Minneapolis, May 17, 2006

Light-Front Hamiltonian Calculation of Masses of Heavy Quarkonia

Stanisław D. Głazek

Institute of Theoretical Physics, Warsaw University

Method: Weak-coupling expansion for QCD in the LF Fock space

Result: α_λ, m_λ $\longrightarrow$ masses of $c\bar{c}$ or $b\bar{b}$

boost-invariant rotationally symmetric spectrum and states

SDG, Jarosław Młynik - IFT/08/06

stglazek@fuw.edu.pl

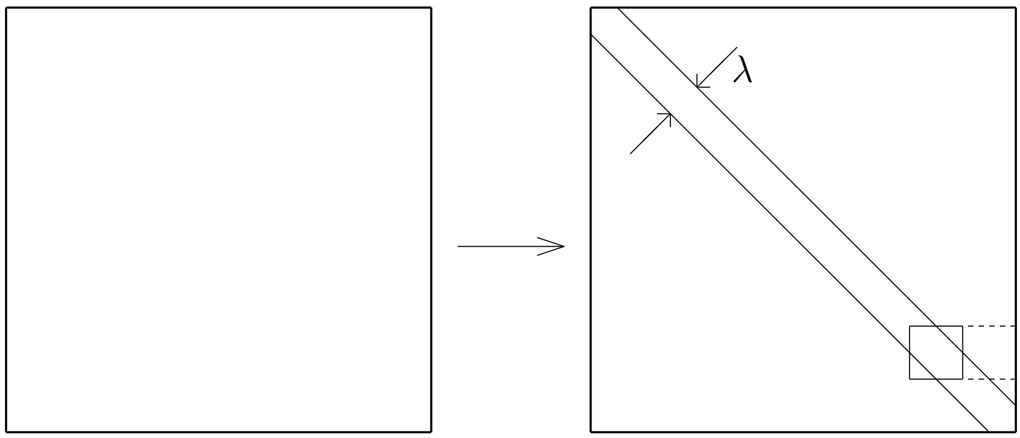
Calculations do not involve:

scattering states for quarks or gluons, Feynman diagrams, path integral, euclidicity postulate, lattice, or vacuum expectation values.

Instead:

Renormalization Group Procedure for Effective Particles in QFT (RGPEP in LFQCD).

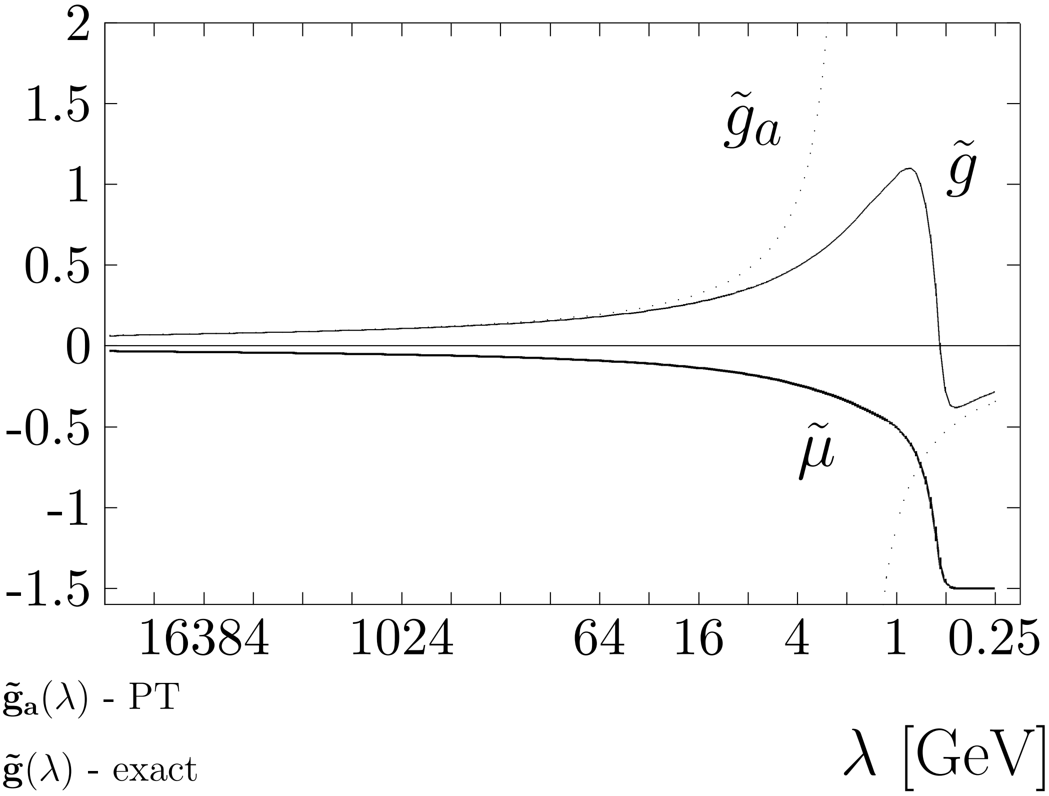
stglazek@fuw.edu.pl



RGPEP has roots in “similarity” RG procedure:

SDG and KGW (’93,’94,’98)

width λ in Hamiltonian matrices $\rightarrow$ form factors f_λ in RGPEP



$\tilde{\mathbf{g}}_{\mathbf{a}}(\lambda)$ - PT

$\tilde{\mathbf{g}}(\lambda)$ - exact

$\tilde{\mu}(\lambda)$ - matrix element with bound state eigenvalue

stglazek@fuw.edu.pl

.

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^{\mu\nu a}F^a_{\mu\nu} \, .$$

$$x^\pm = x^0 \pm x^3, \qquad t \to x^+, \qquad z \to x^-, \qquad x^\perp \, .$$

$$A^+ = 0 \, .$$

$$H_{can} = H_{\psi^2} + H_{A^2} + H_{A^3} + H_{A^4} + H_{\psi A \psi}$$
$$+ H_{\psi A A \psi} + H_{[\partial A A]^2} + H_{[\partial A A](\psi\psi)} + H_{(\psi\psi)^2} \, .$$

stglazek@fuw.edu.pl

.

$$\mathcal{H}_{\psi^2} = \frac{1}{2}\bar{\psi}\gamma^+ \frac{-\partial^{\perp\,2} + m^2}{i\partial^+}\psi \,,$$

$$\mathcal{H}_{A^2} = -\frac{1}{2}A^\perp(\partial^\perp)^2 A^\perp \,,$$

$$\mathcal{H}_{\psi A\psi} = g\,\bar{\psi}\not{A}\psi \,,$$

$$\mathcal{H}_{(\psi\psi)^2} = \frac{1}{2}g^2\,\bar{\psi}\gamma^+ t^a\psi \frac{1}{(i\partial^+)^2}\bar{\psi}\gamma^+ t^a\psi \,.$$

At $x^+ = 0$:

$$\psi = \sum_{\sigma c} \int [k] \left[\chi_c u_{k\sigma} b_{k\sigma c} e^{-ikx} + \chi_c v_{k\sigma} d^\dagger_{k\sigma c} e^{ikx} \right] ,$$

$$A^\mu = \sum_{\sigma c} \int [k] \left[t^c \varepsilon^\mu_{k\sigma} a_{k\sigma c} e^{-ikx} + t^c \varepsilon^{\mu *}_{k\sigma} a^\dagger_{k\sigma c} e^{ikx} \right] .$$

Regularization (boost invariant, 7 Poincaré symmetries)

E. g., three-prong vertices $Y \to rY$, $r_3 = x^\delta \exp\left(-\kappa^{\perp\,2}/\Delta^2\right)$.

Counterterms.

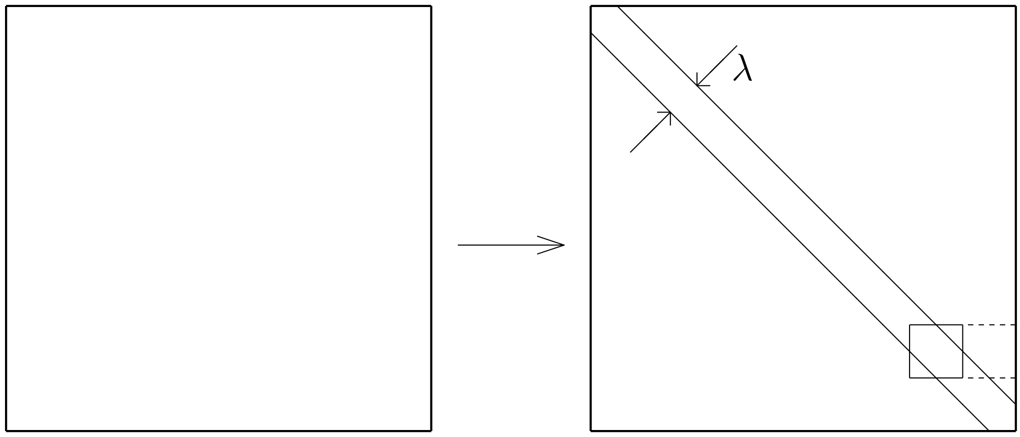
$$H = [H_{can} + H_{CT}]_{reg} \ .$$

RGPEP:

$$q_\lambda = U_\lambda \, q_{can} \, U_\lambda^\dagger \, ,$$

$$\hat{O}'_\lambda = [\mathcal{T}, \hat{O}_\lambda] \, , \quad \mathcal{T} = U'_\lambda U_\lambda^\dagger \, ,$$

$$\hat{O}_\infty = \hat{O}_{can(reg)} + \hat{O}_{CT} \quad \to \quad \hat{O}_\lambda \, .$$



The window Hamiltonian is independent of the UV regularization r_Δ.

stglazek@fuw.edu.pl

$$H|P\rangle = M^2|P\rangle \quad \to \quad H_\lambda|P\rangle = M^2|P\rangle \,.$$

$$|P\rangle = |Q_\lambda \bar{Q}_\lambda\rangle + |Q_\lambda \bar{Q}_\lambda g_\lambda\rangle + \,\ldots\,.$$

$$[H_\lambda] = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & H_3 & fY \\ \cdot & fY^\dagger & H_2 \end{bmatrix} \to \begin{bmatrix} T_3 + \mu^2 & fY \\ fY^\dagger & T_2 + V_2 \end{bmatrix} .$$

$$H_{Q\bar{Q}\lambda} = T_{2\lambda} + V_{2\lambda} + f_\lambda Y_\lambda^\dagger \frac{1}{T_3 + \mu^2} f_\lambda Y_\lambda \,.$$

$$H_\lambda|P\rangle = M^2|P\rangle \to H_{Q\bar{Q}\lambda}|P\rangle = M^2|P\rangle \,.$$

$$\boxed{f_\lambda f_\lambda \, \frac{4m^2}{q_z^2} \, \frac{\mu^2}{q^2+\mu^2} \quad \longrightarrow \quad harmonic\ force\ in\ H_{Q\bar{Q}\lambda} \,.}$$

stglazek@fuw.edu.pl

Perturbative derivation of counterterms: $rY\,T^{-1}rY$

$$H_\lambda|Q\rangle = m^2|Q\rangle \to m^2_{CT}(\delta,\Delta)\,.$$

1 quark (gap $\mu^2_{g/Q} \neq$ 0 when $x_g \to 0$): $fY(T+\mu^2_{q/Q})^{-1}fY$

$$H_\lambda|Q\rangle = m^2_Q(\delta)|Q\rangle\,, \qquad m^2_Q(\delta) \to \infty\,.$$

$Q\bar{Q}$ **(gap $\mu^2_{g/Q\bar{Q}} \to$ 0 when $x_g \to 0$):** $fY(T+\mu^2_{g/Q\bar{Q}})^{-1}fY$

$$(T + V_{Coulomb} + \Sigma + V)|Q\bar{Q}\rangle \;=\; M^2|Q\bar{Q}\rangle\,.$$

stglazek@fuw.edu.pl

$$fY(T + \mu^2_{g/Q\bar{Q}})^{-1} fY$$

$$(T + V_{Coulomb} + \Sigma + V)|Q\bar{Q}\rangle = M^2|Q\bar{Q}\rangle.$$

$$\Sigma\,\psi(\vec{p}) = \alpha\left[\int d^3q\,\frac{ff}{q_z^2}\,\frac{\mu^2(q_z)}{\mu^2(q_z)+q^2}\right]\psi(\vec{p})\,,$$

$$V\,\psi = -\alpha\int d^3q\,\frac{ff}{q_z^2}\,\frac{\mu^2(q_z)}{\mu^2(q_z)+q^2}\,\psi(\vec{p}-\vec{q})\,.$$

$$f = \exp\left\{-[m\vec{q}^{\,2}/(q_z\lambda^2)]^2\right\},$$

$$q_z \to 0 \longrightarrow |\vec{q}\,| \to 0\,,$$

$$\psi(\vec{p}-\vec{q}) = \psi(\vec{p}) - \vec{q}\,\vec{\partial}\,\psi(\vec{p}) + \frac{1}{2}q^iq^j\,\partial^i\partial^j\psi(\vec{p}) + \ \ldots\,,$$

$$-\alpha\int d^3q\,\frac{ff}{q_z^2}\,\frac{\mu^2(q_z)}{\mu^2(q_z)+\vec{q}^{\,2}}\frac{1}{2}q^iq^j = -\frac{1}{2}\,k\,\delta^{ij}\quad INDEPENDENT\ of\ \mu^2.$$

stglazek@fuw.edu.pl

.

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & H_4 & fY_1 & fY_2 \\ \cdot & fY_1^\dagger & H_3 & fY \\ \cdot & fY_2^\dagger & fY^\dagger & H_2 \end{bmatrix} \longrightarrow \begin{bmatrix} T_4 + \mu^2 & fY_1 & fY_2 \\ fY_1^\dagger & H_3 & fY \\ fY_2^\dagger & fY^\dagger & H_2 \end{bmatrix}$$

$$\mu^2 \longrightarrow \left(1 - \frac{\alpha^2}{\alpha_0^2}\right) \mu^2$$

$$|M, P^+, P^\perp\rangle = \int [ij]\,\tilde{\delta}\,P^+\,\frac{\bar{u}_i \Psi_{ij} v_j}{-4m^2}\,|ij\rangle\,.$$

$$|ij\rangle = b^\dagger_{\lambda i} d^\dagger_{\lambda j}\,|0\rangle\,, \qquad \frac{\bar{u}_i \Psi_{ij} v_j}{-4m^2} \to \chi_i^\dagger\,\phi(\vec{k}_{ij})\chi_j \quad \leftarrow Melosh.$$

$$0 = \left[\vec{p}^{\,2} - k_p\,\Delta_p - x\right]\phi(\vec{p}) - 2\int \frac{d^3k}{(2\pi)^3}\,\mathcal{V}\,\phi(\vec{k}\,)\,,$$

$$k_p = \frac{9}{128\sqrt{2\pi}}\,\left(\frac{\lambda^2}{\alpha m^2}\right)^3\,,$$

$$\mathcal{V} = f\,\frac{4\pi}{(\vec{p}-\vec{k}\,)^2}\,(1+BF) \quad \leftarrow scaling, \lambda \sim \sqrt{\alpha}\,m\,, (N=?)$$

$$f = \exp\left\{-\left[\frac{\mathcal{M}^2(p)-\mathcal{M}^2(k)}{\lambda^2}\right]^2\right\},$$

$$M = 2m\sqrt{1 + x\,\left(\frac{2}{3}\alpha\right)^2}\,.$$

stglazek@fuw.edu.pl

Example of J/ψ or Υ:

$$\phi(\vec{k}\,) = \left[a(\vec{k}\,) + \vec{b}(\vec{k}\,)\vec{\sigma}\,\right]$$

$$a(\vec{k}\,) = 0\,, \quad b^m(\vec{k}\,) = \left[\delta^{mn}\,\frac{S(k)}{k} + \frac{1}{\sqrt{2}}\,\left(\delta^{mn} - 3\frac{k^m k^n}{k^2}\right)\,\frac{D(k)}{k}\right]\,s^n\,.$$

$$0 = \begin{bmatrix} p^2 - k_p\partial_p^2 - x\,, & 0 \\ 0\,, & p^2 - k_p\partial_p^2 + k_p\frac{6}{p^2} - x \end{bmatrix} \begin{bmatrix} S(p) \\ D(p) \end{bmatrix} - \frac{2}{\pi}\int_0^\infty dk\, f\, pk \begin{bmatrix} \mathcal{W}_{ss},\ \mathcal{V} \\ \mathcal{W}_{ds},\ \mathcal{V} \end{bmatrix}$$

$$\mathcal{W}_{ss} = J_0 + \frac{\alpha^2}{3}\left[(p^2+k^2)\,J_0 - 16/9\right]\,,$$

$$\mathcal{W}_{sd} = \frac{\alpha^2}{3}\left[p^2\,(J_2 - J_0) + 4/3\right]\frac{\sqrt{2}}{3}\,, \quad \mathcal{W}_{ds} = \frac{\alpha^2}{3}\left[k^2\,(J_2 - J_0) + 4/3\right]\frac{\sqrt{2}}{3}\,,$$

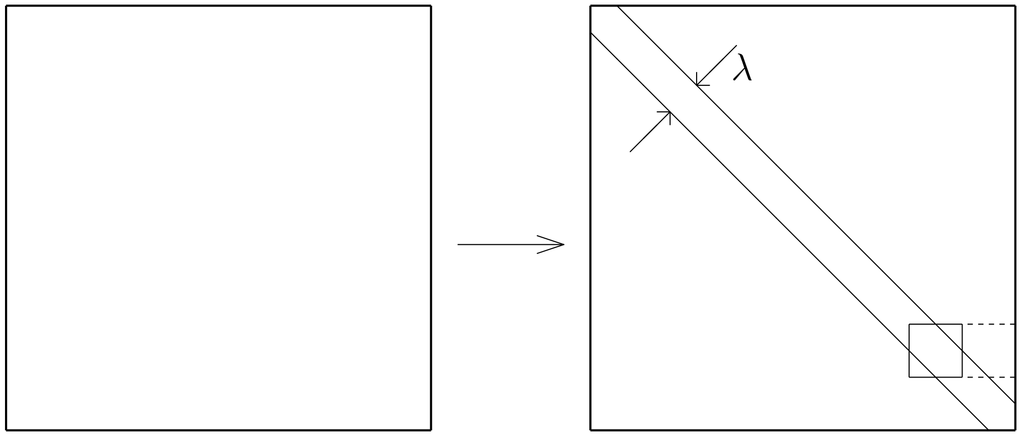
$$\mathcal{W}_{dd} = J_2 + (J_2 - J_0)/2 + \frac{\alpha^2}{3}\left\{(p^2+k^2)\,\left[J_0 - (J_2 - J_0)/6\right] - 20/9\right\}\,.$$

δ-functions with f_λ. J_0, J_2 are known functions.

stglazek@fuw.edu.pl

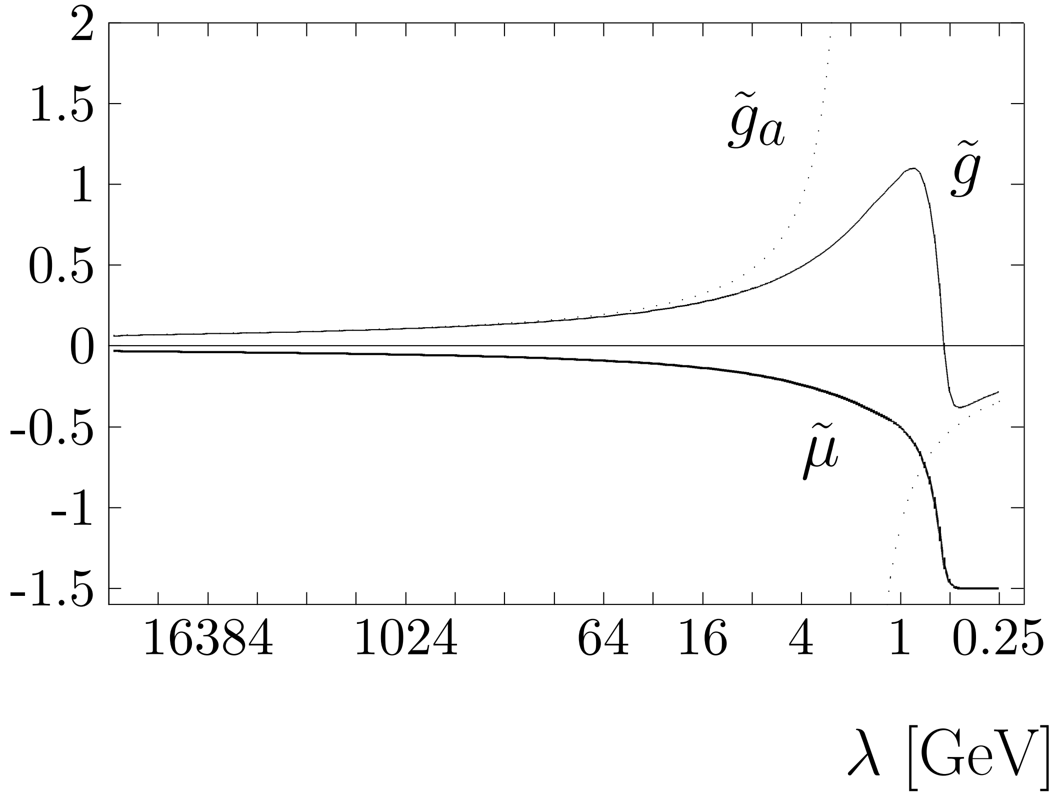
In principle: 2 states $\rightarrow \alpha_\lambda$, m_λ.

$\lambda = ?$



Two examples for $c\bar{c}$ and $b\bar{b}$.

stglazek@fuw.edu.pl



2
1.5
1
0.5
0
-0.5
-1
-1.5
$\tilde{g}_a$
$\tilde{g}$
$\tilde{\mu}$
16384
1024
64
16
4
1
0.25
λ [GeV]

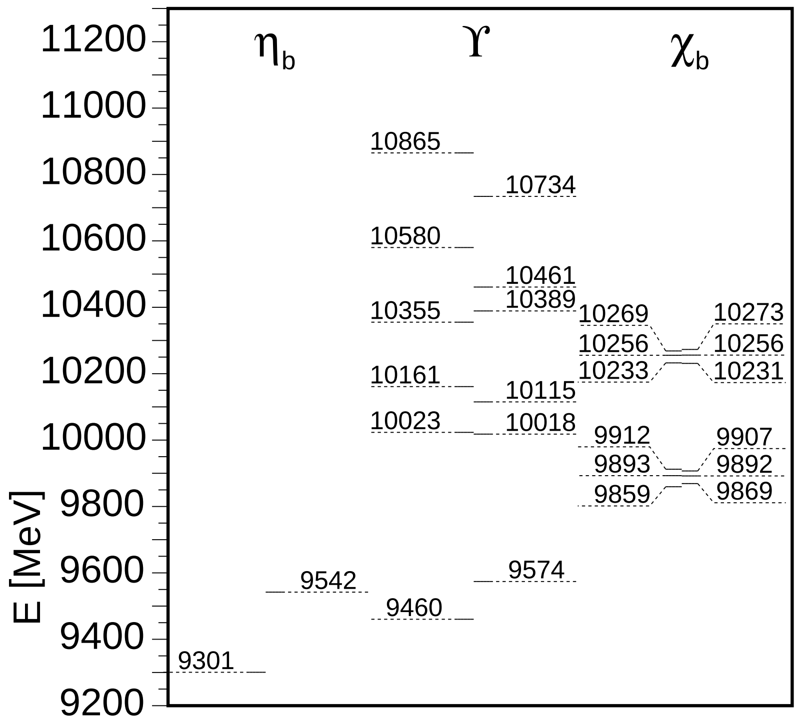


FIG. 1: 7 $b\bar{b}$ middle states: $\lambda = 3779.8$ MeV, $\alpha_\lambda = 0.28839$, $m = 4835.9$ MeV, ($\omega = 184.62$ MeV).

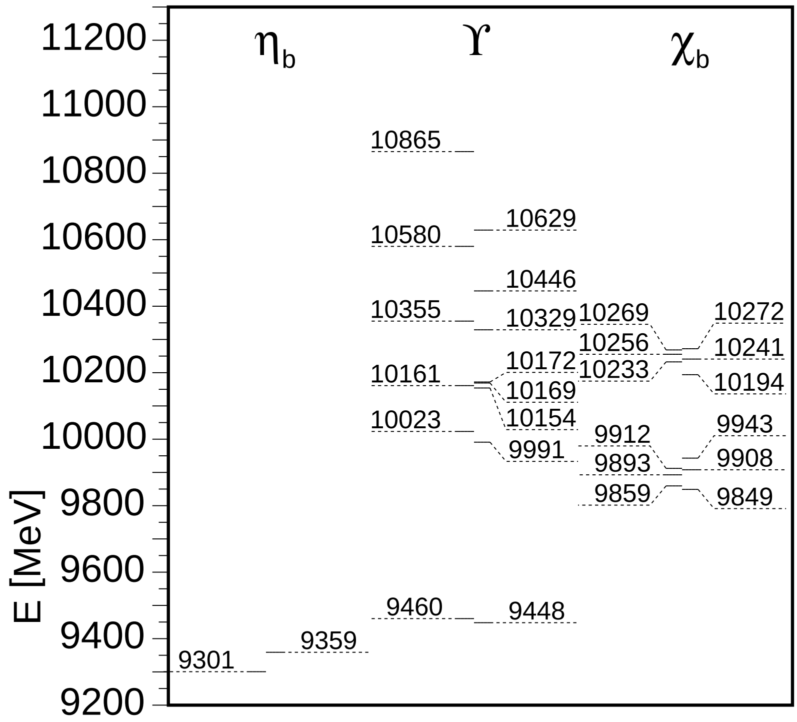


FIG. 2: 12 $b\bar{b}$ states: $\lambda = 3252.3$ MeV, $\alpha_\lambda = 0.50738$, $m = 4979.7$ MeV, ($\omega = 147.11$ MeV).

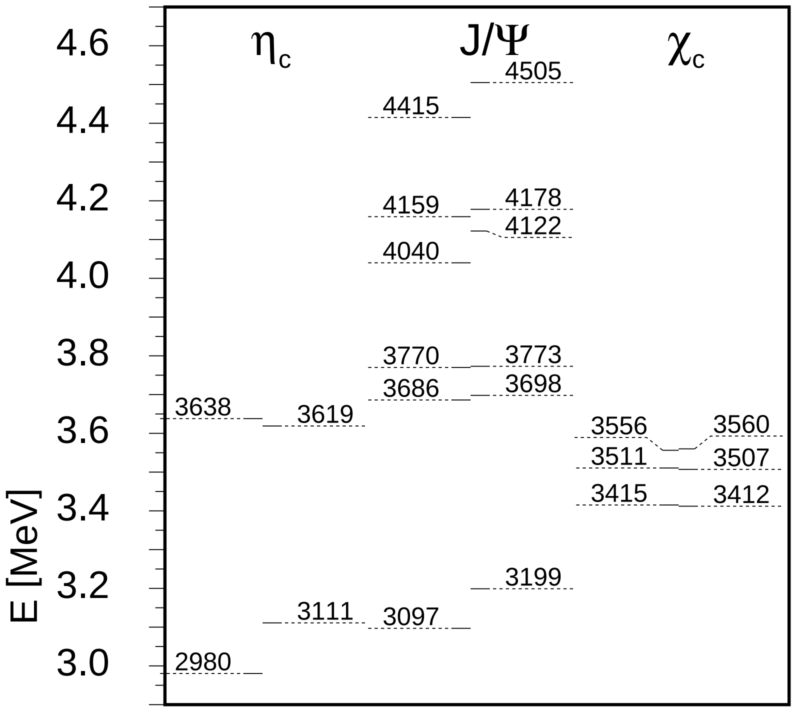


FIG. 3: 3 middle $c\bar{c}$ states: $\lambda = 1990.0$ MeV, $\alpha_\lambda = 0.34335$, $m = 1553.3$ MeV, ($\omega = 284.93$ MeV).

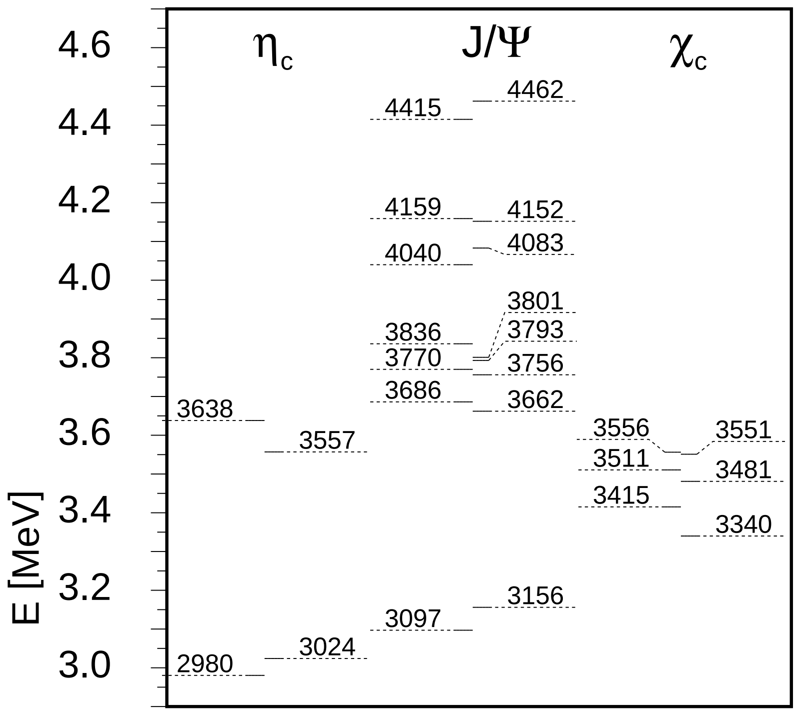
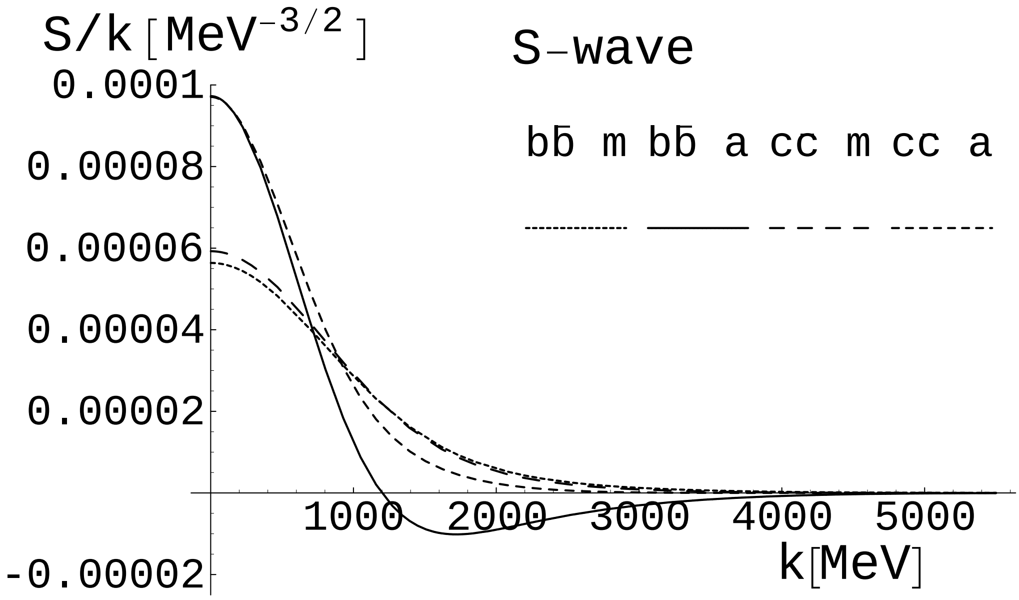
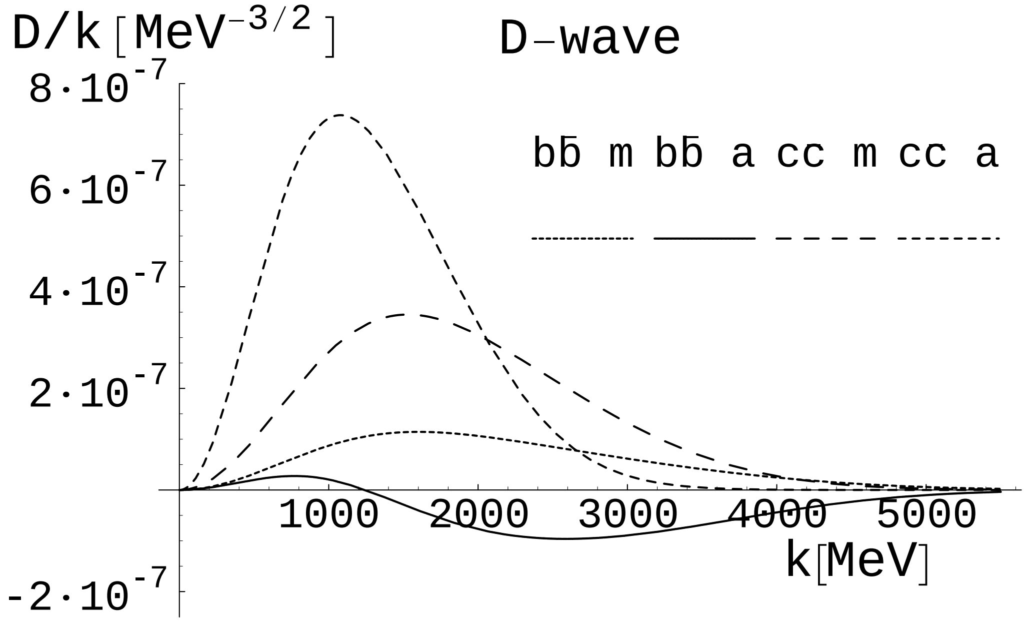


FIG. 4: 11 $c\bar{c}$ states: $\lambda = 1934.2$ MeV, $\alpha_\lambda = 0.41443$, $m = 1577.4$ MeV, ($\omega = 278.72$ MeV).



S/k [MeV$^{-3/2}$]
S−wave
$b\bar{b}$ m $b\bar{b}$ a cc m cc a
0.0001
0.00008
0.00006
0.00004
0.00002
-0.00002
1000
2000
3000
4000
5000
k[MeV]



D/k [MeV^{-3/2}]
D−wave
8·10^{-7}
6·10^{-7}
4·10^{-7}
2·10^{-7}
-2·10^{-7}
bb̄ m bb̄ a cc m cc a
1000
2000
3000
4000
5000
k[MeV]

$b\bar{b}$, fix masses of $\chi_2(1P)$ and $\chi_2(2P)$:

$$\alpha(\lambda) = 0.25 + (3.85 - \lambda/\mathrm{GeV})/3\,,$$

$$m(\lambda) = [4.83 + (3.85 - \lambda/\mathrm{GeV})/5]\ \mathrm{GeV}\,.$$

Magnitudes as expected.

?! m_λ stable, α_λ varies about 10 times faster

than α_λ in 3rd order $H_{\lambda LFQCD}$, PRD**63**, 116006 (2001).

$$\mathcal{V} = f\,\left(\frac{4\pi}{p^2} + \alpha^2 R\right)(1+\alpha^2 S)(1+\alpha^2 M)\,.$$

Optimizing f.

Conclusion:

- There exists an approximate constituent picture for heavy quarkonia in 1 flavor QCD: relativistic, simple, usable for fast mesons.
- Fits meson masses reasonably well for reasonable α_λ, and m_λ.
- Provides very specific boost-invariant wave functions in the LF Fock space. Decays? Production? Unequal masses? Exclusive processes? Large velocities.
- Explicit LF bound states with J=0, J=1, J=2, waves S, P, D, F.
- Systematically improvable within RGPEP ($\alpha \sim 1/3$).

Will it improve in 4th order?

$$\frac{1}{2}\,\frac{m}{2}\,\omega^2\,r^2+\mathcal{V}\,.$$

$$\omega=\sqrt{\frac{4}{3}\,\frac{\alpha}{\pi}}\;\lambda\;\left(\frac{\lambda}{m}\right)^2\;\left(\frac{\pi}{1152}\right)^{1/4}\;.$$

$$\mathcal{V}=f\;\left(\frac{4\pi}{p^2}+\alpha^2 R\right)(1+\alpha^2 S)(1+\alpha^2 M)\,.$$

.

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & H_4 & Y_1 & Y_2 \\ \cdot & Y_1^\dagger & H_3 & Y \\ \cdot & Y_2^\dagger & Y^\dagger & H_2 \end{bmatrix} \longrightarrow \begin{bmatrix} T_4 + \mu^2 & Y_1 & Y_2 \\ Y_1^\dagger & H_3 & Y \\ Y_2^\dagger & Y^\dagger & H_2 \end{bmatrix} .$$

P. A. M. Dirac (1977):

"Working with a front is a process that is unfamiliar to physicists. But still I feel that the mathematical simplification that it introduces is all-important. I consider the method to be promising and have recently been making an extensive study of it. It offers new opportunities, while the familiar instant form seems to be played out."

K. G. Wilson (2004):

"...the time is ripe for a few accomplished theorists to switch into light-front theory and help build a growing research effort in this area."