LC2006,

Minneapolis, USA

May 18, 2006

On derivation of the LFD fermion form factors from the Feynman approach

V.A. Karmanov[‡]

In collaboration with
J.-F. Mathiot[†] and A.V. Smirnov[‡]

[‡] *Lebedev Physical Institute, Moscow, Russia*

[†] *Laboratoire de Physique Corpusculaire, Clermont-Ferrand, France*

Perturbative calculation of EM form factors, in Yukawa model, in LFD, with emphasis on dependence on the regularization procedure.

• Plan

- Introduction (motivation)
- Calculating self energy in Yukawa model.
- Calculating EM form factors in Yukawa model.
- Dependence on the regularization procedure.
- Conclusions

• Motivation

It is two-fold:

- ffb vertex enters our non-perturbative equations (talk by J.-F. Mathiot)
  To study its properties and vertex counter terms.
- The EM vertex γff is calculated with our LFWF.
  To study its structure and counter terms.

• References

N.E. Ligrerink and B.L.G. Bakker, Phys. Rev. **D52**, 5954 (1995)
C.-H. Ji, B.L.G Bakker and H.-M. Choi, LC2005, hep-ph/0510210.
B.L.G. Bakker, M.A. DeWitt, C.-H. Ji, and Yu. Mishchenko, Phys. Rev. **D 72**, 076005 (2005)
. . .

The difference: we use Explicitly Covariant version of LFD.
Analysis from this point of view is instructive.

• Explicitly covariant LFD

V.A. Karmanov, JETP, **44** (1976) 201.

J. Carbonell, B. Desplanques, V.A. Karmanov and J.-F. Mathiot, Phys. Reports, **300** (1998) 215

- Take LF of general orientation:
  $\omega\cdot x = \omega_0 t - \vec{\omega}\cdot\vec{x} = 0$, $\omega = (\omega_0, \vec{\omega})$ such that $\omega^2 = 0$.
- Construct LFD, using this general LF plane.

Standard approach: $\omega = (1, 0, 0, -1)$.
In LF coordinates: $\omega_- = 2$, $\omega_+ = 0$, $\vec{\omega}_\perp = 0$.

• Advantages

- Explicit covariance
- When an amplitude depends on LF plane orientation
  $\Rightarrow$
  this dependence is explicit (v.s. ω.)
- Observables (e.g., EM vertex and form factors) must not depend on LF plane orientation.
  $\Rightarrow$
  Analysis of their dependence on ω
  because of cutoff and/or approximation.
- Etc.

• Calculating self energy

Yukawa model, LFD

$$\Sigma = -\frac{g^2}{(2\pi)^3}\int \frac{(\hat{p}-\hat{k}+m)}{\left(\frac{k_\perp^2+\mu^2}{x}+\frac{k_\perp^2+m^2}{1-x}-p^2\right)}\frac{d^2k_\perp dx}{2x}$$

$\vec{k}_\perp \perp \vec{\omega} \quad \Rightarrow \quad$ dependence of Σ on ω

General decomposition:

$$\Sigma(p) = A(p^2) + \frac{\hat{p}}{m}B(p^2) + \frac{\hat{\omega}}{\omega\cdot p}C(p^2)$$

$$C = \frac{1}{4}Tr\left[\Sigma\left(\hat{p} - \frac{p^2}{\omega\cdot p}\hat{\omega}\right)\right]$$

• Calculating Σ from Feynman graph

$$-i\Sigma = (-ig)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - \mu^2 + i\epsilon} \frac{i(\hat{p} - \hat{k} + m)}{(p-k)^2 - m^2 + i\epsilon}$$

Introduce the LF variables:

$$k_- = k_0 - k_z, \quad k_+ = k_0 + k_z, \quad \vec{k}_\perp = (k_x, k_y)$$

and put $k_+ = xp_+$:

$$\begin{aligned} \Sigma &= ig^2 \int \frac{p_+ dk_- dx d^2 k_\perp}{2(2\pi)^4} \frac{1}{[k_- p_+ x - k_\perp^2 - \mu^2 + i\epsilon]} \\ &\times \frac{(\hat{p} - \hat{k} + m)}{[(p_- - k_-)p_+(1-x) - (\vec{p}_\perp - \vec{k}_\perp)^2 - m^2 + i\epsilon]} \end{aligned}$$

• Three contributions

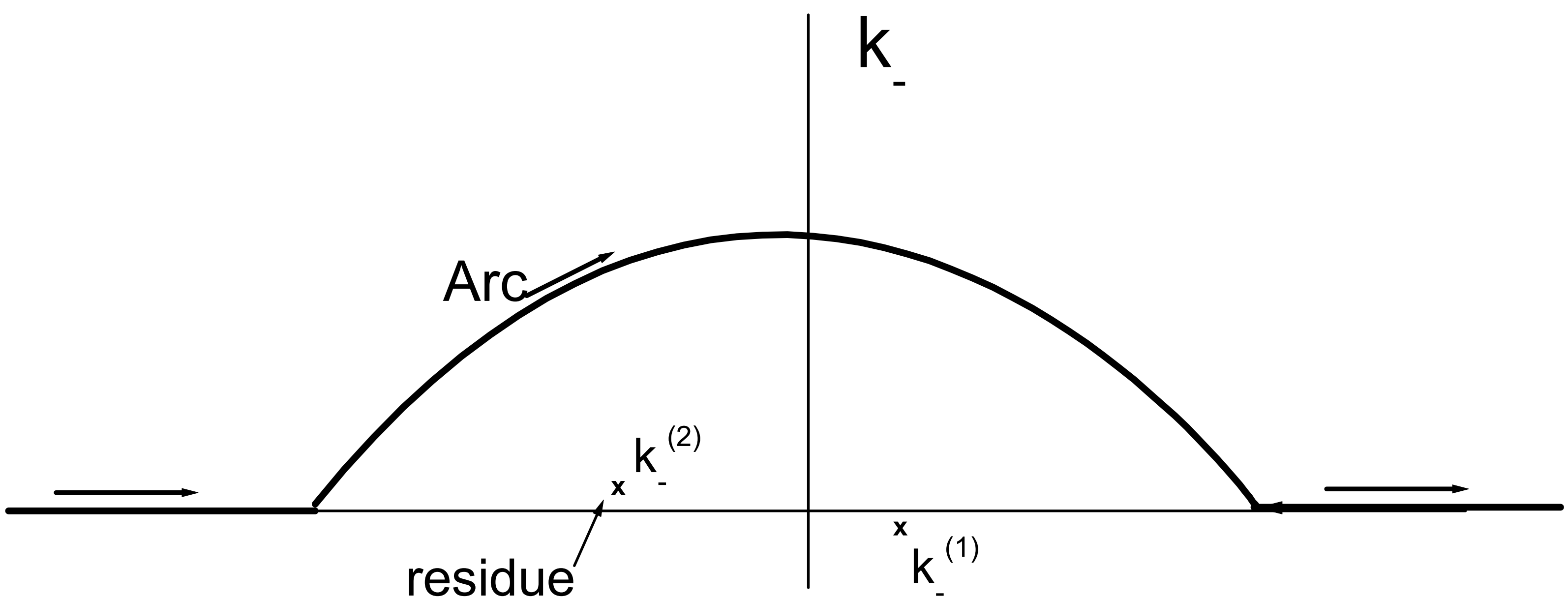
The integral over k_- can be calculated directly, by primitive. But it is useful to separate it in three contributions. (Ligterink, Bakker, Ji).

- Residues.
- Integral over arc.
- End point singularities (zero modes).

• Residue contribution

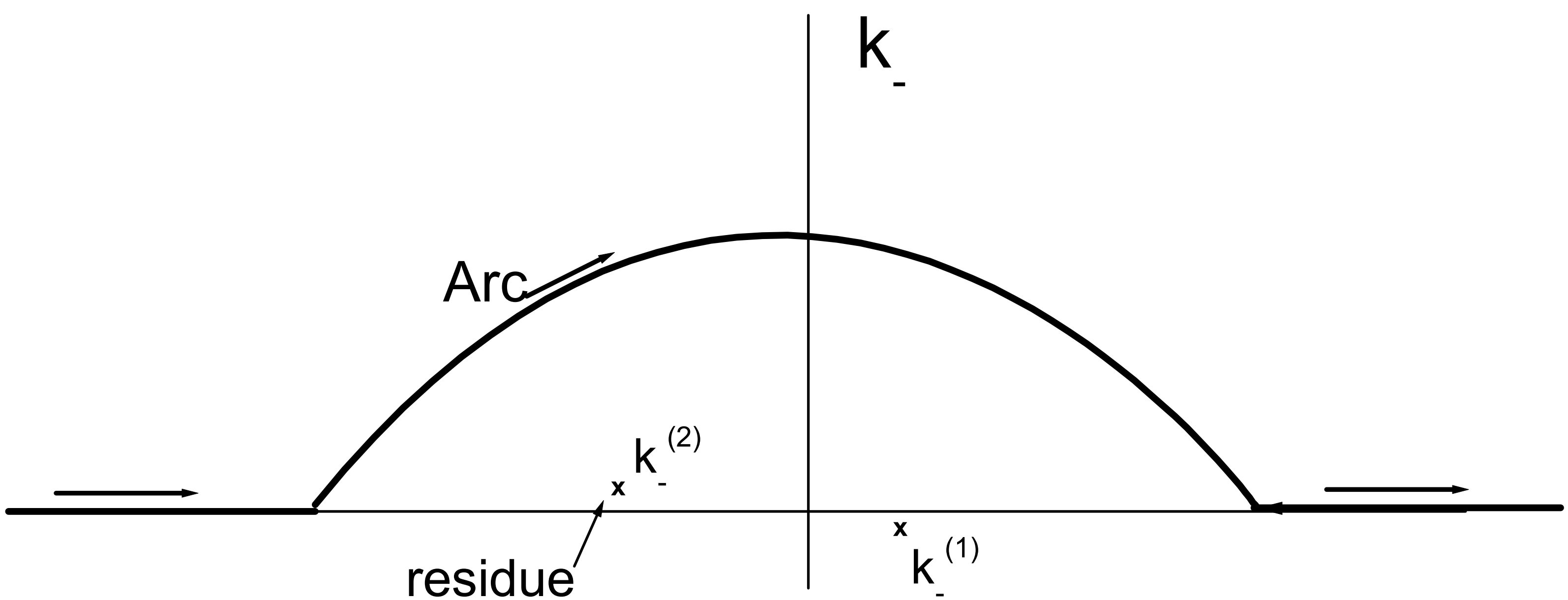
The poles are at different sides of contour if $0 < x < 1$

$$C^{res} = g^2 \int \frac{d^2 k_\perp}{4(2\pi)^3} \log \frac{k_\perp^2 + \mu^2}{k_\perp^2 + m^2} - \frac{g^2}{32\pi^2} \log \delta \int d^2 k_\perp$$



• Arc contribution

$$C^{Arc} \sim \int^{\infty} \ldots \frac{dk_-}{k_-} = \int_{\pi}^{0} \ldots id\phi, \quad k_- = Re^{i\phi}, \quad R \to \infty$$



• Zero modes

$$\Sigma = ig^2 \int_{-L}^{L} dk_- \int \frac{p_+ dx d^2 k_\perp}{2(2\pi)^4} \frac{1}{[k_- p_+ x - k_\perp^2 - \mu^2]} \frac{\frac{1}{2}\gamma_+ k_-}{(k_- p_+)}\Bigg|_{x\to 0}$$

$$= -ig^2 \gamma_+ \int_{-L}^{L} dk_- \int_{-\epsilon}^{\epsilon} dx \int \frac{d^2 k_\perp}{4(2\pi)^4} \frac{1}{[k_\perp^2 + \mu^2]}$$

When $x \to 0$ (and $x \to 1$), integral over k_- diverges:

$$\Sigma \sim \gamma_+ \int_{-L}^{L} dk_- \ldots \sim \gamma_+ L \to \infty$$

$$\int_{-L}^{L} \frac{dk_-}{xk_- - b - xp + i0} \sim \delta(x) \quad \Leftarrow$$ zero modes

• Full results

$$C^{res} = g^2 \int \frac{d^2k_\perp}{4(2\pi)^3} \log \frac{k_\perp^2 + \mu^2}{k_\perp^2 + m^2} - \frac{g^2}{32\pi^2} \log \delta \int d^2k_\perp$$

$$C^{Arc} = + \frac{g^2}{32\pi^2} \log \delta \int d^2k_\perp$$

$$C^{ZM} = -g^2 \int \frac{d^2k_\perp}{4(2\pi)^3} \log \frac{k_\perp^2 + \mu^2}{k_\perp^2 + m^2}$$

$$C = C^{res} + C^{Arc} + C^{ZM} = 0$$

Resumé

Arc and zero mode contributions kill the ω-dependent contribution in self energy.

However, they *do not* kill the ω-dependent contribution in EM vertex (see below).

• Regularization by the Pauli-Villars

meson μ_1 and fermion m_1

$$
\begin{aligned}
C^{PV} &= \Big[C(m,\mu) - C(m,\mu_1)\Big] \\
&- \Big[C(m_1,\mu) - C(m_1,\mu_1)\Big]
\end{aligned}
$$

$$
\begin{aligned}
C^{res,PV} &= 0 \\
C^{Arc,PV} &= 0 \\
C^{ZM,PV} &= 0
\end{aligned}
$$

$$C^{PV} = C^{res,PV} + C^{Arc,PV} + C^{ZM,PV} = 0$$

A, B are standard.

• EM vertex and form factors

General decomposition
(V.A.K. and J.-F. Mathiot, Nucl. Phys. **A 602** (1996) 338)

$$\begin{aligned} \bar{u}'\Gamma_\rho(p,p')u &= \bar{u}'\left[F_1\gamma_\rho + \frac{iF_2}{2m}\sigma_{\rho\nu}q_\nu\right. \\ &+ \left. B_1\left(\frac{\not{\omega}}{\omega\cdot p}P_\rho - 2\gamma_\rho\right) + B_2\frac{m\omega_\rho}{\omega\cdot p} + B_3\frac{m^2\not{\omega}\omega_\rho}{(\omega\cdot p)^2}\right]u \end{aligned}$$

FF's are found by: $F_1 = \frac{1}{2}c_5, \quad F_2 = \frac{1}{2\eta}(c_5 - c_4),$

with $c_4 = m\text{Tr}\left[O_\rho\right]\omega_\rho/(\omega\cdot p), \quad c_5 = m^2\text{Tr}\left[O_\rho^{LF}\not{\omega}\right]\omega_\rho/(\omega\cdot p)^2$

Similarly for B_{1-3}.

• Calculating Γ_ρ from Feynman graph

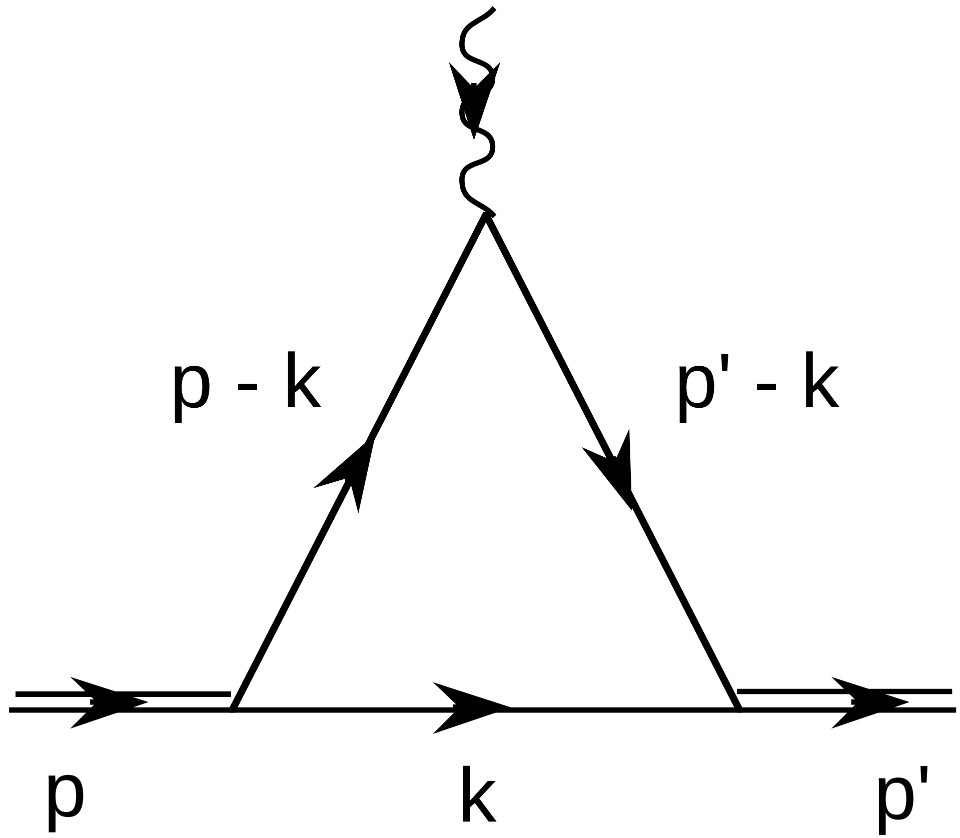


Figure 3: Feynman diagram for the EM vertex.

• Form factors F_1, F_2

Arc integrals and zero modes do not contribute in F_1, F_2.
The only contribution results from residues.

$$F_1^{Feyn} = \frac{g^2}{16\pi^2}\log\frac{L}{m} - \frac{g^2}{4\pi^2}\left(\log\frac{\mu}{m} + \frac{15}{16}\right) + \mathcal{O}(\Delta^2)$$

$$F_1^{LF,res} = \frac{g^2}{16\pi^2}\log\frac{\Lambda_\perp}{m} - \frac{g^2}{4\pi^2}\left(\log\frac{\mu}{m} + \frac{14}{16}\right) + \mathcal{O}(\Delta^2)$$

There is a difference: $F_1^{Feyn} - F_1^{LF,res} = -\frac{g^2}{64\pi^2}$

$$F_2^{Feyn} = F_2^{res} = \frac{3g^2}{16\pi^2} + \mathcal{O}(\Delta^2)$$

• Form factor B_1

Arc integral does not contribute.
Zero modes contribute.

$$\begin{aligned} B_1^{res} &\approx \frac{g^2}{16\pi^2}\log\frac{\Lambda_\perp}{m} - \frac{g^2}{64\pi^2} - \frac{g^2}{192\pi^2}\frac{\Delta^2}{m^2} + \mathcal{O}(\Delta^4) \\ B_1^{Arc} &= 0 \\ B_1^{ZM} &= -\frac{g^2}{16\pi^2}\log\frac{\Lambda_\perp}{m} + \frac{g^2}{192\pi^2}\frac{\Delta^2}{m^2} + \mathcal{O}(\Delta^4) \end{aligned}$$

$$B_1 = B_1^{res} + B_1^{Arc} + B_1^{ZM} = -\frac{g^2}{64\pi^2}$$

• Form factor B_2

Arc integral does not contribute.
Zero modes contribute.

$$B_2^{res} = -\frac{g^2}{8\pi^2}\log\frac{\Lambda_\perp}{m} - \frac{g^2}{32\pi^2} + \frac{g^2}{96\pi^2}\frac{\Delta^2}{m^2} + \mathcal{O}(\Delta^4)$$

$$B_2^{ZM} = 0$$

$$B_2^{ZM} = \frac{g^2}{8\pi^2}\log\frac{\Lambda_\perp}{m} - \frac{g^2}{96\pi^2}\frac{\Delta^2}{m^2} + \mathcal{O}(\Delta^4)$$

$$B_2 = B_2^{res} + B_2^{Arc} + B_2^{ZM} = -\frac{g^2}{32\pi^2}$$

• Form factor B_3

Both arc integral and zero modes contribute.

$$B_3^{res} = -\frac{g^2}{32\pi^2}\Lambda_\perp^2 \log\epsilon - \frac{3g^2}{16\pi^2}\log\frac{\Lambda_\perp}{m} - \frac{3g^2}{96\pi^2}\left[3\log\frac{\Lambda_\perp}{m} - 1\right]\frac{\Delta^2}{m^2}$$

$$B_3^{Arc} = \frac{g^2}{32\pi^2}\Lambda_\perp^2 \log\epsilon$$

$$B_3^{ZM} = \frac{g^2}{32\pi^2} + \frac{3g^2}{16\pi^2}\log\frac{\Lambda_\perp}{m} + \frac{3g^2}{96\pi^2}\left[3\log\frac{\Lambda_\perp}{m} - 1\right]\frac{\Delta^2}{m^2}$$

$$B_3 = B_3^{res} + B_3^{Arc} + B_3^{ZM} = \frac{g^2}{32\pi^2}$$

Without zero-modes B_1, B_2, B_3 depend on Δ and cannot be eliminated by the counter terms.

• Form factors: summary

$$F_1 = \frac{g^2}{16\pi^2}\log\frac{\Lambda_\perp}{m} - \frac{g^2}{4\pi^2}\left(\log\frac{\mu}{m} + \frac{14}{16}\right) + \mathcal{O}(\Delta^2)$$

$$F_2 = \frac{3g^2}{16\pi^2} + \mathcal{O}(\Delta^2)$$

$$B_1 = -\frac{g^2}{64\pi^2}, \quad B_2 = -\frac{g^2}{32\pi^2}, \quad B_3 = \frac{g^2}{32\pi^2}$$ $\Leftarrow$ for any Δ and $\Lambda_\perp$

We find finite non-zero form factors B_1, B_2, B_3 !
It looks very paradoxical. How can it be?

• Example

Cutoff $k < L$: $\quad I_{ij} = \int \frac{k_i k_j}{(k^3 + m^2)^{5/2}} \theta(L^2 - k^2)\, d^3k = F\delta_{ij}$

$$F = \frac{4\pi}{3} \log \frac{L}{m} + \frac{4\pi}{3}(\log 8 - 4)$$

Cutoff $k_\perp < \Lambda_\perp$, $\vec{k}_\perp \perp \vec{n}$:

$$I_{ij} = \int_{-\infty}^{\infty} dk_{||} \int \frac{k_i k_j \theta(\Lambda^2 - k_\perp^2)}{(k^3 + m^2)^{5/2}}\, d^2k_\perp = F\delta_{ij} + B n_i n_j$$

$$F = \frac{4\pi}{3}\left[\log \frac{\Lambda_\perp}{m} - \frac{1}{2}\right], \quad B = \frac{2\pi}{3}$$

B is finite when $\Lambda_\perp \to \infty$.

• Calculating B

Cutoff $k < L$:

$$B = \frac{1}{2}\int_{sphere} \frac{(3\cos\theta^2 - 1)k^2}{(k^3 + m^2)^{5/2}} d^3k = 0$$

Cutoff $k_\perp < \Lambda_\perp$:

$$B = -\frac{1}{2}\int_{cylinder} \frac{(3\cos\theta^2 - 1)k^2}{(k^3 + m^2)^{5/2}} d^3k = \frac{2\pi}{3}$$

B is generated by "non-covariant" $k_\perp$ cutoff.
The same in LFD. The only (but very important) that comes from LFD is the LF plane.

• Puali-Villars regularization

$$
\begin{aligned}
F_1^{res,PV} &= \frac{g^2}{16\pi^2}\log\frac{\mu_1}{m} - \frac{g^2}{4\pi^2}\left(\log\frac{\mu}{m} + \frac{15}{16}\right) + \mathcal{O}(\Delta^2) \\
F_2^{res,PV} &= \frac{3g^2}{16\pi^2} + \mathcal{O}(\Delta^2) + \mathcal{O}(\Delta^2) \\
B_1^{res,PV} &= B_2^{res,PV} = B_3^{res,PV} = 0
\end{aligned}
$$

PV meson is enough for F_1, F_2, B_1, B_2.
PV meson and fermion are needed for B_3.

• Spin 1

$$
\begin{aligned}
G^{\rho}_{\lambda'\lambda} &= e^{*\lambda'}_{\mu}(p')\Big\{\{P_\rho\left[\mathcal{F}_1(q^2)g^{\mu\nu} + \mathcal{F}_2(q^2)\frac{q^\mu q^\nu}{2M^2}\right] \\
&+ \mathcal{G}_1(q^2)(g^\mu_\rho q^\nu - g^\nu_\rho q^\mu) \\
&+ B^{\mu\nu}_\rho\Big\}e^\lambda_\nu(p)
\end{aligned}
$$

• Eight form factors B's

$$
\begin{aligned}
B_\rho^{\mu\nu} &= \frac{M^2}{2(\omega\cdot p)}\,\omega_\rho\left[B_1 g^{\mu\nu} + B_2\frac{q^\mu q^\nu}{M^2} + B_3 M^2\frac{\omega^\mu\omega^\nu}{(\omega\cdot p)^2}\right.\\
&+ \left. B_4\frac{q^\mu\omega^\nu - q^\nu\omega^\mu}{2\omega\cdot p}\right] + B_5 P_\rho M^2\frac{\omega^\mu\omega^\nu}{(\omega\cdot p)^2} + B_6 P_\rho\frac{q^\mu\omega^\nu - q^\nu\omega^\mu}{2\omega\cdot p}\\
&+ B_7 M^2\frac{g_\rho^\mu\omega^\nu + g_\rho^\nu\omega^\mu}{\omega\cdot p} + B_8 q_\rho\frac{q^\mu\omega^\nu + q^\nu\omega^\mu}{2\omega\cdot p}
\end{aligned}
$$

V.A.K. and A.V. Smirnov, Nucl. Phys. **A546** (1992) 691.

• Finding form factors

$$G^{+}_{++} = -\mathcal{F}_1 + \eta\mathcal{F}_2 \,, \quad \text{(e.g.: } G^{+}_{++} = G^{\rho=+}_{\lambda'=+1,\lambda=+1}\text{)}$$

$$G^{+}_{+-} = -\eta\mathcal{F}_2 \,,$$

$$G^{+}_{+0} = -\sqrt{2\eta}(\mathcal{F}_1 - \eta\mathcal{F}_2 + \mathcal{G}_1/2) + \sqrt{\eta/2}B_6 \,,$$

$$G^{+}_{00} = -(1-2\eta)\mathcal{F}_1 - 2\eta^2\mathcal{F}_2 + 2\eta\mathcal{G}_1 - 2\eta B_6 + B_5 + B_7 \,.$$

Angular condition

$$\Delta(Q^2) \equiv (1+2\eta)G^{+}_{++} + G^{+}_{+-} - 2\sqrt{2\eta}G^{+}_{+0} - G^{+}_{00} = -(B_5 + B_7) \,.$$

V.A.K., Nucl. Phys. **A608** (1996) 316.

• Comparison with JBC

C.-H. Ji, B.L.G. Bakker and H.-M. Choi, LC2005, hep-ph/0510210.:
F_1, F_2, F_3
Also: a few papers and talk at LC06 by J.P.C. de Melo, T. Frederico et al.

$$F_1 = -\frac{1}{2p_+}\mathcal{F}_1, \quad F_2 = -\frac{1}{2p_+}\mathcal{G}_1, \quad F_3 = -\frac{1}{2p_+}\mathcal{F}_2.$$

$$(F_2 + 2F_1)^{+0} = \frac{1}{p_+}\left(\frac{1}{2\eta}G^+_{+0} + G^+_{+-}\right)$$

$$(F_2 + 2F_1)^{00} = \frac{1}{4\eta p_+}\left[(1+2\eta)G^+_{++} - G^+_{00} + (1+4\eta)G^+_{+-}\right]$$

• Translation of JBC in explicitly cov. LFD

JBC: $(F_2+2F_1)^{00}$ and $(F_2+2F_1)^{+0}$ disagree.
transl.: $(F_2+2F_1)^{00}-(F_2+2F_1)^{+0}=-\frac{1}{4\eta p_+}(B_5+B_7)$
disagree by $\sim (B_5+B_7)$.

JBC: (F_2+2F_1) depends on the renormalization method.
transl.: The values of B's depends on the renormalization method.

JBC: $(F_2+2F_1)^{00}$ and $(F_2+2F_1)^{+0}$ are the same for PV reg.
transl.: $B_i=0$ for PV reg.

JBC: F_3 (found from G^{+}_{+-}) does not depend on the renormalization method.
transl.: $G^{+}_{+-}=-2\eta p_+ F_3$ does not contain B and therefore it does not depend on the renormalization method.

• Resumè

- Properties B's are general, since they are determined by the "cylindrical" ($k_\perp$) cutoff.
- They were established for spin 1/2 e.m. vertex.
- They are confirmed by JBC for spin 1.

• Extracting form factors

$$\mathcal{F}_1 = G^{\mu\nu}_{\rho} \frac{\omega^{\rho}}{2\omega\cdot p} \left[g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2} - \frac{P_{\mu}\omega_{\nu} + P_{\nu}\omega_{\mu}}{2\omega\cdot p} + P^2 \frac{\omega_{\mu}\omega_{\nu}}{4(\omega\cdot p)^2} \right] ,$$

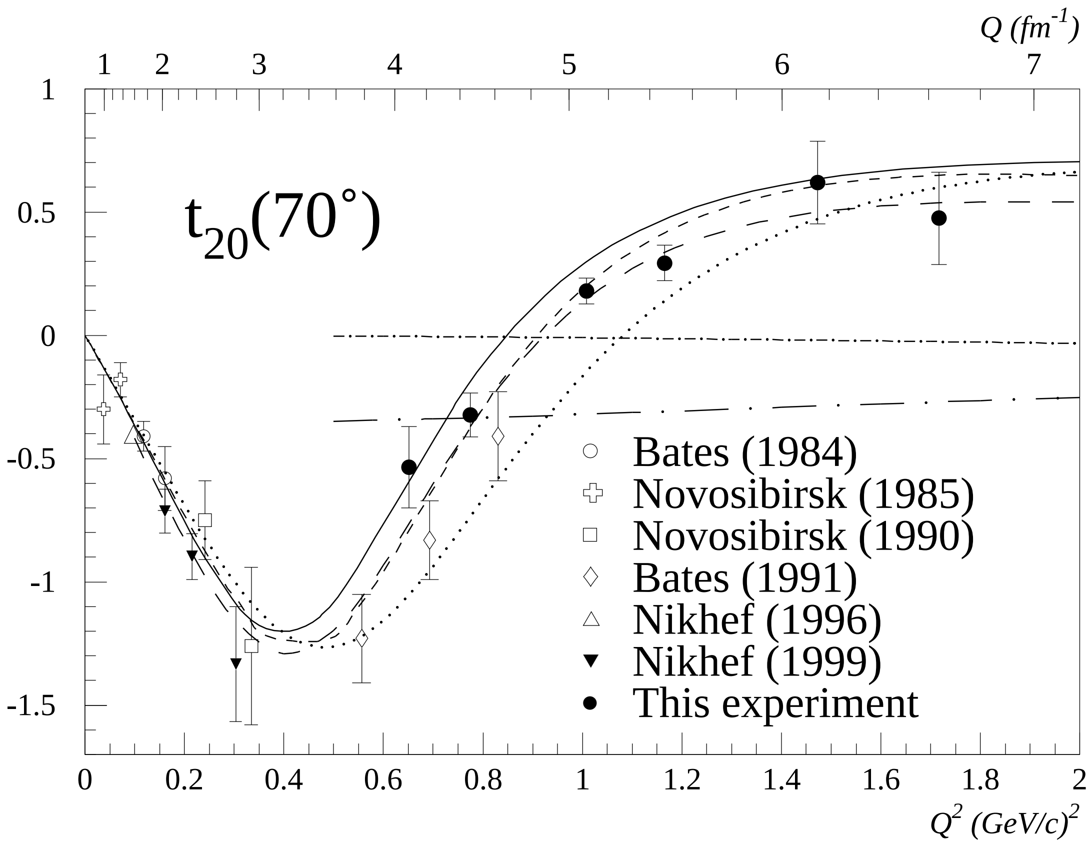
$$\frac{\mathcal{F}_2}{2M^2} = -G^{\mu\nu}_{\rho} \frac{\omega^{\rho}}{2(\omega\cdot p)q^2} \left[g_{\mu\nu} \right.$$

$$\left. -2\frac{q_{\mu}q_{\nu}}{q^2} - \frac{P_{\mu}\omega_{\nu} + P_{\nu}\omega_{\mu}}{2\omega\cdot p} + M^2 \frac{\omega_{\mu}\omega_{\nu}}{(\omega\cdot p)^2} - \frac{q_{\mu}\omega_{\nu} - q_{\nu}\omega_{\mu}}{2\omega\cdot p} \right] ,$$

$$\mathcal{G}_1 = \frac{1}{4} G^{\mu\nu}_{\rho} \left[2\frac{g^{\rho}_{\mu}q_{\nu} - g^{\rho}_{\nu}q_{\mu}}{q^2} + \frac{g^{\rho}_{\mu}\omega_{\nu} + g^{\rho}_{\nu}\omega_{\mu}}{\omega\cdot p} + ldots \right]$$

V.A.K. and A.V. Smirnov, Nucl. Phys. **A546** (1992) 691.

• Deuteron



Deuteron polarization observable t_{20}

Our calculation (J. Carbonell and V.A.K., Eur. Phys. J.) has been published two years before experiment!

• Question

Can one invent an invariant regularization (à la PV), but without extra PV particles?

A promising candidate:
Approach presented at LC05 and LC06 by P. Grangé.

Since Pierre Grangé claims:

- regularization is covariant;
- no PV particles (no need to increase Fock state basis)

• Conclusions

- For the non-renormalized, cutoff-dependent Feynman and LFD amplitudes (invariant cutoff L for Feynman, $\Lambda_\perp$ cutoff for LFD) the coincidence, in general, cannot be achieved.
  - Zero-modes improve but do not save the situation.
  - With the invariant regularization (Pauli-Villars) the Feynman and LFD amplitudes coincide.

- The renormalized, finite Feynman and LFD amplitudes coincide with each other. However, the difficulty of calculation depends on the type of cutoff (PV or $\Lambda_\perp$).
  - With $\Lambda_\perp$ and without zero modes one should introduce infinite number of ω-dependent counter terms.
  - With $\Lambda_\perp$ and with zero modes one should introduce the ω-dependent counter terms, finite number.
  - With PV regularization the zero modes are zero. Normal counter terms are enough.

Replace the PV regularization by other invariant regularization without PV particles?