REGULATING QCD
BOUND-STATE CALCULATIONS

OUTLINE

TO USEFULLY SOLVE $P^-|s\rangle = M^2|s\rangle$ WE MUST

- REGULATE AND RENORMALIZE
- INCLUDE THE INDUCED OPERATORS

OUTLINE

REGULATION

- GENERAL PHILOSOPHY
- ST. PETERSBURG REGULATORS

SAMPLE EQUATION

TAKING THE LIMITS

POSSIBLE IMPROVEMENTS

INDUCED OPERATORS

SUMMARY

REGULARIZATION AND RENOMALIZATION

GENERAL IDEA

- ADD PAULI-VILLARS FIELDS TO REGULARIZE
- PRESERVE AS MANY SYMMETRIES AS POSSIBLE
- ADD COUNTER TERMS IF NECESSARY
- GIVES A FINITE, "SYMMETRIC" TARGET

NOW TRUNCATE THE REPRESENTATION SPACE

- NOW SOLVE $P^-|s\rangle = M^2|s\rangle$
- WILL BREAK THE SYMMETRIES
- QUESTION IS MORE ACCURACY THAN SYMMETRY
- UNCANCELLED DIVERGENCES $\rightarrow$ Λ FINITE

ST. P. REGULATORS

$$L = -\frac{1}{4}\sum_{j=0,1}(-1)^j f_j^{a,\mu\nu}\left(1+\frac{\partial_\parallel^2}{\Lambda_j^2}-\frac{\partial_\perp^2}{\Lambda^2}\right) f^a_{j,\mu\nu} + c_3 f^{abc}A^a_\mu A^b_\nu \partial^\mu A^{c,\nu}$$
$$+\sum_{l=0}^{3}\frac{1}{v_l}\overline{\psi}_l\left(i\gamma^\mu\partial_\mu - M_l\right)\psi_l + c_9 A^a_\mu\overline{\psi}\gamma^\mu\frac{\lambda^a}{2}\psi$$
$$+A^a_\mu A^b_\nu A^c_\gamma A^d_\delta (c_4 f^{abe}f^{cde}g^{\mu\gamma}g^{\nu\delta}+\delta^{ab}\delta^{cd}\left(c_5 g^{\mu\gamma}g^{\nu\delta}+c_6 g^{\mu\nu}g^{\gamma\delta})\right)$$

$$f_{j,\mu\nu} = \partial_\mu A_{j,\nu} - \partial_\nu A_{j,\mu}, \quad v_0 = 1, \quad \sum_{l=0}^{3} v_l = 0, \quad \sum_{l=0}^{3} v_l M_l = 0, \quad \sum_{l=0}^{3} v_l M_l^2 = 0,$$

$$A_\mu = A_{0,\mu} + A_{1,\mu}, \quad \psi = \sum_{l=0}^{3}\psi_l, \quad \frac{1}{\Lambda_j^2} = \begin{cases} 1/\Lambda^2, & j=0, \\ 1/\Lambda^2 + 1/\mu^2, & j=1, \end{cases},$$

ALSO: $|p_-| \geq \epsilon$; $p_\perp^2 \geq \mu^2$; $\{\epsilon \to 0\,;\ \mu \to 0\,;\ M_l \to \infty\}$

PERTURBATIVE CALCULATIONS: $\Lambda \to \infty$

NONPERTURBATIVE CALCULATIONS: Λ FINITE

ALGEBRA GIVES NEGATIVE NORMED STATES

A^- IS A DEGREE OF FREEDOM

NO FOUR POINT INTERACTIONS

COUNTER TERMS

$$
\begin{aligned}
& c_0\partial_\mu A^a_\nu \partial^\mu A^{a,\nu} + c_{01}\partial_\mu A^a_\nu \, N^{\mu\alpha}\, \partial_\alpha A^{a,\nu} + c_1 \partial_\mu A^{a,\mu}\, \partial_\nu A^{a,\nu} + c_{11}\, N^{\mu\alpha}\, \partial_\mu A^a_\alpha \, \partial_\nu A^{a,\nu} \\
& + c_{12}\, N^{\mu\alpha}\, \partial_\mu A^a_\alpha \, N^{\nu\beta}\, \partial_\nu A^a_\beta + c_2 A^a_\mu A^{a,\mu} + c_{31} f^{abc} A^a_\mu A^b_\nu \, N^{\alpha\mu}\, \partial_\alpha A^{c,\nu} \\
& + c_7 \overline{\psi}\gamma^\mu i\partial_\mu \psi + c_{71}\overline{\psi}\gamma_\mu \, N^{\mu\nu}\, i\partial_\nu \psi + c_{72}\overline{\psi}\gamma_\mu \, N^{\nu\mu}\, i\partial_\nu\psi - c_8\overline{\psi}\psi \\
& + c_{91} A^a_\alpha \, N^{\mu\alpha}\, \overline{\psi}\gamma_\mu \frac{\lambda^a}{2}\psi
\end{aligned}
$$

$$
N^{\alpha\beta} = \frac{n^\alpha n^{*\beta}}{(nn^*)}
$$

GIVES ALL-ORDERS PERTURBATIVE AGREEMENT WITH E.T.

SIMPLEST (ONLY) METHOD KNOWN TO DO SO

FIRST NONPERTURBATIVE CALCULATIONS NEED NO C.T.

METHOD WAS USED IN ANOMALOUS-MOMENT N.P.C.

EQUATION

$$
D_1^{ab}(y,q)\psi_{++}^{ab}(y,q) - \sum_c I_1^{abc}(y,q)\psi_{++}^{cb}(y,q) - \sum_c I_2^{abc}(y,q)\psi_{++}^{ac}(y,q) = -\frac{3g^2}{4\pi^3}\Bigg\{
$$

$$
+ \sum_{c,d,j}\int_0^y \frac{dx\, d^2k_\perp \nu_c\nu_d(-1)^j}{(y-x)^3(1+\frac{\Lambda^2}{\mu^2})^j} \frac{\Lambda^2\psi_{++}^{cd}(x,k)}{\left(M^2 - \frac{m_c^2+k^2}{x} - \frac{m_b^2+q^2}{1-y} - \frac{\Lambda^2+(q-k)^2}{(y-x)(1+\frac{\Lambda^2}{\mu^2})^j}\right)}
$$

$$
+ \sum_{c,d,j}\int_y^1 \frac{dx\, d^2k_\perp \nu_c\nu_d(-1)^j}{(x-y)^3(1+\frac{\Lambda^2}{\mu^2})^j} \frac{\Lambda^2\psi_{++}^{cd}(x,k)}{\left(M^2 - \frac{m_d^2+k^2}{1-x} - \frac{m_a^2+q^2}{y} - \frac{\Lambda^2+(k-q)^2}{(x-y)(1+\frac{\Lambda^2}{\mu^2})^j}\right)}
$$

$$
+ \sum_{c,d,j,\zeta}\int_0^y \frac{dx\, d^2k_\perp \nu_c\nu_d(-1)^\zeta}{2(y-x)(\frac{(q-k)^2}{\mu^2}-1)^j} \frac{\left[\frac{q_-}{1-y} + \frac{(q-k)_-}{y-x}\right]\left[\frac{-k_+}{x} + \frac{(q-k)_+}{y-x}\right]\psi_{++}^{cd}(x,k)}{\left(M^2 - \frac{m_c^2+k^2}{x} - \frac{m_b^2+q^2}{1-y} - \frac{\zeta\Lambda^2+(q-k)^2}{(y-x)(1+\zeta\frac{\Lambda^2}{\mu^2})^j}\right)}
$$

$$
+ \sum_{c,d,j,\zeta}\int_y^1 \frac{dx\, d^2k_\perp \nu_c\nu_d(-1)^\zeta}{2(x-y)(\frac{(q-k)^2}{\mu^2}-1)^j} \frac{\left[\frac{-q_-}{1-y} + \frac{(k-q)_-}{x-y}\right]\left[\frac{k_+}{1-x} + \frac{(k-q)_+}{x-y}\right]\psi_{++}^{cd}(x,k)}{\left(M^2 - \frac{m_d^2+k^2}{1-x} - \frac{m_a^2+q^2}{y} - \frac{\zeta\Lambda^2+(q-k)^2}{(x-y)(1+\zeta\frac{\Lambda^2}{\mu^2})^j}\right)}
$$

$$
+ \sum_{c,d,j,\zeta}\int_0^y \frac{dx\, d^2k_\perp \nu_c\nu_d(-1)^\zeta}{2(y-x)(\frac{(q-k)^2}{\mu^2}-1)^j} \frac{\left[\frac{k_+}{1-x} + \frac{(q-k)_+}{y-x}\right]\left[\frac{-q_-}{y} + \frac{(q-k)_-}{y-x}\right]\psi_{++}^{cd}(x,k)}{\left(M^2 - \frac{m_c^2+k^2}{x} - \frac{m_b^2+q^2}{1-y} - \frac{\zeta\Lambda^2+(q-k)^2}{(y-x)(1+\zeta\frac{\Lambda^2}{\mu^2})^j}\right)}
$$

$$
+ \sum_{c,d,j,\zeta}\int_y^1 \frac{dx\, d^2k_\perp \nu_c\nu_d(-1)^\zeta}{2(x-y)(\frac{(q-k)^2}{\mu^2}-1)^j} \frac{\left[\frac{-k_+}{x} + \frac{(k-q)_+}{x-y}\right]\left[\frac{q_-}{1-y} + \frac{(k-q)_-}{x-y}\right]\psi_{++}^{cd}(x,k)}{\left(M^2 - \frac{m_d^2+k^2}{1-x} - \frac{m_a^2+q^2}{y} - \frac{\zeta\Lambda^2+(q-k)^2}{(x-y)(1+\zeta\frac{\Lambda^2}{\mu^2})^j}\right)}
$$

$$+\sum_{c,d,j,\zeta}\int_0^y \frac{dx\,d^2k_\perp \nu_c\nu_d(-1)^\zeta}{2(y-x)(\frac{(q-k)^2}{\mu^2}-1)^j}\frac{\left[\frac{m_b}{1-y}-\frac{m_d}{1-x}\right]\left[\frac{-q_-}{y}+\frac{(q-k)_-}{y-x}\right]\psi^{cd}_{+-}(x,k)}{\left(M^2-\frac{m_c^2+k^2}{x}-\frac{m_b^2+q^2}{1-y}-\frac{\zeta\Lambda^2+(q-k)^2}{(y-x)(1+\zeta\frac{\Lambda^2}{\mu^2})^j}\right)}$$

$$+\sum_{c,d,j,\zeta}\int_y^1 \frac{dx\,d^2k_\perp \nu_c\nu_d(-1)^\zeta}{2(x-y)(\frac{(q-k)^2}{\mu^2}-1)^j}\frac{\left[\frac{m_d}{1-x}-\frac{m_b}{1-y}\right]\left[\frac{-k_+}{x}+\frac{(k-q)_+}{x-y}\right]\psi^{cd}_{+-}(x,k)}{\left(M^2-\frac{m_d^2+k^2}{1-x}-\frac{m_a^2+q^2}{y}-\frac{\zeta\Lambda^2+(q-k)^2}{(x-y)(1+\zeta\frac{\Lambda^2}{\mu^2})^j}\right)}$$

$$+\sum_{c,d,j,\zeta}\int_0^y \frac{dx\,d^2k_\perp \nu_c\nu_d(-1)^\zeta}{2(y-x)(\frac{(q-k)^2}{\mu^2}-1)^j}\frac{\left[\frac{m_c}{x}-\frac{m_a}{y}\right]\left[\frac{k_+}{1-x}+\frac{(q-k)_+}{y-x}\right]\psi^{cd}_{-+}(x,k)}{\left(M^2-\frac{m_c^2+k^2}{x}-\frac{m_b^2+q^2}{1-y}-\frac{\zeta\Lambda^2+(q-k)^2}{(y-x)(1+\zeta\frac{\Lambda^2}{\mu^2})^j}\right)}$$

$$\sum_{c,d,j,\zeta}\int_y^1 \frac{dx\,d^2k_\perp \nu_c\nu_d(-1)^\zeta}{2(x-y)(\frac{(q-k)^2}{\mu^2}-1)^j}\frac{\left[\frac{m_a}{y}-\frac{m_c}{x}\right]\left[\frac{q_-}{1-y}+\frac{(k-q)_-}{x-y}\right]\psi^{cd}_{-+}(x,k)}{\left(M^2-\frac{m_d^2+k^2}{1-x}-\frac{m_a^2+q^2}{y}-\frac{\zeta\Lambda^2+(q-k)^2}{(x-y)(1+\zeta\frac{\Lambda^2}{\mu^2})^j}\right)}\Bigg\}$$

HERE $k_\pm = k_1 \pm ik_2$

CAN DEFINE

$$f^{ab}_{+-}(x,k)=\psi^{ab}_{+-}(x,k)\ \ ;\ \ f^{ab}_{-+}(x,k)=\psi^{ab}_{-+}(x,k)$$

$$f^{ab}_{++}(x,k)(-k_1+ik_2)=\psi^{ab}_{++}(x,k)\ \ ;\ \ f^{ab}_{--}(x,k)(-k_1-ik_2)=\psi^{ab}_{--}(x,k)$$

NEED TO SIMPLIFY, TAKE LIMITS

ϵ IS PART OF DLCQ GRID

CAN WE TAKE $\mu \to 0$; $M_L \to \infty$?

IMPROVEMENTS?

OTHER GAUGES:

IN QED FEYNMAN GAUGE IS SIMPLER

PROBLEM 1: $(\partial_- - gA_-)^{-1}$

PV FIELDS LEAD TO $(\partial_-)^{-1}$

ALLOWS USE IN QED

PROBLEM 2: DIRAC ALGEBRA

- NONABELIAN MODIFICATIONS TO Π_A
- CONSTRAINT ALGEBRA HAS NOT BEEN SOLVED

PROBLEM 3: GRIBOV COPIES

TECHNICAL FIELD (j=1):

INCLUDE INTEGRATION CONSTANTS FROM

$$\partial_-^2 A^- + \partial_- \partial_i A^i = -e\Psi_+^\dagger \Psi_+$$

FIX SELF ENERGY LOOP IN QED

MIGHT(?) SIMPLIFY REGULATION

THE VACUUM IN LCQ

ZERO-MODE FIELDS DRESS THE BARE VACUUM

THESE FIELDS ARE INTEGRATION CONSTANTS

$$\mathrm{i}\partial_{-}\psi_{-} = \mathrm{i}\gamma_{\perp} \cdot D_{\perp}\gamma^{0}\psi_{+}$$

EFFECTIVE THEORY USES USUAL LQRS (BARE VACUUM)

EXACT EFFECTIVE THEORY

- PHYSICAL STATES ARE USUAL LCO $|\Omega\rangle$
- $\{S_{ET}\} = \{S_{FT}\}$
- $\Psi_{ET} = \mathcal{P}\Psi_{FT}$

ET DYNAMICS INCLUDES INDUCED OPERATORS

INDUCED OPERATORS

INDUCED OPERATORS BREAK CHIRAL SYMMETRY

INDUCED OPERATORS SPLIT THE π AND THE ρ

INDUCED OPERATORS GIVE $M_\pi^2 \sim m_0$

SUMMARY

REGULARIZATION AND RENORMALIZATION AGREES WITH COVARIANT PERTURBATION THEORY

- REMOVE INDUCED OPERATORS
- EXPAND NONPERTURBATIVE ANSWER
- GRAPHS PRESENT ARE STANDARD

REGULARIZATION IS COMPLICATED

SIMPLIFICATIONS MAY BE POSSIBLE

INDUCED OPERATORS INDUCED VACUUM EFFECTS

ADDITIONAL INDUCED OPERATORS REMAIN TO BE DERIVED

INDUCED OPERATORS MAY BE TESTED BY SUM RULES

NUMERICAL RESULTS LATER