The Yukawa model in Covariant Light Front Dynamics

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Motivations

- One knows already a lot on Light Front phenomenology
- If one wants to have a chance to make an (approximate) calculation of the structure of relativistic bound state systems from "first principles" in LFD one has
  - to control violation of rotational invariance
  - to fully determine the structure of possible counterterms
  - to have a systematic renormalization scheme within a truncated Fock space in a non-perturbative way
- The explicitly covariant formulation of LFD may be a systematic tool in order to analyze these questions

Plan

- General overview
- Regularization scheme
  - Pauli-Villars boson and fermion fields
- Renormalization of bare quantities
  - Mass counterterm and bare coupling constants
  - New counterterms depending on the position of the light front
  - Systematic non-perturbative determination within truncated Fock space
- Application to non-perturbative calculations
  - QED in two-body truncation: anomalous magnetic moment of the electron
  - Yukawa model in three-body truncation
- Conclusions

General framework

- We consider a spin 1/2 fermion coupled to a boson

- Explicitly covariant formulation of LFD
  - light front position characterized by a four-vector ω

$$\omega \cdot x = 0 \quad \text{with} \quad \omega^2 = 0$$

(Karmanov, 76)

- Equation of motion given by

$$\hat{P}^2 \, \phi(p) = M^2 \, \phi(p) \qquad \text{with} \qquad \hat{P}_\mu = \hat{P}^0_\mu + \hat{P}^{int}_\mu$$

$$\hat{P}^{int}_\mu = \omega_\mu \int H^{int}(x) \delta(\omega \cdot x) \, d^4x = \omega_\mu \int_{-\infty}^{+\infty} \tilde{H}^{int}(\omega\tau) \frac{d\tau}{2\pi}$$

- **Angular momentum operator**

$$\hat{J}_{\mu\nu} = \hat{J}^0_{\mu\nu} + \hat{J}^{int}_{\mu\nu}$$

$$\hat{J}^{int}_{\mu\nu} = \int H^{int}(x)(x_\mu \omega_\nu - x_\nu \omega_\mu)\delta(\omega{\cdot}x - \sigma)\ d^4x$$

with the condition

$$\hat{J}^{int}_{\mu\nu}\ \phi_\omega(\sigma) = i\left(\omega_\mu \frac{\partial}{\partial \omega^\nu} - \omega_\nu \frac{\partial}{\partial \omega^\mu}\right)\phi_\omega(\sigma)$$

- **Very convenient tool in order to analyze electromagnetic amplitudes and Fock state components**
  - **Nucleon electromagnetic form factor**

$$\Gamma_\rho = F_1 \gamma_\rho + \frac{iF_2}{2M}\sigma_{\rho\nu} q^\nu$$

(V. Karmanov, JFM, 96)

$$+B_1\left(\frac{\not{\omega}}{\omega{\cdot}p} - \frac{1}{(1+\eta)M}\right)P_\rho + B_2 \frac{M}{\omega{\cdot}p}\omega_\rho + B_3 \frac{M^2}{(\omega{\cdot}p)^2}\ \not{\omega}\omega_\rho$$

- **Two body Fock state component in the Yukawa model**

$$\bar{u}(k_1)\Gamma_2 u(p) = b_1 \bar{u}(k_1) u(p) + b_2 \frac{m}{\omega \cdot p} \bar{u}(k_1) \not{\omega}\, u(p)$$

- **Enables a straightforward control on any violation of rotational invariance in any approximate calculation, and at any step of the calculation**

(J. Carbonell et al, Phys. Rep. 300, 98)

- **Decomposition of the state vector**

$$\phi(p) = |1\rangle + |2\rangle + \ldots$$

$$\phi(p) = (2\pi)^{3/2} \int \phi_1(k_1, p, \omega\tau) a^\dagger(\vec{k}_1)|0\rangle \delta^{(4)}(k_1 - p - \omega\tau) 2(\omega.p) d\tau \frac{d^3k_1}{(2\pi)^{3/2}\sqrt{2\varepsilon_{k_1}}}$$

$$+(2\pi)^{3/2} \int \phi_2(k_1, k_2, p, \omega\tau) a^\dagger(\vec{k}_1) b^\dagger(\vec{k}_2)|0\rangle$$

$$\delta^{(4)}(k_1 + k_2 - p - \omega\tau) 2(\omega.p) d\tau \frac{d^3k_1}{(2\pi)^{3/2}\sqrt{2\varepsilon_{k_1}}} \frac{d^3k_2}{(2\pi)^{3/2}\sqrt{2\varepsilon_{k_2}}}$$

❑ Equation of motion

with the momentum conservation law for each Fock component

$$k_1 + k_2 + \cdots + k_n = p + \omega\tau \ .$$

with

$$(\sum k_i)^2 - M^2 = 2(\omega.p)\tau$$

one can rewrite the equation of motion $\hat{P}^2\ \phi(p) = M^2\ \phi(p)$ as

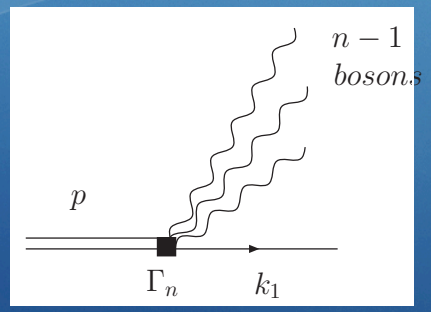
$$\frac{1}{2\pi}\int \tilde{H}^{int}(\omega\tau)\frac{d\tau}{\tau}\mathcal{G}(p) = -\mathcal{G}(p) \equiv -\lambda(M^2)\mathcal{G}(p)$$

with

$$\mathcal{G}(p) = 2(\omega\cdot p)\hat{\tau}\phi(p)$$

and

$$\hat{\tau}\phi_n = \tau_n\phi_n \equiv \bar{u}(k_1)\Gamma_n u(p)$$



- System of coupled integral equations for the vertex functions Γ_n
- No kinetic part anymore thanks to τ

Regularization scheme

- Example of the fermion self energy
  - General decomposition on the Light Front

$$\Sigma = \Sigma(p)|_{p^2=m^2} = A + B\frac{\not p}{m} + C\frac{m\ \not\omega}{\omega\cdot p}$$

  - The coefficient C is a priori non zero!

for a cut off Λ on $k_\perp$

$$A = -g^2\frac{m}{16\pi^2}Log\frac{\Lambda^2}{m^2}$$

$$B = -g^2\frac{m}{32\pi^2}Log\frac{\Lambda^2}{m^2}$$

$$C = -g^2\frac{m}{32\pi^2}Log\frac{\Lambda^2}{m^2}$$

  - This implies the need for a specific counterterm

$$Z_\omega\frac{m}{i\omega\cdot\partial}\bar\Psi\ \not\omega\ \Psi \qquad \text{with} \qquad Z_\omega = C$$

(V. Karmanov, JFM, A. Smirnov, LC2004)

- However C IS zero with a Pauli-Villars regularization provided one has one boson PV AND one fermion PV
  - The counterterm Z_ω is also zero
  - One can do a similar analysis for the 3 point GF and electromagnetic form factors (V. Karmanov)
  - Very well suited for non-perturbative calculations
  - In the following, use of PV regularization by extending the Fock space to include one fermion and one boson PV fields
    - no contact interactions anymore
    - two one-body components and four two-body components,...

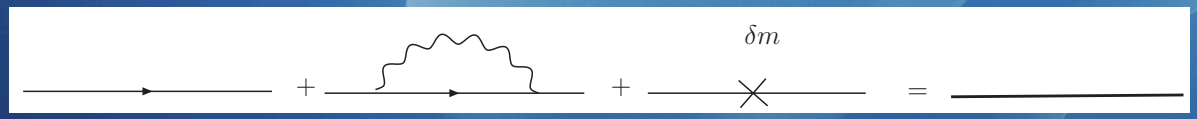
(S. Brodsky, J. Hiller, G. Mc Cartor,...)

Renormalization of bare quantities

- We start with the standard
  - mass counterterm $\delta m \; \bar{\Psi}\Psi$ in order to develop the Fock components in terms of free fields with their physical mass m
  - bare coupling constants $g_0 \; \bar{\Psi}\phi\Psi$

- Analysis in terms of Fock state components (K.Wilson, R.Perry, ... 90)

  ➢ Example of the self energy (V. Karmanov, JFM, A. Smirnov, LC2005)



at $p^2 = m^2$

  - couples two different Fock components
  - one should keep track of the physical content of the counterterm as a function of the number of particles it corresponds to : $\delta m^{(2)}$

  ➢ One should therefore consider a whole set of counterterms deduced from $\delta m \rightarrow \delta m^{(n)}$

  n = total number of particles in which the fermion can fluctuate

- The same is true for the bare coupling constant $g_0 \rightarrow g_0^{(n)}$

- $\delta m^{(n)}$ and $g_0^{(n)}$ calculated by successive solutions of the N=1, N=2 , N=3 ...N systems

  - N=1 $\delta m^{(1)} = 0$
  - N=2 $\delta m^{(2)}$ determined to get $\hat{P}^2\,\phi(p) = M^2\,\phi(p)$

$$\delta m^{(2)} = -\Sigma(p^2 = M^2)$$

$g_0^{(2)}$ determined to get

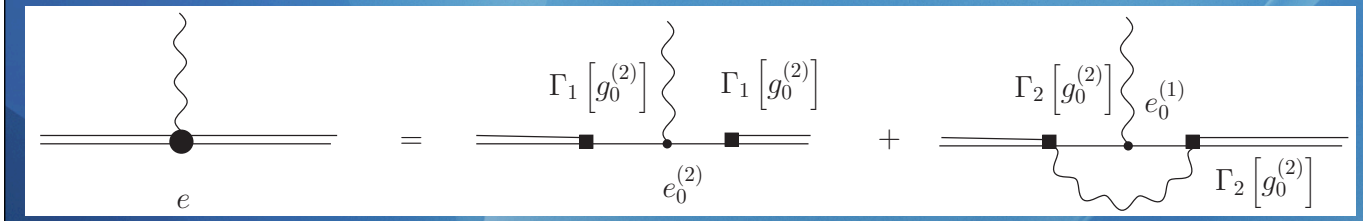
$$\bar{u}(k_1)\Gamma_2(s = M^2)u(p) = g_{phys}\bar{u}(k_1)u(p)$$

➢ This is a systematic, non-perturbative, procedure which should avoid uncancelled divergences

but also true if there are no divergences!

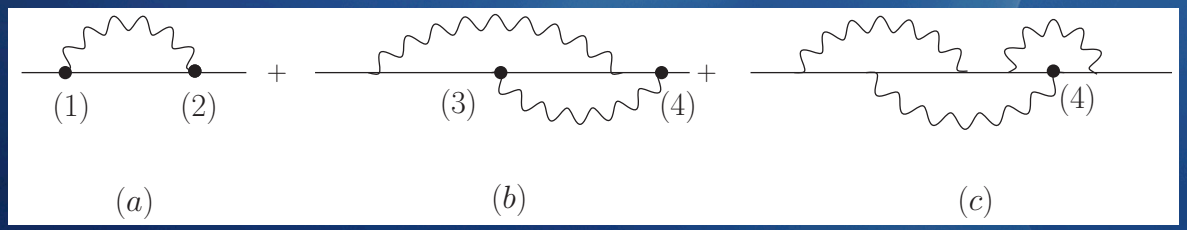
Application to non-perturbative calculations

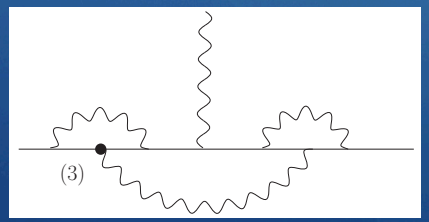
- QED in the two-body truncation (in Feynman gauge)
  - Definition of the electromagnetic form factors



(V. Karmanov, JFM, A. Smirnov, LC2005)

  - Because of Fock state truncation, the renormalization of the external photon coupling e_0 is **NOT** the same as the internal photon coupling g_0





  - One can find in this way analytically $e_0^{(2)} \equiv e$

➢ **We can now proceed to calculate the anomalous magnetic moment of the electron in the two-body truncation**

- Calculate Γ_2 from the equation of motion and the normalization condition

$$\bar{u}(k_1)\Gamma_2 u(p) = e\ \bar{u}(k_1)u(p)$$

- Decompose the electromagnetic operator in all spin structures

$$\Gamma_\rho = F_1\gamma_\rho + \frac{iF_2}{2M}\sigma_{\rho\nu}q^\nu$$
$$+B_1\left(\frac{\not{\omega}}{\omega\cdot p} - \frac{1}{(1+\eta)M}\right)P_\rho + B_2\frac{M}{\omega\cdot p}\omega_\rho + B_3\frac{M^2}{(\omega\cdot p)^2}\ \not{\omega}\omega_\rho$$

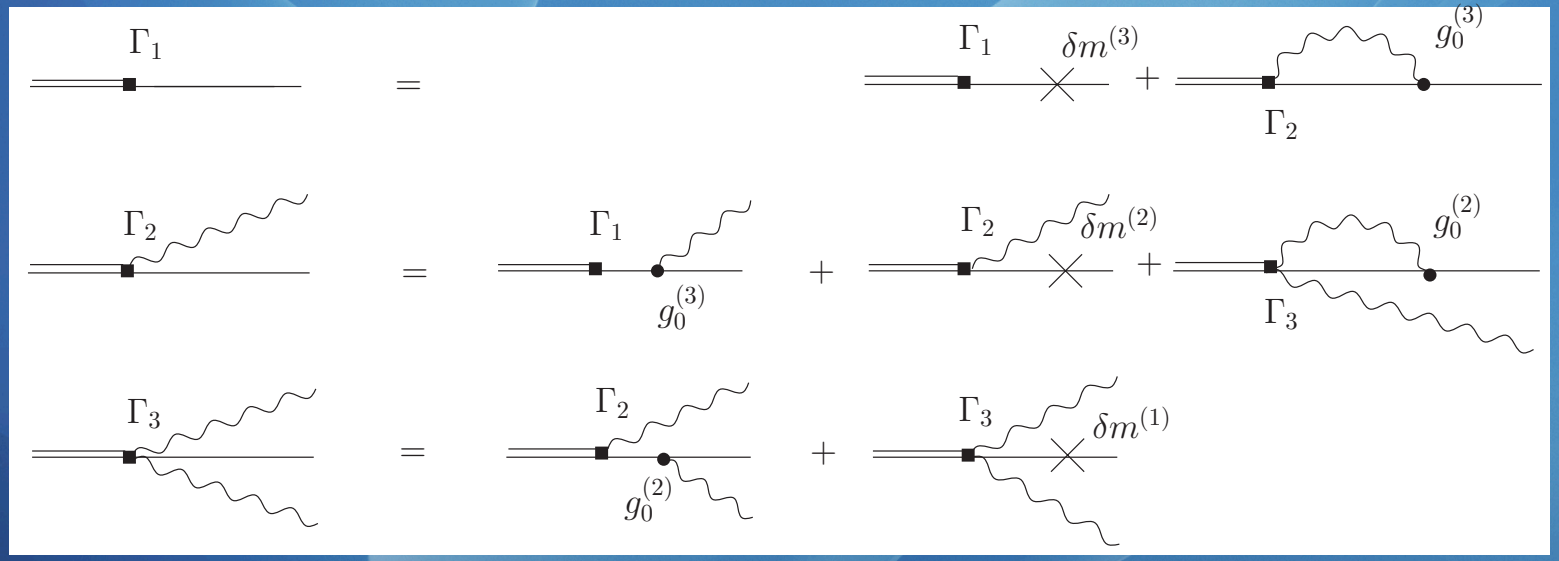
- Calculate the two-body contribution with a regularization scheme
- With a naive cut-off on $k_\perp$: $B_{1,2,3} \neq 0$ and $F_2(0)$ diverges
- With Pauli-Villars regulators as advocated above

$$B_{1,2,3} = 0 \qquad \text{and} \qquad F_2(0) = \frac{\alpha}{2\pi}$$

- Analytical, non-perturbative result without any expansion in e

❑ The Yukawa model in the three-body truncation

➢ Equation of motion



➢ System of ten coupled equations

- **Two one-body components**

$$\bar{u}(p_1)\Gamma_1^i u(p)$$

- **Eight two-body components**

$$b_1^{i,j}\bar{u}(k_1)u(p) + b_2^{i,j}\frac{m}{\omega\cdot p}\bar{u}(k_1)\ \not{\omega} u(p)$$

- **The component Γ_3 is eliminated from the equation with $\delta m^{(1)} = 0$**

- **Determination of the parameters**
  - $\delta m^{(2)}$, $g_0^{(2)}$ determined from the analytic N=2 calculation
  - Unknown $\delta m^{(3)}$, $g_0^{(3)}$ in fact only $\dfrac{\left[g_0^{(3)}\right]^2}{\delta m^{(3)}}$

    fixed to get a solution to the equation of motion (consistency condition) and the two-body component at some physical point
- **Take the limit of infinite PV fermion mass M_{PV} :**
  - Identify the leading contribution in M_{PV} for the one and two-body components
  - Redefine these components in order to absorb this behavior
  - Obtain a new set of equations which is FINITE in the limit

    $M_{PV} \to \infty$
- **It remains to solve numerically this system of equations and investigate the limit of infinite PV boson mass (logarithmic behavior)**

Conclusions

Non-perturbative calculation of physical systems in a truncated Fock space demands :

- **A careful analysis of the counterterms as a function of the position of the light front**
  - need an explicit dependence on ω
- **A systematic determination of bare quantities within the truncated Fock space**
  - Fock state dependent counterterms and bare quantities
- **An appropriate regularization scheme to preserve symetries**
  - Pauli-Villars boson and fermion
  - More general framework ? (P. Grangé / E. Werner)

We have analyzed in our framework

- QED in the two-body truncation
  - Exact and analytic anomalous magnetic moment of the electron
- Yukawa model in the three-body truncation
  - Finite and analytic form for the equations in the limit of infinite fermion Pauli Villars mass
  - Full numerical solution in progress