On derivation of the LFD fermion form factors from the Feynman approach

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Perturbative calculation of EM form factors, in Yukawa model, in LFD, with emphasis on dependence on the regularization procedure.

- **Plan**
  - Introduction (motivation)
  - Calculating self energy in Yukawa model.
  - Calculating EM form factors in Yukawa model.
  - Dependence on the regularization procedure.
  - Conclusions
Motivation

It is two-fold:

- $f f b$ vertex enters our non-perturbative equations (talk by J.-F. Mathiot)
  To study its properties and vertex counter terms.

- The EM vertex $\gamma f f$ is calculated with our LFWF.
  To study its structure and counter terms.
• References

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...

The difference: we use Explicitly Covariant version of LFD.
Analysis from this point of view is instructive.
Explicitly covariant LFD

V.A. Karmanov, JETP, 44 (1976) 201.

Take LF of general orientation:

\[ \omega \cdot x = \omega_0 t - \bar{\omega} \cdot \bar{x} = 0, \omega = (\omega_0, \bar{\omega}) \text{ such that } \omega^2 = 0. \]

Construct LFD, using this general LF plane.

Standard approach: \( \omega = (1, 0, 0, -1) \).
In LF coordinates: \( \omega_- = 2, \omega_+ = 0, \bar{\omega} \perp = 0 \).
• **Advantages**

- Explicit covariance
- When an amplitude depends on LF plane orientation
  
  ⇒
  
  this dependence is explicit (v.s. $\omega$.)

- Observables (e.g., EM vertex and form factors) must not depend on LF plane orientation.
  
  ⇒
  
  Analysis of their dependence on $\omega$ because of cutoff and/or approximation.

- Etc.
Calculating self energy

Yukawa model, LFD

\[ \Sigma = -\frac{g^2}{(2\pi)^3} \int \frac{(\hat{p} - \hat{k} + m)}{\left(\frac{k_{\perp}^2 + \mu^2}{x} + \frac{k_{\perp}^2 + m^2}{1-x} - p^2\right)} \frac{d^2 k_{\perp} dx}{2x} \]

\( \vec{k}_{\perp} \perp \vec{\omega} \) \Rightarrow dependence of \( \Sigma \) on \( \omega \)

General decomposition:

\[ \Sigma(p) = A(p^2) + \frac{\hat{p}}{m} B(p^2) + \frac{\hat{\omega}}{\omega \cdot p} C(p^2) \]

\[ C = \frac{1}{4} Tr \left[ \Sigma \left( \frac{\hat{p}}{\omega \cdot \hat{\omega}} - \frac{p^2}{\omega \cdot \hat{\omega}} \right) \right] \]
Calculating $\Sigma$ from Feynman graph

\[-i\Sigma = (-ig)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - \mu^2 + i\epsilon} \frac{i(\hat{p} - \hat{k} + m)}{(p - k)^2 - m^2 + i\epsilon}\]

Introduce the LF variables:

\[k_- = k_0 - k_z, \quad k_+ = k_0 + k_z, \quad \vec{k}_\perp = (k_x, k_y)\]

and put $k_+ = xp_+$:

\[\Sigma = ig^2 \int \frac{p_+ dk_- dx d^2k_\perp}{2(2\pi)^4} \frac{1}{[k_-p_+x - k_\perp^2 - \mu^2 + i\epsilon]} \times \frac{(\hat{p} - \hat{k} + m)}{[(p_- - k_-)p_+(1 - x) - (\vec{p}_\perp - \vec{k}_\perp)^2 - m^2 + i\epsilon]}\]
Three contributions

The integral over $k_-$ can be calculated directly, by primitive. But it is useful to separate it in three contributions. (Ligterink, Bakker, Ji).

- Residues.
- Integral over arc.
- End point singularities (zero modes).
\section*{Residue contribution}

The poles are at different sides of contour if $0 < x < 1$

\[ C^{res} = g^2 \int \frac{d^2k_\perp}{4(2\pi)^3} \log \frac{k_\perp^2 + \mu^2}{k_\perp^2 + m^2} - \frac{g^2}{32\pi^2} \log \delta \int d^2k_\perp \]
• Arc contribution

\[ C^{Arc} \sim \int_{\infty}^{\infty} \ldots \frac{dk_-}{k_-} = \int_{\pi}^{0} \ldots id\phi, \quad k_- = Re^{i\phi}, \quad R \to \infty \]
Zero modes

\[ \Sigma = ig^2 \int_{-L}^{L} dk_- \int \frac{p_+ dx d^2 k_\perp}{2(2\pi)^4} \left\{ \frac{1}{[k_- p_+ x - k_\perp^2 - \mu^2]} \frac{\frac{1}{2} \gamma_+ k_-}{(k_- p_+)} \right\}_{x \to 0} \]

\[ = -ig^2 \gamma_+ \int_{-L}^{L} dk_- \int_{-\epsilon}^{\epsilon} dx \int \frac{d^2 k_\perp}{4(2\pi)^4} \frac{1}{[k_\perp^2 + \mu^2]} \]

When \( x \to 0 \) (and \( x \to 1 \)), integral over \( k_- \) diverges:

\[ \Sigma \sim \gamma_+ \int_{-L}^{L} dk_- \ldots \sim \gamma_+ L \to \infty \]

\[ \int_{-L}^{L} \frac{dk_-}{x k_- - b - xp + i0} \sim \delta(x) \quad \Leftarrow \quad \text{zero modes} \]
\[ C^{res} = g^2 \int \frac{d^2 k_\perp}{4(2\pi)^3} \log \frac{k_\perp^2 + \mu^2}{k_\perp^2 + m^2} - \frac{g^2}{32\pi^2} \log \delta \int d^2 k_\perp \]

\[ C^{Arc} = + \frac{g^2}{32\pi^2} \log \delta \int d^2 k_\perp \]

\[ C^{ZM} = -g^2 \int \frac{d^2 k_\perp}{4(2\pi)^3} \log \frac{k_\perp^2 + \mu^2}{k_\perp^2 + m^2} \]

\[ C = C^{res} + C^{Arc} + C^{ZM} = 0 \]
Resumé

Arc and zero mode contributions kill the $\omega$-dependent contribution in self energy.

However, they *do not* kill the $\omega$-dependent contribution in EM vertex (see below).
Regularization by the Pauli-Villars meson $\mu_1$ and fermion $m_1$

\[
C^{PV} = \left[ C(m, \mu) - C(m, \mu_1) \right] - \left[ C(m_1, \mu) - C(m_1, \mu_1) \right]
\]

\[
C^{\text{res},PV} = 0 \\
C^{\text{Arc},PV} = 0 \\
C^{\text{ZM},PV} = 0
\]

\[
C^{PV} = C^{\text{res},PV} + C^{\text{Arc},PV} + C^{\text{ZM},PV} = 0
\]

$A, B$ are standard.
**EM vertex and form factors**

General decomposition

\[
\bar{u}' \Gamma_\rho (p, p') u = \bar{u}' \left[ F_1 \gamma_\rho + \frac{i F_2}{2m} \sigma_{\rho \nu} q_\nu \right]
+ B_1 \left( \frac{\varphi}{\omega \cdot p} P_\rho - 2 \gamma_\rho \right)
+ B_2 \frac{m \omega_\rho}{\omega \cdot p}
+ B_3 \frac{m^2 \varphi \omega_\rho}{(\omega \cdot p)^2} \right] u
\]

FF’s are found by: \( F_1 = \frac{1}{2} c_5, \quad F_2 = \frac{1}{2 \eta} (c_5 - c_4), \)

with \( c_4 = m \text{Tr} [O_\rho] \omega_\rho / (\omega \cdot p), \quad c_5 = m^2 \text{Tr} \left[ O_\rho^{LF} \varphi \right] \omega_\rho / (\omega \cdot p)^2 \)

Similarly for \( B_{1-3}. \)
• Calculating $\Gamma_\rho$ from Feynman graph

Figure 3: Feynman diagram for the EM vertex.
Arc integrals and zero modes do not contribute in $F_1, F_2$. The only contribution results from residues.

\[
F_1^{\text{Feyn}} = \frac{g^2}{16\pi^2} \log \frac{L}{m} - \frac{g^2}{4\pi^2} \left( \log \frac{\mu}{m} + \frac{15}{16} \right) + O(\Delta^2)
\]

\[
F_1^{\text{LF, res}} = \frac{g^2}{16\pi^2} \log \frac{\Lambda_\perp}{m} - \frac{g^2}{4\pi^2} \left( \log \frac{\mu}{m} + \frac{14}{16} \right) + O(\Delta^2)
\]

There is a difference:

\[
F_1^{\text{Feyn}} - F_1^{\text{LF, res}} = -\frac{g^2}{64\pi^2}
\]

\[
F_2^{\text{Feyn}} = F_2^{\text{res}} = \frac{3g^2}{16\pi^2} + O(\Delta^2)
\]
Form factor $B_1$

Arc integral does not contribute.
Zero modes contribute.

\[
B_1^{\text{res}} \approx \frac{g^2}{16\pi^2} \log \frac{\Lambda}{m} - \frac{g^2}{64\pi^2} \Delta^2 + O(\Delta^4)
\]

\[
B_1^{\text{Arc}} = 0
\]

\[
B_1^{\text{ZM}} = -\frac{g^2}{16\pi^2} \log \frac{\Lambda}{m} + \frac{g^2}{192\pi^2} \Delta^2 + O(\Delta^4)
\]

\[
B_1 = B_1^{\text{res}} + B_1^{\text{Arc}} + B_1^{\text{ZM}} = -\frac{g^2}{64\pi^2}
\]
**Form factor** $B_2$

Arc integral does not contribute. Zero modes contribute.

\[
B_{2}^{\text{res}} = - \frac{g^2}{8\pi^2} \log \frac{\Lambda_\perp}{m} - \frac{g^2}{32\pi^2} + \frac{g^2}{96\pi^2} \frac{\Delta^2}{m^2} + O(\Delta^4)
\]

\[
B_{2}^{ZM} = 0
\]

\[
B_{2}^{ZM} = \frac{g^2}{8\pi^2} \log \frac{\Lambda_\perp}{m} - \frac{g^2}{96\pi^2} \frac{\Delta^2}{m^2} + O(\Delta^4)
\]

\[
B_{2} = B_{2}^{\text{res}} + B_{2}^{\text{Arc}} + B_{2}^{ZM} = - \frac{g^2}{32\pi^2}
\]
Form factor $B_3$

Both arc integral and zero modes contribute.

\[
B_{3}^{\text{res}} = -\frac{g^2}{32\pi^2} \Lambda_{\perp}^2 \log \epsilon - \frac{3g^2}{16\pi^2} \log \frac{\Lambda_{\perp}}{m} - \frac{3g^2}{96\pi^2} \left[ 3 \log \frac{\Lambda_{\perp}}{m} - 1 \right] \frac{\Delta^2}{m^2}
\]

\[
B_{3}^{\text{Arc}} = \frac{g^2}{32\pi^2} \Lambda_{\perp}^2 \log \epsilon
\]

\[
B_{3}^{ZM} = \frac{g^2}{32\pi^2} + \frac{3g^2}{16\pi^2} \log \frac{\Lambda_{\perp}}{m} + \frac{3g^2}{96\pi^2} \left[ 3 \log \frac{\Lambda_{\perp}}{m} - 1 \right] \frac{\Delta^2}{m^2}
\]

\[
B_{3} = B_{3}^{\text{res}} + B_{3}^{\text{Arc}} + B_{3}^{ZM} = \frac{g^2}{32\pi^2}
\]

Without zero-modes $B_1, B_2, B_3$ depend on $\Delta$ and cannot be eliminated by the counter terms.
**Form factors: summary**

\[
F_1 = \frac{g^2}{16\pi^2} \log \frac{\Lambda_\perp}{m} - \frac{g^2}{4\pi^2} \left( \log \frac{\mu}{m} + \frac{14}{16} \right) + \mathcal{O}(\Delta^2)
\]

\[
F_2 = \frac{3g^2}{16\pi^2} + \mathcal{O}(\Delta^2)
\]

\[
B_1 = -\frac{g^2}{64\pi^2}, \quad B_2 = -\frac{g^2}{32\pi^2}, \quad B_3 = \frac{g^2}{32\pi^2} \quad \Leftarrow \text{for any } \Delta \text{ and } \Lambda_\perp
\]

We find **finite** non-zero form factors \(B_1, B_2, B_3\)!

It looks very paradoxical. How can it be?
• Example

Cutoff $k < L$: \[ I_{ij} = \int \frac{k_i k_j}{(k^3 + m^2)^{5/2}} \theta(L^2 - k^2) \, d^3k = F \delta_{ij} \]

\[ F = \frac{4\pi}{3} \log \frac{L}{m} + \frac{4\pi}{3} (\log 8 - 4) \]

Cutoff $k_\perp < \Lambda_\perp$, $\vec{k}_\perp \perp \vec{n}$:

\[ I_{ij} = \int_{-\infty}^{\infty} dk_\parallel \int \frac{k_i k_j \theta(\Lambda^2 - k_\perp^2)}{(k^3 + m^2)^{5/2}} \, d^2k_\perp = F \delta_{ij} + B n_i n_j \]

\[ F = \frac{4\pi}{3} \left[ \log \frac{\Lambda_\perp}{m} - \frac{1}{2} \right], \quad B = \frac{2\pi}{3} \]

$B$ is finite when $\Lambda_\perp \to \infty$. 
• Calculating $B$

Cutoff $k < L$:

$$B = \frac{1}{2} \int_{sphere} \frac{(3 \cos \theta^2 - 1)k^2}{(k^3 + m^2)^{5/2}} d^3k = 0$$

Cutoff $k_\perp < \Lambda_\perp$:

$$B = -\frac{1}{2} \int_{cylinder} \frac{(3 \cos \theta^2 - 1)k^2}{(k^3 + m^2)^{5/2}} d^3k = \frac{2\pi}{3}$$

$B$ is generated by "non-covariant" $k_\perp$ cutoff.
The same in LFD. The only (but very important) that comes from LFD is the LF plane.
• Puali-Villars regularization

\[ F_{1}^{res, PV} = \frac{g^2}{16\pi^2} \log \frac{\mu_1}{m} - \frac{g^2}{4\pi^2} \left( \log \frac{\mu}{m} + \frac{15}{16} \right) + \mathcal{O}(\Delta^2) \]

\[ F_{2}^{res, PV} = \frac{3g^2}{16\pi^2} + \mathcal{O}(\Delta^2) + \mathcal{O}(\Delta^2) \]

\[ B_{1}^{res, PV} = B_{2}^{res, PV} = B_{3}^{res, PV} = 0 \]

PV meson is enough for \( F_1, F_2, B_1, B_2 \).
PV meson and fermion are needed for \( B_3 \).
\[
G^{\rho}_{\lambda\lambda'} = e^{*\lambda'}_{\mu}(p') \left\{ \left\{ P_{\rho} \left[ F_{1}(q^2)g^{\mu\nu} + F_{2}(q^2)\frac{q^\mu q^\nu}{2M^2} \right] \right\} G_{1}(q^2)(g_{\rho\mu}q^\nu - g_{\rho\nu}q^\mu) + B_{\rho}^{\mu\nu} \right\} e^{\lambda}_{\nu}(p)
\]
\[ B_{\rho}^{\mu\nu} = \frac{M^2}{2(\omega \cdot p)} \omega_{\rho} \left[ B_1 g_{\mu\nu} + B_2 \frac{q^\mu q^\nu}{M^2} + B_3 M^2 \frac{\omega^\mu \omega^\nu}{(\omega \cdot p)^2} \right. \]
\[ + \frac{B_4 \left( q^\mu \omega^\nu - q^\nu \omega^\mu \right)}{2\omega \cdot p} \left. \right] + B_5 P_\rho M^2 \frac{\omega^\mu \omega^\nu}{(\omega \cdot p)^2} + B_6 P_\rho \frac{q^\mu \omega^\nu - q^\nu \omega^\mu}{2\omega \cdot p} \]
\[ + B_7 M^2 \frac{g_{\rho}^\mu \omega^\nu + g_{\rho}^\nu \omega^\mu}{\omega \cdot p} + B_8 q_\rho \frac{q^\mu \omega^\nu + q^\nu \omega^\mu}{2\omega \cdot p} \]

Finding form factors

\[ G_{++}^+ = -\mathcal{F}_1 + \eta \mathcal{F}_2 , \quad (\text{e.g.: } G_{++}^+ = G_{\lambda'=+1,\lambda=+1}^\rho) \]
\[ G_{+-}^+ = -\eta \mathcal{F}_2 , \]
\[ G_{+0}^+ = -\sqrt{2\eta}(\mathcal{F}_1 - \eta \mathcal{F}_2 + G_1/2) + \sqrt{\eta/2} B_6 , \]
\[ G_{00}^+ = -(1 - 2\eta) \mathcal{F}_1 - 2\eta^2 \mathcal{F}_2 + 2\eta G_1 - 2\eta B_6 + B_5 + B_7 . \]

Angular condition

\[ \Delta(Q^2) \equiv (1 + 2\eta)G_{++}^+ + G_{+-}^+ - 2\sqrt{2\eta}G_{+0}^+ - G_{00}^+ = -(B_5 + B_7) . \]

Comparison with JBC


Also: a few papers and talk at LC06 by J.P.C. de Melo, T. Frederico et al.

\[
F_1 = -\frac{1}{2p_+} F_1, \quad F_2 = -\frac{1}{2p_+} G_1, \quad F_3 = -\frac{1}{2p_+} F_2.
\]

\[
(F_2 + 2F_1)^{+0} = \frac{1}{p_+} \left( \frac{1}{2\eta} G^{+0} + G^{+-} \right)
\]

\[
(F_2 + 2F_1)^{00} = \frac{1}{4\eta p_+} \left[ (1 + 2\eta) G^{++} - G^{00} + (1 + 4\eta) G^{+-} \right]
\]
Translation of JBC in explicitly cov. LFD

JBC: \((F_2 + 2F_1)^{00}\) and \((F_2 + 2F_1)^{+0}\) disagree.

transl.: \((F_2 + 2F_1)^{00} - (F_2 + 2F_1)^{+0} = -\frac{1}{4\eta p_+}(B_5 + B_7)\)

disagree by \(\sim (B_5 + B_7)\).

JBC: \((F_2 + 2F_1)\) depends on the renormalization method.

transl.: The values of \(B\)'s depends on the renormalization method.

JBC: \((F_2 + 2F_1)^{00}\) and \((F_2 + 2F_1)^{+0}\) are the same for PV reg.

transl.: \(B_i = 0\) for PV reg.

JBC: \(F_3\) (found from \(G^{+}_{++}\)) does not depend on the renormalization method.

transl.: \(G^{+}_{+-} = -2\eta p_+ F_3\) does not contain \(B\) and therefore it does not depend on the renormalization method.
Resumè

- Properties $B$’s are general, since they are determined by the "cylindrical" ($k_\perp$) cutoff.
- They were established for spin 1/2 e.m. vertex.
- They are confirmed by JBC for spin 1.
Extracting form factors

\[
\mathcal{F}_1 = G^\mu_\nu \frac{\omega^\rho}{2\omega \cdot p} \left[ g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} - \frac{P_\mu \omega_\nu + P_\nu \omega_\mu}{2\omega \cdot p} + P^2 \frac{\omega_\mu \omega_\nu}{4(\omega \cdot p)^2} \right],
\]

\[
\frac{\mathcal{F}_2}{2M^2} = -G^\mu_\nu \frac{\omega^\rho}{2(\omega \cdot p) q^2} \left[ g_{\mu\nu} - \frac{2q_\mu q_\nu}{q^2} - \frac{P_\mu \omega_\nu + P_\nu \omega_\mu}{2\omega \cdot p} + M^2 \frac{\omega_\mu \omega_\nu}{(\omega \cdot p)^2} - \frac{q_\mu \omega_\nu - q_\nu \omega_\mu}{2\omega \cdot p} \right],
\]

\[
G_1 = \frac{1}{4} G^\mu_\nu \left[ 2 \frac{g^\rho_\mu q_\nu - g^\rho_\nu q_\mu}{q^2} + \frac{g^\rho_\mu \omega_\nu + g^\rho_\nu \omega_\mu}{\omega \cdot p} + l\text{dots} \right].
\]

Deuteron polarization observable $t_{20}$

Our calculation (J. Carbonell and V.A.K., Eur. Phys. J.) has been published two years before experiment!
Can one invent an invariant regularization (à la PV), but without extra PV particles?

A promising candidate: Approach presented at LC05 and LC06 by P. Grangé.

Since Pierre Grangé claims:
- regularization is covariant;
- no PV particles (no need to increase Fock state basis)
Conclusions

- For the non-renormalized, cutoff-dependent Feynman and LFD amplitudes (invariant cutoff $L$ for Feynman, $\Lambda_\perp$ cutoff for LFD) the coincidence, in general, cannot be achieved.
- Zero-modes improve but do not save the situation.
- With the invariant regularization (Pauli-Villars) the Feynman and LFD amplitudes coincide.
The renormalized, finite Feynman and LFD amplitudes coincide with each other. However, the difficulty of calculation depends on the type of cutoff (PV or $\Lambda_\perp$).

- With $\Lambda_\perp$ and without zero modes one should introduce infinite number of $\omega$-dependent counter terms.
- With $\Lambda_\perp$ and with zero modes one should introduce the $\omega$-dependent counter terms, finite number.
- With PV regularization the zero modes are zero. Normal counter terms are enough.

Replace the PV regularization by other invariant regularization without PV particles?