Hadron Optics : Diffraction Patterns in Deeply Virtual Compton Scattering

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- Light-front wave functions and DVCS
- Simple example : electron at one loop
- Fourier Transform; DVCS amplitude in σ space
- Simulated bound state calculation

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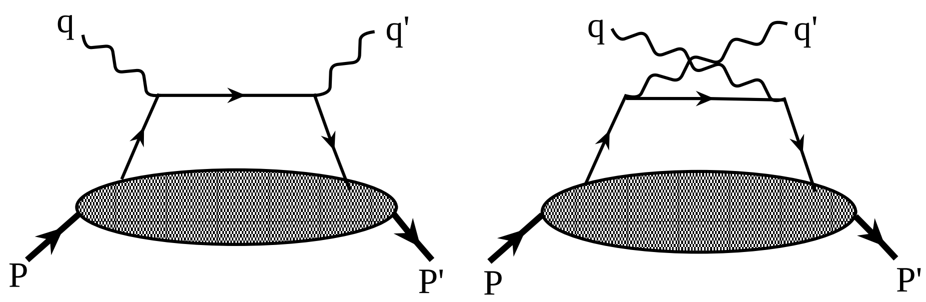
Deeply Virtual Compton Scattering (DVCS) Amplitude

Deeply virtual Compton scattering :



Incident photon highly virtual, final photon real; momentum transfer $\Delta = P - P'$, $t = \Delta^2$

Momenta of initial and final proton :

$$P = \left(P^+ \ , \ \vec{0}_\perp \ , \ \frac{M^2}{P^+} \right) \ , P' = \left((1-\zeta)P^+ \ , \ -\vec{\Delta}_\perp \ , \ \frac{M^2 + \vec{\Delta}_\perp^2}{(1-\zeta)P^+} \right)$$

$$t \ = \ 2P \cdot \Delta \ = \ -\frac{\zeta^2 M^2 + \vec{\Delta}_\perp^2}{1-\zeta}, \qquad \frac{Q^2}{2P \cdot q} = \zeta$$

ζ : skewness variable

For DVCS, $-q^2 = Q^2$ is large compared to the masses and $\mid t \mid$

Choose a frame where the incident space-like photon carries $q^+ = 0$

DVCS (contd.)

DVCS amplitude

$$M^{IJ}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) = \epsilon^I_\mu \, \epsilon^{*J}_\nu \, M^{\mu\nu}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) = -e_q^2 \, \frac{1}{2\bar{P}^+} \int_{\zeta-1}^{1} \mathrm{d}x$$

$$\times \quad \left\{ t^{IJ}(x,\zeta) \, \bar{U}(P') \left[H(x,\zeta,t) \, \gamma^+ + E(x,\zeta,t) \, \frac{i}{2M} \, \sigma^{+\alpha}(-\Delta_\alpha) \right] U(P) \right\},$$

where $\bar{P} = \frac{1}{2}(P' + P)$,

Do not consider $\frac{1}{Q}$ contributions ; x is the fraction of the proton momentum carried by the active quark

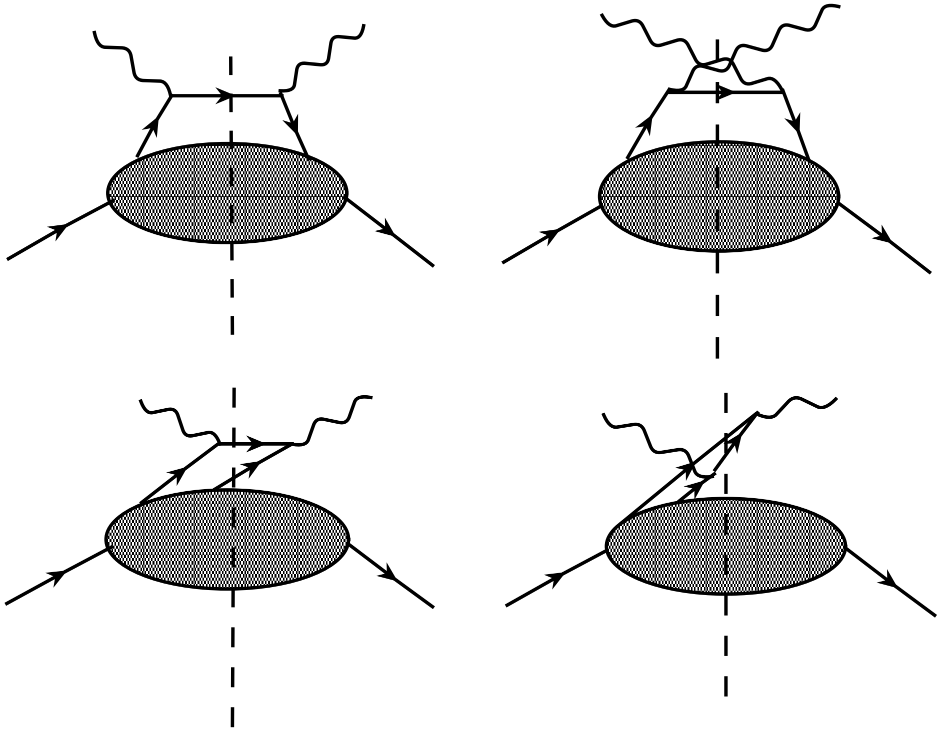
For circularly polarized initial and final photons

$$t^{\uparrow\uparrow}(x,\zeta) = \quad t^{\downarrow\downarrow}(x,\zeta) = \frac{1}{x - i\epsilon} + \frac{1}{x - \zeta + i\epsilon}$$

$$F_{\lambda,\lambda'} = \int \frac{dy^-}{8\pi} \, e^{ixP^+y^-/2} \, \langle P', \lambda' | \bar{\psi}(0) \, \gamma^+ \, \psi(y) \, | P, \lambda \rangle \Big|_{y^+=0, y_\perp=0}$$

$$= \quad \frac{1}{2\bar{P}^+} \, \bar{U}(P', \lambda') \left[H(x,\zeta,t) \, \gamma^+ + E(x,\zeta,t) \, \frac{i}{2M} \, \sigma^{+\alpha}(-\Delta_\alpha) \right] U(P, \lambda),$$

DVCS (contd.)



Light-front time ordered contributions to DVCS at leading twist

Overlap Representation

• The target state is expanded in terms of multiparticle light-front wave functions in Fock space; choose light-front gauge

DVCS amplitude is given in terms of overlaps of the light-front wave functions

Diehl, Feldman, Jacob, Kroll (2001);
Brodsky, Diehl, Huang (2001)

• Diagonal parton number conserving $n \to n$ overlap in the kinematical regime $\zeta < x < 1$ and $\zeta - 1 < x < 0$

Off-diagonal $n+1 \to n-1$ overlap for $0 < x < \zeta$ where the parton number is decreased by two.

• Consider a dressed electron state instead of a proton

State is expanded in Fock space : $\mid e^- \gamma \rangle$ and $\mid e^- e^- e^+ \rangle$ contribute to $O(\alpha)$

• Generalized form of QED : mass M to the external electrons, m to the internal electron lines λ to the internal photon lines $\to$ composite fermion state with mass M : a fermion and a vector ‘diquark" constituents

Brodsky, Drell (1980)

• Two and three particle LFWFs are systematically evaluated in perturbation theory

In the kinematical region $\zeta < x < 1$ one has $2 \to 2$ contribution

$$
\begin{aligned}
F^{22}_{++} &= \frac{\sqrt{1-\zeta}}{1-\frac{\zeta}{2}}\, H_{(2\to 2)}(x,\zeta,t) \;-\; \frac{\zeta^2}{4(1-\frac{\zeta}{2})\sqrt{1-\zeta}} E_{(2\to 2)}(x,\zeta,t) \\
&= \int \frac{\mathrm{d}^2\vec{k}_\perp}{16\pi^3} \Big[\psi^{\uparrow *}_{+\frac{1}{2}+1}(x',\vec{k}'_\perp)\psi^{\uparrow}_{+\frac{1}{2}+1}(x,\vec{k}_\perp) + \psi^{\uparrow *}_{+\frac{1}{2}-1}(x',\vec{k}'_\perp)\psi^{\uparrow}_{+\frac{1}{2}-1}(x,\vec{k}_\perp) \\
&\qquad + \psi^{\uparrow\ *}_{-\frac{1}{2}+1}(x',\vec{k}'_\perp)\psi^{\uparrow}_{-\frac{1}{2}+1}(x,\vec{k}_\perp) \Big],
\end{aligned}
$$

$$
\begin{aligned}
F^{22}_{+-} &= \frac{1}{\sqrt{1-\zeta}} \frac{(\Delta^1 - i\Delta^2)}{2M} E_{(2\to 2)}(x,\zeta,t) \\
&= \int \frac{\mathrm{d}^2\vec{k}_\perp}{16\pi^3} \Big[\psi^{\uparrow *}_{+\frac{1}{2}-1}(x',\vec{k}'_\perp)\psi^{\downarrow}_{+\frac{1}{2}-1}(x,\vec{k}_\perp) + \psi^{\uparrow *}_{-\frac{1}{2}+1}(x',\vec{k}'_\perp)\psi^{\downarrow}_{-\frac{1}{2}+1}(x,\vec{k}_\perp) \Big],
\end{aligned}
$$

where

$$
x' = \frac{x-\zeta}{1-\zeta}, \quad \vec{k}'_\perp = \vec{k}_\perp - \frac{1-x}{1-\zeta}\,\vec{\Delta}_\perp .
$$

DVCS in QED at one loop (contd.)

In the kinematical region $0 < x < \zeta$ contribution comes from $3 \to 1$ overlap :

$$
\begin{aligned}
F^{31}_{++} &= \frac{\sqrt{1-\zeta}}{1-\frac{\zeta}{2}}\, H_{(3\to 1)}(x,\zeta,t) - \frac{\zeta^2}{4(1-\frac{\zeta}{2})\sqrt{1-\zeta}} E_{(3\to 1)}(x,\zeta,t) \\
&= \sqrt{1-\zeta}\int \frac{\mathrm{d}^2\vec{k}_\perp}{16\pi^3}\Big[\psi^{\uparrow}_{+\frac{1}{2}\;+\frac{1}{2}\;-\frac{1}{2}}(x, 1-\zeta, \zeta - x,\; \vec{k}_\perp, -\vec{\Delta}_\perp, \vec{\Delta}_\perp - \vec{k}_\perp) \\
&\qquad + \psi^{\uparrow}_{-\frac{1}{2}\;+\frac{1}{2}\;+\frac{1}{2}}(x, 1-\zeta, \zeta - x, \vec{k}_\perp, -\vec{\Delta}_\perp, \vec{\Delta}_\perp - \vec{k}_\perp)\Big]
\end{aligned}
$$

$$
\begin{aligned}
F^{31}_{+-} &= \frac{1}{\sqrt{1-\zeta}}\,\frac{(\Delta^1 - i\Delta^2)}{2M}\, E_{(3\to 1)}(x,\zeta,t) \\
&= \sqrt{1-\zeta}\int \frac{\mathrm{d}^2\vec{k}_\perp}{16\pi^3}\Big[\psi^{\downarrow}_{+\frac{1}{2}\;+\frac{1}{2}\;-\frac{1}{2}}(x, 1-\zeta, \zeta - x,\; \vec{k}_\perp, -\vec{\Delta}_\perp, \vec{\Delta}_\perp - \vec{k}_\perp) \\
&\qquad + \psi^{\downarrow}_{-\frac{1}{2}\;+\frac{1}{2}\;+\frac{1}{2}}(x, 1-\zeta, \zeta - x, \vec{k}_\perp, -\vec{\Delta}_\perp, \vec{\Delta}_\perp - \vec{k}_\perp)\Big]
\end{aligned}
$$

third region $\zeta - 1 < x < 0$ does not contribute : corresponds to the emission and reabsorption of a positron from the physical electron

Real and Imaginary Parts of the Amplitude

- Real and imaginary parts of the DVCS amplitude are calculated separately using the principal value prescription for the propagator and using the explicit form of the two-and three-particle LFWFs

Imaginary part of the DVCS amplitude :

$$\begin{aligned}\mathrm{Im}[M_{\lambda,\lambda'}](\zeta,\Delta^\perp) &= -i\pi e^2\Big[\int_0^\zeta dx\, F^{31}_{\lambda,\lambda'}\,[\,\delta(x-\zeta)\,]+\int_\zeta^1 dx\, F^{22}_{\lambda,\lambda'}\,[\,\delta(x-\zeta)\,]\Big]\\ &= -i\pi e^2\Big[F^{22}_{\lambda,\lambda'}(x=\zeta,\Delta^\perp)+F^{31}_{\lambda,\lambda'}(x=\zeta,\Delta^\perp)\Big].\end{aligned}$$

Real part of the DVCS amplitude :

$$\begin{aligned}\mathrm{Re}\,[M_{\lambda,\lambda'}]\,(\zeta,\Delta^\perp) &= -e^2\int_\varepsilon^{\zeta-\varepsilon_1} dx\, F^{31}_{\lambda,\lambda'}(x,\zeta,\Delta^\perp)\left[\frac{1}{x}+P\Big(\frac{1}{x-\zeta}\Big)\right]\\ &\quad -e^2\int_{\zeta+\varepsilon_1}^{1-\varepsilon} dx\, F^{22}_{\lambda,\lambda'}(x,\zeta,\Delta^\perp)\left[\frac{1}{x}+P\Big(\frac{1}{x-\zeta}\Big)\right].\end{aligned}$$

Note : $x=0$ is an end point for a dressed electron
We have used cutoffs at $\varepsilon=\varepsilon_1=0.001$ for the numerical calculation; divergence at $x=\zeta$ gets canceled between $2\to 2$ and $3\to 1$ contributions.

Fourier Transform

• In order to obtain the DVCS amplitude in y^- space, we take a Fourier transform in ζ as,

$$A_{++}(\sigma, \Delta^\perp) = \frac{1}{2\pi} \int_{\varepsilon_2}^{1-\varepsilon_2} d\zeta \, e^{i\sigma\zeta} \, M_{++}(\zeta, \Delta^\perp),$$
$$A_{+-}(\sigma, \Delta^\perp) = \frac{1}{2\pi} \int_{\varepsilon_2}^{1-\varepsilon_2} d\zeta \, e^{i\sigma\zeta} \, M_{+-}(\zeta, \Delta^\perp),$$

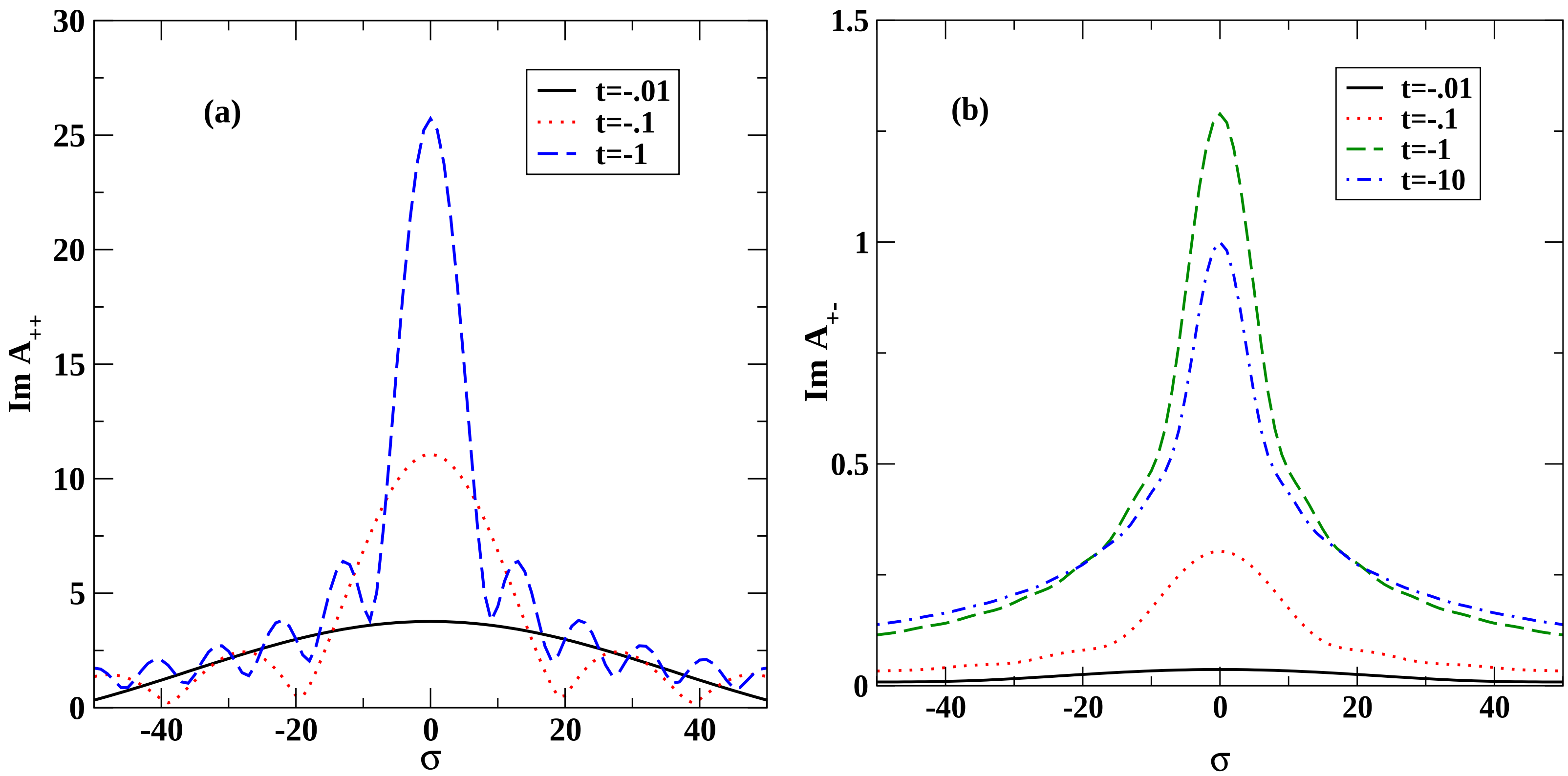
where $\sigma = \frac{1}{2} P^+ y^-$ is the (boost invariant) longitudinal distance on the light cone

We take the cutoff $\varepsilon_2 = 0.002$

• Fourier transforms have been performed by numerically calculating the Fourier sine and cosine transforms and then calculating the resultant by squaring them, adding and taking the square root

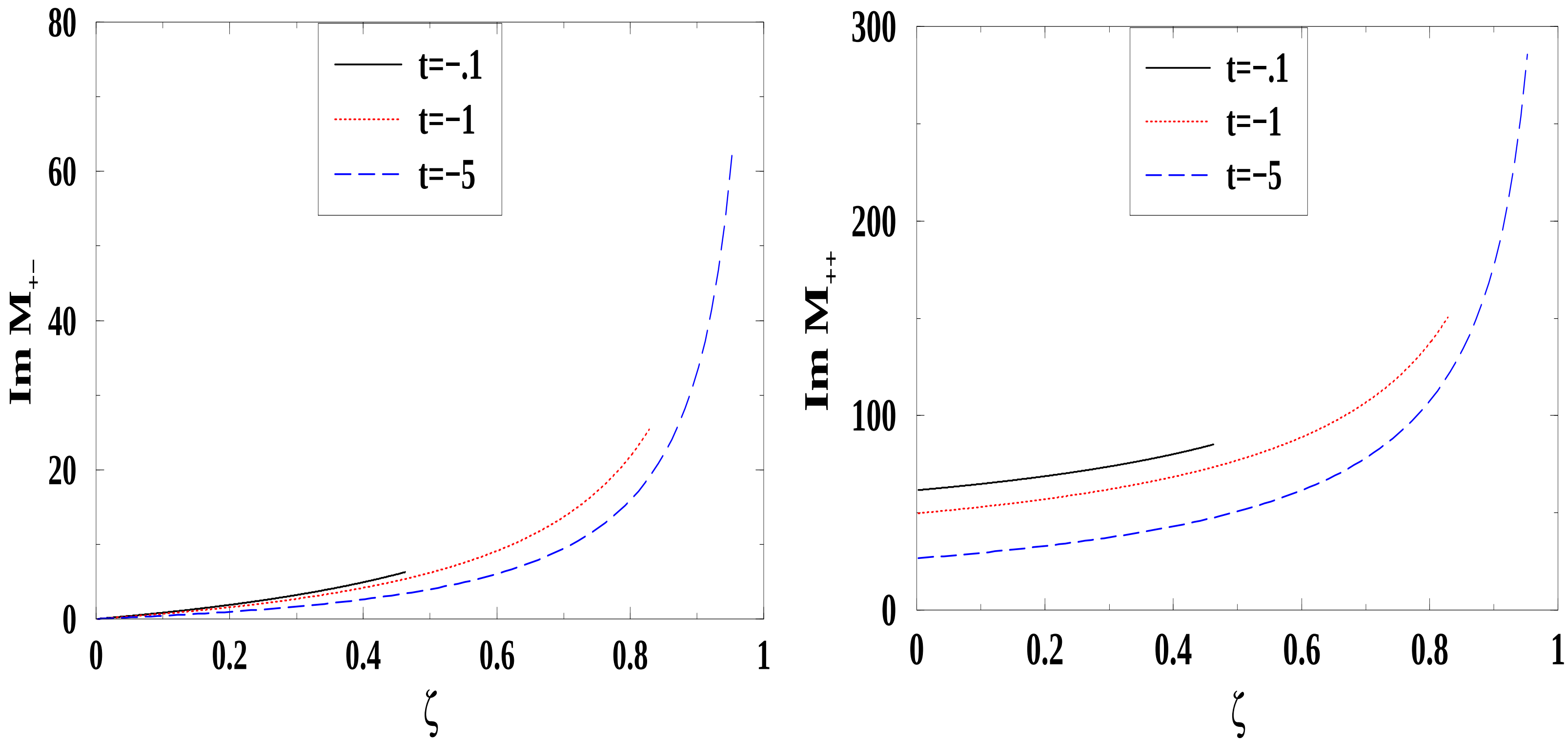
• Helicity non-flip part of the amplitude depends on the scale λ. We took $\Lambda = Q = 10$ MeV.

Imaginary Part of the DVCS Amplitude in σ Space



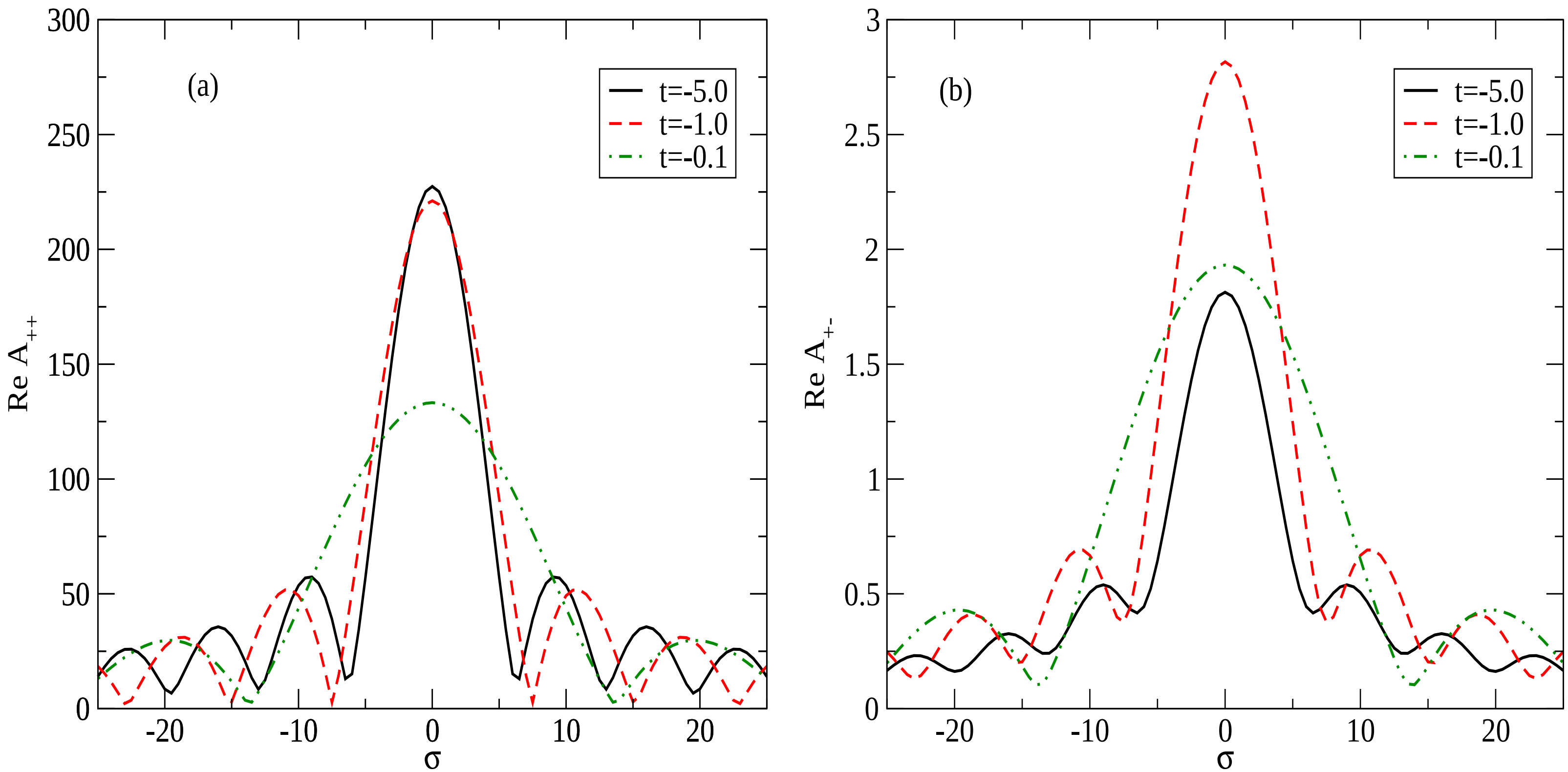
- (a) When the electron helicity is not flipped, (b) helicity is flipped
- $M = 0.5$ MeV, $m = 0.8$ MeV, $\lambda = 0.02$ MeV, t is in MeV^2
- No diffraction pattern in (b), diffraction pattern in σ in (a)

Imaginary Part of the DVCS Amplitude in ζ



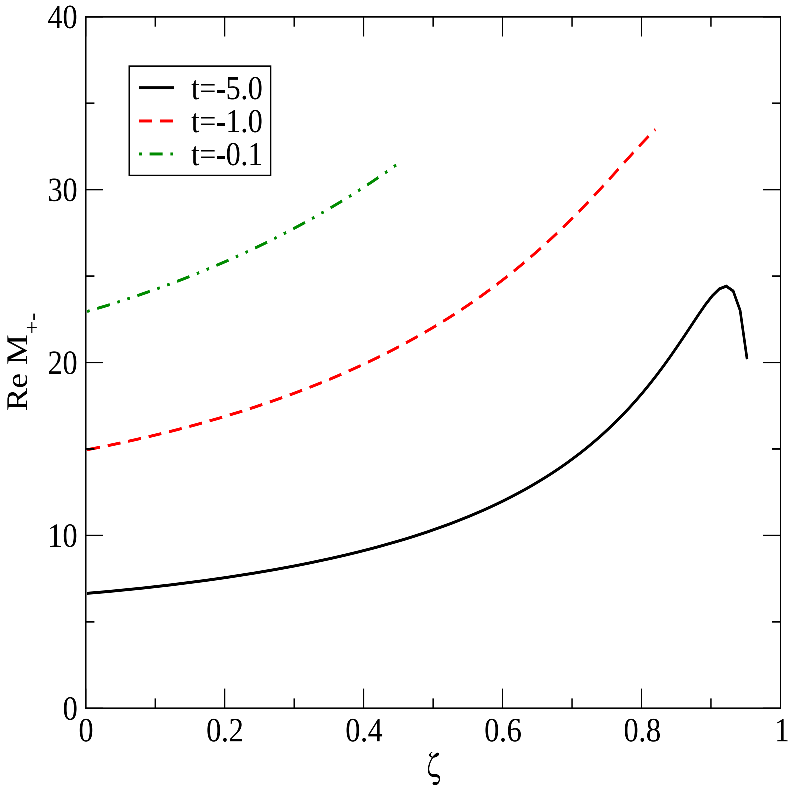
- LHS: When the electron helicity is flipped, RHS: helicity is not flipped
- Imaginary part of the helicity flip amplitude vanishes as $\zeta \longrightarrow 0$
- In both cases, amplitude decreases as $\mid t \mid$ increases for fixed ζ, allowed range of ζ increases

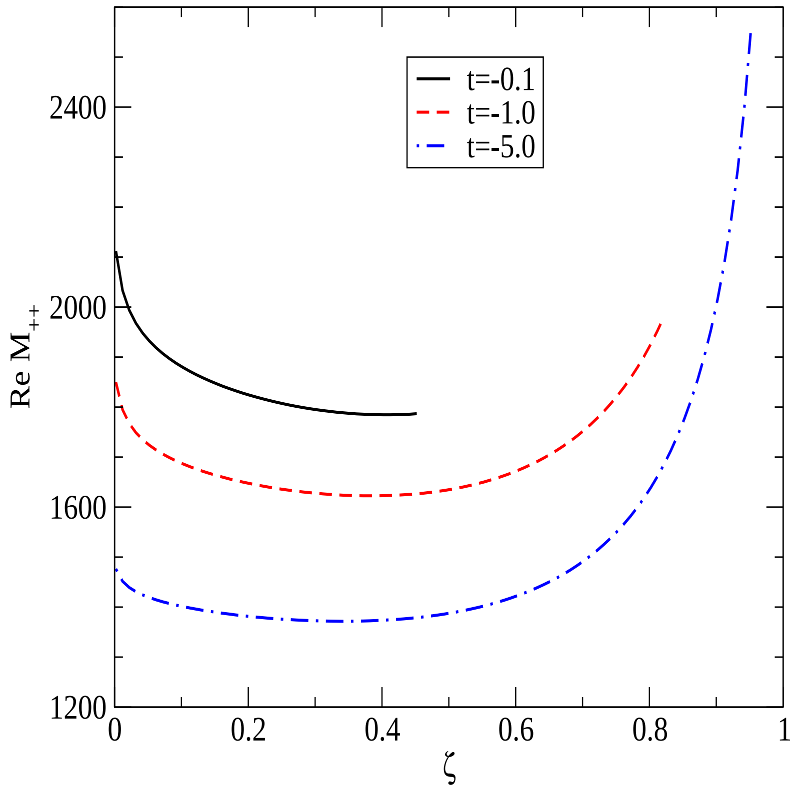
Real Part of the DVCS Amplitude in σ



- (a) When the electron helicity is not flipped, (b) helicity is flipped
- $M = 0.51$ MeV, $m = 0.5$ MeV, $\lambda = 0.02$ MeV, t is in MeV^2
- Diffraction pattern in σ in both cases
- As $\mid t \mid$ increases the first minima move in

Real Part of the DVCS Amplitude in ζ



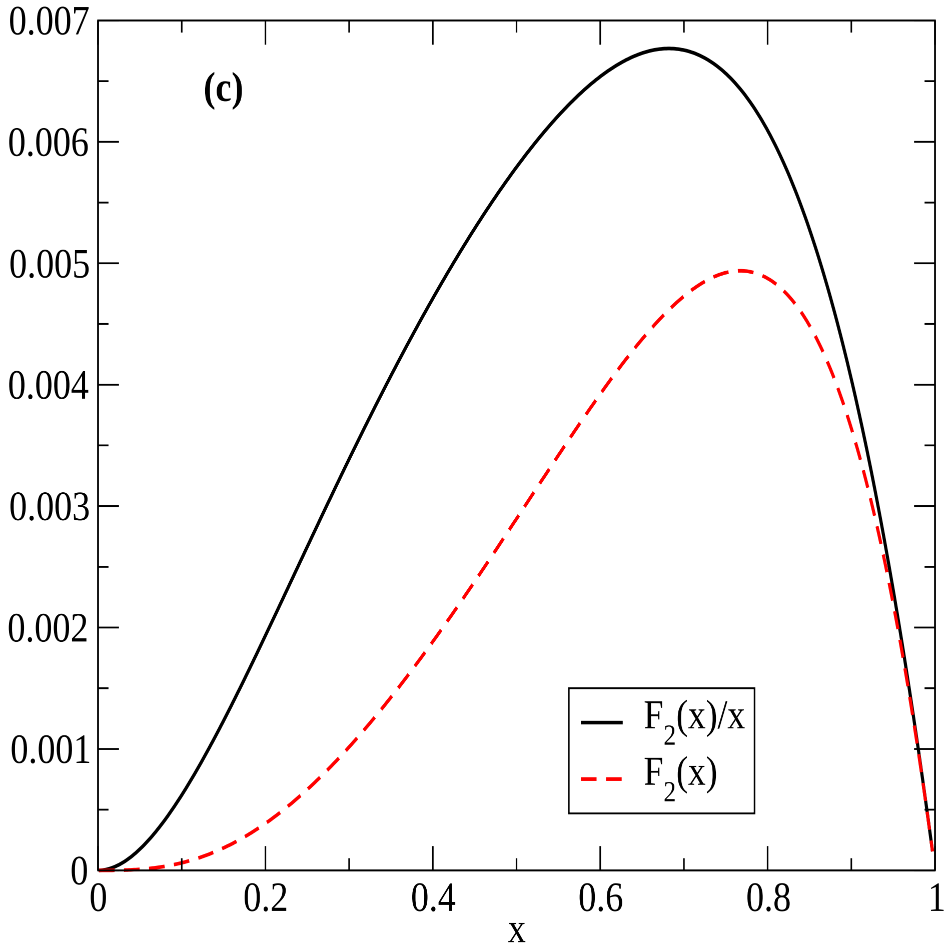


- LHS: When the electron helicity is flipped, RHS: helicity is not flipped
- Both are finite at ζ close to 0
- In both cases, amplitude decreases as $\mid t \mid$ increases for fixed ζ, allowed range of ζ increases

Simulated model for a hadron

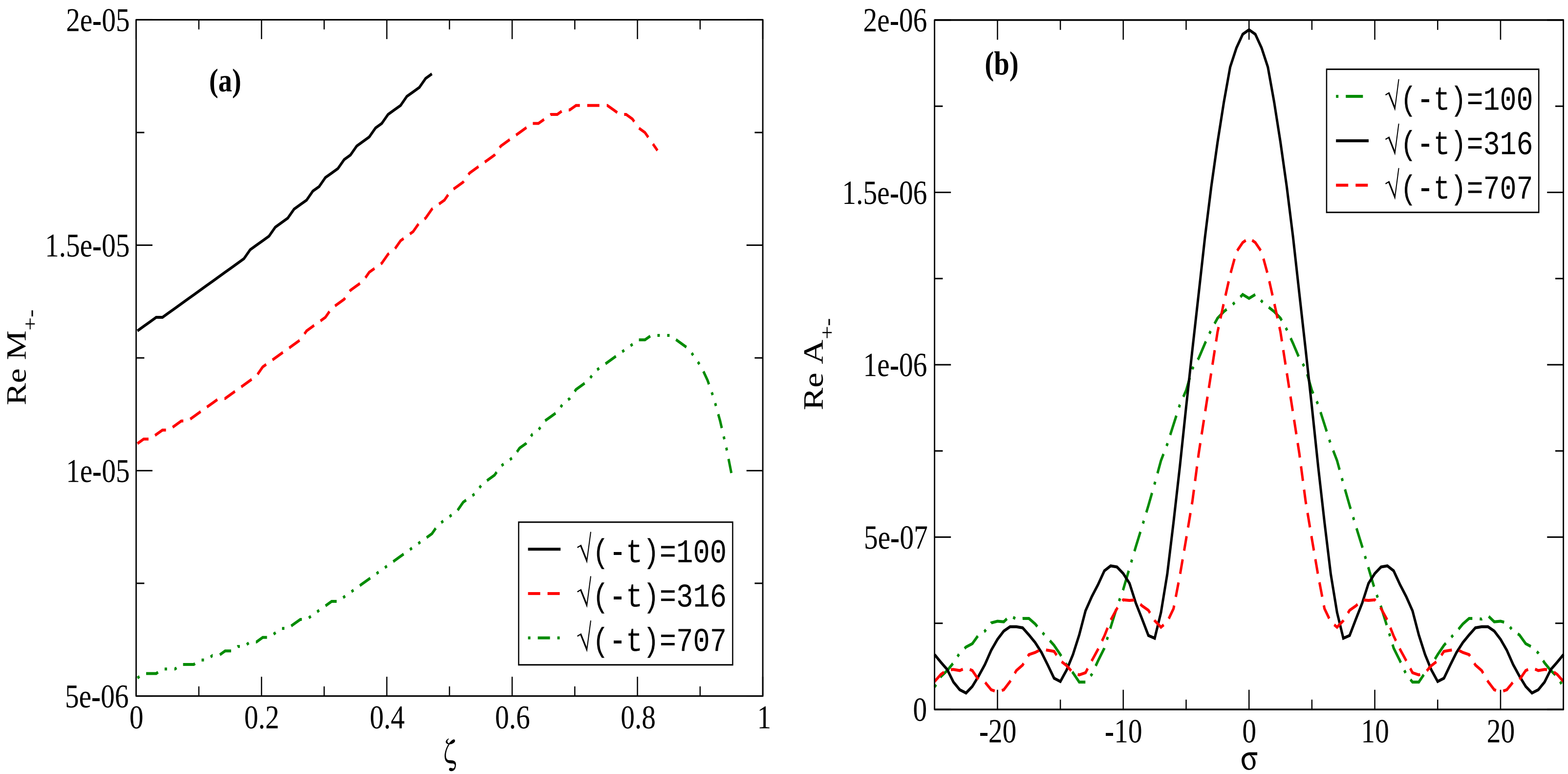
- Dressed electron calculation : used cutoffs at $x = 0, 1$, real part of DVCS amplitude depends on these cutoffs
- Wave function depends on bound state mass square M^2 : differentiation wrt M^2 increases the fall-off of the LFWFs near the end points and simulates the wave functions of a hadron
- Convolutions of these wave functions gives the DVCS amplitude for the simulated hadron model
- Note : Differentiation of the single particle wave function gives zero : so the $3 \longrightarrow 1$ overlap vanishes in this model
- 2 particle wave function is normalized to 1
- The equivalent but easier way is to differentiate the DVCS amplitude with respect to the initial and final state masses.

Simulated model for a hadron : Structure Function



- Mass parameters $M = 150, m = \lambda = 300$ MeV

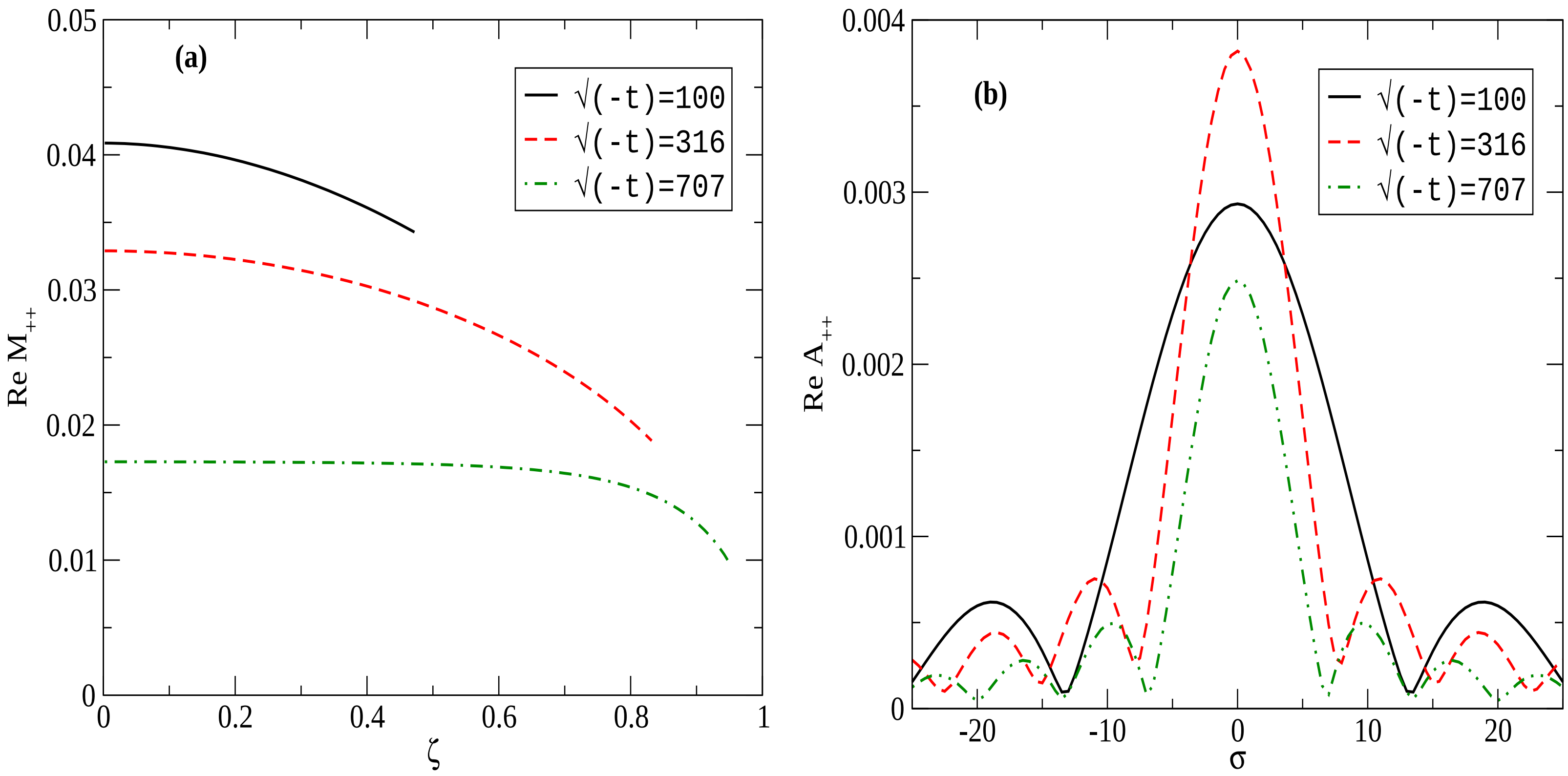
Helicity Flip DVCS Amplitude in the Simulated Model



• Differentiation wrt M^2 brings an extra factor of $x - \zeta$ in the numerator : imaginary part of the DVCS amplitude vanishes in this model

• This can never happen in forward virtual Compton scattering amplitude : imaginary part is still non zero there and gives the structure function

Helicity Non-flip DVCS Amplitude in the Simulated Model



- Mass parameters $M = 150, m = \lambda = 300$ MeV
- Helicity non-flip amplitude decreases as ζ increases for fixed $\mid t \mid$
- Diffraction pattern in σ

Optics Analog

- In DVCS, we are effectively looking at the interference pattern between the plane wave of the initial virtual photon and the plane wave of the outgoing real photon. The final-state proton wavefunction is modified relative to the incident proton wavefunction because of the momentum transferred to the quark in the hard Compton scattering. The change in quark momentum along the longitudinal direction ζ can be Fourier transformed to a boost-invariant distribution in the longitudinal light-front coordinate $\sigma = \frac{1}{2} y^- P^+$.

- In the case of the optical diffraction pattern obtained in a single-slit experiment, the size of the central maximum is inversely proportional to the width of the slit. Deeply virtual Compton scattering is analogous to the diffractive scattering of an electromagnetic wave in optics, where the diffractive pattern in σ reflects the size of the scattering center in units of the target's Compton scale.

- We can study the diffraction pattern in σ as function of t or $\Delta_\perp^2$ in order to register the effect of a change in transverse momentum resulting from the Compton scattering. In the case of deeply virtual Compton scattering on the quantum fluctuations of a lepton target in QED, and also in the corresponding hadronic model, one sees that the diffractive patterns in σ sharpen and the positions of the first minima typically move in with increasing momentum transfer. Thus the invariant longitudinal size of the parton distribution becomes longer and the shape of the conjugate light-cone momentum distribution becomes narrower with increasing $\mid t \mid$.

Brodsky, Chakrabarti, Harindranath, AM, Vary, hep-ph/0604262

Summary and Conclusions

• Fourier transform of the Deeply Virtual Compton Scattering (DVCS) amplitude with respect to the skewness variable ζ provides a unique way to visualize the light-front wavefunctions (LFWFs) of the target state in the boost-invariant longitudinal coordinate space variable ($\sigma = \frac{P^+ y^-}{2}$).

• As a specific example, we consider a fermion state at one loop in QED. We then simulate the wavefunction for a hadron by differentiating the above LFWFs with respect to M^2 and study the corresponding DVCS amplitudes in σ space.

• Results are analogous to the diffractive scattering of a wave in optics in which the dependence of the amplitude on σ measures the physical size of the scattering center of a one-dimensional system.

• If one combines this longitudinal transform with the Fourier transform of the DVCS amplitude with respect to the transverse momentum transfer $\Delta^\perp$, one can obtain a complete three-dimensional description of hadron optics at fixed light-front time $\tau = t + z/c$.