Holographic and Quark-Hadron Duality for Form Factors

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Hadronic form factors: \((1/Q^2)^{n_q-1}\) counting rules

Expectation: some fundamental/easily visible reason

Most natural suspect: scale invariance

Implementation: hard exchange in a theory with dimensionless coupling constant.

QCD: \((\alpha_s/Q^2)^{n_q-1}\)

Suppression: \(F_\pi(Q^2) = (2\alpha_s/\pi)s_0/Q^2\)

\[
\left[ s_0 = 4\pi^2 f_\pi^2 \approx 0.7 \text{ GeV}^2 \right]
\]

Looks like \(\mathcal{O}(\alpha_s)\) correction to VMD’s \(F^{\text{VMD}}_\pi(Q^2) \sim 1/(1 + Q^2/m_\rho^2)\)

Known: \(\alpha_s/\pi \sim 0.1\) is penalty for an extra loop

Growing consensus: pQCD gives small correction, dominant contribution comes from soft terms modeled by GPDs \(\mathcal{F}(x, Q^2)\) with exponential fall-off \(e^{-Q^2g(x)}\) for fixed \(x\)
AdS/CFT claims: nonperturbative explanation of quark counting rules

Reason: conformal invariance & short-distance behavior of normalizable modes $\Phi(\zeta)$

Form factor in AdS/CFT:

$$ F(Q^2) = \int_0^{1/\Lambda} \frac{d\zeta}{\zeta^3} \Phi_P'(\zeta) J(Q, \zeta) \Phi_P(\zeta) $$

Nonnormalizable mode: $J(Q, \zeta) = \zeta Q K_1(\zeta Q) \equiv \mathcal{K}_1(\zeta Q)$

For large $Q$: $\mathcal{K}_1(\zeta Q) \sim e^{-\zeta Q} \Rightarrow$ only small $\zeta \lesssim 1/Q$ are important

Normalizable modes: $\Phi(\zeta) = C \zeta^2 J_L(\beta_L, \kappa \zeta \Lambda)$
In light-cone formalism:

\[ F(2)(Q^2) = \int_0^1 dx \int d^2 b_\perp e^{i(1-x)b_\perp \cdot q_\perp} \left| \Psi_2(x, b_\perp) \right|^2 \]

\[ \Rightarrow \]

\[ 2\pi \int_0^1 dx \int_0^\infty bdb J_0(\bar{x}bQ) \left| \Psi_2(x, b) \right|^2 \]

\[ \Rightarrow \]

\[ 2\pi \int_0^1 \frac{dx}{x} \int_0^\infty zdz J_0 \left( \sqrt{\frac{x}{\bar{x}}} z Q \right) \left| \phi(x, z) \right|^2 \]

\( \bar{x} \equiv 1 - x, \ z \equiv \sqrt{x\bar{x}b} \)

Observation: \( \int_0^1 dx J_0 \left( \sqrt{\frac{x}{\bar{x}}} z Q \right) = K_1(zQ) \)

Need: \( |\phi(x, z)|^2 = x\bar{x}\chi^2(z) = x\bar{x}\Phi^2(\zeta)/\zeta^4 \)

“SJB/GdT” correspondence \( \Rightarrow \) Holographic LFWF
Normalized to 1:

\[ \int_0^1 dx \int d^2 b_\perp |\Psi_{\text{eff}}(x, b_\perp)|^2 = 1 \]

⇒ Effective wave functions

Lowest meson state: \( M = \beta_{0,1} \Lambda \)

\[ \Psi_M(x, b) = \frac{M \sqrt{x \bar{x}}/\pi}{\beta_{0,1} J_1(\beta_{0,1})} J_0(\sqrt{x \bar{x} M} b) \theta(\sqrt{x \bar{x} b} \leq \beta_{0,1}/M) \]
In \( k_\perp \)-representation:

\[
\tilde{\Psi}_M(x, k_\perp) = \frac{M}{\sqrt{\pi x \bar{x}}} \frac{J_0(\beta_{0,1} k_\perp / \sqrt{x \bar{x} M})}{M^2 - k_\perp^2 / x \bar{x}}
\]

Note: No singularity for \( k_\perp^2 / x \bar{x} = M^2 \),
zeros when \( k_\perp^2 / x \bar{x} = (\text{mass})^2 \) of higher state

Oscillates for large \( k_\perp \), magnitude decreases as \( 1/k_\perp^{5/2} \)

Below first zero: \( \sim \exp(-k_\perp^2 / 2x \bar{x} M^2) \)
Lowest state form factor:

\[ F_M(Q^2) = \frac{2M^2}{Q^2\gamma^2} \int_0^{Q/\Lambda} \xi d\xi \mathcal{K}_1(\xi) J_0^2(\xi M/Q) \]

\[(\gamma \equiv \beta_{0,1} J_1(\beta_{0,1}) \approx 1.2)\]

Asymptotic behavior:

\[ F_M(Q^2) = \frac{4M^2}{Q^2\gamma^2} \left[ 1 - \frac{4M^2}{Q^2} + \frac{9}{8} \left( \frac{4M^2}{Q^2} \right)^2 + \mathcal{O}(M^6/Q^6) \right] \]

\[ \sim \frac{0.64}{1 + Q^2/4M^2} \]
Note: Unlike VMD $Q^2/(1 + Q^2/m_{\rho}^2)$ (blue), 
$Q^2 F_M(Q^2)$ (red) is not constant
in accessible region $Q^2 \lesssim 10$ GeV$^2$

Question: What mechanism generates these huge corrections?

Observation: if $\zeta$ is interpreted as $\sqrt{x\bar{x}}b$,
then $\zeta = 0$ may correspond to $x = 1$
Use Drell-Yan formula:

\[ F_M(Q^2) = \int_0^1 dx \int d^2k_\perp \tilde{\Psi}_M^*(x, k_\perp + \bar{x}q_\perp) \tilde{\Psi}_M(x, k_\perp) \]

Two possible asymptotic regimes:

- finite \( x \) and small \( |k_\perp| \), e.g., region \( |k_\perp| \ll \bar{x}|q_\perp| \), where \( \tilde{\Psi}_M(x, k_\perp) \) is maximal. Then

\[ F_M(Q^2) \sim 2 \int_0^1 dx |\tilde{\Psi}_M^*(x, \bar{x}q_\perp) \varphi(x)| \]

\( \Rightarrow \) form factor repeats large-\( k_\perp \) behavior of WF

\( \Rightarrow \) second possibility:

- \( x \) is close to 1, so that \( |\bar{x}q_\perp| \sim |k_\perp| \), and \( |k_\perp| \) is small

\( \Rightarrow \) both \( \tilde{\Psi}_M(x, k_\perp) \) and \( \tilde{\Psi}_M^*(x, k_\perp + \bar{x}q_\perp) \) are maximal.
DY estimate: dominant contribution comes from
$$\bar{x}|q_\perp| \lesssim m = \text{const}$$

$$\Rightarrow$$ large-$Q^2$ behavior of form factor reflects phase space available for such configurations

Dedicated estimate: represent form factor as $x$-integral of GPD

$$\mathcal{F}_M(x, Q^2) = \frac{2}{\beta_{0,1}^2 J_1^2(\beta_{0,1})} \int_0^{\beta_{0,1}} y dy J_0\left(\sqrt{\frac{\bar{x}}{x}} \frac{Q}{M} y\right) J_0^2(y)$$

$$\equiv \mathcal{G}(\sigma), \quad \sigma \equiv \bar{x}Q^2/xM^2$$

$$F_M(Q^2) = \frac{M^2}{Q^2} \int_0^\infty d\sigma \frac{\mathcal{G}(\sigma)}{(1 + \sigma M^2/Q^2)^2}$$

$$\Rightarrow$$ Asymptotic $1/Q^2$ term given by $0^{\text{th}}$ $\sigma$-moment of $\mathcal{G}(\sigma)$
Blue line: $G(\sigma)$. Red line: $e^{-\sigma/3} = e^{-\bar{x}Q^2/3xM^2}$

⇒ comes from $\sigma \lesssim 10 \Rightarrow \bar{x} \lesssim 10M^2/Q^2$

Note: Asymptotic estimate applicable only if $Q^2 \gg 4M^2$

Conclusion: large-$Q^2$ asymptotics of $F_M(Q^2)$ is governed by soft Feynman mechanism
$\Rightarrow$ Power law is determined by $x \rightarrow 1$ behavior of

$$f(x) = \mathcal{F}(x, Q^2 = 0) = \int d^2 b_\perp |\Psi(x, b_\perp)|^2 = 1,$$

$f(x)$ = parton distribution function of the model

Extra $\bar{x}^N \Rightarrow F_M(Q^2) \sim (\Lambda^2/Q^2)^{N+1}$

Question: Is interpretation of

holographic variable $\zeta$ as product

$\sqrt{\bar{x}\bar{x}b}$ of light-cone variables justified?
**MEROMORPHIZATION**

Erlich et al.: some results of holographic approach are reproduced in Migdal’s program

Meromorphization substitutes correlator

\[ \Pi(p^2) = \frac{1}{\pi} \int_0^\infty \frac{\rho(s)}{s - p^2} \, ds \]

by

\[ \Pi_M(p^2) = \Pi(p^2) - \frac{1}{\pi Q(p^2)} \int_0^\infty \frac{\rho(s) Q(s)}{s - p^2} \, ds \]

⇒ cut of \( \Pi(p^2) \) is eliminated

zeros of \( Q(p^2) \) ⇒ poles of \( \Pi_M(p^2) \)

⇒ hadronic bound states

Explicit Padé construction: \( Q(p^2) \Rightarrow J_0(\beta_{0,1} \sqrt{p^2/M}) \)

for \( j = \varphi \varphi \) and \( j_\mu = \bar{\psi}\gamma_\mu \psi, j_{5\mu} = \bar{\psi}\gamma_5\gamma_\mu \psi \)
Coupling of the lowest state

\[ f_M^2 = \lim_{p^2 \to M^2} (M^2 - p^2) \Pi_M(p^2) = \frac{1}{\pi Q'(M^2)} \int_0^\infty \frac{\rho(s) Q(s)}{s - M^2} \, ds \]

Explicit calculation with \( \rho(s) = \rho_0 \theta(s) \)

\[ f_M^2 = \frac{2\rho_0 M^2}{\pi \beta_{0,1} J_1(\beta_{0,1})} \int_0^\infty \frac{J_0(\beta_{0,1} \sqrt{s}/M)}{M^2 - s} \, ds = \frac{4\rho_0 M^2}{\pi [\beta_{0,1} J_1(\beta_{0,1})]^2} \]

\( \rho_0 = 1/16\pi \) for \( j = \varphi \varphi \), and \( \rho_0 = N_c/12\pi \) for \( \bar{u} \gamma_\mu (\gamma_5) d \)
Dosch et al.: Meromorphize 3-point function to get form factors

Lowest order: triangle diagram has only double spectral density \( \rho(s_1, s_2, Q^2) \) \( \Rightarrow \) build function

\[
\mathcal{T}(p_1^2, p_2^2, Q^2) = T(p_1^2, p_2^2, Q^2)
\]

\[
+ \frac{1}{\pi^2 Q(p_1^2)Q(p_2^2)} \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{\rho(s_1, s_2, Q^2) Q(s_1) Q(s_2)}{(s_1 - p_1^2)(s_2 - p_2^2)}
\]

\( \Rightarrow \) removes cuts & has poles at \( \Pi(p^2) \) locations

Elastic form factor of the lowest state:

\[
f_M^2 F_M(Q^2) = \frac{1}{\pi^2 [Q'(M^2)]^2} \int_0^\infty ds_1 \int_0^\infty ds_2
\]

\[
\times \frac{\rho(s_1, s_2, Q^2) Q(s_1) Q(s_2)}{(s_1 - M^2)(s_2 - M^2)}
\]
Spectral densities: use Cutkosky rules

\[
\rho(s_1, s_2, Q^2) = \rho_0 \int_0^1 dx \frac{n(x)}{xx} \int d^2k_\perp \\
\times \delta \left( s_1 - \frac{k_\perp^2}{xx} \right) \delta \left( s_2 - \frac{(k_\perp + \bar{x}q_\perp)^2}{xx} \right)
\]

\[n(x) = 1 \text{ for } j = \varphi \varphi , \ j^\mu = i\varphi \overset{\leftrightarrow}{\partial^\mu} \varphi\]
Form factor:

$$f_M^2 F_M^{\text{scalar}}(Q^2) = \frac{\rho_0}{\pi^2 [Q'(M^2)]^2} \int_0^1 \frac{dx}{x \bar{x}} \int d^2 k_\perp$$

$$\times \frac{Q(k_\perp^2/x \bar{x})}{M^2 - k_\perp^2/x \bar{x}} \frac{Q((k_\perp + \bar{x}q_\perp)^2/x \bar{x})}{M^2 - (k_\perp + \bar{x}q_\perp)^2/x \bar{x}}$$

⇒ has structure of Drell-Yan formula
Using $Q(s) = J_0(\beta_{0,1} \sqrt{s}/M)$ and

$$f_M^2 = \frac{4\rho_0 M^2}{\pi[\beta_{0,1} J_1(\beta_{0,1})]^2}$$

gives

$$\tilde{\Psi}^{\text{scalar}}_M (x, k_\perp) = \frac{M}{\sqrt{\pi xx}} \frac{J_0(\beta_{0,1} k_\perp/\sqrt{x\bar{x}} M)}{M^2 - k^2_\perp/xx}$$

⇒ Same expression as in holographic model

⇒

Meromorphization supports interpretation of $\zeta$

in terms of light-cone variables $x, b$
Spinor case: vector currents \( j_\alpha, j_\beta \) for hadronized vertices

\[ \Rightarrow \] tensor amplitude \( T^\mu_{\alpha\beta}(p_1, p_2) \)

\[ \Rightarrow \] choose tensor structure to meromorphize

Simplest projection: \( T^\mu_{\alpha\beta} n_\mu n^\alpha n^\beta \), with \( n^2 = 0 \), \( (nq) = 0 \)

picks out
\[ F_1(Q^2) + \kappa F_2(Q^2) - \kappa^2 F_3(Q^2), \quad \kappa \equiv Q^2/2m_\rho^2 \]

Compare to leading pQCD:

\[ F_{LL}(Q^2) = F_1(Q^2) - \kappa F_2(Q^2) + (\kappa^2 + 2\kappa) F_3(Q^2) \]

(expects \( F_1(Q^2) \sim F_2(Q^2) \sim 1/Q^4 \) and \( F_3(Q^2) \sim 1/Q^6 \))

For this projection: \( n(x) = 6x\bar{x} \)
Extra $\bar{x}$ factor in GPD $\mathcal{F}(x, Q^2)$ should result in $1/Q^4$ asymptotics.

Note: Extra $6x\bar{x}$ gives

$$6 \int_0^1 dx \, xx \, J_0 \left( \sqrt{\frac{1-x}{x}} \, z \, Q \right)$$

$$= \frac{3}{2} z^2 Q^2 K_2(zQ) - \frac{1}{4} z^3 Q^3 K_3(zQ) \equiv \mathcal{K}_2(zQ)$$

instead of nonnormalizable mode $\mathcal{K}_1(zQ) = zQ K_1(zQ)$

Still, $\mathcal{K}_2(\xi) \sim e^{-\xi}$, and $z \sim 1/Q$ dominates for large $Q$

⇒ Apparently, we should get $1/Q^2$ again!
Resolution of paradox:

\[ F_M^{\text{spinor}}(Q^2) = \frac{2M^2}{Q^2 \gamma^2} \int_0^\infty d\xi \, \xi \mathcal{K}_2(\xi) \left[ 1 - \frac{1}{2} \xi^2 \frac{M^2}{Q^2} + \frac{3}{32} \xi^4 \frac{M^4}{Q^4} \right. \]

\[ - \frac{5}{576} \xi^6 \frac{M^6}{Q^6} + \mathcal{O}(M^8/Q^8) \bigg] \]

\[ = \frac{2M^2}{Q^2 \gamma^2} \left[ 0 + 24 \frac{M^2}{Q^2} - 288 \frac{M^4}{Q^4} + 2400 \frac{M^6}{Q^6} + \mathcal{O}(M^8/Q^8) \right] \]

⇒ First term vanishes because \( \int_0^\infty d\xi \, \xi \mathcal{K}_2(\xi) = 0 \)

⇒ leading term has \( 1/Q^4 \) behavior

But: \( 1/Q^6 \) correction exceeds it up to \( Q^2 = 12M^2 \sim 7 \text{ GeV}^2 \)

⇒ \( 1/Q^4 \) asymptotics establishes above \( 20 \text{ GeV}^2 \)
Now, delayed $1/Q^4$ asymptotics is good news!

Here: VMD-like $Q^2/(1 + Q^2/m_p^2)$, holographic (scalar meromorphization) $Q^2 F_M(Q^2)$ spinor meromorphization $Q^2 F_M^{\text{spinor}}(Q^2)$

$\Rightarrow$ In the region of a few GeV$^2$ $F_M^{\text{spinor}}(Q^2)$ shows “power counting” $1/Q^2$ behavior more successfully than $F_M(Q^2)$ that displays its nominal $1/Q^2$ asymptotics outside the few GeV$^2$ region.
Miraculous cancellation of moments for $K_2(\xi)$ and $K_3(\xi)$ producing $F_M^{\text{spinor}}(Q^2) \sim M^4/Q^4$ can be traced to 1/$Q^4$ asymptotic behavior of double spectral density $\rho(s_1, s_2, Q^2)$.

Consider double Borel transform of 3-point function

$$
\Phi(\tau_1, \tau_2, Q^2) = \frac{1}{\pi^2} \int_0^\infty ds_1 \int_0^\infty ds_2 \rho(s_1, s_2, Q^2) e^{-s_1 \tau_1 - s_2 \tau_2}
$$

For triangle diagram

$$
\Phi(\tau_1, \tau_2, Q^2) = \frac{N_c}{2\pi^2(\tau_1 + \tau_2)} \int_0^1 dx \, x \bar{x} \exp \left[ -Q^2 \frac{\bar{x} \, \tau_1 \tau_2}{x(\tau_1 + \tau_2)} \right]
$$

Note: $x \bar{x}$ factor (absent for scalar quarks)

$\Rightarrow$ 1/$Q^4$ behavior of $\Phi(\tau_1, \tau_2, Q^2)$

$\Rightarrow$ 1/$Q^4$ behavior of spectral density

$$
\rho(s_1, s_2, Q^2) = N_c \theta(s_1) \theta(s_2) (s_1 + s_2)/2Q^4 + \ldots
$$
LOCAL QUARK-HADRON DUALITY

Combines, in simplified form, some ideas of Migdal’s program and QCD sum rule approach

QCD Sum Rules: only lowest state is narrow
⇒ model spectrum: first resonance plus perturbative “continuum” from $s = s_0$
⇒ Transform correlator

$$\Pi(p^2) = \frac{1}{\pi} \int_{0}^{\infty} ds \frac{\rho_{\text{pert}}(s)}{s - p^2}$$

into

$$\Pi^{\text{LD}}(p^2) = \frac{F_M^2}{M^2 - p^2} + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\rho_{\text{pert}}(s)}{s - p^2} ds$$

Then: try to reach the best possible agreement
in deep spacelike region $p^2 \equiv -P^2$
For axial currents: \( F_M \to f_\pi, \quad M \to m_\pi \approx 0, \quad \rho^{\text{pert}}(s) = 1/4\pi \)

\[
\Pi(p^2) - \Pi^{\text{LD}}(p^2) = \frac{f_\pi^2}{p^2} + \frac{1}{4\pi^2} \int_0^{s_0} ds \frac{\rho^{\text{pert}}(s)}{s - p^2}
\]

Eliminate leading \( 1/p^2 \) term \( \Rightarrow \) \( f_\pi^2 = s_0/4\pi^2 \)

\( \Rightarrow \) Local duality relation

\[
\int_0^{s_0} \rho_\pi(s) \, ds = \int_0^{s_0} \rho^{\text{pert}}(s) \, ds
\]
Three-point function ⇒ local duality relation for pion form factor

\[ f_\pi^2 F^{\text{LD}}_\pi(Q^2) = \frac{1}{\pi^2} \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 \rho^{\text{pert}}(s_1, s_2, Q^2) \]

⇒ Drell-Yan-type formula

\[ F^{\text{LD}}_\pi(Q^2) = \frac{6}{\pi s_0} \int_0^1 dx \int d^2k_\perp \]

\[ \times \theta \left( k^2_\perp \leq x\bar{x}s_0 \right) \theta \left( (k_\perp + \bar{x}q_\perp)^2 \leq x\bar{x}s_0 \right) \]

with effective local duality wave function

\[ \tilde{\Psi}^{\text{LD}}_\pi(x, k_\perp) = \sqrt{6/\pi s_0} \theta(k^2_\perp \leq x\bar{x}s_0) \]

In the impact parameter representation

\[ \Psi^{\text{LD}}_\pi(x, b_\perp) = \sqrt{6x\bar{x}/\pi} J_1(b\sqrt{x\bar{x}s_0})/b \]

\( b \to 0 \) limit: “LD” distribution amplitude \( \varphi^{\text{LD}}_\pi(x) = 6f_\pi xx \) coincides with asymptotic pion DA.
“Holographize” $\Psi_{\pi}^{LD}(x, b_\perp)$ by cut-off $\theta(b\sqrt{x x_s} \leq \beta_{1,1})$

First zero of $\tilde{\Psi}_{\pi}^{LD}(x, k_\perp)$ is located at $k_\perp^2 / x x_s \approx (1.26 \text{ GeV})^2$, “unexpectedly close” to $A_1$ position.
Explicit expression for LD form factor:

\[ F_{\pi}^{LD}(Q^2) = 1 - \frac{1 + 6s_0/Q^2}{(1 + 4s_0/Q^2)^{3/2}} \]

\[
= \frac{6s_0^2}{Q^4} - \frac{40s_0^3}{Q^6} + \frac{210s_0^4}{Q^8} - \frac{1008s_0^4}{Q^8} + \mathcal{O}(s_0^5/Q^{10})
\]

As expected, has \(1/Q^4\) asymptotics

But: \(1/Q^4\) estimate should not be used at accessible \(Q^2\)

At accessible \(Q^2\), successfully imitates \(1/Q^2\) behavior

and is close to data
Right panel: Pion form factor in local quark-hadron duality model

\( Q^2 F^\text{LD}_\pi \): lowest-order, (green)

\( Q^2 F^\text{LD}_\pi(\alpha_s) \): \( \mathcal{O}(\alpha_s) \) correction (red)

total contribution (blue)
Higher orders and transition to pQCD: include higher order $\alpha_s$ corrections to spectral densities

There appear gluon-exchange diagrams with large-$Q^2$ behavior determined by hard pQCD mechanism
As a result:

\[
\rho_{s}(s_1, s_2, Q^2) = 2\pi\alpha_s \frac{C_F}{N_c} \int_0^1 dx \int_0^1 dy \frac{\rho(x, s_1)\rho(y, s_2)}{xyQ^2}
\]

\[+\mathcal{O}(1/Q^4)\]

\(\rho(x, s_1)\): \(x\)-unintegrated 2-point spectral density,

\[
\rho(x, s_1) = \frac{N_c}{2\pi^2} \int \delta\left(s_1 - \frac{k_{1\perp}^2}{x\bar{x}}\right) d^2k_{1\perp} = \frac{N_c}{2\pi} \theta(s_1) x\bar{x}
\]

Its integral over duality interval \(0 \leq s \leq s_0\) gives \(\varphi^{LD}_{\pi}(x) = 6f_\pi x\bar{x}\) for the pion DA

\(\Rightarrow\) Local duality at \(\mathcal{O}(\alpha_s)\) order gives pQCD result

\[
F^{pQCD}_\pi(Q^2) = 8\pi\alpha_s f_\pi^2 / Q^2
\]

calculated for asymptotic shape of pion DA
Writing

\[ F^\text{pQCD}_\pi(Q^2) = 2(s_0/Q^2)(\alpha_s/\pi) \]

reveals its nature as \( \alpha_s \) correction to soft contribution

Note: \( F^{\text{LD}(\alpha_s)}_\pi(0) = \alpha_s/\pi \) from Ward identity

\( \Rightarrow \) Interpolation:

\[ F^{\text{LD}(\alpha_s)}_\pi(Q^2) = (\alpha_s/\pi)/(1 + Q^2/2s_0) \]

(very accurate)
**SUMMARY**

Studied large-$Q^2$ behavior of meson form factor $F_M(Q^2)$ constructed using holographic model

Observed that, despite its $1/Q^2$ asymptotic behavior, combination $Q^2 F_M(Q^2)$ is not flat in accessible region $Q^2 \lesssim 10$ GeV$^2$

Found that asymptotic $1/Q^2$ result for $F_M(Q^2)$ is governed by Feynman mechanism

Using (scalar) meromorphization approach reproduced wave functions of holographic model

For spin-$1/2$ quarks, demonstrated that extra $(1 - x)$ factor, results in $F_M^{\text{spinor}}(Q^2) \sim 1/Q^4$ asymptotic behavior

Due to late onset of the asymptotic pattern $Q^2 F_M^{\text{spinor}}(Q^2)$ is flat in few GeV$^2$ region
Presented results for pion form factor in local quark-hadron duality model: the lowest-order term again has nominally $1/Q^4$ asymptotics, but it imitates $1/Q^2$ behavior in the few GeV$^2$ region.

Showed that $\mathcal{O}(\alpha_s)$ term of $\rho(s_1, s_2, Q^2)$ brings in hard pQCD contribution having the dimensional counting $1/Q^2$ behavior.

Argued that at accessible $Q^2$, the $\mathcal{O}(\alpha_s)$ term is a small fraction of total result, because of small $\alpha_s/\pi \sim 0.1$ factor associated with each higher order correction.

Did not discuss nucleons form factors. But want to mention that the lowest-order local duality result for $G_M^p(Q^2)$ closely follows the dipole shape of the data up to $Q^2 \sim 15$ GeV$^2$. 
Local duality predictions is shown by dashed lines
CONCLUSIONS

Power of \((1 - x)\) in perturbative versions of relevant parton densities \(f(x)\) increases with number of quarks like \(f_n(x) \sim (1 - x)^{n-1}\)

\(\Rightarrow\) Probability that total momentum of \(n - 1\) spectators is \(x_{sp}\) goes like \(x_{sp}^{n-1}\), which looks OK

Feynman mechanism then gives \((1/Q^2)^n\) asymptotic behavior for form factors

Because of late onset of asymptotic regime, form factors imitate \((1/Q^2)^{n-1}\) behavior in a rather wide preasymptotic region

In this scenario, quark counting rules (if any) is approximate and transitional phenomenon dominated by nonperturbative, long-distance aspects of hadron dynamics