Lessons Learned from Discrete Symmetries on the Light-Front

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Motivation

QCD naturally violates CP symmetry through the interaction

$$\mathcal{L}_{CP} = \theta \frac{g^2}{32\pi^2} G^{\mu\nu} \tilde{G}_{\mu\nu} - i\bar{q}(\mathrm{Im}\, M_q)\gamma_5 q$$

and gives the neutron a **permanent electric dipole moment** d, though none has been observed:
$|d^n| < 3.0 \cdot 10^{-26}$ e-cm (90% CL) [Baker et al., hep-ex/0602020]
A complete analysis of QCD with light-front methods demands the analysis of CP-violating observables as well.

Here we analyze T-odd and/or P-odd electromagnetic form factors in the light-front formalism of QCD.

We begin with the construction of discrete symmetry transformations appropriate to a light front formalism.

Discrete Symmetries on the Light Front

Particles remain on their energy shell in analogy to the on-mass-shell condition of the equal-time formalism.
Consider transformations on $\mathbf{k}_\perp$; thus $|\mathbf{k}_\perp|^2$, k^+, k^- are unchanged.

Parity $\mathcal{P}_\perp$:

A vector d^μ transforms as $d^R \to -d^L$, $d^L \to -d^R$, $d^\pm \to d^\pm$.
Note $d^{R,L} \equiv d^1 \pm i d^2$ so that $d^1 \to -d^1$, $d^2 \to d^2$.

$\mathcal{P}_\perp$ is unitarity; it flips the spin as well:

$$\begin{aligned} \mathcal{P}_\perp a^\lambda_{p^L,p^R} \mathcal{P}_\perp^\dagger &= \eta_a a^{-\lambda}_{-p^R,-p^L} , \\ \mathcal{P}_\perp b^\lambda_{p^L,p^R} \mathcal{P}_\perp^\dagger &= \eta_b b^{-\lambda}_{-p^R,-p^L} , \end{aligned}$$

$$\mathcal{P}_\perp \psi^\dagger(x) \mathcal{P}_\perp^\dagger = \eta_a^* \gamma^1 \gamma_5 \psi^\dagger(x^+, x^-, -x^R, -x^L) ,$$

so that, e.g.,

$$\begin{aligned} \mathcal{P}_\perp \bar\psi\psi(x) \mathcal{P}_\perp^\dagger &= \bar\psi\psi(x^+, x^-, -x^R, -x^L) \\ \mathcal{P}_\perp i\bar\psi\gamma_5\psi(x) \mathcal{P}_\perp^\dagger &= -i\bar\psi\gamma_5\psi(x^+, x^-, -x^R, -x^L) \end{aligned}$$

Discrete Symmetries on the Light Front

Time Reversal $\mathcal{T}_\perp$:

Momentum q^μ transforms as $q^R \to -q^L$, $q^L \to -q^R$, $q^\pm \to q^\pm$.
Thus $x^\mu = (x^+, x^-, x^L, x^R) \to (-x^+, -x^-, x^R, -x^L)$.

$\mathcal{T}_\perp$ is antiunitarity, but it does not flip the spin.

$$\begin{aligned} \mathcal{T}_\perp a^\lambda_{p^L,p^R} \mathcal{T}_\perp^\dagger &= \tilde{\eta}_a a^\lambda_{-p^R,-p^L} \,, \\ \mathcal{T}_\perp b^\lambda_{p^L,p^R} \mathcal{T}_\perp^\dagger &= \tilde{\eta}_b b^\lambda_{-p^R,-p^L} \,, \end{aligned}$$

$$\mathcal{T}_\perp \psi^\dagger(x) \mathcal{T}_\perp^\dagger = \tilde{\eta}_a^* \sigma^{12} \psi^\dagger(-x^+, -x^-, x^R, x^L)$$

so that, e.g.,

$$\begin{aligned} \mathcal{T}_\perp \bar{\psi}\psi(x) \mathcal{T}_\perp^\dagger &= \bar{\psi}\psi(-x^+, -x^-, x^R, x^L) \\ \mathcal{T}_\perp i\bar{\psi}\gamma_5\psi(x) \mathcal{T}_\perp^\dagger &= -i\bar{\psi}\gamma_5\psi(-x^+, -x^-, x^R, x^L) \end{aligned}$$

With $\mathcal{C}$ as usual all scalar fermion bilinears are invariant under $\mathcal{C}\mathcal{P}_\perp\mathcal{T}_\perp$.

Electric Dipole Form Factor on the Light Front

We consider the electric dipole form factor $F_3(q^2)$ in the light-front formalism of QCD, to complement earlier studies of the Dirac and Pauli form factors. [Drell, Yan, PRL 1970; West, PRL 1970; Brodsky, Drell, PRD 1980]

Recall

$$\langle P', S'_z | J^\mu(0) | P, S_z \rangle =$$
$$\bar{U}(P', \lambda') \left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2M}\sigma^{\mu\alpha}q_\alpha + F_3(q^2)\frac{-1}{2M}\sigma^{\mu\alpha}\gamma_5 q_\alpha \right] U(P, \lambda)$$

We ignore the anapole form factor and define

$$\kappa = \frac{e}{2M}\left[F_2(0)\right] , \qquad d = \frac{e}{M}\left[F_3(0)\right]$$

We will find a close connection between κ and d, as long recognized. [Bigi, Uralstev, NPB 1991]

Electromagnetic Form Factors on the Light Front

We work in the Drell ($q^+ = 0$) frame:

$$q = (q^+, q^-, \mathbf{q}_\perp) = (0, -q^2/P^+, \mathbf{q}_\perp)$$

$$P = (P^+, P^-, \mathbf{P}_\perp) = (P^+, M^2/P^+, \mathbf{0}_\perp)$$

$$F_1(q^2) = \left\langle P+q, \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \uparrow \right\rangle = \left\langle P+q, \downarrow \left| \frac{J^+(0)}{2P^+} \right| P, \downarrow \right\rangle$$

$$\frac{F_2(q^2)}{2M} = \frac{1}{2} \left[-\frac{1}{q^L} \left\langle P+q, \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \downarrow \right\rangle + \frac{1}{q^R} \left\langle P+q, \downarrow \left| \frac{J^+(0)}{2P^+} \right| P, \uparrow \right\rangle \right]$$

$$\frac{F_3(q^2)}{2M} = \frac{i}{2} \left[-\frac{1}{q^L} \left\langle P+q, \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \downarrow \right\rangle - \frac{1}{q^R} \left\langle P+q, \downarrow \left| \frac{J^+(0)}{2P^+} \right| P, \uparrow \right\rangle \right]$$

$q^{R/L} \equiv q^1 \pm iq^2$.

Both $F_2(q^2)$ and $F_3(q^2)$ are helicity-flip form factors.

Properties under $\mathcal{P}_\perp, \mathcal{T}_\perp$

$\mathcal{P}_\perp$

$$-\frac{1}{q^L}\langle P+q,\uparrow|J^+(0)|P,\downarrow\rangle \overset{\mathcal{P}_\perp}{\leftrightarrow} \frac{1}{q^R}\langle P+q,\downarrow|J^+(0)|P,\uparrow\rangle$$

$F_2(q^2)$ is even and $F_3(q^2)$ is odd under $\mathcal{P}_\perp$. Also $F_1(q^2)$ is even.

$\mathcal{T}_\perp$

$$\begin{aligned}\langle P+q,\uparrow|J^+(0)|P,\downarrow\rangle &\overset{\mathcal{T}_\perp}{\rightarrow} (\langle P+\tilde{q},\uparrow|J^+(0)|P,\downarrow\rangle)^* \\ &= -\langle P+q,\uparrow|J^+(0)|P,\downarrow\rangle\,, \\ \langle P+q,\downarrow|J^+(0)|P,\uparrow\rangle &\overset{\mathcal{T}_\perp}{\rightarrow} (\langle P+\tilde{q},\downarrow|J^+(0)|P,\uparrow\rangle)^* \\ &= -\langle P+q,\downarrow|J^+(0)|P,\uparrow\rangle\,,\end{aligned}$$

with $\tilde{q} = (q^+, q^-, \tilde{\mathbf{q}}_\perp)$ and $\tilde{\mathbf{q}}_\perp = (-q^1, q^2)$.
Re(F_2) and Im(F_3) are even and Re(F_3) and Im(F_2) are odd under $\mathcal{T}_\perp$.

Also Re(F_1) is even and Im(F_1) is odd.

Electromagnetic Form Factors on the Light Front

Interaction picture for $J^+(0)$ and assumed simple vacuum imply:

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right],$$

$$\frac{F_3(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{i}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) - \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right],$$

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

for the struck constituent j and

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$$

for each spectator ($i \neq j$). $q^+ = 0 \Longrightarrow$ only $n' = n$.

Wave Function Model for $\mathcal{P}_\perp$, $\mathcal{T}_\perp$-Odd Observables

In a quark-scalar-diquark ($q(qq)_0$) model

$$\begin{cases} \psi^{\uparrow}_{+\frac{1}{2}}(x,\mathbf{k}_\perp) = f(x)\varphi(x,k_\perp^2)\, e^{i\alpha_1}\, e^{+i\beta_1} \ , \\ \psi^{\uparrow}_{-\frac{1}{2}}(x,\mathbf{k}_\perp) = -(k^1+\mathrm{i}k^2)g(x)\varphi(x,k_\perp^2)\, e^{i\alpha_2}\, e^{+i\beta_2} \ , \end{cases}$$

$$\begin{cases} \psi^{\downarrow}_{+\frac{1}{2}}(x,\mathbf{k}_\perp) = (k^1-\mathrm{i}k^2)g(x)\varphi(x,k_\perp^2)\, e^{i\alpha_2}\, e^{-i\beta_2} \ , \\ \psi^{\downarrow}_{-\frac{1}{2}}(x,\mathbf{k}_\perp) = f(x)\varphi(x,k_\perp^2)\, e^{i\alpha_1}\, e^{-i\beta_1} \ , \end{cases}$$

α_1, α_2, β_1, and β_2 are real constants.

$$\mathcal{A} = \int \frac{\mathrm{d}x\, \mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\, e\, \varphi(x,{k'_\perp}^2)\varphi(x,k_\perp^2)f(x)g(x)$$

$$\frac{F_2(q^2)}{2M} = \mathcal{A}\cos\beta\,[(1-x)\exp(i(\alpha_1-\alpha_2)) + 2i\sin(\alpha_1-\alpha_2)] \ .$$

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$$[F_3(q^2)]_a = (\tan\beta_a)[F_2(q^2)]_a$$

A Universal Relation for $F_2(q^2)$ and $F_3(q^2)$

β_a violates $\mathcal{P}_\perp$ and $\mathcal{T}_\perp$.

$$\psi_a^\uparrow(x_i, \mathbf{k}_{\perp\, i}, \lambda_i) = \phi_a^\uparrow(x_i, \mathbf{k}_{\perp\, i}, \lambda_i) e^{+i\beta_a/2} \,,$$
$$\psi_a^\downarrow(x_i, \mathbf{k}_{\perp\, i}, \lambda_i) = \phi_a^\downarrow(x_i, \mathbf{k}_{\perp\, i}, \lambda_i) e^{-i\beta_a/2}$$

$$\frac{F_2(q^2)}{2M} = \sum_a \cos(\beta_a)\Xi_a$$
$$\frac{F_3(q^2)}{2M} = \sum_a \sin(\beta_a)\Xi_a \,,$$

$$\Xi_a = \int \frac{\left[\mathrm{d}^2\vec{k}_\perp \mathrm{d}x\right]}{16\pi^3} \sum_j e_j \frac{1}{-q^1 + iq^2} \left[\phi_a^{\uparrow *}(x_i, \vec{k}'_{\perp\, i}, \lambda_i)\, \phi_a^\downarrow(x_i, \vec{k}_{\perp\, i}, \lambda_i)\right] .$$

For Fock component a:

$$[F_3(q^2)]_a = (\tan\beta_a)[F_2(q^2)]_a$$
$$d_a = (\tan\beta_a) 2\kappa_a \quad \text{or} \quad d_a = 2\kappa_a\beta_a \quad \text{as} \quad q^2 \to 0$$

Implications for Models of CP Violation

Estimates of hadronic electric dipole moments depend on the hadron's non-perturbative structure.

For example, in the SM (CKM mechanism of CP violation), long-distance effects (π-loop) give for the neutron

$d_n^{\rm KM} \simeq 10^{-32}$e-cm [Gavela et al., PLB 1982; Khriplovich & Zhitnitsky, PLB 1982]

whereas a LL computation in three-loops yields

$d_d^{\rm KM} \simeq 10^{-34}$e-cm. [Czarnecki & Krause, PRL 1997]

Evaluating d_n and d_p is also important to interpreting the ^2H EDM.

[Lebedev et al., PRD 2004]

CP violation via a QCD $\bar{\theta}$-term.

In a $q(qq)_0$ model of the nucleon

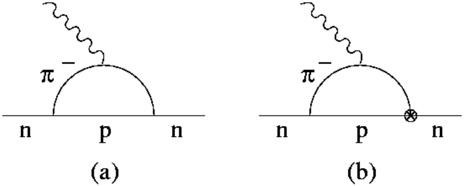
$$d^n \approx e\beta^n\kappa^n(2\cdot 10^{-14}\,\text{cm})\,, \quad d^p \approx e\beta^p\kappa^p(2\cdot 10^{-14}\,\text{cm})\,,$$

Since δL_{CP} is isoscalar, $\beta^n = \beta^p$ and
$(d^n + d^p)/(d^p - d^n) = (\kappa^n + \kappa^p)/(\kappa^p - \kappa^n) \approx -0.12/3.70 \approx -0.03$

Smaller than leading-order QCD sum rule estimate. [Pospelov and Ritz, PRL 1999]

Implications for Models of CP Violation

In a chiral Lagrangian framework [Baluni, PRD 1979; Crewther et al., PLB 1979; Pich & de Rafael, NPB 1991]



d^n (and d^p) determined from b) as π-loop is logarithmically enhanced.
Here $d^n = -d^p$; no isoscalar component.
Can we estimate d^n?
Assume Fock-state sum saturated by $udd u\bar{u}$ Fock component:

$$|\beta_a| \approx 2\frac{|\bar{g}_{\pi NN}|}{|g_{\pi NN}|}\log(M_N/M_\pi) \approx 4\left(\frac{0.027}{13.4}\right)|\bar{\theta}|$$

$\bar{g}_{\pi NN}$ is the CP-violating $g_{\pi NN}$ coupling constant

$$d^n \sim \bar{\theta}e(3 \cdot 10^{-16}\text{cm})$$

Summary and Outlook

We have used the light-front formalism of QCD to analyze the electromagnetic form factors, extending the earlier Brodsky-Drell-Yan-West work to the analysis of $\mathcal{P}_\perp$ and $\mathcal{T}_\perp$-odd observables.

We have used the light-front formalism to find a general equality between the anomalous magnetic and electric dipole moments.

- Relation holds for spin-$1/2$ systems, in general, not only neutron and is independent of the mechanism of CP violation.
- Both the EDM and anomalous magnetic moment should be calculated in a given model, to test for consistency.
- We argue that d^n and d^p (in the SM) should echo the isospin structure of the anomalous magnetic moments.

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