parity splittings in M’s and Γ’s (?) are interesting, as well as Γ/M versus n and J, are doable at large n and J NOW!
- Leading terms are known at large J; further check is desirable at large n; next-to-leading corrections is an immediate target