Poincaré invariance with fields as OPVD in the Epstein and Glaser context

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”The SMOOTH OPERATOR”:

(a hit for Evening Jazz Sessions ?) [a]

[a] (what about end-of-day LC Sessions ?)

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- Applications to QFT dating back to the work of Epstein-Glaser,Stora (1973) with recent revival from many contributors (Scharf,Dutsch,Fredenhagen,Estrada,Garcia-Bondia,Prange,Pinter....
- Provides a comprehensive handling of UV and IR behaviour: see LC2004(Amsterdam) and LC2005(Cairns)

Field as OPVD, mathematical apparatus

- Scalar Field (or Fermi as well) as OPVD
  OPVD defines a functional with respect to a test function $\rho(x)$, C^∞ with compact support,

$$\Phi(\rho) \equiv < \varphi, \rho > = \int d^{(D)}y \varphi(y)\rho(y).$$

  More general interpretation: functional $\Phi(x,\rho)$ evaluated at $x = 0$.
  The translated functional is a well defined object such that

$$T_x\Phi(\rho) = < T_x\varphi, \rho > = < \varphi, T_{-x}\rho > = \int d^{(D)}y \varphi(y)\rho(x-y).$$

  Due to the properties of ρ , $T_x\Phi(\rho)$ obeys the EQM (KG or Dirac) and is taken as the physical field $\phi(x)$

Fourier decomposition of $\rho(x-y)$

$$\rho(x-y) = \int \frac{d^{(D)}q}{(2\pi)^D} e^{iq(x-y)} f(q_0^2, \vec{q}^2)$$

quantized form for $\phi(x)$ follows:

In Minkowskian metric:

$$\phi(x) = \int \frac{d^{(D-1)}p}{(2\pi)^{D-1}} \frac{f(\omega_p^2, \vec{p}^2)}{(2\omega_p)} [a_p^+ e^{ipx} + a_p e^{-ipx}].$$

or in Euclidean metric:

$$\phi(x) = \int \frac{d^{(D)}p}{(2\pi)^D} [a_p^+ e^{-ipx} + a_p e^{ipx}] f(p^2).$$

- Mathematical apparatus: 4 theorems

  i) Open covering of a topological space M

  $\exists(O_i)_{i\in I}$ open subsets that cover M : $M = \bigcup_{i\in I} O_i$;

  (α_i) a family of functions on M $M\overset{\alpha_i}{\to}R$;

  α_i is locally finite: for any point $P \in M$ only a finite number of α's are $\neq 0$*ie* $\sum_{i\in I} \alpha_i$ is finite at every point.Then

  $\beta_j = \frac{\alpha_j}{\sum_{j\in I} \alpha_j}$ is a partition of unity on M as$\sum_{j\in I} \beta_j = 1$

  ii) Paracompact spaces and decomposition of unity subordinate to the open covering,

  M is paracompact if to an arbitrary open covering $(O_i)_{i\in I}$ one can find a partition of unity $\sum_{i\in I} \beta_i$ such that β_i vanishes outside O_i: β_i is said to be subordinate to O_i.

iii)Euclidean manifolds are paracompact:

one may use partition of unity when needed. Well known case: definition of integrals of differential forms (Spivak,Felsager,Kobayashi-Nomizu....)

F a differential form, (β_i) is used to cut F into *small* pieces: $F_i = \beta_i F$ and $\sum F_i = F$. for $\Omega \subset M$ one defines: $\int_\Omega F := \sum_i \int_{\Omega_i} \beta_i F$.

result is indepedent of the choice of coordinates (atlas) on Ω_i and of the partition of unity.

iv) Localisation of distributions: J. Dieudonné's GPT-theorem (Glueing-Pieces-Together) establishes the above properties for distribution functionals on Euclidean manifolds

Consequences :

i) the test function $f(p^2)$ can be taken as a partition of unity and the integral defining the physical field $\phi(x)$ is independant of its construction.

ii) The translated functional built out of the physical field $\phi(x)$ is again $\phi(x)$ itself

$$T_x\phi(\rho) = \int d^{(D)}y\phi(y)\rho(x-y) = \int \frac{d^{(D)}p}{(2\pi)^D}[a_p^+ e^{-ipx} + a_p e^{ipx}]f^2(p^2).$$

here $f^2(p^2)$ is still a decomposition of unity (another) and $T_x\phi(\rho) \equiv \phi(x)$.

the test function $f(p^2)$ ensures convergence of otherwise diverging integrals but plays no role on the reverse.(it may taken as 1 everywhere in this case)

- Partition of unity and Poincaré commutator algebra (scalar field) Itzykson-Zuber conventions $(p115)$

$$d\Omega_k = \frac{d^3k}{(2\pi)^3 2\omega_k}; \omega_k = \sqrt{k^2+m^2}; [a_{\vec{k}}, a^+_{\vec{k}'}] = (2\pi)^3 2\omega_k \delta(\vec{k}-\vec{k}')$$

$$\varphi(x) = \int d\Omega_k [a_{\vec{k}} e^{-ik.x} + a^+_{\vec{k}} e^{ik.x}] f(\omega_k^2, \vec{k}^2);$$

$$\Pi(x) = -i \int d\Omega_k \omega_k [a_{\vec{k}} e^{-ik.x} - a^+_{\vec{k}} e^{ik.x}] f(\omega_k^2, \vec{k}^2);$$

$$\theta^{\mu\nu} = \partial^\mu\varphi\partial^\nu\varphi - \frac{1}{2} g^{\mu\nu}[(\partial\varphi)^2 - m^2\varphi^2]$$

Purpose: Check that $i[P^\mu, \varphi(x)] = \partial^\mu\varphi(x)$

$$P^0 = H = \int d\Omega_k \omega_k a^+_{\vec{k}} a_{\vec{k}} f^2(\omega_k^2, \vec{k}^2)$$

$$P^j = \frac{1}{2} \int d\Omega_k \omega_k k^j [a^+_{\vec{k}} a_{\vec{k}} + a_{\vec{k}} a^+_{\vec{k}}] f^2(\omega_k^2, \vec{k}^2)$$

$$i[P^j,\varphi] = i\int d\Omega_k k^j [a^+_{\vec{k}} e^{ikx} - a_{\vec{k}} e^{-ikx}] f^3(\omega^2,\vec{k}^2) \equiv \partial^j\varphi$$

since f^3 is also a partition of unity with same support as f^2

$$M^{\mu\nu} = \int d^3x [x^\mu \theta^{0\nu} - x^\nu \theta^{0\mu}]$$

$$M_{0j} = -\int d^3x x_j \theta_{00} = i\int d\Omega_k \omega_k a^+_{\vec{k}} f(\omega_k^2,\vec{k}^2)\frac{\partial}{\partial k^j}(a_{\vec{k}} f(\omega_k^2,\vec{k}^2))$$

$$M_{jl} = i\int d\Omega_k a^+_{\vec{k}} f(\omega_k^2,\vec{k}^2)[k_j\frac{\partial}{\partial k^l} - k_l\frac{\partial}{\partial k^j}](a_{\vec{k}} f(\omega_k^2,\vec{k}^2))$$

The commutations of the a's and a^+'s give the usual result

$$[M^{\mu\nu},P^\lambda] = i(g^{\mu\nu}P^\lambda - g^{\nu\lambda}P^\mu)$$

since in the RHS f^2 in $P^\nu(P^\mu)$ is replaced by f^4 with the same conclusion as before. Holds also for $[M^{\mu\nu},M^{\lambda\sigma}]$

Lorentz invariant extension of singular distributions : the double magic of the partition of unity and Lagrange's formula

- $E-G$'s analysis of singular distributions

$f(X)$: $\mathbb{C}^\infty(\mathbb{R}^d)$ test function $\in \mathbb{S}(\mathbb{R}^d)$

$T(X)$ distribution $\in \mathbb{S}'(\mathbb{R}^d - \{0\})$

singular order k of $T(X)$ at the origin of $(\mathbb{R}^d)$ such that

$$k = inf\{s : \lim_{\lambda\to 0} \lambda^s T(\lambda X) = 0\} - d$$

$E-G$'s extension consists in performing an "educated" Taylor surgery on the original test function by throwing away the the weigthed k-jet of $f(X)$. Call $R_1^k f(X)$ (Taylor's remainder) Notation:

$$\alpha! = \alpha^1! \cdots \alpha^d!; |\alpha| = \alpha^1 + \cdots + \alpha^d; \partial^\alpha = \partial_{x_1}^{\alpha^1} \cdots \partial_{x_d}^{\alpha^d}$$

$$R_1^k f(X) = f(X) - \sum_{n=0}^{k} \sum_{|\alpha|=n} \frac{X^\alpha}{\alpha!} \partial^\alpha f(X)|_{X=0}$$

Call $\widetilde{T}(X)$ extension of $T(X)$ by transposition such that

$$< \widetilde{T}, f >:=< T, R_1^k f >$$

but $R_1^k f(X) \notin \mathbb{S}(\mathbb{R}^d)$ E-G's remedy: introduce a weight with properties $w(0) = 1$, $w^{(\alpha)}(0) = 0$,$0 <| \alpha |\leq k$ such that

$$R_w^k f(X) = f(X) - w(X) \sum_{n=0}^{k} \sum_{|\alpha|=n} \frac{X^\alpha}{\alpha!} \partial^\alpha f(X)|_{X=0}$$

$$< \widetilde{T}, f >:=< T, R_w^k f >$$

However under action of element Λ of the Lorentz group derivatives transform as

$$x^\alpha \partial_\alpha (\Lambda f) = x^\alpha [\Lambda^{-1}]_\alpha^\beta (\partial_\beta f) \circ \Lambda^{-1}$$

$= (\Lambda^{-1} X)^\beta (\partial_\beta f) \circ \Lambda^{-1};$ and in the Taylor expansion

$x^\alpha \partial_\alpha (\Lambda f)(0) = (\Lambda^{-1} X)^\beta \partial_\beta f(0) \Rightarrow$ Lorentz covariance violation

Restauration: $\widetilde{T}(X)$ determined up to $\sum_{|\alpha|\leq k}(-1)^{\alpha}\frac{a_{\alpha}}{\alpha!}\delta^{(\alpha)}(X)$;

since $\delta^{(\alpha)}(\Lambda X)=[\Lambda^{-1}X]^{\alpha}_{\beta}\delta^{(\beta)}(X)$ scheme is : determine a_{α} to correct for the violation due to derivatives.

•Lorentz covariance with test function as partition of unity and Lagrange's formula for Taylor's remainder $R^k_1 f(X)$

with a p. of u. $f^{(\alpha)}(0)=0 \quad \forall\alpha\geq 0$ it holds that
$f(X)=R^k_1 f(X) \quad \forall k>0$:no violation due to derivatives (they are just not there) and $\Lambda R^k_1 f(X)=R^k_1\Lambda f(X)$;$R^k_1 f(X)$ given by Lagrange and $\widetilde{T}(X)$ by partial integration (*cf* LC05)

$$f^{<}(X)=R^k_1 f(X)=(k+1)\sum_{|\beta|=k+1}\partial^{\beta}[\frac{X^{\beta}}{\beta!}\int_0^1 dt(1-t)^k\partial^{\beta}_{(tX)}f^{<}(tX)]\}$$

$$\widetilde{T}^{<}(X)=(-)^{k+1}(k+1)\sum_{|\beta|=k+1}\partial^{\beta}_X[\frac{X^{\beta}}{\beta!}\int_{\tilde{\mu}\|X\|}^1 dt \quad T(\frac{X}{t}) \quad \frac{(1-t)^k}{t^{(k+d+1)}}]$$

•Link with BPHZ renormalization

Denote by $\hat{f} = \mathcal{F}(f)$ the Fourier transform (normalized with $(2\pi)^{\frac{d}{2}}$) of $f(X)$; Then

$$\widehat{(X^\mu f)}(p) = (-i)^{|\mu|}\partial^\mu \hat{f}(p); \widehat{(X^\mu)}(p) = (-i)^{|\mu|}(2\pi)^{\frac{d}{2}}\delta^{(\mu)}(p);$$
$$\partial^\mu f(0) = (-i)^{|\mu|}(2\pi)^{-\frac{d}{2}} < p^\mu, \hat{f} >$$

which implies

$$< T, f >=< T, R_1^k f >=< \mathcal{F}(T), \mathcal{F}^{-1}(R_1^k f) >$$
$$=< \mathcal{F}(T), R_1^k(\mathcal{F}^{-1}(f)) >=< R_1^k\mathcal{F}(T), \mathcal{F}^{-1}(f) >$$

that is

$\mathcal{F}(\tilde{T}) = R_1^k\mathcal{F}(T)$ *ie* BPHZ in momentum space

UV divergences in Minkowskian metric

- Recall UV extension of $T(X)$ at $\|X\| \approx h$ Taylor remainder is :

$$f(X) \equiv f^{>}(X) \equiv -(k+1)\sum_{|\beta|=k+1} [\frac{X^\beta}{\beta!}\int_1^\infty dt(1-t)^k \partial^\beta_{(tX)} f(tX)]\forall k$$

$$< T, f^{>} > = \int d^d X T(X)\{-(k+1)\sum_{|\beta|=k+1} [\frac{X^\beta}{\beta!}\int_1^{\mu^2} dt \frac{(1-t)^k}{t^{(k+1)}} \partial^\beta_X f^{>}(tX)]\}$$

$$= \; < \widetilde{T}^{>}, 1 > \Longrightarrow \widetilde{T}^{>}(X) \quad \text{after partial integration}$$

$$\widetilde{T}^{>}(X) = (-)^k (k+1) \sum_{|\beta|=k+1} \partial^\beta_X [\frac{X^\beta}{\beta!}\int_1^{\mu^2} dt \frac{(1-t)^k}{t^{(d+k+1)}} T(\frac{X}{t})]$$

Evaluation of $\int d^D p \frac{f(p_0^2,p^2)}{p_0^2-p^2-m^2+2i\epsilon\omega_p}$ at $D=2$ and $D=4$

Do first the p_0-integration with the result

$$\int_{-\infty}^{\infty} dp_0 \frac{f(p_0^2,p^2)}{p_0 \pm \omega_p \mp i\epsilon} = \pm i\pi f(\omega_p^2,p^2)$$

by application of Lagrange's formula and integration by part

$$PP\int_{-\infty}^{\infty} dp_0 \frac{f(p_0^2,p^2)}{p_0 \pm \omega_p} = PP\int_{-\infty}^{\infty} dp_0 \frac{1}{p_0 \pm \omega_p}\{-p_0 \frac{d}{dp_0}\int_1^{\mu^2} \frac{dt}{t} f(p_0^2 t^2,p^2)\} = 0$$

at D=2 the remaining integral is

$$-i\pi \int_{-\infty}^{\infty} dp \frac{f(\omega_p^2,p^2)}{\omega_p} = -i\pi < \frac{1}{\omega_p}, f(\omega_p^2,p^2) >= -i\pi < \widetilde{(\frac{1}{\omega_p})}, 1 >$$

here $d=1, \omega=0, k=0 \rightarrow \widetilde{(\frac{1}{\omega_p})} = \frac{d}{dp}[p\int_1^{\mu^2} \frac{dt}{t} \frac{1}{\sqrt{p^2+m^2t^2}}]$

$$= \frac{1}{\sqrt{p^2+m^2}} - \frac{1}{\sqrt{p^2+m^2\mu^4}}$$

$$2i\pi\int_0^\infty dp\left[\frac{1}{\sqrt{p^2+m^2}}-\frac{1}{\sqrt{p^2+m^2\mu^2}}\right]=-2i\pi\log[\mu^2]$$ as expected

one may rewrite under Pauli-Villars form

$$\int d^2p\frac{f(p_0^2,p^2)}{p^2-m^2+i\epsilon}\equiv\int d^2p\left[\frac{1}{p^2-m^2+i\epsilon}-\frac{1}{p^2-m^2\mu^4+i\epsilon}\right]$$

At $D=4$ the p_0-integral is the same and with $X=p^2$

$$\int d^4p\frac{f(p_0^2,p^2)}{p_0^2-p^2-m^2+i\epsilon}=-4i\pi^2\int_0^\infty\frac{XdXf(X)}{\sqrt{X(X+m^2)}}$$

here $d=2,\omega=0,k=1$ giving

$$\left(\widetilde{\frac{1}{\sqrt{X(X+m^2)}}}\right)=-2\frac{\partial^2}{\partial X^2}\left[\frac{X^2}{2!}\int_1^{\mu^2}\frac{dt}{t^3}\frac{(1-t)}{\sqrt{X(X+m^2t)}}\right]$$

$$=-\frac{3}{4}\int_1^{\mu^2}\frac{dt(1-t)}{t^3}\underbrace{\left[\frac{1}{\sqrt{X(X+m^2t)}}-\frac{2\sqrt{X}}{(X+m^2t)^{3/2}}+\frac{X^{3/2}}{(X+m^2t)^{5/2}}\right]}_{\text{PV-type of substraction } I(X,t)}$$

$$\int_0^\infty X dX I(X,t) = \frac{4}{6} m^2 t; \int_1^{\mu^2} \frac{dt(1-t)}{t^2} = -\frac{1}{\mu^2}(1-\mu^2+\mu^2\log(\mu^2))$$

Caracterisation of $I(X,t)$ as PV-type of substraction: see transparencies for technical details.

Equivalence with dispersion relations

• Recall first E-G's construction of retarded/advanced parts of $T(X)$

$\chi(t)$ a "smooth" extension of the usual step-function (necessary for multiplication with a distribution) such that

$$\chi(t) = \begin{cases} 0 & \text{for } t \leq 0 \\ < 1 & \text{for } 0 < t < 1 \\ 1 & \text{for } t \geq 1 \end{cases}$$

Pick up a vector $v = (v_1, \cdots, v_{d-1}) \in \Gamma^+ \Longrightarrow v.x = 0$ defines hyperplane separating causal support . Define retarded part $T_r(X)$ of $T(X)$ as the existing limit

$$T_r(X) = \lim_{\delta \to 0} \chi(\frac{v.X}{\delta}) T(X) := \Theta(v.X) T(X) \quad \text{and}$$

$$<T_r(X), f(X)> = <T(X), \chi(v.X)(k+1) \sum_{|\beta|=k+1} [\frac{X^\beta}{\beta!} \int_0^1 dt \frac{(1-t)^k}{t^{(k+1)}} \partial_X^\beta f(tX)]>$$

$\widetilde{T}_r(X)$ is obtained after integration by part

$$\widetilde{T}_r(X)=(-)^{k+1}(k+1)\sum_{|\beta|=k+1}\partial_X^\beta\left[\frac{X^\beta}{\beta!}\int_0^1 dt\frac{(1-t)^k}{t^{d+k+1}}\chi\left(\frac{v.X}{t}\right)T\left(\frac{X}{t}\right)\right]$$

Take Fourier-transforms:

$\mathcal{F}(\chi(v.X)) = \hat{\chi}_v(k) = (2\pi)^{(d/2-1)}\frac{i}{k_0+i\epsilon}\delta^{(d-1)}(\vec{k}-k_0\vec{v})$ etc....

Then

$$\widetilde{\hat{T}}_r(p) = \frac{(k+1)}{(2\pi)^{d/2}}\sum_{|\beta|=k+1}\frac{p^\beta}{\beta!}\int_0^1 dt(1-t)^k\int dq\hat{\chi}_v(q)\partial^\beta_{(pt)}\hat{T}(pt-q)$$

may evaluate in special coordinate system

$p = (p_1^0, 0, 0, ...); v = (1, 0, 0, ...)$ to give

$$\widetilde{\hat{T}}_r(p_1^0) = \frac{i}{2\pi}\frac{(p_1^0)^{(k+1)}}{k!}\int_0^1 dt(1-t)^k\int_{-\infty}^{\infty}\frac{dq_1^0}{p_1^0t-q_1^0+i\epsilon}\frac{\partial^{k+1}}{\partial(q_1^0)^{(k+1)}}\hat{T}(q_1^0,..)$$

after partial integration on q_1^0 and on t and with the final integration variable $s = \frac{q_1^0}{p_1^0}$ one finds for any p a dispersion relation for the retarded part

$$\tilde{\hat{T}}_r(p) = \frac{i}{2\pi}\int_{-\infty}^{\infty} ds \frac{\hat{T}(ps)}{(s - i\epsilon)^{(k+1)}(1 - s + i\epsilon)}$$

advanced part is obtained in a similar way with $i\epsilon \to -i\epsilon$

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- Towards a finite LCQFT for the S-matrix repesented in terms of the light-front time $\sigma = \omega.x$ (counterterms avoided)

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