The Yukawa model in Covariant Light Front Dynamics

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One knows already a lot on Light Front phenomenology.

If one wants to have a chance to make an (approximate) calculation of the structure of relativistic bound state systems from “first principles” in LFD one has:

- to control violation of rotational invariance
- to fully determine the structure of possible counterterms
- to have a systematic renormalization scheme within a truncated Fock space in a non-perturbative way

The explicitly covariant formulation of LFD may be a systematic tool in order to analyze these questions.
Plan

➢ General overview

➢ Regularization scheme
  • Pauli-Villars boson and fermion fields

➢ Renormalization of bare quantities
  • Mass counterterm and bare coupling constants
  • New counterterms depending on the position of the light front
  • Systematic non-perturbative determination within truncated Fock space

➢ Application to non-perturbative calculations
  • QED in two-body truncation: anomalous magnetic moment of the electron
  • Yukawa model in three-body truncation

➢ Conclusions
General framework

- We consider a spin 1/2 fermion coupled to a boson

- Explicitly covariant formulation of LFD
  - light front position characterized by a four-vector \( \omega \)
    \[ \omega \cdot x = 0 \quad \text{with} \quad \omega^2 = 0 \]  
    (Karmanov, 76)

- Equation of motion given by
  \[
  \hat{P}^2 \phi(p) = M^2 \phi(p) \quad \text{with} \quad \hat{P}_\mu = \hat{P}_\mu^0 + \hat{P}_\mu^{\text{int}}
  \]
  \[
  \hat{P}_\mu^{\text{int}} = \omega_\mu \int H^{\text{int}}(x) \delta(\omega \cdot x) \, d^4 x = \omega_\mu \int_{-\infty}^{+\infty} \tilde{H}^{\text{int}}(\omega \tau) \frac{d\tau}{2\pi}
  \]
Angular momentum operator

\[ \hat{J}_{\mu \nu} = \hat{J}^0_{\mu \nu} + \hat{J}^{int}_{\mu \nu} \]

\[ \hat{J}^{int}_{\mu \nu} = \int H^{int}(x)(x_\mu \omega_\nu - x_\nu \omega_\mu) \delta(\omega \cdot x - \sigma) \, d^4x \]

with the condition

\[ \hat{J}^{int}_{\mu \nu} \phi_\omega(\sigma) = i \left( \omega_\mu \frac{\partial}{\partial \omega_\nu} - \omega_\nu \frac{\partial}{\partial \omega_\mu} \right) \phi_\omega(\sigma) \]

Very convenient tool in order to analyze electromagnetic amplitudes and Fock state components

- Nucleon electromagnetic form factor

\[ \Gamma_\rho = F_1 \gamma_\rho + \frac{i F_2}{2M} \sigma_{\rho \nu} q^\nu \]

\[ \quad + B_1 \left( \frac{\omega \cdot p}{\omega \cdot p} - \frac{1}{(1 + \eta)M} \right) P_\rho + B_2 \frac{M}{\omega \cdot p} \omega_\rho + B_3 \frac{M^2}{(\omega \cdot p)^2} \omega_\rho \]

(V. Karmanov, JFM, 96)
• Two body Fock state component in the Yukawa model

\[ \tilde{u}(k_1) \Gamma_2 u(p) = b_1 \tilde{u}(k_1) u(p) + b_2 \frac{m}{\omega \cdot p} \tilde{u}(k_1) \phi u(p) \]

- Enables a straightforward control on any violation of rotational invariance in any approximate calculation, and at any step of the calculation

(J. Carbonell et al, Phys. Rep. 300, 98)

- Decomposition of the state vector

\[ \phi(p) = |1\rangle + |2\rangle + \ldots \]

\[ \phi(p) = (2\pi)^{3/2} \int \phi_1(k_1, p, \omega \tau) a^\dagger(\vec{k}_1) |0\rangle \delta^{(4)}(k_1 - p - \omega \tau) 2(\omega \cdot p) d\tau \frac{d^3 k_1}{(2\pi)^{3/2} \sqrt{2\varepsilon k_1}} \]

\[ + (2\pi)^{3/2} \int \phi_2(k_1, k_2, p, \omega \tau) a^\dagger(\vec{k}_1) b^\dagger(\vec{k}_2) |0\rangle \delta^{(4)}(k_1 + k_2 - p - \omega \tau) 2(\omega \cdot p) d\tau \frac{d^3 k_1}{(2\pi)^{3/2} \sqrt{2\varepsilon k_1}} \frac{d^3 k_2}{(2\pi)^{3/2} \sqrt{2\varepsilon k_2}} \]
Equation of motion

with the momentum conservation law for each Fock component

\[ k_1 + k_2 + \cdots + k_n = p + \omega \tau. \]

with

\[ (\sum k_i)^2 - M^2 = 2(\omega \cdot p) \tau \]

one can rewrite the equation of motion \( \hat{P}^2 \phi(p) = M^2 \phi(p) \) as

\[ \frac{1}{2\pi} \int \tilde{H}^{\text{int}}(\omega \tau) \frac{d\tau}{\tau} G(p) = -\hat{G}(p) \equiv -\lambda(M^2) G(p) \]

with

\[ \hat{G}(p) = 2(\omega \cdot p) \hat{\tau} \phi(p) \]

and

\[ \hat{\tau} \phi_n = \tau_n \phi_n \equiv \tilde{u}(k_1) \Gamma_n u(p) \]

System of coupled integral equations for the vertex functions \( \Gamma_n \)

No kinetic part anymore thanks to \( \tau \)
Regularization scheme

- Example of the fermion self energy
  - General decomposition on the Light Front

\[ \Sigma = \Sigma(p)\big|_{p^2=m^2} = A + B \frac{\not{p}}{m} + C \frac{m \not{\phi}}{\omega \cdot p} \]

- The coefficient \( C \) is a priori non zero!

  for a cut off \( \Lambda \) on \( k_\perp \)

\[ A = -g^2 \frac{m}{16\pi^2} \log \frac{\Lambda^2}{m^2} \]
\[ B = -g^2 \frac{m}{32\pi^2} \log \frac{\Lambda^2}{m^2} \]
\[ C = -g^2 \frac{m}{32\pi^2} \log \frac{\Lambda^2}{m^2} \]

- This implies the need for a specific counterterm

\[ Z_\omega \frac{m}{i\omega \cdot \partial} \bar{\Psi} \not{\phi} \Psi \quad \text{with} \quad Z_\omega = C \]

(V. Karmanov, JFM, A. Smirnov, LC2004)
However, $C$ IS zero with a Pauli-Villars regularization provided one has one boson PV AND one fermion PV.

- The counterterm $Z_\omega$ is also zero.
- One can do a similar analysis for the 3 point GF and electromagnetic form factors. (V. Karmanov)
- Very well suited for non-perturbative calculations.
- In the following, use of PV regularization by extending the Fock space to include one fermion and one boson PV fields:
  - no contact interactions anymore
  - two one-body components and four two-body components,...

(S. Brodsky, J. Hiller, G. Mc Cartor,...)
Renormalization of bare quantities

- We start with the standard
  - mass counterterm $\delta m \bar{\Psi} \Psi$ in order to develop the Fock components in terms of free fields with their physical mass $m$
  - bare coupling constants $g_0 \bar{\Psi} \phi \Psi$

- Analysis in terms of Fock state components
  - Example of the self energy
    - couples two different Fock components
    - one should keep track of the physical content of the counterterm as a function of the number of particles it corresponds to: $\delta m^{(2)}$
    - One should therefore consider a whole set of counterterms deduced from $\delta m \rightarrow \delta m^{(n)}$
      - $n =$ total number of particles in which the fermion can fluctuate

*(K. Wilson, R. Perry, ... 90)*

*(V. Karmanov, JFM, A. Smirnov, LC2005)*

*at* $p^2 = m^2$
The same is true for the bare coupling constant $g_0 \rightarrow g_0^{(n)}$

$\delta m^{(n)}$ and $g_0^{(n)}$ calculated by successive solutions of the $N=1$, $N=2$, $N=3$ ... $N$ systems

- **N=1**
  \[ \delta m^{(1)} = 0 \]

- **N=2**
  \[ \delta m^{(2)} \] determined to get
  \[ \hat{P}^2 \phi(p) = M^2 \phi(p) \]
  \[ \delta m^{(2)} = -\sum(p^2 = M^2) \]
  \[ g_0^{(2)} \] determined to get
  \[ \bar{u}(k_1)\Gamma_2(s = M^2)u(p) = g_{phys} \bar{u}(k_1)u(p) \]

This is a systematic, non-perturbative, procedure which should avoid uncancelled divergences

but also true if there are no divergences!
Application to non-perturbative calculations

- QED in the two-body truncation (in Feynman gauge)
  - Definition of the electromagnetic form factors

Because of Fock state truncation, the renormalization of the external photon coupling $e_0$ is NOT the same as the internal photon coupling $g_0$.

One can find in this way analytically $e_0^{(2)} \equiv e$.

(W. Karmanov, JFM, A. Smirnov, LC2005)
We can now proceed to calculate the anomalous magnetic moment of the electron in the two-body truncation

- Calculate \( \Gamma_2 \) from the equation of motion and the normalization condition

\[
\bar{u}(k_1)\Gamma_2 u(p) = e \bar{u}(k_1)u(p)
\]

- Decompose the electromagnetic operator in all spin structures

\[
\Gamma_\rho = F_1 \gamma_\rho + \frac{iF_2}{2M} \sigma_{\rho\nu} q^\nu + B_1 \left( \frac{\phi}{\omega \cdot p} - \frac{1}{(1 + \eta)M} \right) P_\rho + B_2 \frac{M}{\omega \cdot p} \omega_\rho + B_3 \frac{M^2}{(\omega \cdot p)^2} \phi \omega_\rho
\]

- Calculate the two-body contribution with a regularization scheme

- With a naive cut-off on \( k_\perp \): \( B_{1,2,3} \neq 0 \) and \( F_2(0) \) diverges

- With Pauli-Villars regulators as advocated above

\[
B_{1,2,3} = 0 \quad \text{and} \quad F_2(0) = \frac{\alpha}{2\pi}
\]

- Analytical, non-perturbative result without any expansion in \( \epsilon \)
The Yukawa model in the three-body truncation

Equation of motion

System of ten coupled equations

- Two one-body components
  \[ \bar{u}(p_1) \Gamma_1^i u(p) \]
- Eight two-body components
  \[ b_{1,j}^i \bar{u}(k_1) u(p) + b_{2,j}^i \frac{m}{\omega \cdot p} \bar{u}(k_1) \varphi u(p) \]
- The component \( \Gamma_3 \) is eliminated from the equation with \( \delta m^{(1)} = 0 \)
Determination of the parameters

- \( \delta m^{(2)} , g_{0}^{(2)} \) determined from the analytic N=2 calculation
- Unknown \( \delta m^{(3)} , g_{0}^{(3)} \) in fact only

fixed to get a solution to the equation of motion (consistency condition) and the two-body component at some physical point

Take the limit of infinite PV fermion mass \( M_{PV} \):

- Identify the leading contribution in \( M_{PV} \) for the one and two-body components
- Redefine these components in order to absorb this behavior
- Obtain a new set of equations which is FINITE in the limit

\[ M_{PV} \to \infty \]

It remains to solve numerically this system of equations and investigate the limit of infinite PV boson mass (logarithmic behavior)
Conclusions

Non-perturbative calculation of physical systems in a truncated Fock space demands:

- A careful analysis of the counterterms as a function of the position of the light front
  - need an explicit dependence on $\omega$

- A systematic determination of bare quantities within the truncated Fock space
  - Fock state dependent counterterms and bare quantities

- An appropriate regularization scheme to preserve symmetries
  - Pauli-Villars boson and fermion
  - More general framework? (P. Grangé / E. Werner)
We have analyzed in our framework

- **QED in the two-body truncation**
  - Exact and analytic anomalous magnetic moment of the electron

- **Yukawa model in the three-body truncation**
  - Finite and analytic form for the equations in the limit of infinite fermion Pauli Villars mass
  - Full numerical solution in progress