Dyson-Schwinger Equations: an update

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Dichotomy of the Pion

How does one make an almost massless particle from two massive constituent-quarks?
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Not Allowed to do it by fine-tuning

Must exhibit \( m_\pi^2 \propto m_q \)

Current Algebra \ldots 1968
Dichotomy of the Pion

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**Current Algebra . . . 1968**

The **correct understanding** of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a **well-defined and valid chiral limit**, and an **accurate realisation** of dynamical chiral symmetry breaking.
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Requires detailed understanding of Connection between Current-quark and Constituent-quark masses
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Using DSEs, we've provided this.
Dyson-Schwinger Equations
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A Modern Method for Relativistic Quantum Field Theory
Dyson-Schwinger Equations

A Modern Method for Relativistic Quantum Field Theory

Simplest level: Generating Tool for Perturbation Theory

........................ Materially Reduces Model Dependence
Dyson-Schwinger Equations

- A Modern Method for Relativistic Quantum Field Theory
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- NonPerturbative, Continuum approach to QCD
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Materially Reduces Model Dependence

NonPerturbative, Continuum approach to QCD

Hadrons as Composites of Quarks and Gluons
Dyson-Schwinger Equations

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- Hadrons as Composites of Quarks and Gluons
  Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
  - Quark & Gluon Confinement
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  - Understanding InfraRed (long-range)
  - behaviour of $\alpha_s(Q^2)$
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- Method yields Schwinger Functions $\equiv$ Propagators
**Dyson-Schwinger Equations**

- A Modern Method for Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  
  Materially Reduces Model Dependence

- NonPerturbative, Continuum approach to QCD

  - Hadrons as Composites of Quarks and Gluons
  
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  ⟹ Understanding InfraRed (long-range)

  ............................................................ behaviour of $\alpha_s(Q^2)$

- Method yields Schwinger Functions $\equiv$ Propagators

  Cross-Sections built from Schwinger Functions
Contemporary Reviews

- Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD

- The IR behavior of QCD Green’s functions: Confinement, DCSB, and hadrons . . .

- Dyson-Schwinger equations: A Tool for Hadron Physics
Persistent Challenge

Infinitely Many Coupled Equations

\[ \Sigma = \Gamma \]

\[ \Sigma \rightarrow \Gamma \]

\[ \gamma \rightarrow S \rightarrow \Gamma \]
Persistent Challenge

- Infinitely Many Coupled Equations
- Solutions are Schwinger Functions
  (Euclidean Green Functions)
- Same VEVs measured in Lattice-QCD simulations
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  - Weak coupling expansion $\Rightarrow$ Perturbation Theory
**Persistent Challenge**

- Infinitely Many Coupled Equations
- Solutions are Schwinger Functions (Euclidean Green Functions)
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- Coupling between equations necessitates truncation
- Weak coupling expansion $\Rightarrow$ Perturbation Theory
  Not useful for the nonperturbative problems in which we’re interested.
Persistent Challenge

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- Has Enabled Proof of EXACT Results in QCD
- And Formulation of Practical Phenomenological Tool to Make Predictions with Readily Quantifiable Errors
Perturbative Dressed-quark Propagator
Perturbative Dressed-quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

Gap Equation
Perturbative Dressed-quark Propagator

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dressed-quark propagator

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\[ S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} \]
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**dressed-quark propagator**

Gap Equation

Weak Coupling Expansion

Reproduces Every Diagram in Perturbation Theory
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- dressed-quark propagator

**Gap Equation**

\[ \Sigma \rightarrow = \]

- Weak Coupling Expansion
  - Reproduces Every Diagram in Perturbation Theory
  - But in Perturbation Theory

\[ B(p^2) = m \left( 1 - \frac{\alpha}{\pi} \ln \left[ \frac{p^2}{m^2} \right] + \ldots \right) \quad m \rightarrow 0 \quad 0 \]
**Perturbative Dressed-quark Propagator**

\[
S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}
\]

- **dressed-quark propagator**

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**Gap Equation**

**Weak Coupling Expansion**

Reproduces Every Diagram in Perturbation Theory

- **But in Perturbation Theory**

\[
B(p^2) = m \left(1 - \frac{\alpha}{\pi} \ln \left[\frac{p^2}{m^2}\right] + \ldots\right) \xrightarrow{m \to 0} 0
\]

No DCSB Here!
Dressed-Quark Propagator
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\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

Gap Equation

\[ \Sigma = \gamma \Gamma \]

\[ \Sigma \]

\[ \Gamma \]
Dressed-Quark Propagator

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Gap Equation’s Kernel Enhanced on IR domain

⇒ IR Enhancement of \( M(p^2) \)
\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

Gap Equation’s Kernel Enhanced on IR domain

\[ \Rightarrow \text{IR Enhancement of } M(p^2) \]

Euclidean Constituent–Quark Mass: \( M_f^E : p^2 = M(p^2)^2 \)

<table>
<thead>
<tr>
<th>flavour</th>
<th>( u/d )</th>
<th>( s )</th>
<th>( c )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{M_f^E}{m_\zeta} )</td>
<td>( \sim 10^2 )</td>
<td>( \sim 10 )</td>
<td>( \sim 1.5 )</td>
<td>( \sim 1.1 )</td>
</tr>
</tbody>
</table>
Dressed-Quark Propagator

Longstanding Prediction of Dyson-Schwinger Equation Studies
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Quenched-QCD

Dressed-Quark Propagator

M(p)

Z(p)

“data:” Quenched Lattice Meas.

– Bowman, Heller, Leinweber, Williams: [he-lat/0209129](https://arxiv.org/abs/he-lat/0209129)
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current-quark masses: 30 MeV, 50 MeV, 100 MeV
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Curves: Quenched DSE Cal.
- Bhagwat, Pichowsky, Roberts, Tandy nu-th/0304003
**Quenched-QCD**

**Dressed-Quark Propagator**

2002

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Linear extrapolation of lattice data to chiral limit is inaccurate
Kernel of Gap Equation: $D_{\mu\nu}(p - q) \Gamma_{\nu}(q)$

Dressed-gluon propagator and dressed-quark-gluon vertex

Reliable DSE studies of Dressed-gluon propagator:

Kernel of Gap Equation: $D_{\mu\nu}(p - q) \Gamma_\nu(q)$

Dressed-gluon propagator and dressed-quark-gluon vertex

Reliable DSE studies of Dressed-gluon propagator:


- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:


Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex
Dressed-gluon Propagator

\[ D_{\mu\nu}(k) = \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{Z(k^2)}{k^2} \]

Suppression means \( \exists \) IR gluon mass-scale \( \approx 1 \text{ GeV} \)

Naturally, this scale has the same origin as \( \Lambda_{\text{QCD}} \)
Dressed-gluon Propagator

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Hadrons

- Established understanding of two- and three-point functions
Hadrons

- Established understanding of two- and three-point functions
- What about bound states?
• Without bound states, Comparison with experiment is impossible
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• They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions
• Without bound states, Comparison with experiment is impossible

• Bethe-Salpeter Equation

QFT Generalisation of Lippman-Schwinger Equation.
Hadrons

- Without bound states, Comparison with experiment is impossible

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QFT Generalisation of Lippman-Schwinger Equation.

- What is the kernel, $K$?
• Without bound states, Comparison with experiment is impossible

• Bethe-Salpeter Equation

QFT Generalisation of Lippman-Schwinger Equation.

• What is the kernel, $K$?

or What is the long-range potential in QCD?
**Bethe-Salpeter Kernel**

Axial-vector Ward-\textit{Takahashi} identity

\[
P_\mu \Gamma^l_{\bar{5}\mu}(k; P) = S^{-1}(k_+) \left( \frac{1}{2} \lambda_f i\gamma_5 + \frac{1}{2} \lambda_f i\gamma_5 \right) S^{-1}(k_-)
\]

\[
- M_\zeta i\Gamma^l_5(k; P) \quad \text{QFT Statement of Chiral Symmetry}
\]
Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma_{5\mu}^l (k; P) = S^{-1}(k_+) \frac{1}{2} \lambda_f i \gamma_5 + \frac{1}{2} \lambda_f i \gamma_5 \frac{1}{2} \lambda_f i \gamma_5 S^{-1}(k_-) \]

\[ -M_\zeta \, i \Gamma_{5}^l (k; P) - i \Gamma_{5}^l (k; P) M_\zeta \]

Satisfies BSE \hspace{1cm} \text{Satisfies DSE}
Axial-vector Ward-Takahashi identity

\[ P_\mu \, \Gamma^{l}_{5\mu}(k; P) = S^{-1}(k_+) \frac{1}{2} \chi_f i \gamma_5 + \frac{1}{2} \chi_f i \gamma_5 \left( S^{-1}(k_-) - M_\zeta i \Gamma^l_5(k; P) - i \Gamma^l_5(k; P) M_\zeta \right) \]

Satisfies BSE

Satisfies DSE

Kernels must be intimately related
Axial-vector Ward-Takahashi identity

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\[ \text{Kernels must be \textit{intimately} related} \]

\[ \text{Relation \textbf{must} be preserved by truncation} \]

\[ \text{Nontrivial constraint} \]
Bethe-Salpeter Kernel

Axial-vector Ward-Takahashi identity

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\[ -M_\zeta i \Gamma_5^l (k; P) - i \Gamma_5^l (k; P) M_\zeta \]

Satisfies BSE  \hspace{2cm} \text{Satisfies DSE}

\[ \text{Kernels must be \textit{intimately} related} \]

- Relation \textbf{must} be preserved by truncation
- Failure \implies Explicit Violation of QCD’s Chiral Symmetry
Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

\[ \Gamma_{\pi j}(k; P) = \tau^{\pi j} \gamma_5 \left[ iE_{\pi}(k; P) + \gamma \cdot PF_{\pi}(k; P) \right. \]

\[ \left. + \gamma \cdot k \cdot P G_{\pi}(k; P) + \sigma_{\mu\nu} k_{\mu} P_{\nu} H_{\pi}(k; P) \right] \]
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\[ f_\pi E_{\pi}(k; P = 0) = B(p^2) \]
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- Dressed-quark Propagator: \( S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} \)

- Axial-vector Ward-Takahashi identity

\[
\Rightarrow 
\frac{f_\pi E_\pi(k; P = 0)}{F_R(k; 0) + 2 f_\pi F_\pi(k; 0)} = B(p^2) \\
G_R(k; 0) + 2 f_\pi G_\pi(k; 0) = A(k^2) \\
H_R(k; 0) + 2 f_\pi H_\pi(k; 0) = 2A'(k^2) \\
\]
Goldberger-Treiman for pion

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\[ \Gamma_{\pi^j}(k; P) = \tau^{\pi^j} \gamma_5 \left[ iE_{\pi}(k; P) + \gamma \cdot P F_{\pi}(k; P) \right. \]

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- Dressed-quark Propagator: \[ S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} \]

- Axial-vector Ward-Takahashi identity

\[ f_{\pi} E_{\pi}(k; P = 0) = B(p^2) \]

\[ F_R(k; 0) + 2 f_{\pi} F_{\pi}(k; 0) = A(k^2) \]

\[ G_R(k; 0) + 2 f_{\pi} G_{\pi}(k; 0) = 2A'(k^2) \]

\[ H_R(k; 0) + 2 f_{\pi} H_{\pi}(k; 0) = 0 \]

Pseudovector components necessarily nonzero

Exact in Chiral QCD
Andreas Krassnigg

FWF “Erwin Schrödinger Fellow,” ANL 2003-2005
Future President . . . almost Blood Relative of Arnold
Radial Excitations
& Chiral Symmetry

\[ f_H \ m_H^2 = - \ \rho^H_\zeta \ M_H \]
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

\[ f_H \quad m_H^2 = - \rho^H_\zeta \quad M_H \]

- Mass\(^2\) of pseudoscalar hadron
Radial Excitations & Chiral Symmetry

\[ f_H m_H^2 = -\rho_H^\zeta M_H \]

\[ M_H := \text{tr}_{\text{flavour}} \left[ M_{(\mu)} \left\{ T^H, (T^H)^t \right\} \right] = m_{q_1} + m_{q_2} \]

- Sum of constituents’ current-quark masses
- e.g., \( T^{K^+} = \frac{1}{2} (\lambda^4 + i\lambda^5) \)
Radial Excitations & Chiral Symmetry

\[ m_H^2 = - \rho^H_\zeta M_H \]

\[ f_H p_\mu = Z_2 \int_0^\Lambda \frac{1}{2} \text{tr} \left\{ \left( T^H \right)^t \gamma_5 \gamma_\mu S(q+) \Gamma_H(q; P) S(q-) \right\} \]

- Pseudovector projection of BS wave function at \( x = 0 \)
- Pseudoscalar meson’s leptonic decay constant

\[ \vec{\pi} \quad -f_\pi k^\mu \quad \vec{A}_5^\mu \quad \text{equiv} \quad \vec{\Gamma}_5 \quad i (\tau/2) \gamma^\mu \gamma_5 \]
Radial Excitations & Chiral Symmetry

\[ f_H \quad m_H^2 = - \rho^H_\zeta \mathcal{M}_H \]

\[ i \rho^H_\zeta = Z_4 \int^\Lambda_q \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 S(q_+) \Gamma_H(q; P)S(q_-) \right\} \]

- Pseudoscalar projection of BS wave function at \( x = 0 \)
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

\[ f_H \ m_H^2 = - \ \rho_H^H \ \mathcal{M}_H \]

- Light-quarks; i.e., \( m_q \sim 0 \)

\[ f_H \rightarrow f_H^0 \quad \text{and} \quad \rho_H^\zeta \rightarrow \frac{-\langle \bar{q}q \rangle_0^\zeta}{f_H^0}, \quad \text{Independent of} \quad m_q \]

Hence \( m_H^2 = \frac{-\langle \bar{q}q \rangle_0^\zeta}{(f_H^0)^2} m_q \) \( \ldots \) GMOR relation, a corollary
Radial Excitations & Chiral Symmetry

\[ f_H m_H^2 = - \rho^H \zeta M_H \]

- **Light-quarks**: i.e., \( m_q \sim 0 \)
  
  \[ f_H \rightarrow f_H^0 \quad \text{and} \quad \rho^H_\zeta \rightarrow -\frac{\langle \bar{q}q \rangle^0_\zeta}{f_H^0}, \quad \text{Independent of} \quad m_q \]

  \[ m_H^2 = \frac{-\langle \bar{q}q \rangle^0_\zeta}{(f_H^0)^2} m_q \quad \text{...GMOR relation, a corollary} \]

- **Heavy-quark + light-quark**
  
  \[ f_H \propto \frac{1}{\sqrt{m_H}} \quad \text{and} \quad \rho^H_\zeta \propto \sqrt{m_H} \]

  Hence, \( m_H \propto m_q \)

... QCD Proof of Potential Model result
$$f_H \quad m_H^2 = - \quad \rho^H \quad \mathcal{M}_H$$

Valid for ALL Pseudoscalar mesons
Valid for **ALL** Pseudoscalar mesons

\[ f_H \ m_H^2 = - \ \rho_H^H \ \mathcal{M}_H \]

- \( \rho_H \) \( \Rightarrow \) finite, nonzero value in chiral limit, \( \mathcal{M}_H \rightarrow 0 \)
Radial Excitations & Chiral Symmetry

Valid for \textbf{ALL} Pseudoscalar mesons

\( \rho_H \) \Rightarrow \text{finite, nonzero value in chiral limit, } M_H \to 0

“radial” excitation of \( \pi \)-meson,

\( m_{\pi_n \neq 0}^2 > m_{\pi_n = 0}^2 = 0, \text{ in chiral limit} \)
Radial Excitations & Chiral Symmetry

\[ f_H \ m_H^2 = - \ \rho H \ \mathcal{M}_H \]

- Valid for ALL Pseudoscalar mesons
- \( \rho_H \Rightarrow \) finite, nonzero value in chiral limit, \( \mathcal{M}_H \rightarrow 0 \)
- “radial” excitation of \( \pi \)-meson,
  \[ m_{\pi_n \neq 0}^2 > m_{\pi_n = 0}^2 = 0, \text{ in chiral limit} \]
- \( \Rightarrow f_H = 0 \)

ALL pseudoscalar mesons except \( \pi(140) \) in chiral limit
Valid for ALL Pseudoscalar mesons

\[ \rho_H \Rightarrow \text{finite, nonzero value in chiral limit, } M_H \to 0 \]

“radial” excitation of \( \pi \)-meson,

\[ m_{\pi n \neq 0}^2 > m_{\pi n = 0}^2 = 0, \text{ in chiral limit} \]

\[ \Rightarrow f_H = 0 \]

ALL pseudoscalar mesons except \( \pi(140) \) in chiral limit

Dynamical Chiral Symmetry Breaking

– Goldstone’s Theorem –

impacts upon every pseudoscalar meson
Radial Excitations

- Spectrum contains 3 pseudoscalars $[I^G(J^P)L = 1^-(0^-)S]$

  masses below 2 GeV: $\pi(140)$; $\pi(1300)$; and $\pi(1800)$
Radial Excitations

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  $S_{\bar{Q}Q} = 1 \oplus L_F = 1 \Rightarrow J = 0$

  & $L_F = 1 \Rightarrow ^3S_1 \oplus ^3S_1 (\bar{Q}Q)$ decays suppressed?
Radial Excitations

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- NSAC Long-Range Plan, 2002: . . . an understanding of confinement “remains one of the greatest intellectual challenges in physics”
Radial Excitations & Chiral Symmetry
Fundamental properties of QCD
Fundamental properties of QCD

If chiral symmetry is dynamically broken, then in the chiral limit every pseudoscalar meson is blind to the weak interaction except $\pi(140)$. 

![Graph showing $f_\pi$ as a function of $m_q$]
Fundamental properties of QCD

- If chiral symmetry is dynamically broken, then in the chiral limit every pseudoscalar meson is blind to the weak interaction except $\pi(140)$.
- If chiral symmetry is not broken, then NO pseudoscalar meson experiences the weak interaction.
Two-photon Couplings of Pseudoscalar Mesons


\[ T_{\pi^0}^{\pi^0}(k_1, k_2) = \alpha i \varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma} G_{\pi^0}^{\pi^0}(k_1, k_2) \]

Define: \[ T_{\pi^0}^{\pi^0}(P^2, Q^2) = G_{\pi^0}^{\pi^0}(k_1, k_2) \bigg|_{k_1^2 = Q^2 = k_2^2} \]

This is a transition form factor.

Physical Processes described by couplings:
\[ g_{\pi^0\gamma\gamma} := T_{\pi^0}^{\pi^0}(-m_{\pi^0}^2, 0) \]

Width: \[ \Gamma_{\pi^0\gamma\gamma} = \alpha_{em}^2 \frac{m_{\pi^0}^3}{16\pi^3} g_{\pi^0\gamma\gamma}^2 \]
Two-photon Couplings: Goldstone Mode


\[ \pi_0^0(P) \]

\[ T_{\mu\nu}^{\pi_0^0}(k_1, k_2) = \frac{\alpha}{\pi} i \epsilon_{\mu\nu\rho\sigma} k_1^\rho k_2^\sigma G^{\pi_0^0}(k_1, k_2) \]

Chiral limit, model-independent and algebraic result

\[ g_{\pi_0^0\gamma\gamma} := T_{\pi_0^0}(-m_{\pi_0^0}^2 = 0, 0) = \frac{1}{2} \frac{1}{f_{\pi_0}} \]

So long as truncation preserves chiral symmetry and the pattern of its dynamical breakdown

The most widely known consequence of the Abelian anomaly
Two-photon Couplings: Transition Form Factor


\[ T_{\mu\nu}^{\pi_0}(k_1, k_2) = \frac{\alpha}{\pi} i\varepsilon_{\mu\nu\rho\sigma} k_1^\rho k_2^\sigma G_{\pi_0}^{\pi_0}(k_1, k_2) \]

So long as truncation preserves chiral symmetry and the pattern of its dynamical breakdown, and the one-loop renormalisation group properties of QCD: model-independent result – \( \forall n \):

\[ T_{\pi_0}^{\pi_0}(P^2, Q^2) = G_{\pi_0}^{\pi_0}(k_1, k_2) \bigg|_{k_1^2 = Q^2 = k_2^2} \quad \frac{Q^2 \gg \Lambda_{QCD}^2}{4\pi^2} \frac{4\pi^2}{3} \frac{f_{\pi_0}}{Q^2} \]

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excited state pseudoscalar mesons,”
nu-th/0503043

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- BUT, $f_{\pi_n} \equiv 0, \forall n!$
- Model-independent result, in chiral limit: $\forall n \geq 1$

$$\lim_{\hat{m} \to 0} T_{\pi_0}(-m_{\pi_n}^2, Q^2)$$

$$Q^2 \gg \Lambda^2_{\text{QCD}} \quad \frac{4\pi^2}{3} \quad F_n^{(2)}(-m_{\pi_n}^2) \quad \frac{\ln \gamma Q^2/\omega_{\pi_n}^2}{Q^4} \bigg|_{\hat{m}=0}$$

where:
- $\gamma$ is an anomalous dimension
- $\omega_{\pi_n}$ is a width mass-scale

both determined, in part, by properties of the meson’s Bethe-Salpeter wave function.
Höll, Krassnigg, Maris, et al.,
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both determined, in part, by properties of the meson’s
Bethe-Salpeter wave function.

Importantly, $F_{n}^{(2)} (-m_{\pi_n}^2) \not\propto f_{\pi_n}$. Instead, it is determined by
DCSB mass-scales for $\pi_n$ that do not vanish in the chiral limit.

\[
m_u(1 \text{ GeV}) = m_d(1 \text{ GeV}) = 0
\]
Transition Form Factor (Chiral): 
RGI Rainbow-Ladder

Höll, Krassnigg, Maris, et al.,
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\[
|T_{\pi n}(Q^2)| \propto \left( \frac{4\pi^2 f_{\pi}}{3Q^2} \right) \left( \frac{0.22 \text{ GeV}}{Q^2} \right)^{3/4}
\]

- \( m_u(1 \text{ GeV}) = m_d(1 \text{ GeV}) = 0 \)
- Again, Predicted UV-behaviour is abundantly clear
- precise for \( Q^2 > 120 \text{ GeV}^2 \)

Transition Form Factor (Chiral): RGI Rainbow-Ladder

\[ F_{1}^{(2)}(-m_{\pi}^{2}) \ln \frac{Q^{2}}{\omega_{\pi}^{2}} \rvert_{\hat{m}=0} \approx (0.22 \text{ GeV})^{3} \approx -\langle \bar{q}q \rangle^{0} \] (3)

\[ m_{u}(1 \text{ GeV}) = m_{d}(1 \text{ GeV}) = 0 \]

Again, Predicted UV-behaviour is abundantly clear

precise for \( Q^{2} > 120 \text{ GeV}^{2} \)
Are we there yet?
Maris & Tandy ... series of five papers ... excellent description of light pseudoscalar and vector mesons ... basket of 31 masses/couplings/radii with r.m.s. error of 15% ... moreover, prediction of $F_\pi(Q^2)$ measured in Hall A.
Nucleon Properties

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Pieter Maris

Peter Tandy
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Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks
Another Direction ... Also want/need information about three-quark systems
Nucleon Properties

- Another Direction . . . Also want/need information about three-quark systems
- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.
Another Direction ... Also want/need information about three-quark systems

With this problem ... current expertise at approximately same point as studies of mesons in 1995.

Namely ... Model-building and Phenomenology, constrained by the DSE results outlined already.
Proton Form Factors:
Modern Experiment
Rosenbluth and Polarization-Transfer Extractions of Ratio of Proton’s Electric and Magnetic Form Factors
Proton Form Factors: Modern Experiment

If Pol. Trans. Correct, then Completely Unexpected Result:
In the Proton – On Relativistic Domain
– Distribution of Quark-Charge Not Equal
Distribution of Quark-Current!
Closing in on something
Nucleon EM Form Factors: A Précis


- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons

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  ⇒ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)

Easily obtained:

\[
\left( \frac{1}{N_H} \sum_H \frac{[M_H^{\text{exp}} - M_H^{\text{calc}}]^2}{[M_H^{\text{exp}}]^2} \right)^{1/2} = 2\% 
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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

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Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons ⇒ Covariant dressed-quark Faddeev Equation

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Cloudy Bag: \( \delta M_+^{\pi \text{-loop}} = -300 \) to \(-400 \text{ MeV}\)!
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  - Cloudy Bag: \( \delta M^\pi_{+}^{\text{loop}} = -300 \text{ to } -400 \text{ MeV!} \)
  - Critical to anticipate pion cloud effects

Roberts, Tandy, Thomas, et al., nu-th/02010084
Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$


Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$


Pion cloud effects are large in the low $Q^2$ region.

*Ratio of the M1 form factor in $\gamma N \rightarrow \Delta$ transition and proton dipole form factor $G_D$. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.*
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Quark Core

- Responsible for only 2/3 of result at small $Q^2$
- Dominant for $Q^2 > 2 – 3 \text{ GeV}^2$
Faddeev equation
Faddeev equation

\[ \Psi^a \begin{array}{c} P \\ \Gamma^a \end{array} \begin{array}{c} p_q \\ p_d \end{array} = \begin{array}{c} \Gamma^a \begin{array}{c} p_q \\ q \end{array} \\ \Gamma^b \end{array} \begin{array}{c} p_q \\ p_d \end{array} \Psi^b \begin{array}{c} P \\ \Gamma^b \end{array} \]
Faddeev equation

\[ \Psi^a_{p_q} \Gamma^{a\Gamma} p_p q_q \Psi^b_{p_d} \Gamma^{b\Gamma} \]

Linear, Homogeneous Matrix equation

- Yields wave function (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon

- Scalar and Axial-Vector Diquarks . . . In Nucleon’s Rest Frame Amplitude has . . . s—, p— & d—wave correlations
Parametrising diquark properties
Parametrising diquark properties

- Dressed-quark ... fixed by DSE and Meson Studies
Parametrising diquark properties

Bethe-Salpeter-Like Amplitudes

\[ \Gamma^{0+}(k; K) = \frac{1}{N^{0+}} H^a C i\gamma_5 i\tau_2 \mathcal{F}(k^2/\omega_{0+}^2), \]

\[ t^i \Gamma^{1+}_\mu(k; K) = \frac{1}{N^{1+}} H^a i\gamma_\mu C t^i \mathcal{F}(k^2/\omega_{1+}^2). \]
Parametrising diquark properties

• Bethe-Salpeter-Like Amplitudes

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\[
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\]

• Colour matrices:

\[
\{H^1 = i\lambda^7, H^2 = -i\lambda^5, H^3 = i\lambda^2\}, \epsilon_{c_1c_2c_3} = (H^{c_3})_{c_1c_2}
\]
Parametrising diquark properties

- Bethe-Salpeter-Like Amplitudes

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\Gamma^0^+ (k; K) = \frac{1}{\mathcal{N}^0^+} H^a C \gamma_5 i \tau_2 \mathcal{F}(k^2 / \omega^2_0^+),
\]

\[
t^i \Gamma^1^+_{\mu} (k; K) = \frac{1}{\mathcal{N}^1^+} H^a i \gamma_{\mu} C t^i \mathcal{F}(k^2 / \omega^2_1^+).
\]

- Two parameters: \(\omega_0^+, \omega_1^+\)
  - \(\sim\) Inverse of diquarks’ configuration-space size
Parametrising diquark properties

Pseudoparticle Propagators

\[ \Delta^0_{+}(K) = \frac{1}{m_{0+}^2} \mathcal{F}(K^2/\omega_{0+}^2), \]

\[ \Delta^{1+}_{\mu\nu}(K) = \left( \delta_{\mu\nu} + \frac{K_{\mu}K_{\nu}}{m_{1+}^2} \right) \frac{1}{m_{1+}^2} \mathcal{F}(K^2/\omega_{1+}^2) \]

\[ \mathcal{F}(x) = \frac{1 - \exp(-x)}{x} \]

Absence of a Spectral Representation

Realisation of Confinement
Parametrising diquark properties

Pseudoparticle Propagators

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\]

- Two parameters: \( m_{0+}, m_{1+} \)
- ~Inverse of diquarks’ configuration-space correlation length
**Parametrising diquark properties**

- Total of four parameters

... reduce that via Normalisation Condition

\[
\frac{d}{dK^2} \left( \frac{1}{m_{JP}^2} \mathcal{F}(K^2/\omega_{JP}^2) \right)^{-1} \bigg|_{K^2=0} = 1 \Rightarrow \omega_{JP}^2 = \frac{1}{2} m_{JP}^2,
\]

Accentuates free-particle-like propagation characteristics of the diquarks within hadron.


**Parametrising diquark properties**

- Total of four parameters
  
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- Two Parameter Faddeev Equation Model of Nucleon
Parametrising diquark properties

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Accentuates free-particle-like propagation characteristics of the diquarks within hadron.

- Two Parameter Faddeev Equation Model of Nucleon
- Solve Faddeev Equation
- Vary \( m_{0^+} \) and \( m_{1^+} \) to obtain desired masses for \( N \) and \( \Delta \)
Results: Nucleon and $\Delta$ Masses
**Results: Nucleon and $\Delta$ Masses**

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and $\Delta$ masses

- **Set A** – fit to the actual masses was required; whereas for
- **Set B** – fitted mass was offset to allow for “$\pi$-cloud” contributions

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$m_{1+} \rightarrow \infty: M_N^A = 1.15 \text{ GeV}; M_N^B = 1.46 \text{ GeV}$
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\( m_1^+ \rightarrow \infty: M_N^A = 1.15 \text{ GeV} ; M_N^B = 1.46 \text{ GeV} \)

Axial-vector diquark provides significant attraction
Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and ∆ masses

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$m_{1^+} \rightarrow \infty: M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV

Constructive Interference: $1^{++}$-diquark + $\partial_\mu \pi$
Nucleon-Photon Vertex
Nucleon-Photon Vertex

constructed systematically . . . current conserved automatically for on-shell nucleons described by Faddeev Amplitude
6 terms . . . constructed systematically . . . current conserved automatically for on-shell nucleons described by Faddeev Amplitude

Nucleon-Photon Vertex

\[ \Psi_f \rightarrow \Gamma \rightarrow \Psi_i \]

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\[ \Psi_f \rightarrow \Gamma \rightarrow \Psi_i \]
Form Factor Ratio: \( GE/GM \)
Combine these elements . . .
Combine these elements ... 

Dressed-Quark Core 

\[
\frac{G_E^p}{G_M^p}\]

Rosenbluth precision Rosenbluth polarization transfer 

Rosenbluth polarization transfer
Combine these elements . . .

- **Dressed-Quark Core**
- **Ward-Takahashi**
  Identity preserving current

![Graph showing the ratio of form factors](image)

**Form Factor Ratio:** GE/GM
Combine these elements . . .

- Dressed-Quark Core
- Ward-Takahashi Identity preserving current
- Anticipate and Estimate Pion Cloud’s Contribution

Form Factor Ratio: \( \frac{G_E}{G_M} \)
Combine these elements . . .

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- **Anticipate and Estimate Pion Cloud’s Contribution**

![Graph showing the form factor ratio \( \mu_p G_E^P/G_M^P \) as a function of \( Q^2 \) in GeV^2.](image)
Combine these elements . . .

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- Ward-Takahashi Identity preserving current
- Anticipate and Estimate Pion Cloud’s Contribution

All parameters fixed in other applications . . . Not varied.
Combine these elements . . .

- **Dressed-Quark Core**
- **Ward-Takahashi**
  Identity preserving current
- **Anticipate and Estimate Pion Cloud’s Contribution**

All parameters fixed in other applications . . . **Not varied.**

Agreement with Pol. Trans. data at $Q^2 \gtrsim 2 \text{ GeV}^2$
Combine these elements . . .

- Dressed-Quark Core
- Ward-Takahashi Identity preserving current
- Anticipate and Estimate Pion Cloud’s Contribution

All parameters fixed in other applications . . . Not varied.

- Agreement with Pol. Trans. data at $Q^2 \gtrsim 2 \text{ GeV}^2$
- Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement
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All parameters fixed in other applications . . . Not varied.

- Agreement with Pol. Trans. data at $Q^2 \sim 2 \text{ GeV}^2$
- Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement
- Predict Zero at $Q^2 \approx 6.5 \text{ GeV}^2$
Epilogue
Epilogue
Dyson-Schwinger Equations

Provide Understanding of
Dynamical Chiral Symmetry Breaking:

$\pi$ is quark-antiquark Bound State
AND QCD’s Goldstone Mode
Dyson-Schwinger Equations

- Provide Understanding of Dynamical Chiral Symmetry Breaking:
  \[ \pi \text{ is quark-antiquark Bound State} \]
  \[ \text{AND} \quad \text{QCD’s Goldstone Mode} \]

- Foundation for Proof of Exact Results in QCD
  e.g., Quark Goldberger-Treiman Properties of Pseudoscalar Mesons
Epilogue

Dyson-Schwinger Equations

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  e.g., Quark Goldberger-Treiman Properties of Pseudoscalar Mesons

- Renormalisation-Group-Improved Rainbow-Ladder
  \[ \Rightarrow \] Practical Phenomenological Tool Corrections Quantifiable
Poincaré Covariant Faddeev Equation

Epilogue
Epilogue

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Quenched-QCD

Dressed-quark-gluon Vertex

- Bhagwat, et al.:  
  - nu-th/0304003  
  - nu-th/0403012  
  - hep-ph/0407163

share 65 citations
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Light-Cone QCD and Nonperturbative Hadron Physics, 15-19/05/06 – p. 38/49
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Parameter Free DSE Prediction confirms
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Light-Cone QCD and Nonperturbative Hadron Physics, 15-19/05/06 – p. 38/49
Colour-singlet

Bethe-Salpeter equation

Detmold et al., nu-th/0202082

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Bethe-Salpeter equation

- Coupling-modified dressed-ladder vertex

\[ \Gamma_\mu(k,p) = \Gamma_\mu + C \Gamma_\mu + C^2 \Gamma_\mu + \ldots \]
Colour-singlet

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\[ \Gamma_M = \sum_n \left[ \Gamma_\mu + \Gamma_\nu + \Lambda^{(n)}_{\mu \nu} \right] \]

- Bethe-Salpeter kernel... recursion relation

\[ -\frac{1}{8C} \Lambda^{(n)}_{\mu \nu} = \Gamma_{\mu}^{n-1} + \Gamma_{\nu}^{n-1} + \Gamma_M + \Lambda^{(n-1)}_{\mu \nu} \]
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Detmold et al., nu-th/0202082

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\[ \Gamma_{\mu}(k,p) = \Gamma_{\mu} + \Gamma_{\mu}^2 + \cdots \]

- BSE consistent with vertex

\[ \Gamma_M = \sum G \Gamma_M + \text{integer terms} \]

- Bethe-Salpeter kernel \ldots \text{recursion relation}

\[ -\frac{1}{8C} \]

- Kernel necessarily non-planar, even with planar vertex
\( \pi \) and \( \rho \) mesons
**π and ρ mesons**

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<tr>
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- \( \pi \) massless in chiral limit ... **NO** Fine Tuning
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- **π massless** in chiral limit . . . **NO** Fine Tuning
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Calculated Transition Form Factor:

Höll, Krassnigg, Maris, et al.,
“Electromagnetic properties of ground and excited state pseudoscalar mesons,”
uu-th/0503043

$m_u(1\ \text{GeV}) = m_d(1\ \text{GeV}) = 5.5\ \text{MeV}$
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RGI Rainbow-Ladder

Höll, Krassnigg, Maris, et al.,  
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\[
m_u(1 \text{ GeV}) = m_d(1 \text{ GeV}) = 5.5 \text{ MeV}
\]

\[
T_{\pi_1^0}(-m_{\pi_1^0}^2, Q^2) < 0, \quad Q^2 \geq -m_{\pi_1^0}^2 / 4;
\]

viz., it is negative on the entire kinematically accessible domain.
Calculated Transition Form Factor: RGI Rainbow-Ladder


\[ \frac{1}{(2 f_{\pi_0})} \]

\[ m_u(1 \text{ GeV}) = m_d(1 \text{ GeV}) = 5.5 \text{ MeV} \]

\[ \mathcal{T}_{\pi_1^0}(-m_{\pi_1}^2, Q^2) < 0, \quad Q^2 \geq -m_{\pi_1}^2/4; \]

viz., it is negative on the entire kinematically accessible domain.

\[ \Gamma_{\pi_0^0 \gamma\gamma} = 7.9 \text{ eV}, \quad \Gamma_{\pi_1^0 \gamma\gamma} = 240 \text{ eV} \]
Calculated Transition Form Factor: RGI Rainbow-Ladder


$|\tau_{\pi_n}(Q^2)| \left[ \text{GeV}^{-1} \right]$

$m_u(1 \text{ GeV}) = m_d(1 \text{ GeV}) = 5.5 \text{ MeV}$
Calculated Transition Form Factor:
**RGI Rainbow-Ladder**


\[ |T_{\pi_n}(Q^2)| \quad [\text{GeV}^{-1}] \]

\[
m_u(1 \text{ GeV}) = m_d(1 \text{ GeV}) = 5.5 \text{ MeV}
\]

Predicted **UV**-behaviour is abundantly clear

precise for \( Q^2 > 120 \text{ GeV}^2 \)
Electromagnetic Charge Radii – RGI
Rainbow-Ladder

\[ m_{u,d}(1 \text{ GeV}) = 5.5 \text{ MeV} \]
Electromagnetic Charge Radii – RGI

Rainbow-Ladder

- \( m_{u,d}(1 \text{ GeV}) = 5.5 \text{ MeV} \)

- Reminder:
  - MT-model has one IR-mass-scale – \( \omega \)
  - \( r_\alpha := 1/\omega \)
    - gauges the range of strong attraction

\[ r_\pi \text{ [fm]} \]

\[ \begin{align*}
\text{linear fit: } & 0.61 + 0.11 \omega \\
\text{linear fit: } & 0.09 + 1.76 \omega
\end{align*} \]
Electromagnetic Charge Radii – RGI

Rainbow-Ladder

Höll, Krassnigg, Maris, et al., nu-th/0503043

- $m_{u,d}(1 \text{ GeV}) = 5.5 \text{ MeV}$

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- Goldstone Mode’s properties are insensitive to $r_\alpha$

- Expected cf. $T \neq 0$, Goldstone mode’s properties do not change until very near chiral symmetry restoration temperature.
$m_{u,d}(1 \text{ GeV}) = 5.5 \text{ MeV}$

Reminder:
MT-model has one
IR-mass-scale – $\omega$

$r_a := 1/\omega$
gauges the range
of strong attraction

1st excited state:
orthogonal to Goldstone mode

Not protected ... properties very sensitive to $r_a$
Electromagnetic Charge Radii – RGI

Höll, Krassnigg, Maris, et al., nu-th/0503043

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- Best estimate \( r_{\pi_1} = 1.4 r_{\pi_0} \)

- But \( r_{\pi_1} < r_{\pi_0} \) is possible if confinement force is very strong
**Electromagnetic Charge Radii – RGI**

**Rainbow-Ladder**

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  - \( r_a := 1/\omega \) gauges the range of strong attraction
- Radial excitations are plainly useful to map out the long-range part of interaction between light-quarks.

\[ r_\pi \quad [\text{fm}] \]

\[ 0.3 \quad 0.32 \quad 0.34 \quad 0.36 \quad 0.38 \quad 0.4 \]

\[ \omega \quad [\text{GeV}] \]

\[ 0.60 \quad 0.65 \quad 0.70 \quad 0.75 \quad 0.80 \]

\[ n = 0 \ (\text{ground state}) \]
\[ n = 1 \ (\text{radial excitation}) \]

Linear fit:
- \( 0.61 + 0.11 \)
- \( 0.09 + 1.76 \)

Light-Cone QCD and Nonperturbative Hadron Physics, 15-19/05/06 – p. 43/49
Electromagnetic Charge Radii – RGI

Rainbow-Ladder

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- Same is true of orbital excitations; e.g., axial-vector mesons.
Electromagnetic Charge Radii – RGI
Rainbow-Ladder

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- Same is true of orbital excitations; e.g., axial-vector mesons.
- Hall-D at JLab
Deep-inelastic scattering
Deep-inelastic scattering

Looking for Quarks
Deep-inelastic scattering

Looking for Quarks
Deep-inelastic scattering

Looking for Quarks

Signature Experiment for QCD:

Discovery of Quarks at SLAC
Deep-inelastic scattering

- Looking for Quarks

Signature Experiment for QCD:

Discovery of Quarks at SLAC

Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x), g(x)$
Pion’s valence quark distn
**Pion’s valence quark distn**

- $\pi$ is Two-Body System: “Easiest” Bound State in QCD
- However, NO $\pi$ Targets!
\( \pi \) is Two-Body System: “Easiest” Bound State in QCD

However, NO \( \pi \) Targets!

Proved on

22/July/2002, ANL
Pion’s valence quark distn

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- However, NO $\pi$ Targets!
- Existing Measurement Inferred from Drell-Yan:
  
  $$\pi N \rightarrow \mu^+ \mu^- X$$
**Pion’s valence quark distn**

- \( \pi \) is Two-Body System: “Easiest” Bound State in QCD
- However, NO \( \pi \) Targets!
- Existing Measurement Inferred from Drell-Yan:
  \[ \pi N \rightarrow \mu^+ \mu^- X \]
- Proposal (Holt & Reimer, ANL, nu-ex/0010004)

\[ e^{-5\text{GeV}} - p_{25\text{GeV}} \text{ Collider } \rightarrow \text{ Accurate “Measurement”} \]
Proposal at JLab

(Holt, Reimer, Wijesooriya, et al., JLab at 12 GeV)
Handbag diagrams
\[ W_{\mu\nu}(q; P) = \frac{1}{2\pi} \text{Im} \left[ T_{\mu\nu}^+(q; P) + T_{\mu\nu}^-(q; P) \right] \]

\[ T_{\mu\nu}^+(q, P) = \text{tr} \int \frac{d^4k}{(2\pi)^4} \tau_- \bar{\Gamma}_\pi(k_{-\frac{1}{2}}; -P) S(k_{-0}) ieQ \Gamma_\nu(k_{-0}, k) \]
\[ \times S(k) i\epsilon Q \Gamma_\mu(k, k_{-0}) S(k_{-0}) \tau_+ \Gamma_\pi(k_{-\frac{1}{2}}; P) S(k_{--}) \]
**Bjorken Limit:** \(q^2 \to \infty , \quad P \cdot q \to -\infty\) but \(x := -\frac{q^2}{2P \cdot q}\) fixed.

**Numerous algebraic simplifications**

\[
\begin{align*}
W_{\mu\nu}(q; P) &= \frac{1}{2\pi} \text{Im} \left[ T_{\mu\nu}^+(q; P) + T_{\mu\nu}^-(q; P) \right] \\
T_{\mu\nu}^+(q, P) &= \text{tr} \int \frac{d^4k}{(2\pi)^4} \tau_- \bar{\Gamma}_{\pi}(k_{-\frac{1}{2}}; -P) S(k_{-0}) i e Q \Gamma_{\nu}(k_{-0}, k) \\
&\quad \times S(k) i e Q \Gamma_{\mu}(k, k_{-0}) S(k_{-0}) \tau_+ \Gamma_{\pi}(k_{-\frac{1}{2}}; P) S(k_{-0})
\end{align*}
\]
Extant theory vs. experiment

K. Wijersooriya, P. Reimer and R. Holt, nu-ex/0509012 ... Phys. Rev. C (Rapid)
Form Factor Ratio: $Q \ast F_2/F_1$
Form Factor Ratio: $Q^* F_2/F_1$

![Graph showing the form factor ratio $Q^2 F_{2p} / (\kappa p F_{1p})$ vs. $Q^2$ with data points and error bars from SLAC, JLab1, and JLab2. The graph includes a fit curve and shaded regions representing uncertainty.]
Perhaps $\approx$ constant for $2 < Q^2 < 6 \text{GeV}^2$
Formln Factor Ratio: alternative

\[ F2/F1 \]
Formln Factor Ratio: alternative

\[ \frac{F_2}{F_1} \]

\[
\left( \frac{Q}{\ln \left( \frac{Q^2}{\Lambda^2} \right)} \right)^2 \frac{F_{2p}}{\kappa_p F_{1p}}
\]

set B

SLAC

JLab 1

JLab 2

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Formln Factor Ratio: alternative

\[ \frac{Q^2}{[\ln Q^2/\Lambda^2]^2} \frac{F_2(Q^2)}{F_1(Q^2)} = \text{constant, } Q^2 \gg \Lambda^2 \approx M_N^2 \]

**Suggestive**

NB. Framework constructed to give quark-counting, i.e., “pQCD” *but* with wrong anomalous dimensions *but* they’re ignored in ln-power “2” of this ratio.