Mapping AdS/CFT Results for Holographic QCD to the Light Front

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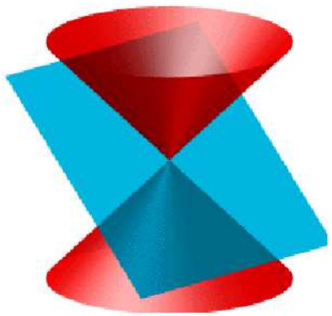


Light-Cone QCD and Nonperturbative Hadron Physics

Minneapolis, May 15-19, 2006

Motivation: Holographic QCD

- Strings describe extended objects (no quarks), QCD degrees of freedom are pointlike confined particles: how can they be related?
- More precisely: how can we map string states into partons?
- Precise mapping of string amplitudes to light-front wavefuntions of hadrons for strongly coupled QCD in the conformal limit.
- Infinite tension limit of strings $\longrightarrow$ effective gravity description
- Holographic duality requires a higher dimensional warped space. Space with negative curvature and a 4-dim boundary: AdS_5
- Eigenvalues of normalizable modes inside AdS give the hadronic spectrum. AdS modes represent also the probability amplitude for distribution of quarks at a given scale
- Non-normalizable modes are related to external currents: they probe the cavity interior

Outline

- Mapping String States to Partons

  Light-Front Wavefunctions in QCD

  The QCD Form Factor in the Light-Front Frame

  The Form Factor in AdS Space

  Holographic Mapping of AdS Modes to QCD LFWF

- Wave Equations in AdS and their Partonic Interpretation

  Scalar and Vector AdS Fields

  Meson Spectrum

  Spinor AdS Fields

  Baryon Spectrum

- Hadronic Form Factors in AdS/QCD

  Meson Form Factors

  Baryon Form Factors

- Outlook

Mapping States to Partons

Light-Front Wave Functions in QCD

- Hadronic bound state expanded in n-particle Fock eigenstates $|\psi_h\rangle = \sum_n \psi_{n/h}|\psi_h\rangle$ of the LC Hamiltonian $H_{LC} = P^2 = P^+P^- - \mathbf{P}_\perp^2$ at fixed light-cone time $x^+ = x^0 + x^3$

  Dirac '49; Brodsky, Pauli and Pinsky, Phys. Rept. 1988.

- Fock components

$$\psi_{n/h}(x_i, \mathbf{k}_{\perp i}) = \langle n;\ x_i, \mathbf{k}_{\perp i}, \big|\psi_h(P^+, \mathbf{P}_\perp)\rangle\,,$$

  frame independent and encode hadron properties and behavior in high momentum-transfer collisions.

- Momentum fraction $x_i = k_i^+/P^+$ and $\mathbf{k}_{\perp i}$ are the relative coordinates of parton i in Fock-state n

$$\sum_{i=1}^{n} x_i = 1 \quad \sum_{i=1}^{n} \mathbf{k}_{\perp i} = 0.$$

- Define transverse position coordinates $\quad x_i \mathbf{r}_{\perp i} = x_i \mathbf{R}_\perp + \mathbf{b}_{\perp i}$

$$\sum_{i=1}^{n} \mathbf{b}_{\perp i} = 0, \qquad \sum_{i=1}^{n} x_i \mathbf{r}_{\perp i} = \mathbf{R}_\perp.$$

The QCD Form Factor in the Light-Front Frame

- LFWF $\psi_n(x_j, \mathbf{k}_{\perp j})$ expanded in terms of $n-1$ independent coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$

$$\psi_n(x_j, \mathbf{k}_{\perp j}) = (4\pi)^{\frac{n-1}{2}} \prod_{j=1}^{n-1} \int d^2\mathbf{b}_{\perp j} \exp\left(i \sum_{j=1}^{n-1} \mathbf{b}_{\perp j} \cdot \mathbf{k}_{\perp j}\right) \widetilde{\psi}_n(x_j, \mathbf{b}_{\perp j}).$$

- Normalization

$$\sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} |\widetilde{\psi}_n(x_j, \mathbf{b}_{\perp j})|^2 = 1.$$

- The form factor has the exact representation (DYW)

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \exp\left(i\mathbf{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) |\widetilde{\psi}_n(x_j, \mathbf{b}_{\perp j})|^2,$$

corresponding to a change of transverse momentum $x_j \mathbf{q}_\perp$ for each of the $n-1$ spectators and elementary coupling to the struck parton.

- Define effective single particle transverse density (Soper '77)

$$F(q^2) = \int_0^1 dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp\cdot\vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp).$$

- From DYW expression for FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j}\, \delta\big(1 - x - \sum_{j=1}^{n-1} x_j\big) \delta^{(2)}\big(\sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} - \vec{\eta}_\perp\big) \left|\widetilde{\psi}_n(x_j, \mathbf{b}_{\perp j})\right|^2.$$

- Integration over the $n-1$ spectator partons, and $x = x_n$ is the coordinate of the active charged quark.

- $\vec{\eta}_\perp = \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}$ is the x-weighted transverse position coordinate of the $n-1$ spectators.

The Form Factor in AdS Space

- Non-conformal metric dual to a confining gauge theory

$$ds^2 = \frac{R^2}{z^2} e^{2A(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right),$$

where $A(z) \to 0$ as $z \to 0$ (Polchinski and Strassler, hep-th/0109174).

- Hadronic matrix element for EM coupling with string mode $\Phi(x,z)$, $x^\ell = (x^\mu, z)$

$$ig_5 \int d^4x \, dz \, \sqrt{g} \, A^\ell(x,z) \Phi^*_{P'}(x,z) \overleftrightarrow{\partial}_\ell \Phi_P(x,z).$$

- Electromagnetic probe polarized along Minkowski coordinates,

$$A_\mu = \epsilon_\mu e^{-iQ\cdot x} J(Q,z), \quad A_z = 0,$$

with

$$J(Q,z) = zQK_1(zQ), \quad J(Q=0,z) = J(Q,z=0) = 1$$

- Hadronic modes are plane waves along the Poincaré coordinates with four-momentum P^μ and invariant mass $P_\mu P^\mu = \mathcal{M}^2$

$$\Phi(x,z) = e^{-iP\cdot x} f(z), \quad f(z) \to z^\Delta, \quad z \to 0.$$

- Form factor in AdS is the overlap of normalizable modes dual to the incoming and outgoing hadrons Φ_P and $\Phi_{P'}$ with the non-normalizable mode $J(Q,z)$ dual to the external source

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) J(Q,z) \Phi_P(z).$$

Polchinski and Strassler, hep-th/0209211

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta).$$

- Transversality variable

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|.$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta)$!

- Hadronic QCD transverse density $\tilde{\rho}$ is identified with the string mode density $|\Phi|^2$ in AdS space!

$$\boxed{\tilde{\rho}(x,\zeta) = \frac{R^3}{2\pi}\,\frac{x}{1-x}\,e^{3A(\zeta)}\,\frac{|\Phi(\zeta)|^2}{\zeta^4}}$$

- The variable ζ represents the invariant separation between point-like constituents and it is also the holographic variable: $\zeta = z$.

- For two-partons

$$\tilde{\rho}(x,\zeta) = \frac{1}{(1-x)^2}\left|\widetilde{\psi}\,(x,\zeta)\right|^2.$$

- Two-parton bound state LFWF

$$\left|\widetilde{\psi}(x,\zeta)\right|^2 = \frac{R^3}{2\pi}\,x(1-x)\,e^{3A(\zeta)}\,\frac{|\Phi(\zeta)|^2}{\zeta^4}.$$

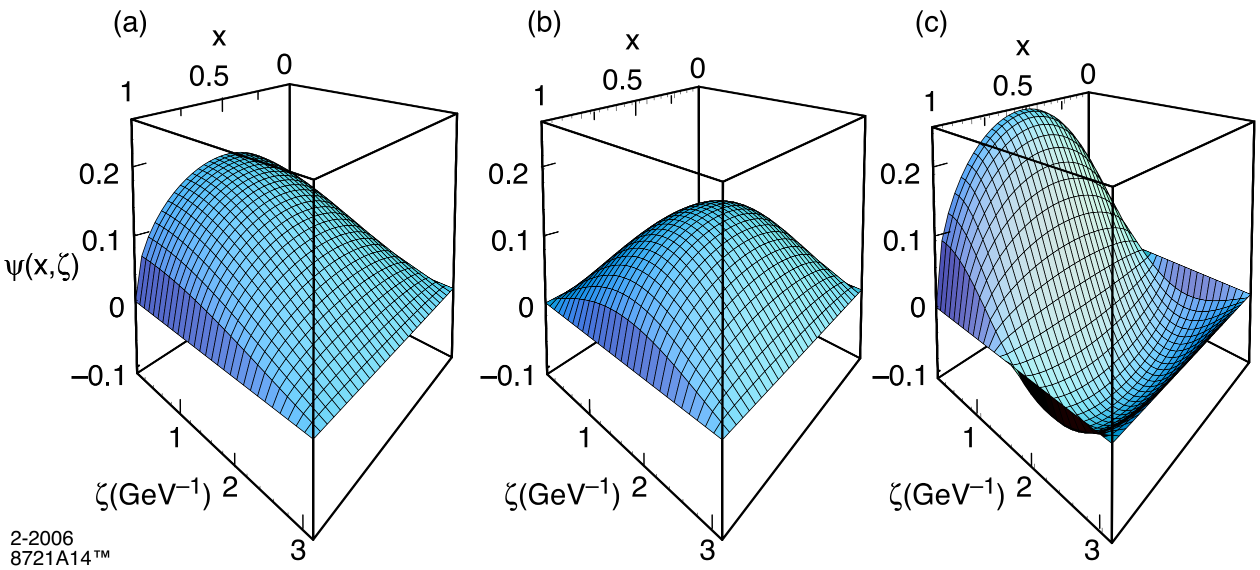
Brodsky and de Teramond, arXiv:hep-ph/0602252

- Short distance behavior of LFWF: $\widetilde{\psi}(x,\mathbf{b}_\perp) \sim (\mathbf{b}_\perp^2)^{\Delta-2}$.

Example:

- Two parton LFWF bound state:

$$\widetilde{\psi}_{\overline{q}q/\pi}(x,\zeta) = B_{L,k}\sqrt{x(1-x)}J_L\left(\zeta\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\big(z \le \Lambda^{-1}_{\rm QCD}\big),$$



(a) ground state $L=0$, $k=1$, (b) first orbital $L=1$, $k=1$, (c) first radial $L=0$, $k=2$.

Summary

- Effective Schrödinger equation in terms of the transverse impact variable ζ

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta),$$

  with effective potential

$$V(\zeta) \to -\frac{1-4L^2}{4\zeta^2},$$

  in the conformal limit.

- Conformal Analytical AdS machinery to extend the hadron into the fifth dimensions and back!

$$3+1 \quad \to \quad \mathrm{AdS}_5 \;\; (z) \quad \to \quad 3+1 \;\; (z \to \zeta)$$

- A different approach: Match the AdS results with Migdal '77 regularization approach to UV correlators (Padé approx) to recover zeros of Bessel functions !

  Two point function: Erlich, Kribs and Low (hep-th/0602110).

  Three point function: Radyushkin (hep-ph/0605116).

Wave Equations in AdS and their Partonic Interpretation

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space $SO(1,5)$

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

  $x^\mu \to \lambda x^\mu,\ z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- Different values of z correspond to different scales at which the hadron is examined.
- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.
- There is a maximum separation of quarks and a maximum value of z at the IR boundary.
- Truncated AdS/CFT model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale.
- Conformal behavior at short distances and color confinement at large interquark separation.

Scalar and Vector AdS Fields

- Consider a scalar wave equation on AdS$_{d+1}$

$$\left[z^2\partial_z^2 - (d-1)z\,\partial_z + z^2\,\mathcal{M}^2 - (\mu R)^2\right]\Phi(z) = 0,$$

  with solution

$$\Phi(z) \sim z^{d/2} J_{\Delta-\frac{d}{2}}(z\mathcal{M}), \quad (\mu R)^2 = \Delta(\Delta - d).$$

- For $d = 4$, lowest stable solution determined by the Breitenlohner-Freedman (B-F) stability bound, $(\mu R)^2 \geq -4$, on the fifth dimensional mass.

- Orbital excitations correspond to higher values of μ. Allowed values determined by the condition that conformal dimensions are spaced by integers, according to the spectral relation

$$(\mu R)^2 = \Delta(\Delta - d) = \kappa(\kappa + d).$$

- If lowest stable state corresponds to the lowest orbital, $L = 0$, then $\kappa = L - 2$, $\Delta = 2 + L$ and

$$(\mu R)^2 = -4 + L^2.$$

- Wave equation in AdS for bound state of two scalar partons with conformal dimension $\Delta = 2 + L$

$$\left[z^2 \partial_z^2 - 3z\,\partial_z + z^2\,\mathcal{M}^2 - L^2 + 4\right] \Phi(z) = 0,$$

with solution

$$\Phi(z) = C e^{-iP\cdot x} z^2 J_L(z\mathcal{M}).$$

- For spin-carrying constituents: $\Delta \to \tau = \Delta - \sigma, \;\; \sigma = \sum_{i=1}^n \sigma_i.$
- The twist τ is equal to the number of partons $\tau = n$.
- Same form of equation for vector wave equation in AdS with lowest stable solution $(\mu R)^2 \geq -1$ and

$$(\mu R)^2 = (\Delta - 1)(\Delta - d - 1) = \kappa(\kappa + d - 2).$$

- Two-quark vector meson described by wave equation

$$\left[z^2\,\partial_z^2 - 3z\,\partial_z + z^2\mathcal{M}^2 - L^2 + 4\right] \Phi_\mu(z) = 0,$$

with solution

$$\Phi_\mu(x, z) = C e^{-iP\cdot x} z^2 J_L(z\mathcal{M})\,\epsilon_\mu.$$

Meson Spectrum

- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z_0 = 1/\Lambda_{\rm QCD}$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$.

- Normalizable AdS modes $\Phi(z)$

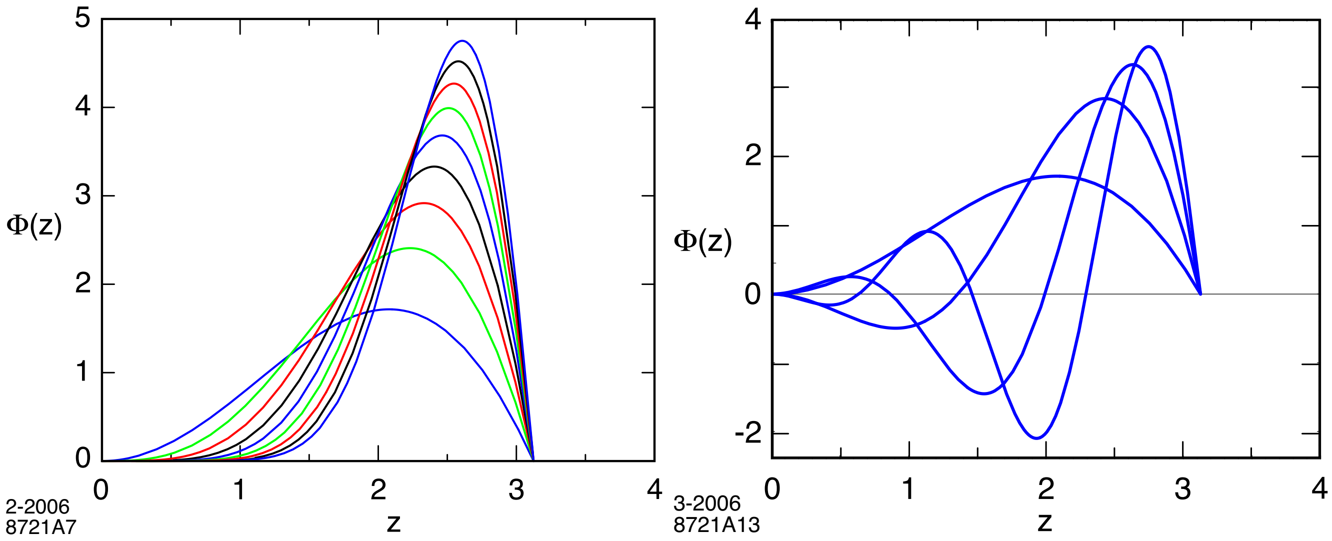


Fig: Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

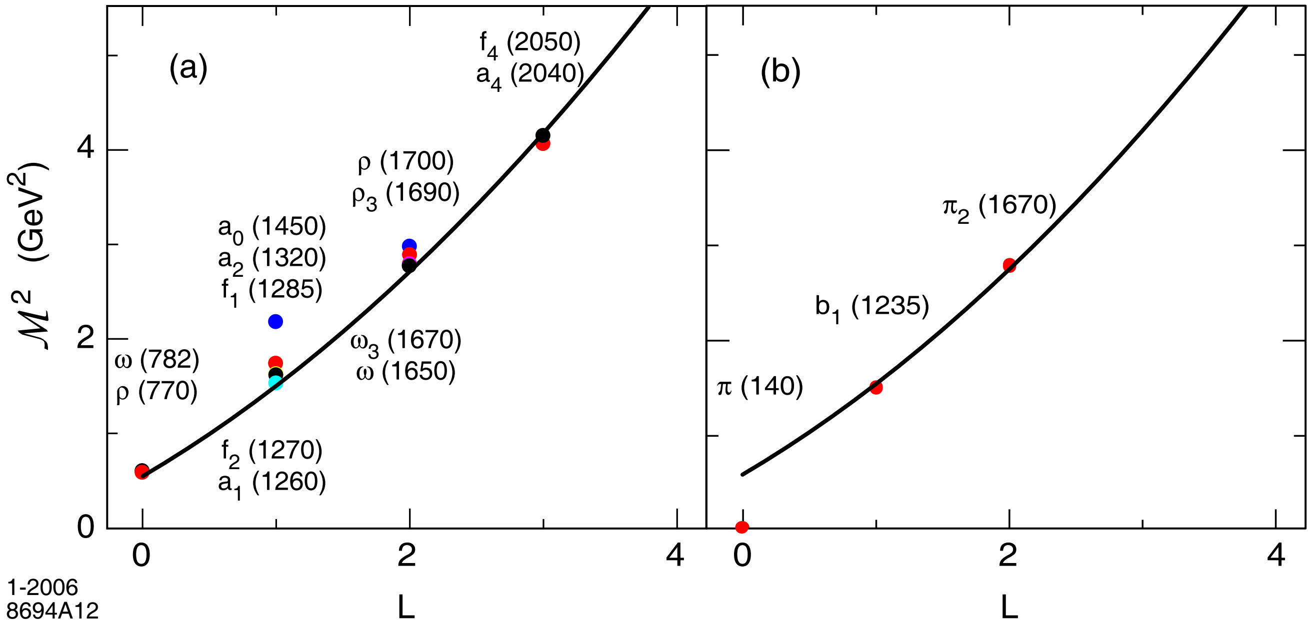


Fig: Light meson orbital spectrum $\Lambda_{QCD} = 0.32\ \mathrm{GeV}$

Spinor AdS Fields

- Baryon: twist-three, dimension $\Delta = \frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Solve full 10-dim Dirac Eq., $\not{D}\hat{\Psi} = 0$, since baryons are charged under $SU(4) \sim SO(6)$. Baryon number conservation?

- $\hat{\Psi}$ is expanded in terms of eigenfunctions $\eta_\kappa(y)$ of the Dirac operator on compact space X with eigenvalues λ_κ:

$$\hat{\Psi}(x, z, y) = \sum_\kappa \Psi_\kappa(x, z) \eta_\kappa(y).$$

- From the 10-dim Dirac equation, $\not{D}\hat{\Psi} = 0$:

$$\left[z^2\, \partial_z^2 - d\, z\, \partial_z + z^2 \mathcal{M}^2 - (\lambda_\kappa + \mu)^2 R^2 + \frac{d}{2}\left(\frac{d}{2} + 1\right) + (\lambda_\kappa + \mu) R\, \hat{\Gamma}\right] f(z) = 0,$$

$$i\not{D}_X \eta(y) = \lambda\, \eta(y),$$

where $\Psi(x, z) = e^{-iP\cdot x}\, f(z)$, $P_\mu P^\mu = \mathcal{M}^2$ and $\hat{\Gamma} u_\pm = \pm u_\pm$ (For $d = 4$, $\hat{\Gamma} = \gamma_5$).

- Spinor field in AdS:

$$\Psi(x,z) = Ce^{-iP\cdot x} z^{\frac{d+1}{2}} \left[J_{(\mu+\lambda_\kappa)R-\frac{1}{2}}(z\mathcal{M})\, u_+(P) + J_{(\mu+\lambda_\kappa)R+\frac{1}{2}}(z\mathcal{M})\, u_-(P) \right],$$

with $\Delta = \frac{d}{2} + |(\mu + \lambda_\kappa)R|$ and spinors $u_\pm(P)$ defined along 4-dim coordinates.

See: Muck and Viswanathan, hep-ph/9805945.

- μ determined asymptotically by spectral comparison with orbital excitations in the boundary: $\mu = k/R$ and λ_κ are the eigenvalues of the Dirac equation on S^{d+1}:

$$\lambda_\kappa R = \pm \left(\kappa + \frac{d}{2} + \frac{1}{2} \right), \quad \kappa = 0, 1, 2...$$

See: Camporesi and Higuchi: gr-gc/9505009.

- Spin-$\frac{3}{2}$ Rarita-Schwinger eq. in AdS similar to spin-$\frac{1}{2}$ in the $\Psi_z = 0$ gauge for polarization along Minkowski coordinates, Ψ_μ. See: Volovich, hep-th/9809009.

Baryon Spectrum

- For spin-carrying constituents: $\Delta \to \tau = \Delta - \sigma, \;\; \sigma = \sum_{i=1}^n \sigma_i.$

- For a three quark state $\Delta \to \Delta - 3/2$. Change compensated in μ by the shift $k \to L - 1$ and $\Psi(z) \to z^{-\frac{1}{2}} \Psi(z)$.

- Three-quark baryon described by wave equation ($d = 4$, $\kappa = 0$)

$$\boxed{\left[z^2\, \partial_z^2 - 3z\, \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_\pm^2 + 4\right] f_\pm(z) = 0}$$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x, z) = Ce^{-iP\cdot x} z^2 \Big[J_{1+L}(z\mathcal{M})\, u_+(P) + J_{2+L}(z\mathcal{M})\, u_-(P)\Big].$$

- 4-d mass spectrum $\Psi(x, z_o)^\pm = 0 \quad \Longrightarrow$ parallel Regge trajectories for baryons !

$$\mathcal{M}_{\alpha,k}^+ = \beta_{\alpha,k}\Lambda_{QCD}, \quad \mathcal{M}_{\alpha,k}^- = \beta_{\alpha+1,k}\Lambda_{QCD}.$$

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions !

- $SU(6)$ multiplet structure for N and Δ orbital states, including internal spin S and L.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$

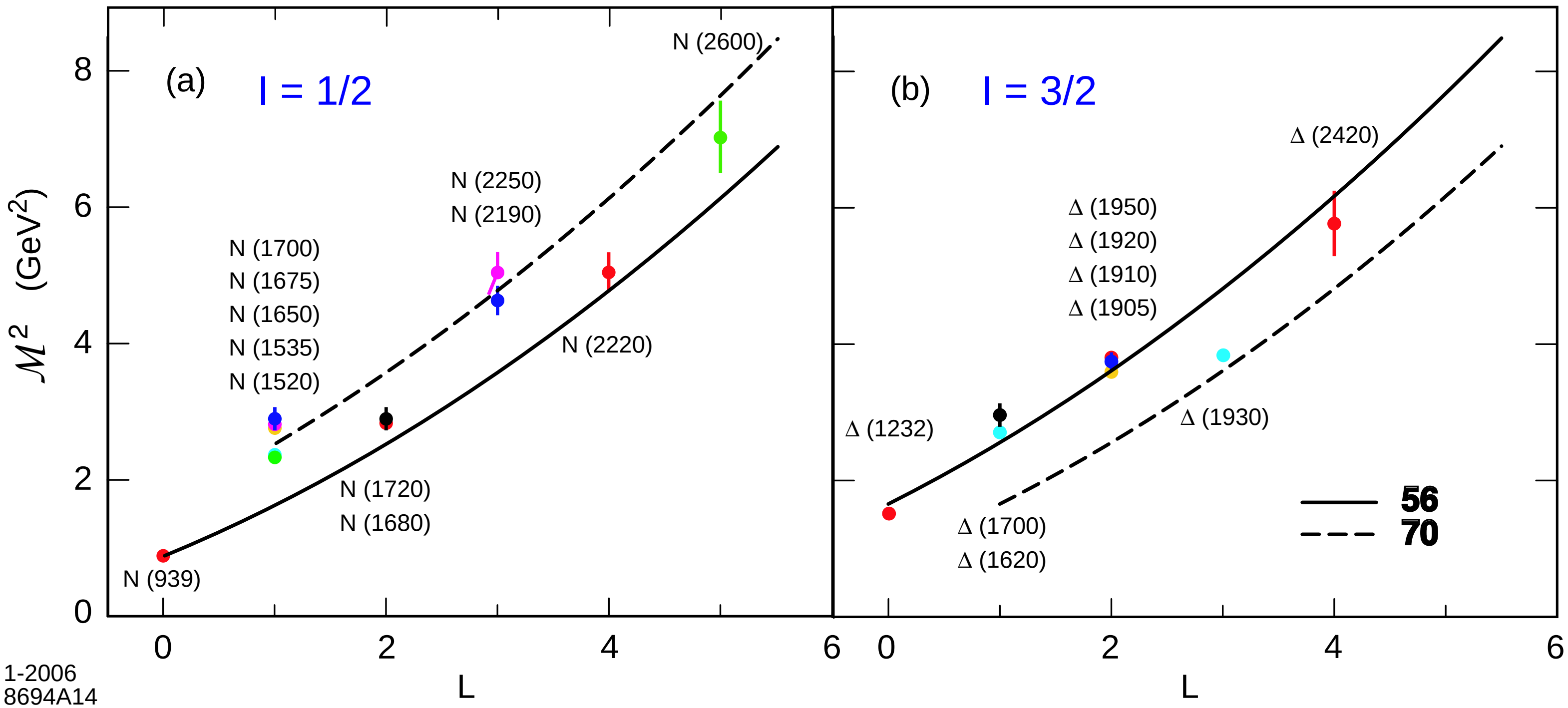


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

Gaussian Deformed Metric Background (In Progress)

Karch, Katz, Son and Stephanov : Non-constant dilaton $\Rightarrow V(z) \sim z^2$ (arXiv:hep-ph/0602229)

- Two-dim harmonic oscillator in terms of transverse variable $\zeta = z$

$$\left[z^2 \partial_z^2 - 3z\, \partial_z + z^2\, \mathcal{M}^2 - (\mu R)^2 - \kappa^4 z^4\right] \Phi(z) = 0.$$

- Normalizable solutions exist if $(\mu R)^2 = -4 + L^2$ (obtained in truncated model by B-F bound)

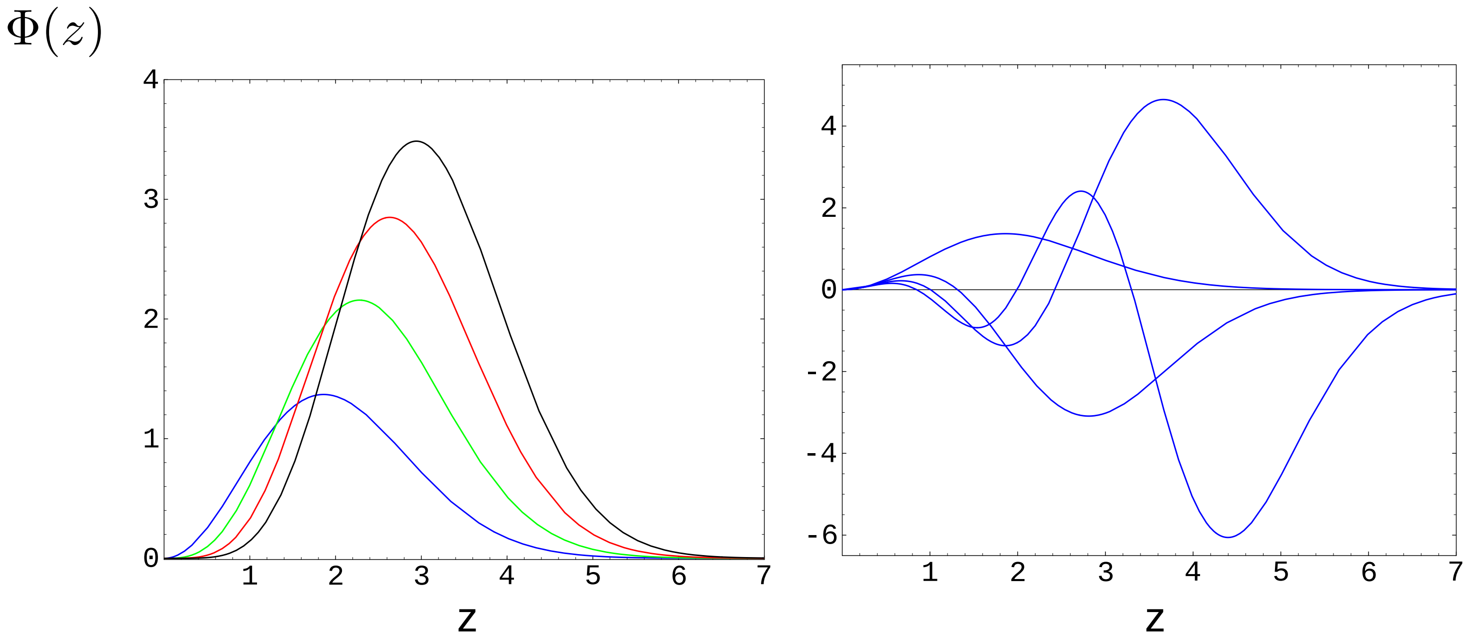
$$\Phi(z) = \frac{\kappa^{L+1}}{R^{3/2}} \sqrt{\frac{2n!}{(n+L)!}}\, z^{2+L} e^{-\kappa^2 z^2/2} L_n^L\left(\kappa^2 z^2\right),$$

with eigenvalues

$$\mathcal{M}^2 = 2\kappa^2(2n + L + 1).$$

- Equivalent problem: $Z(z) = e^{\kappa^2 z^2/2}\Phi(z)$ and metric $e^{A(z)} = e^{-\kappa^2 \zeta^2/3}$ with no potential!

Andreev: Gaussian metric term (arXiv:hep-th/0603170)



Orbital and radial modes for a Gaussian deformed metric asymptotic to AdS_5.

- LFWF $\widetilde{\psi}(x, \vec{b}_\perp)$ for the gaussian example:

$$\widetilde{\psi}(x, \vec{b}_\perp) = \kappa^{L+1} \sqrt{\frac{n!}{\pi(n+L)!}} \, [x(1-x)]^{\frac{1}{2}+L} |\vec{b}_\perp|^L e^{\pm i L \varphi} e^{-\frac{1}{2}\kappa^2 x(1-x)\vec{b}_\perp^2} L_n^L\left(\kappa^2 x(1-x)\vec{b}_\perp^2\right).$$

Hadronic Form Factors in AdS/QCD (In Progress)

- Form factor in AdS (Polchinski and Strassler, hep-th/0209211).

$$F(Q^2)_{A \to B} \simeq R^3 \int_0^{\Lambda_{QCD}^{-1}} \frac{dz}{z^3} \, \Phi_B(z) \, J(Q,z) \, \Phi_A(z).$$

- At large Q, important contribution is from the boundary conformal region $z \sim 1/Q$ where $\Phi \sim z^\Delta$:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\Delta - 1}$$

Constituent counting rule for hard scattering !

(Brodsky and Farrar '73; Matveev *et al.* '73)

- Spin carrying constituents $\Delta \to \tau$.

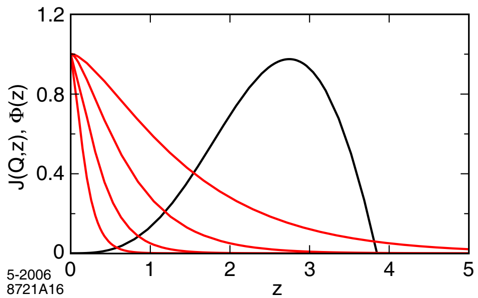
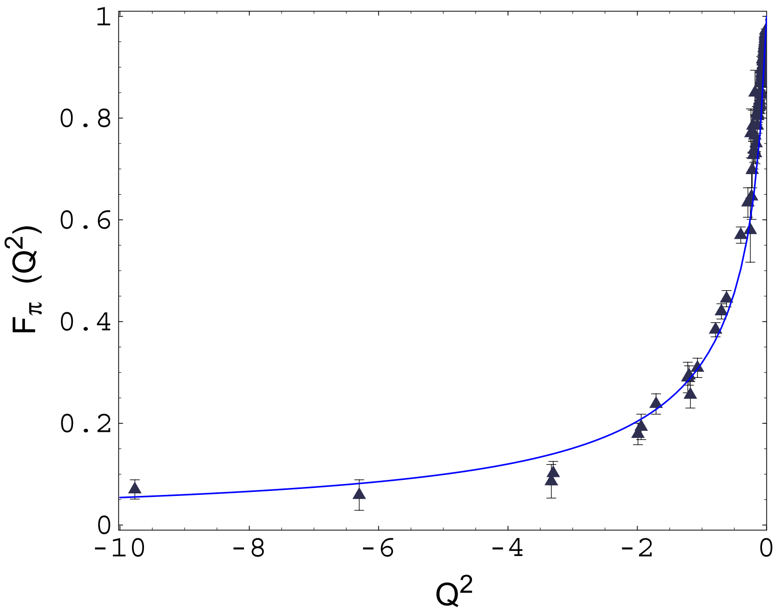


Fig: Suppression of external perturbations for large Q inside AdS.

Meson Form Factor

(Valence Approximation)



Space-like pion form factor in holographic model for $\Lambda_{QCD} = 0.2$ GeV.

Baryon Form Factors

- Coupling of the extended AdS mode with an external gauge field $A^\mu(x,z)$

$$ig_5 \int d^4x\, dz\, \sqrt{g}\, A_\mu(x,z)\, \overline{\Psi}(x,z)\gamma^\mu \Psi(x,z),$$

where

$$\Psi(x,z) = e^{-iP\cdot x}\left[\psi_+(z)u_+(P) + \psi_-(z)u_-(P)\right],$$

$$\psi_+(z) = Cz^2 J_1(zM), \qquad \psi_-(z) = Cz^2 J_2(zM),$$

and

$$u(P)_\pm = \frac{1\pm\gamma_5}{2}\, u(P).$$

- In the large P^+ limit

$$\psi_+(z) \equiv \psi^\uparrow(z), \quad \psi_-(z) \equiv \psi^\downarrow(z),$$

the LC $\pm$ spin projection along $\hat{z}$.

- Constant C determined by charge normalization:

$$C = \frac{\sqrt{2}\Lambda_{\rm QCD}}{R^{3/2}\left[-J_0(\beta_{1,1})J_2(\beta_{1,1})\right]^{1/2}}\,.$$

- Consider the spin non-flip form factors in the infinite wall approximation

$$
\begin{aligned}
F_+(Q^2) &= g_+ R^3 \int \frac{dz}{z^3}\, J(Q,z)\, |\psi_+(z)|^2, \\
F_-(Q^2) &= g_- R^3 \int \frac{dz}{z^3}\, J(Q,z)\, |\psi_-(z)|^2,
\end{aligned}
$$

  where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $s^z = +1/2$. The two AdS solutions $\psi_+(z)$ and $\psi_-(z)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry (proton up)

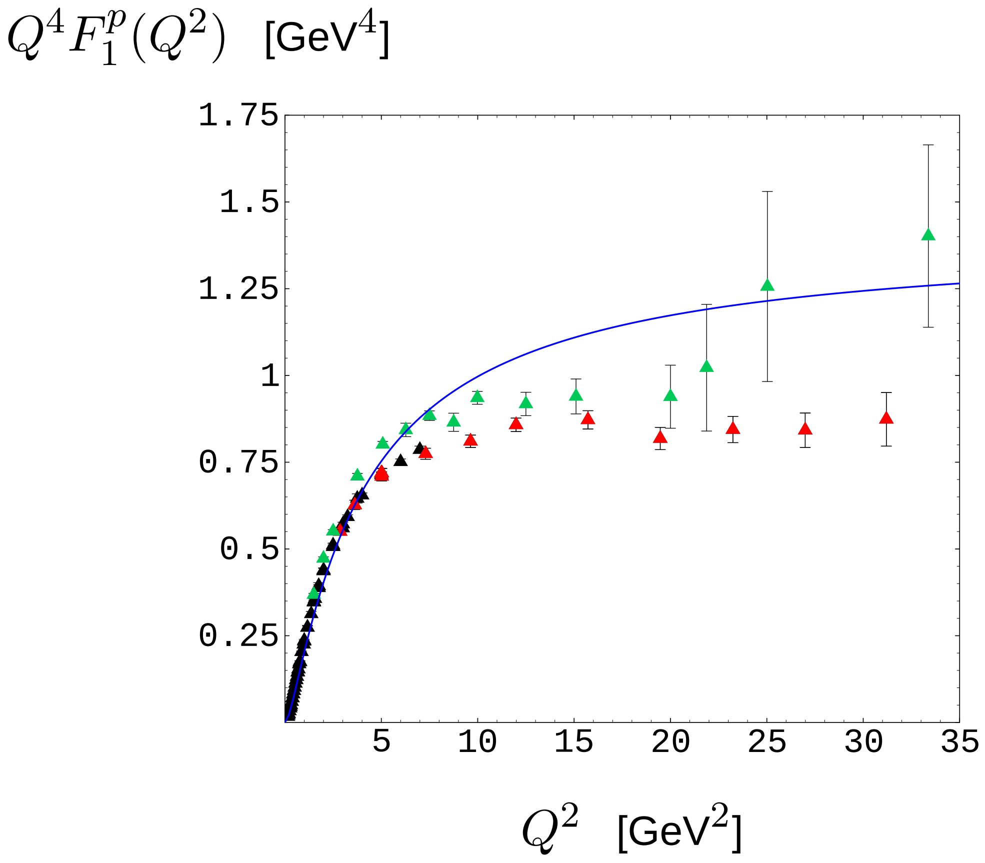
$$
N^{\uparrow}_{u\uparrow} = \frac{5}{3},\; N^{\uparrow}_{u\downarrow} = \frac{1}{3},\; N^{\uparrow}_{d\uparrow} = \frac{1}{3},\; N^{\uparrow}_{d\downarrow} = \frac{2}{3}.
$$

- Final result

$$
\begin{aligned}
F_1^p(Q^2) &= R^3 \int \frac{dz}{z^3}\, J(Q,z) |\psi_+(z)|^2, \\
F_1^n(Q^2) &= -\frac{1}{3} R^3 \int \frac{dz}{z^3}\, J(Q,z) \left[|\psi_+(z)|^2 - |\psi_-(z)|^2 \right],
\end{aligned}
$$

  where $F_1^p(0) = 1,\; F_1^n(0) = 0$.

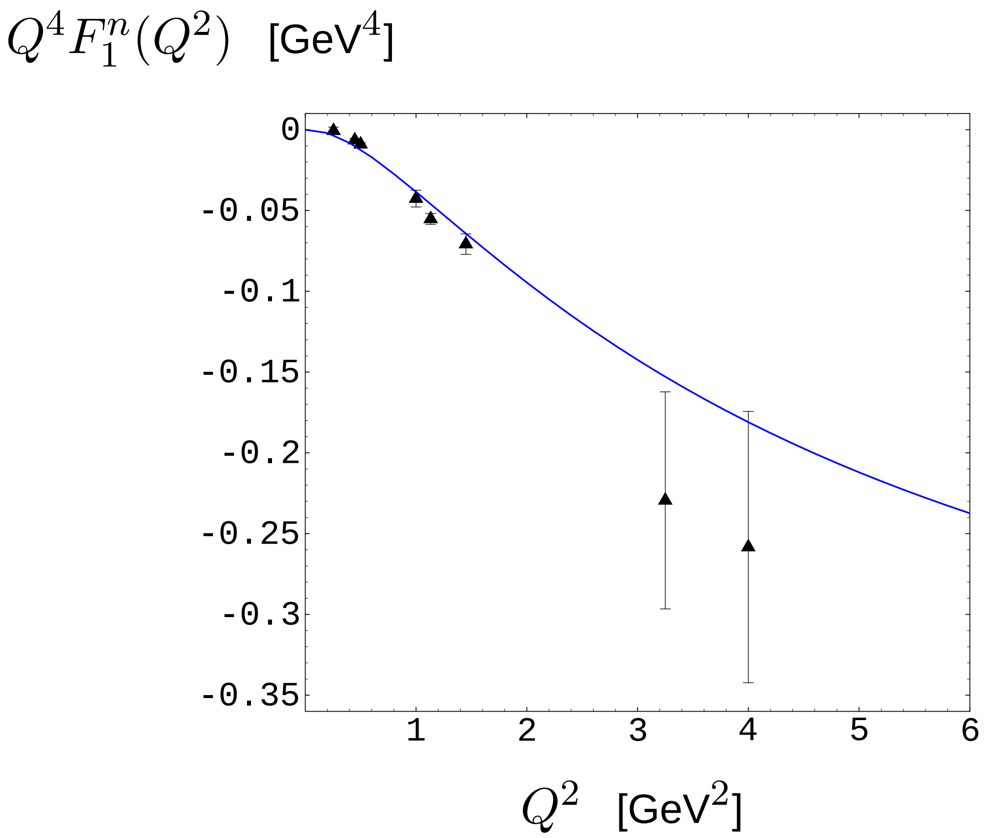
Dirac Proton Form Factor
(Valence Approximation)



Prediction for $Q^4 F_1^p(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Analysis of the data is from Diehl (2005). Red points are from Sill (1993). Superimposed Green points are from Kirk (1973).

Dirac Neutron Form Factor

(Valence Approximation)



Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Example: Evaluation of QCD Matrix Elements

- Pion decay constant f_π defined by the matrix element of EW current J_W^+:

$$\left\langle 0 \left| \overline{\psi}_u \gamma^+ (1-\gamma_5) \psi_d \right| \pi^- \right\rangle = i\sqrt{2} P^+ f_\pi ,$$

with

$$\left|\pi^-\right\rangle = \left|d\overline{u}\right\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left(b^\dagger_{c\, d\downarrow} d^\dagger_{c\, u\uparrow} - b^\dagger_{c\, d\uparrow} d^\dagger_{c\, u\downarrow} \right) \left|0\right\rangle .$$

- Use light-cone expression:

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \, \psi_{\overline{q}q/\pi}(x, k_\perp).$$

Lepage and Brodsky '80

- Find:

$$f_\pi = \frac{\sqrt{3}\Lambda_{\mathrm{QCD}}}{8 J_1(\beta_{0,1})} = 83.4 \,\mathrm{Mev},$$

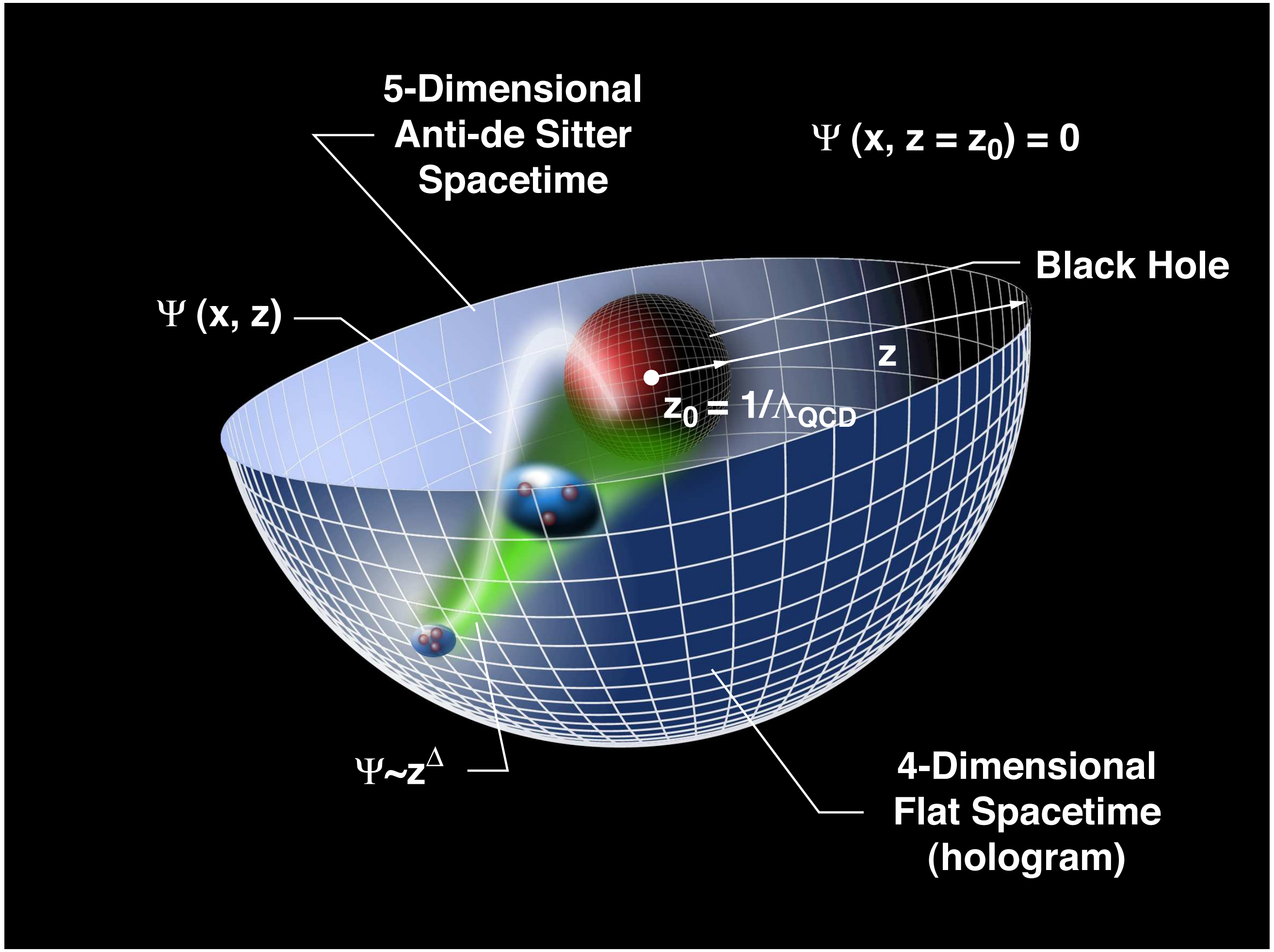
for $\Lambda_{\mathrm{QCD}} = 0.2$ GeV.

Experiment: $f_\pi = 92.4$ Mev.

Outlook

- Only one scale Λ_{QCD} determines hadronic spectrum (slightly different for mesons and baryons).
- Ratio of Nucleon to Delta trajectories determined by zeros of Bessel functions.
- Non-zero orbital angular momentum and higher Fock-states require introduction of quantum fluctuations.
- Initial good approximation for description of the structure of hadronic form factors and other observables.
- Covariant version of the bag model with confinement and conformal symmetry.
- Light-cone frame is the natural frame to establish the AdS/QCD holographic duality.
- Precise mapping of string modes to partonic states. String modes inside AdS represent the probability amplitude for the distribution of quarks at a given scale.
- Write eigenvalue problem in terms of 3+1 QCD degrees of freedom.

... basic features of QCD can be understood in terms of a higher dimensional dual gravity theory which holographically encodes multi parton boundary states into string modes and allows the computation of physical observables at strong coupling ...



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