Poincaré invariance with fields as OPVD in the Epstein and Glaser context

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"The SMOOTH OPERATOR":
(a hit for Evening Jazz Sessions ?)
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- Equivalence with dispersion relations
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- Provides a comprehensive handling of UV and IR behaviour: see LC2004 (Amsterdam) and LC2005 (Cairns)
Scalar Field (or Fermi as well) as OPVD
OPVD defines a functional with respect to a test function \( \rho(x) \), \( C^\infty \) with compact support,
\[
\Phi(\rho) \equiv \langle \varphi, \rho \rangle = \int d^Dy \varphi(y) \rho(y).
\]
More general interpretation: functional \( \Phi(x, \rho) \) evaluated at \( x = 0 \).
The translated functional is a well defined object such that
\[
T_x \Phi(\rho) = \langle T_x \varphi, \rho \rangle = \langle \varphi, T_{-x} \rho \rangle = \int d^Dy \varphi(y) \rho(x - y).
\]
Due to the properties of \( \rho \), \( T_x \Phi(\rho) \) obeys the EQM (KG or Dirac) and is taken as the physical field \( \phi(x) \)
Fourier decomposition of $\rho(x - y)$

$$
\rho(x - y) = \int \frac{d^{(D)}q}{(2\pi)^D} e^{iq(x-y)} f(q_0^2, \vec{q}^2)
$$

quantized form for $\phi(x)$ follows:

In Minkowskian metric:

$$
\phi(x) = \int \frac{d^{(D-1)}p}{(2\pi)^{D-1}} \frac{f(\omega_p^2, \vec{p}^2)}{(2\omega_p)} [a_p e^{ipx} + a_p^* e^{-ipx}].
$$

or in Euclidean metric:

$$
\phi(x) = \int \frac{d^{(D)}p}{(2\pi)^D} [a_p e^{-ipx} + a_p^* e^{ipx}] f(p^2).
$$
Mathematical apparatus: 4 theorems

i) Open covering of a topological space $M$

$\exists (O_i)_{i \in I}$ open subsets that cover $M$: $M = \bigcup_{i \in I} O_i$;

$(\alpha_i)$ a family of functions on $M \rightarrow \mathbb{R}$;

$\alpha_i$ is locally finite: for any point $P \in M$ only a finite number of $\alpha$’s are $0$; $\sum_{i \in I} \alpha_i$ is finite at every point. Then

$$\beta_j = \frac{\alpha_j}{\sum_{j \in I} \alpha_j}$$

is a partition of unity on $M$ as $\sum_{j \in I} \beta_j = 1$

ii) Paracompact spaces and decomposition of unity subordinate to the open covering,

$M$ is paracompact if to an arbitrary open covering $(O_i)_{i \in I}$ one can find a partition of unity $\sum_{i \in I} \beta_i$ such that $\beta_i$ vanishes outside $O_i$: $\beta_i$ is said to be subordinate to $O_i$. 
iii) Euclidean manifolds are paracompact: one may use partition of unity when needed. Well known case: definition of integrals of differential forms (Spivak, Felsager, Kobayashi-Nomizu,...)

$F$ a differential form, $(\beta_i)$ is used to cut $F$ into small pieces: $F_i = \beta_i F$ and $\sum F_i = F$. For $\Omega \subset M$ one defines:
$$\int_{\Omega} F := \sum_i \int_{\Omega_i} \beta_i F.$$ result is independent of the choice of coordinates (atlas) on $\Omega_i$ and of the partition of unity.

iv) Localisation of distributions: J. Dieudonné’s GPT-theorem (Glueing-Pieces-Together) establishes the above properties for distribution functionals on Euclidean manifolds
Consequences:

i) the test function $f(p^2)$ can be taken as a partition of unity and the integral defining the physical field $\phi(x)$ is independant of its construction.

ii) The translated functional built out of the physical field $\phi(x)$ is again $\phi(x)$ itself

$$T_x\phi(\rho) = \int d^{(D)}y \phi(y) \rho(x - y) = \int \frac{d^{(D)}p}{(2\pi)^D} [a_p e^{-ipx} + a_p e^{ipx}] f^2(p^2).$$

here $f^2(p^2)$ is still a decomposition of unity (another) and $T_x\phi(\rho) \equiv \phi(x)$.

the test function $f(p^2)$ ensures convergence of otherwise diverging integrals but plays no role on the reverse.(it may taken as 1 everywhere in this case)
Partition of unity and Poincaré commutator algebra (scalar field) Itzykson-Zuber conventions (p115)

\[ d\Omega_k = \frac{d^3 k}{(2\pi)^3 2\omega_k}; \omega_k = \sqrt{k^2 + m^2}; [a^-_k, a^+_k] = (2\pi)^3 2\omega_k \delta(\vec{k} - \vec{k}') \]

\[ \varphi(x) = \int d\Omega_k [a^-_k e^{-ik \cdot x} + a^+_k e^{ik \cdot x}] f(\omega_k^2, \bar{k}'^2); \]

\[ \Pi(x) = -i \int d\Omega_k \omega_k [a^-_k e^{-ik \cdot x} - a^+_k e^{ik \cdot x}] f(\omega_k^2, \bar{k}'^2); \]

\[ \theta^{\mu\nu} = \partial^\mu \varphi \partial^\nu \varphi - \frac{1}{2} g^{\mu\nu} [(\partial \varphi)^2 - m^2 \varphi^2] \]

Purpose: Check that \( i [P^\mu, \varphi(x)] = \partial^\mu \varphi(x) \)

\[ P^0 = H = \int d\Omega_k \omega_k a^+_k a^-_k f^2(\omega_k^2, \bar{k}'^2) \]

\[ P^j = \frac{1}{2} \int d\Omega_k \omega_k k^j [a^+_k a^-_k + a^-_k a^+_k] f^2(\omega_k^2, \bar{k}'^2) \]
\[ i[P^j, \varphi] = i \int d\Omega_k k^j [a_k^+ e^{ikx} - a_k^- e^{-ikx}] f^3(\omega^2, \vec{k}^2) \equiv \partial^j \varphi \]

since \( f^3 \) is also a partition of unity with same support as \( f^2 \)

\[ M^{\mu\nu} = \int d^3x [x^\mu \theta^{0\nu} - x^\nu \theta^{0\mu}] \]

\[ M_{0j} = - \int d^3x x_j \theta_{00} = i \int d\Omega_k \omega_k a_k^+ f(\omega_k^2, \vec{k}^2) \frac{\partial}{\partial k^j} (a_{\vec{k}} f(\omega_k^2, \vec{k}^2)) \]

\[ M_{jl} = i \int d\Omega_k a_k^+ f(\omega_k^2, \vec{k}^2) [k_j \frac{\partial}{\partial k^l} - k_l \frac{\partial}{\partial k^j}] (a_{\vec{k}} f(\omega_k^2, \vec{k}^2)) \]

The commutations of the \( a \)'s and \( a^+ \)'s give the usual result

\[ [M^{\mu\nu}, P^\lambda] = i(g^{\mu\nu} P^\lambda - g^{\nu\lambda} P^\mu) \]

since in the RHS \( f^2 \) in \( P^\nu (P^\mu) \) is replaced by \( f^4 \) with the same conclusion as before. Holds also for \([M^{\mu\nu}, M^{\lambda\sigma}]\)
Lorentz invariant extension of singular distributions: the double magic of the partition of unity and Lagrange's formula

$E - G$'s analysis of singular distributions

$f(X) : \mathcal{C}^\infty(\mathbb{R}^d)$ test function $\in \mathcal{S}(\mathbb{R}^d)$

$T(X)$ distribution $\in \mathcal{S}'(\mathbb{R}^d - \{0\})$

singular order $k$ of $T(X)$ at the origin of $(\mathbb{R}^d)$ such that

$$k = \inf \{ s : \lim_{\lambda \to 0} \lambda^s T(\lambda X) = 0 \} - d$$

$E - G$'s extension consists in performing an "educated" Taylor surgery on the original test function by throwing away the the weighed k-jet of $f(X)$. Call $R^k_1 f(X)$ (Taylor's remainder) Notation:

$$\alpha! = \alpha^1! \cdots \alpha^d!; |\alpha| = \alpha^1 + \cdots + \alpha^d; \partial^\alpha = \partial_{x_1}^{\alpha_1} \cdots \partial_{x_d}^{\alpha_d}$$

$$R^k_1 f(X) = f(X) - \sum_{n=0}^{k} \sum_{|\alpha| = n} \frac{X^\alpha}{\alpha!} \partial^\alpha f(X)|_{X=0}$$
Call $\tilde{T}(X)$ extension of $T(X)$ by transposition such that

$$< \tilde{T}, f > := < T, R^k_1 f >$$

but $R^k_1 f(X) \notin S(\mathbb{R}^d)$ E-G’s remedy: introduce a weight with properties $w(0) = 1$, $w^{(\alpha)}(0) = 0$, $0 < |\alpha| \leq k$ such that

$$R^k_w f(X) = f(X) - w(X) \sum_{n=0}^{k} \sum_{|\alpha| = n} \frac{X^\alpha}{\alpha!} \partial^\alpha f(X) |_{X=0}$$

$$< \tilde{T}, f > := < T, R^k_w f >$$

However under action of element $\Lambda$ of the Lorentz group derivatives transform as

$$x^\alpha \partial_\alpha (\Lambda f) = x^\alpha [\Lambda^{-1}]^\beta_\alpha (\partial_\beta f) \circ \Lambda^{-1}$$

$$= (\Lambda^{-1} X)^\beta (\partial_\beta f) \circ \Lambda^{-1}; \text{ and in the Taylor expansion}$$

$$x^\alpha \partial_\alpha (\Lambda f)(0) = (\Lambda^{-1} X)^\beta \partial_\beta f(0) \Rightarrow \text{Lorentz covariance violation}$$
Restauration: $\widehat{T}(X)$ determined up to $\sum_{|\alpha|\leq k} (-1)^{\alpha} \frac{a_\alpha}{\alpha!} \delta(\alpha)(X)$; since $\delta(\alpha)(\Lambda X) = [\Lambda^{-1} X]^\alpha_\beta \delta(\beta)(X)$ scheme is : determine $a_\alpha$ to correct for the violation due to derivatives.

- Lorentz covariance with test function as partition of unity and Lagrange’s formula for Taylor’s remainder $R^k_1 f(X)$ with a p. of u. $f^{(\alpha)}(0) = 0 \quad \forall \alpha \geq 0$ it holds that $f(X) = R^k_1 f(X) \quad \forall k > 0$ : no violation due to derivatives (they are just not there) and $\Lambda R^k_1 f(X) = R^k_1 \Lambda f(X); R^k_1 f(X)$ given by Lagrange and $\widehat{T}(X)$ by partial integration (cf LC05)

$$f^<(X) = R^k_1 f(X) = (k + 1) \sum_{|\beta|=k+1} \partial^\beta \left[ \frac{X^\beta}{\beta!} \int_0^1 dt (1 - t)^k \partial^{\beta}(tX) f^<(tX) \right]$$

$$\widehat{T}^<(X) = (-)^{k+1} (k + 1) \sum_{|\beta|=k+1} \partial^\beta \left[ \frac{X^\beta}{\beta!} \int_0^1 dt \frac{X}{t} t^k \frac{(1 - t)^k}{t^{(k+d+1)}} \right]$$
Link with BPHZ renormalization

Denote by \( \hat{f} = \mathcal{F}(f) \) the Fourier transform (normalized with \( (2\pi)^{d/2} \)) of \( f(X) \); Then

\[
(X^\mu f)(p) = (-i)^|\mu| \partial^\mu \hat{f}(p); \quad (X^\mu)(p) = (-i)^|\mu|(2\pi)^{d/2} \delta^\mu(p);
\]

\[
\partial^\mu f(0) = (-i)^|\mu|(2\pi)^{-d/2} < p^\mu, \hat{f} >
\]

which implies

\[
< T, f > = < T, R^k_1 f > = < \mathcal{F}(T), \mathcal{F}^{-1}(R^k_1 f) >
\]

\[
= < \mathcal{F}(T), R^k_1(\mathcal{F}^{-1}(f)) > = < R^k_1 \mathcal{F}(T), \mathcal{F}^{-1}(f) >
\]

that is

\[
\mathcal{F}(\tilde{T}) = R^k_1 \mathcal{F}(T) \quad \text{ie BPHZ in momentum space}
\]
**UV divergences in Minkowskian metric**

- Recall **UV** extension of **T**(*X*) at \(\|X\| \approx h\) Taylor remainder is:

\[
\begin{align*}
    f(X) & \equiv f^>(X) \equiv -(k + 1) \sum_{|\beta|=k+1} \left[ \frac{X^\beta}{\beta!} \int_1^\infty dt (1-t)^k \partial_{(tX)}^\beta f(tX) \right] \forall k \\
\end{align*}
\]

\[
\begin{align*}
    < T, f^> > & = \int d^dX T(X) \left\{ -(k + 1) \sum_{|\beta|=k+1} \left[ \frac{X^\beta}{\beta!} \int_1^\mu t^2 \frac{(1-t)^k}{t(k+1)} \partial_{X}^\beta f^>(tX) \right] \right\} \\
    & = < \tilde{T}^>, 1 > \implies \tilde{T}^>(X) \quad \text{after partial integration} \\
\end{align*}
\]

\[
\begin{align*}
    \tilde{T}^>(X) & = (-)^k(k + 1) \sum_{|\beta|=k+1} \partial_X^\beta \left[ \frac{X^\beta}{\beta!} \int_1^\mu t^2 \frac{(1-t)^k}{t(d+k+1)} T\left(\frac{X}{t}\right) \right] \\
\end{align*}
\]
Evaluation of \( \int d^D p \frac{f(p_0^2, p^2)}{p_0^2 - p^2 - m^2 + 2i\epsilon \omega_p} \) at \( D = 2 \) and \( D = 4 \)

Do first the \( p_0 \)-integration with the result

\[
\int_{-\infty}^{\infty} dp_0 \frac{f(p_0^2, p^2)}{p_0 \pm \omega_p + i\epsilon} = \pm i\pi f(\omega_p^2, p^2)
\]

by application of Lagrange’s formula and integration by part

\[
PP \int_{-\infty}^{\infty} dp_0 \frac{f(p_0^2, p^2)}{p_0 \pm \omega_p} = PP \int_{-\infty}^{\infty} dp_0 \frac{1}{p_0 \pm \omega_p} \{ -p_0 \frac{d}{dp_0} \int_1^{\infty} \frac{dt}{t} f(p_0^2 t^2, p^2) \} = 0
\]

at \( D=2 \) the remaining integral is

\[
-i\pi \int_{-\infty}^{\infty} dp \frac{f(\omega_p^2, p^2)}{\omega_p} = -i\pi < \frac{1}{\omega_p}, f(\omega_p^2, p^2) > = -i\pi < \frac{\sqrt{1}}{\omega_p}, 1 >
\]

here \( d = 1, \omega = 0, k = 0 \rightarrow \left( \frac{1}{\omega_p} \right) = \frac{d}{dp} \left[ p \int_1^{\infty} \frac{dt}{t} \frac{1}{\sqrt{p^2 + m^2 t^2}} \right]
\]

\[
= \frac{1}{\sqrt{p^2 + m^2}} - \frac{1}{\sqrt{p^2 + m^2 \mu^4}}
\]
\[ 2i\pi \int_0^\infty dp \left[ \frac{1}{\sqrt{p^2 + m^2}} - \frac{1}{\sqrt{p^2 + m^2 \mu^2}} \right] = -2i\pi \log[\mu^2] \] as expected

one may rewrite under Pauli-Villars form

\[ \int d^2p \frac{f(p_0^2, p^2)}{p^2 - m^2 + i\epsilon} \equiv \int d^2p \left[ \frac{1}{p^2 - m^2 + i\epsilon} - \frac{1}{p^2 - m^2 \mu^4 + i\epsilon} \right] \]

At \( D = 4 \) the \( p_0 \)-integral is the same and with \( X = p^2 \)

\[ \int d^4p \frac{f(p_0^2, p^2)}{p_0^2 - p^2 - m^2 + i\epsilon} = -4i\pi^2 \int_0^\infty \frac{XdXf(X)}{\sqrt{X(X + m^2)}} \]

here \( d = 2, \omega = 0, k = 1 \) giving

\[ \left( \frac{1}{\sqrt{X(X + m^2)}} \right) = -2 \frac{\partial^2}{\partial X^2} \left[ \frac{X^2}{2!} \int_1^{\mu^2} \frac{dt}{t^3} \frac{(1 - t)}{\sqrt{X(X + m^2t)}} \right] \]

\[ = -\frac{3}{4} \int_1^{\mu^2} \frac{dt}{t^3} \left[ \frac{1}{\sqrt{X(X + m^2t)}} - \frac{2\sqrt{X}}{(X + m^2t)^{3/2}} + \frac{X^{3/2}}{(X + m^2t)^{5/2}} \right] \]

PV-type of substraction \( I(X, t) \)
\[ \int_{0}^{\infty} X dX I(X, t) = \frac{4}{6} m^2 t; \quad \int_{1}^{\mu^2} dt \frac{1 - t}{t^2} = -\frac{1}{\mu^2} (1 - \mu^2 + \mu^2 \log(\mu^2)) \]

Characterisation of \( I(X, t) \) as PV-type of substraction: see transparencies for technical details.
Equivalence with dispersion relations

- Recall first E-G’s construction of retarded/advanced parts of $T(X)$

$\chi(t)$ a "smooth" extension of the usual step-function (necessary for multiplication with a distribution) such that

$$
\chi(t) = \begin{cases} 
0 & \text{for } t \leq 0 \\
< 1 & \text{for } 0 < t < 1 \\
1 & \text{for } t \geq 1 
\end{cases}
$$

Pick up a vector $v = (v_1, \ldots , v_{d-1}) \in \Gamma^+ \implies v.x = 0$ defines hyperplane separating causal support. Define retarded part $T_r(X)$ of $T(X)$ as the existing limit

$$
T_r(X) = \lim_{\delta \to 0} \chi(\frac{v.X}{\delta})T(X) := \Theta(v.X)T(X)
$$

and

$$
<T_r(X), f(X)> = <T(X), \chi(v.X)(k + 1) \sum_{|\beta| = k+1} \frac{X^\beta}{\beta!} \int_0^1 dt \frac{(1-t)^k}{t^{(k+1)}} \partial_x^\beta f(tX)> 
$$
\( \tilde{T}_r(X) \) is obtained after integration by part

\[
\tilde{T}_r(X) = (-)^{k+1}(k+1) \sum_{|\beta| = k+1} \partial^\beta_X \left[ \frac{X^\beta}{\beta!} \int_0^1 dt \frac{(1 - t)^k}{t^{d+k+1}} \chi \left( \frac{v.X}{t} \right) T \left( \frac{X}{t} \right) \right]
\]

Take Fourier-transforms:

\[
\mathcal{F}(\chi(v.X)) = \hat{\chi}_v(k) = (2\pi)^{(d/2)-1} \frac{i}{k_0 + i\epsilon} \delta^{(d-1)}(\vec{k} - k_0 \vec{v}) \text{ etc...}
\]

Then

\[
\tilde{T}_r(p) = \frac{(k+1)}{(2\pi)^{d/2}} \sum_{|\beta| = k+1} \frac{p^\beta}{\beta!} \int_0^1 dt (1 - t)^k \int dq \hat{\chi}_v(q) \partial^\beta_{(pt)} \hat{T}(pt - q)
\]

may evaluate in special coordinate system

\( p = (p_0^1, 0, 0, ....) ; v = (1, 0, 0, ....) \) to give

\[
\tilde{T}_r(p_0^1) = \frac{i}{2\pi} \frac{(p_0^1)^{(k+1)}}{k!} \int_0^1 dt (1 - t)^k \int_{-\infty}^\infty dq^0_1 \frac{d}{dq^0_1 p_1^0 t - q_1^0 + i\epsilon} \frac{\partial^{k+1}}{\partial(q^0_1)^{(k+1)}} \hat{T}(q_0^1, ...)
\]
after partial integration on $q_1^0$ and on $t$ and with the final integration variable $s = \frac{q_1^0}{p_1^0}$ one finds for any $p$ a dispersion relation for the retarded part

$$\tilde{T}_r(p) = \frac{i}{2\pi} \int_{-\infty}^{\infty} ds \frac{\hat{T}(ps)}{(s - i\epsilon)^{(k+1)}(1 - s + i\epsilon)}$$

advanced part is obtained in a similar way with $i\epsilon \rightarrow -i\epsilon$
Final Conclusions

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- Towards a finite LCQFT for the S-matrix represented in terms of the light-front time $\sigma = \omega \cdot \bar{x}$ (counterterms avoided)
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