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Universidade Cruzeiro do Sul
Centro de Ciências Exatas
e Tecnológicas - CETEC

Light-Cone QCD, and
Nonpertubative Hadrons Physics

# Electromagnetic Current of a Composed Vector Particle in the Light-Front

J. Pacheco B. C. de Melo **UNICSUL - CETEC**

**IFT - UNESP**

Tobias Frederico **(CTA-ITA)**

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# Summary

1. Light-Front Formalism
2. Spin-1 Particle: Rho Meson
3. Covariance Restoration in the Light-Front
4. Conclusions

# Light-Front Formalism

- Light-Front Coordinates

$x^+ = t + z \qquad x^+ = x^0 + x^3 \quad \Longrightarrow$ Time

$x^- = t - z \qquad x^- = x^0 - x^3 \quad \Longrightarrow$ **Position**

- **Four-Vector** $\Longrightarrow \quad x^\mu = (x^0, x^1, x^2, x^3) \; = \; (x^+, x^-, x_\perp)$
- **Metric Tensor**

$$g^{\mu\nu} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \text{ and } g_{\mu\nu} = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- **Scalar product**

$$\begin{aligned} x \cdot y = x^\mu y_\mu &= x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 \\ &= \frac{1}{2}(x^+ y^- + x^- y^+) - \vec{x}_\perp \vec{y}_\perp \end{aligned}$$

$$\begin{aligned} p^+ &= p^0 + p^3 \\ p^- &= p^0 - p^3 \\ p^\perp &= (p^1, p^2) \end{aligned}$$

- **Dirac Matrix**

$\gamma^+ = \gamma^0 + \gamma^3 \quad \Longrightarrow$ **Electr. Current** $J^+ = J^0 + J^3$

$\gamma^- = \gamma^0 - \gamma^3 \quad \Longrightarrow$ **Electr. Current** $J^- = J^0 - J^3$

$\gamma^\perp = (\gamma^1, \gamma^2) \quad \Longrightarrow$ **Electr. Current** $J^\perp = (J^1, J^2)$

Fig. 1: Ligh-Front

- $p^\mu x_\mu = \frac{p^+x^- + p^-x^+}{2} - \vec{p_\perp}\vec{x_\perp}$
- $x^+, x^-, \vec{x_\perp} \quad \Longrightarrow p^+, p^-, \vec{p_\perp}$
- $p^-$ $\Longrightarrow$ **Light-Front Energy**
- $p^2 = p^+p^- - p_\perp^2 \Longrightarrow \quad p^- = \frac{p_\perp^2 + m^2}{p^+}$
- Bosons: $\Longrightarrow \quad S_F(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$
- Fermions: $\Longrightarrow \quad S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + \imath\epsilon} + \frac{\gamma^+}{2p^+}$

- Ref: Phys. Rept. 301, (1998) 299-486

  S. J. Brodsky, H.C. Pauli and S.S. Pinsky

# Spin-1 Particle: Rho Meson

- General Electromagnetic Current

$$J^{\mu}_{\alpha\beta} = [F_1(q^2) g_{\alpha\beta} - F_2(q^2) \frac{q_\alpha q_\beta}{2m_\rho^2}] p^\mu - F_3(q^2)(q_\alpha g^\mu_\beta - q_\beta g^\mu_\alpha) \ ,$$

- Polarization Vectors

$$\epsilon^\mu_x = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0) \ , \ \epsilon^\mu_y = (0,0,1,0) \ , \ \epsilon^\mu_z = (0,0,0,1) \ ,$$

$$\epsilon'^{\mu}_x = (\sqrt{\eta}, \sqrt{1+\eta}, 0, 0) \ , \ \epsilon'^{\mu}_y = \epsilon_y \ , \ \epsilon'^{\mu}_z = \epsilon_z$$

where $\eta \ = \ q^2/4m_\rho^2$

- Breit Frame

$p^\mu_i = (p^0, -q_x/2, 0, 0)$ Initial

$p^\mu_f = (p^0, q_x/2, 0, 0)$ Final

where $p^0 = m_\rho \sqrt{1+\eta}.$

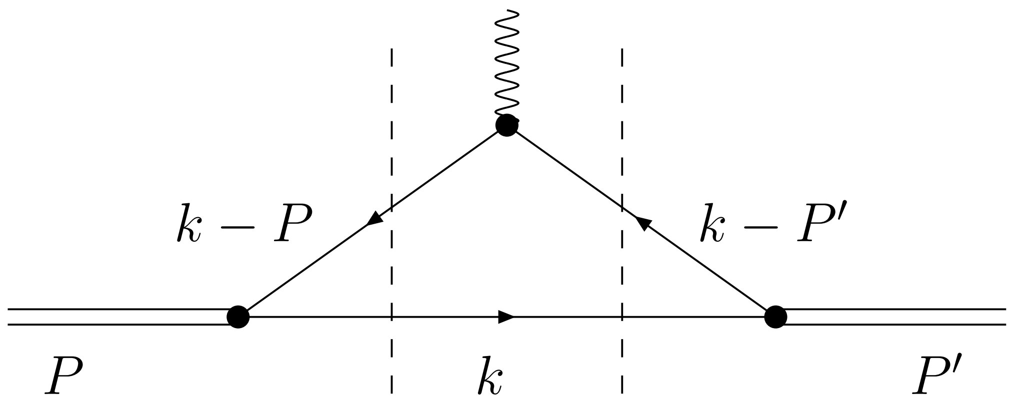


Fig. 2: Light-front time-ordered triangle diagram for the electromagnetic current.

- Plus Component of the Electromagnetic Current

$$J^+_{ji} = \imath \int \frac{d^4k}{(2\pi)^4} \frac{Tr[\epsilon^{'\beta}_j \Gamma_\beta(k,k-p_f)(\not k - \not p_f + m)}{((k-p_i)^2 - m^2 + \imath\epsilon)(k^2 - m^2 + \imath\epsilon)}$$
$$\times \frac{\gamma^+(\not k - \not p_i + m)\epsilon^\alpha_i \Gamma_\alpha(k,k-p_i)(\not k + m)]\Lambda(k,p_f)\Lambda(k,p_i)}{((k-p_f)^2 - m^2 + \imath\epsilon}$$

- Regulator Function

$$\Lambda(k, p_{i(f)}) = N/((p-k)^2 - m_R + \imath\epsilon)^2$$

- $\rho$-Meson Vertex

$$\Gamma^\mu(k,p) = \gamma^\mu - \frac{m_\rho}{2}\frac{2k^\mu - p^\mu}{p.k + m_\rho m - \imath\epsilon}$$

- Mass Squared $\left(x = \frac{k^+}{P^+} \implies 0 < x < 1\right)$

$$M^2(m_a, m_b) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(\vec{p} - \vec{k})^2_\perp + m_b^2}{1-x} - p_\perp^2$$

- Free Mass $M_0^2(m,m)$ and Function $M_R^2(m, m_R)$
- Wave Function

$$\Phi_i(x, \vec{k}_\perp) = \frac{N^2}{(1-x)^2(m_\rho^2 - M_0^2)(m_\rho^2 - M_R^2)^2}\vec{\epsilon}_i.[\vec{\gamma} - \frac{\vec{k}}{\frac{M_0}{2} + m}]$$

- Ref: Phy.Rev. **C55** (1997) 2043

J.P.B. C. de Melo and T. Frederico

- Instant-Form Spin Base

$$J^+ = \frac{1}{2}\begin{pmatrix} J^+_{xx} + J^+_{yy} & \sqrt{2}J^+_{zx} & J^+_{yy} - J^+_{xx} \\ -\sqrt{2}J^+_{zx} & 2J^+_{zz} & \sqrt{2}J^+_{zx} \\ J^+_{yy} - J^+_{xx} & -\sqrt{2}J^+_{zx} & J^+_{xx} + J^+_{yy} \end{pmatrix}$$

- Light-Front

$$I^+ = \begin{pmatrix} I^+_{11} & I^+_{10} & I^+_{1-1} \\ -I^+_{10} & I^+_{00} & I^+_{10} \\ I^+_{1-1} & -I^+_{10} & I^+_{11} \end{pmatrix}$$

- Matrix Elements

$$\begin{aligned} I^+_{11} &= \frac{J^+_{xx} + (1+\eta)J^+_{yy} - \eta J^+_{zz} - 2\sqrt{\eta}J^+_{zx}}{2(1+\eta)} \\ I^+_{10} &= \frac{\sqrt{2\eta}J^+_{xx} + \sqrt{2\eta}J^+_{zz} - \sqrt{2}(\eta-1)J^+_{zx}}{2(1+\eta)} \\ I^+_{1-1} &= \frac{-J^+_{xx} + (1+\eta)J^+_{yy} + \eta J^+_{zz} + 2\sqrt{\eta}J^+_{zx}}{2(1+\eta)} \\ I^+_{00} &= \frac{-\eta J^+_{xx} + J^+_{zz} - 2\sqrt{\eta}J^+_{zx}}{(1+\eta)} \end{aligned}$$

$$
\begin{aligned}
J^+_{xx} &= \frac{1}{1+\eta}[I^+_{11} + 2\sqrt{2\eta}I^+_{10} - \eta I^+_{00} - I^+_{1-1}] \\
J^+_{zx} &= \frac{\sqrt{2}}{1+\eta}[\frac{\sqrt{2\eta}}{2}I^+_{11} + (\eta - 1)I^+_{10} + \sqrt{\frac{\eta}{2}}I^+_{00} \\
& -\frac{\sqrt{2\eta}}{2}I^+_{1-1}] \\
J^+_{yy} &= I^+_{11} + I^+_{1-1} \\
J^+_{zz} &= \frac{1}{1+\eta}[-\eta I^+_{11} + 2\sqrt{2\eta}I^+_{10} + I^+_{00} + \eta I^+_{1-1}]
\end{aligned}
$$

- Angular Condition

$$\Delta(q^2) = (1+2\eta)I^+_{11} + I^+_{1-1} - \sqrt{8\eta}I^+_{10} - I^+_{00} = 0$$

- Ref:


- Sov. J. Nucl. Phys. 39 (1984) 198

I.Grach and L.A. Kondratyku

- Phy. Rev. Lett. 62 (1989) 387

L.L. Frankfurt, I.Grach, L.A. Kondratyku and M. Strikman

- Prescriptions : $\begin{cases} FFS \\ GK \\ CCKP \\ BH \end{cases}$ vs COVARIANT
- Breit Frame $\Longrightarrow P^+ = P'^+,\ P^- = P'^-,\ \vec{P'}_\perp = -\vec{P}_\perp = \vec{q}/2$
- B.F: $q^+ = q^0 + q^3 = 0$
- $J^+_\rho$ $\begin{cases} 4\ \textit{Current Elements} \\ 3\ \textit{Form Factors}\ \ G_0\ ,\ G_1\ \textit{and}\ G_2 \end{cases}$
- Angular Condition: Violation

$$q_x \Longrightarrow J^+_{yy} = J^+_{zz} \begin{cases} Parity \\ + \\ Rotations \end{cases}$$

$$\begin{aligned} \Delta(q^2) &= (1+\eta)(J^+_{yy} - J^+_{zz}) \\ &= (1+2\eta)I^+_{11} + I^+_{1-1} - \sqrt{8\eta}I^+_{10} - I^+_{00} \end{aligned}$$

$$\Delta(q^2) \neq 0$$

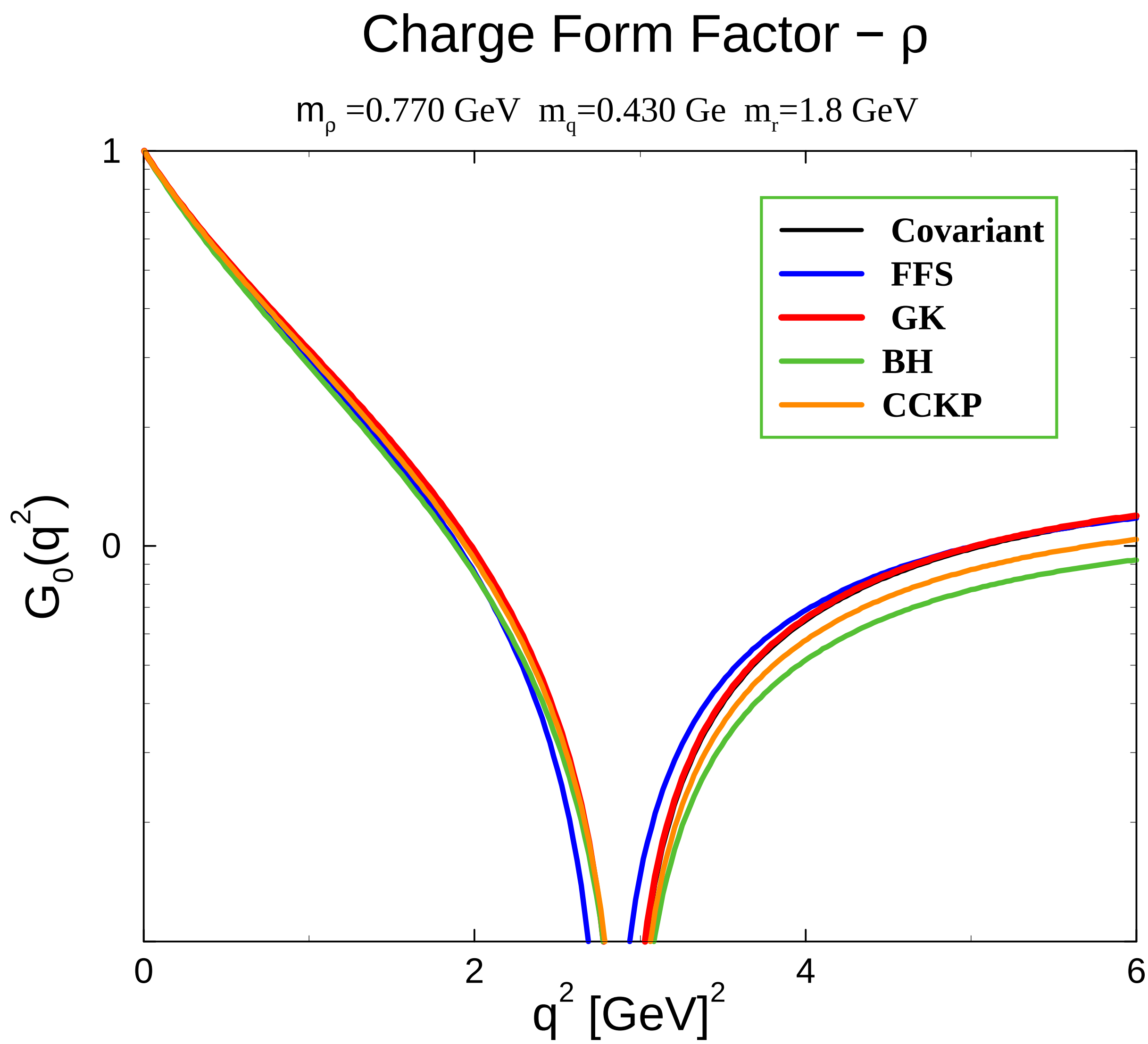


Fig. 3: Rho Meson Charge Form Factor

- GK Sov. J. Nucl. Phys. 39 (1984) 198

I.Grach and L.A. Kondratyku

- CCKP Phy.Rev C37 (1988) 2000

P.L. Chung, F. Coester, B.D. Keister and W.N. Polizou

- FFS Phy.Rev C48 (1993) 2182

L.Frankfurt, T. Frederico and M. Strikman

- BH Phy.Rev D46 (1992) 2141

S.J. Brodsky and J.R. Hiller

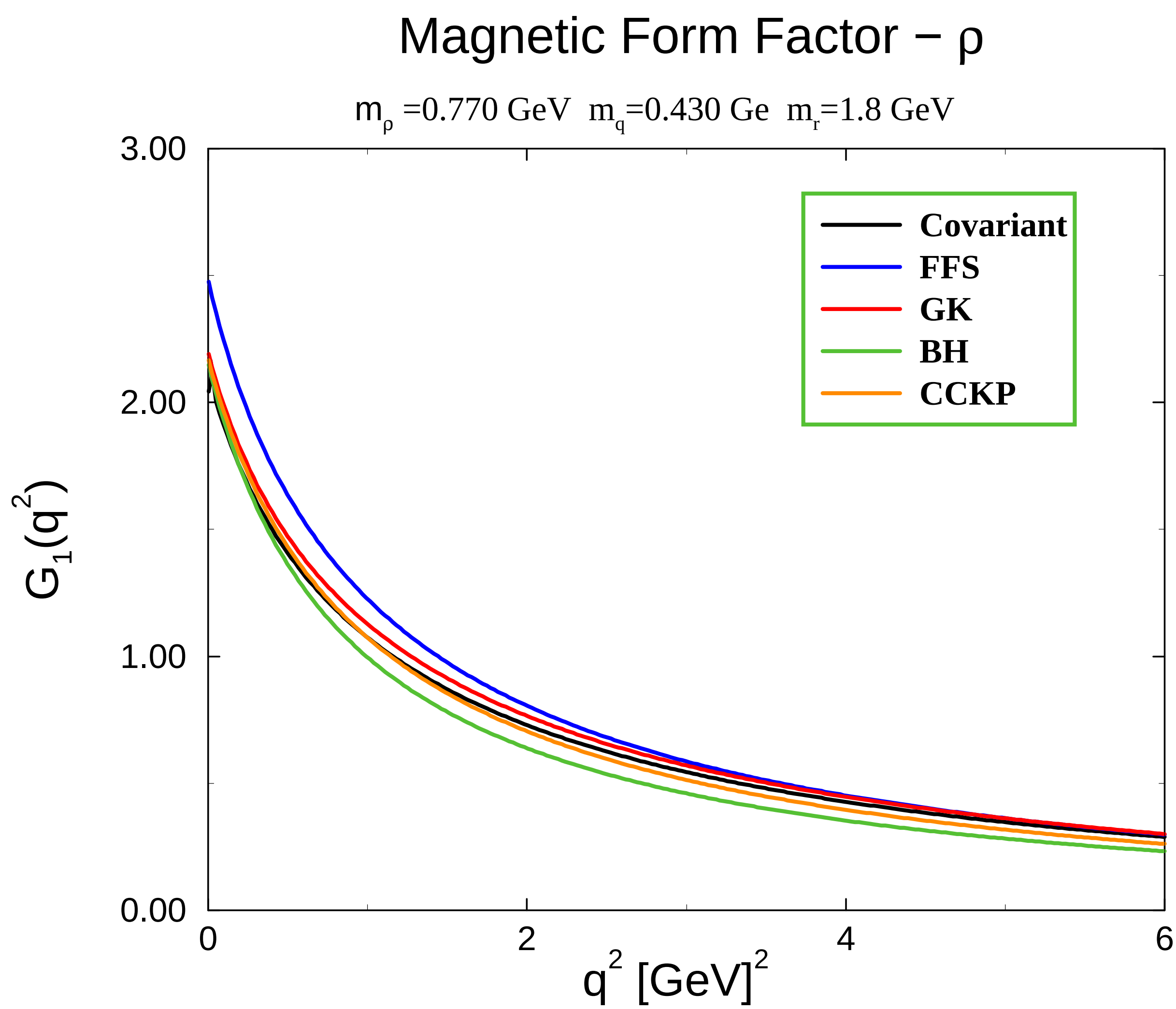


Fig. 4: Rho Meson Magnetic Form Factor

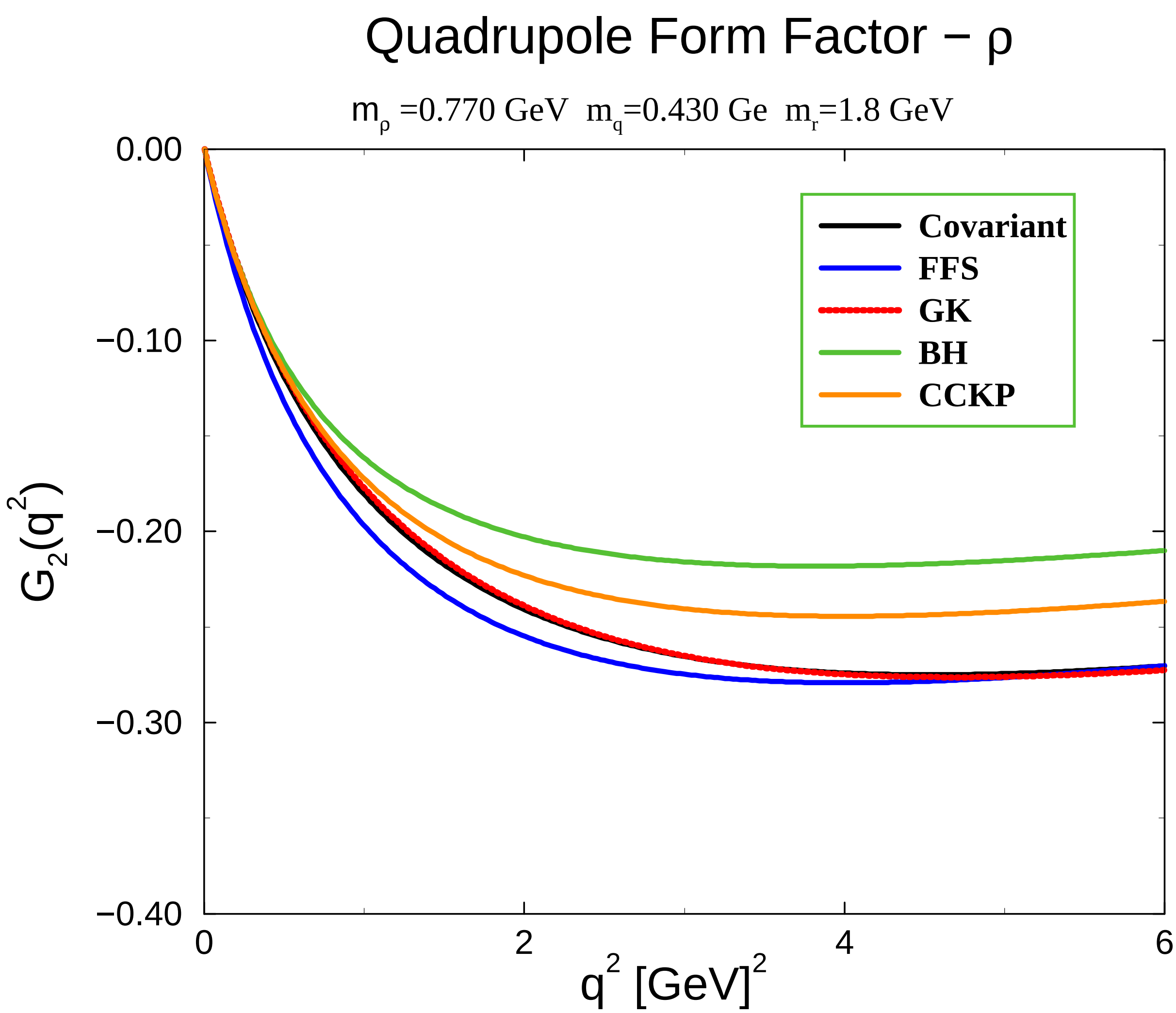


Fig. 5: Rho Meson Quadrupole Form Factor

Table 1: Results for the low-energy electromagnetic $\rho$-meson observables, for the covariant (COV) and light-front calculations. The light-front extraction schemes to obtain the form-factors are given by Refs. (GK), (CCKP) , (FFS) and (BH). In the last column, the results of Ref. [*] are given.

| MODEL | COV | GK | CCKP | BH | FFS | Ref.[*] |
|---|---|---|---|---|---|---|
| $< r^2 > (fm^2)$ | 0.37 | 0.37 | 0.38 | 0.40 | 0.39 | 0.35 |
| $\mu$ | 2.14 | 2.19 | 2.17 | 2.15 | 2.48 | 2.26 |
| $Q_2(fm^2)$ | 0.052 | 0.050 | 0.051 | 0.051 | 0.058 | 0.024 |

$$
\begin{aligned}
< r^2 > &= \lim_{q^2 \to 0} \frac{6(G_0(q^2) - 1)}{q^2} \\
\mu &= \lim_{q^2 \to 0} G_1(q^2) \\
Q_2 &= \lim_{q^2 \to 0} 3\sqrt{2}\frac{G_2(q^2)}{q^2}
\end{aligned}
$$

- Ref.[*]

Phy. Lett. B 349 (1995) 393
F. cardarelli, I.L. Grach, I.M. Narodetskii, E. Pace and G. Salmé

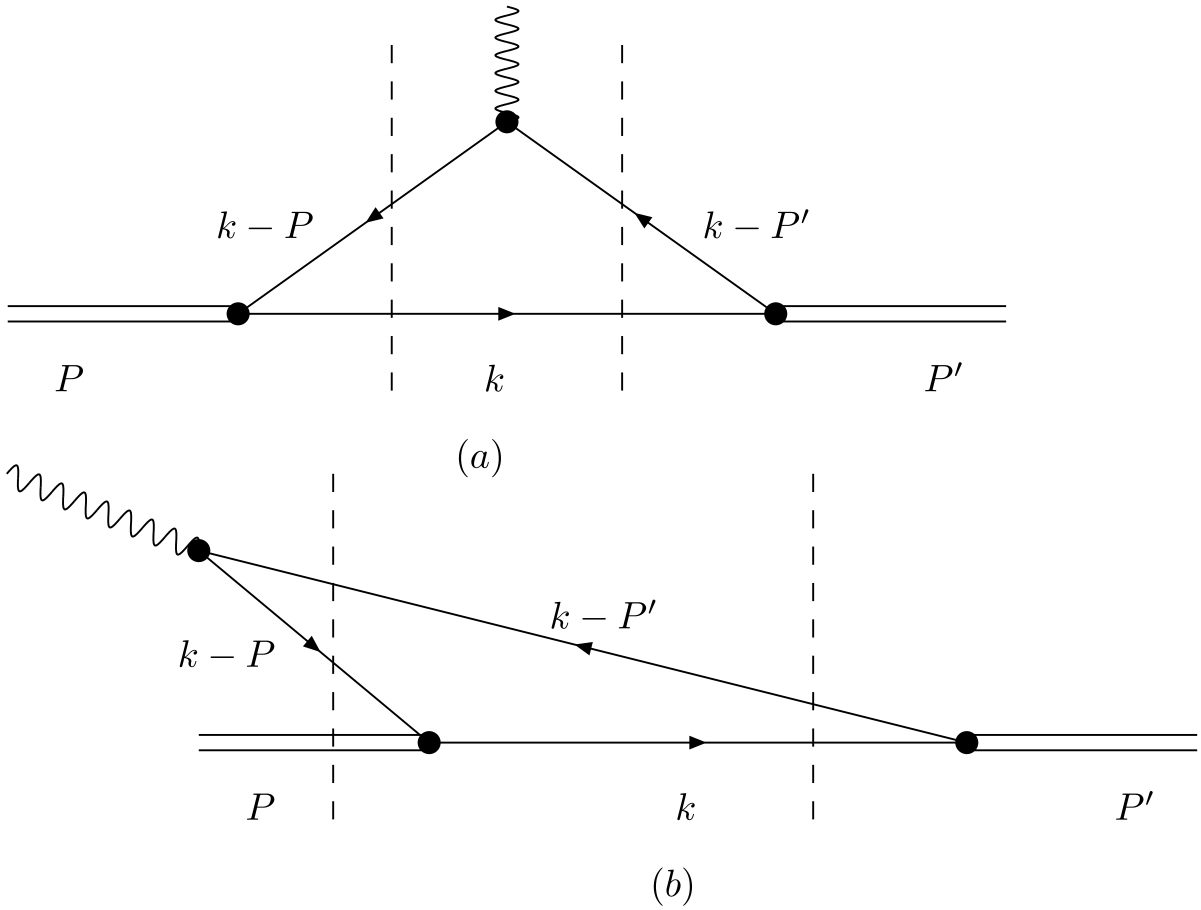


Fig. 6: Light-front time-ordered diagrams for the current: (a) Triangle Diagram and (b) Pair Terms .

# Covariance Restoration in the Light-front

- Vertex $\Gamma(q^\mu, q^\nu)$

- Trace

$$
\begin{aligned}
Tr_{ji} &= Tr[(\not{k} - \not{p}_f + m)\gamma^+(\not{k} - \not{p}_i + m)\frac{\gamma^+}{2}]\; k^- \epsilon'^{\mu}_f (2k^\mu - p'^\mu) \\
&\quad \epsilon^\nu_i (2k^\mu - p^\mu) + \\
&\quad Tr[(\not{k} - \not{p}_f + m)\gamma^+(\not{k} - \not{p}_i + m)(\frac{\gamma^- k^+}{2} - \gamma_\perp k_\perp + m)] \\
&\quad \epsilon'^{\mu}_f (2k^\mu - p'^\mu) \epsilon^\nu_i (2k^\mu - p^\mu)
\end{aligned}
$$

- By Parts:

$$
\begin{aligned}
Tr^A_{ij} &= Tr[(\not{k} - \not{p}_f + m)\gamma^+(\not{k} - \not{p}_i + m)\gamma^+] \\
Tr^B_{iJ} &= Tr[(\not{k} - \not{p}_f + m)\gamma^+(\not{k} - \not{p}_i + m)(\frac{\gamma^- k^+}{2} - \gamma_\perp k_\perp + m)]
\end{aligned}
$$

- Bad Terms (Bad) $\Longrightarrow \;\propto \boldsymbol{k^-}$

$$Tr^{Bad}_{xx} = \frac{k^-3}{2} Tr^A_{ij} + (k^{-2}\eta - k^- q_x \sqrt{\eta}\sqrt{1+\eta}) Tr^B_{ij}$$
$$Tr^{God}_{yy} = (p^+ - k^+)^2$$
$$Tr^{Bad}_{zz} = \frac{k^-3}{2} Tr^A_{ij} + (k^{-2} - k^- k^+) Tr^B_{ij}$$
$$Tr^{Bad}_{zx} = \frac{k^-3\sqrt{\eta}}{2} Tr^A_{ij} + [k^{-2}\sqrt{\eta} - k^- k^+ \sqrt{\eta}(2k_x + \frac{q_x}{2}\sqrt{1+\eta})] Tr^B_{ij}$$

- Integration Interval

i) $0 < k^+ < p^+$

ii) $p^+ < k^+ < p'^+ \longrightarrow \boldsymbol{p'^+ = p^+ + \delta^+}$ (Dislocation Method)

- Poles Contribution

i) $\boldsymbol{k^- = \frac{f_1 + m^2}{k^+}}$

ii) $\boldsymbol{k^- = p^- - \frac{f_3 - \imath\epsilon}{p'^+ - k^+}}$


- XVth Few-Body Confer. Groningem (1997)
Nucl. Phys., A 631 , (1998) 574c

(J.P.B.C. de Melo, J.H.O. Salles,T. Frederico and P.U.Sauer)

- Few-Body Syst., 24, (1998) 99

( H.W. Naus, J.P.B.C. de Melo, T. Frederico and P.U.Sauer)

- Phy. Rev., C59, (1999) 2278

(J.P.B.C. de Melo, H.W. Naus and T. Frederico)

- Definitions - Feynman Propagators

$$
\begin{aligned}
[1] &= k^+(k^- - \frac{f_1 - \imath\epsilon}{k^+}) \\
[2] &= (p^+ - k^+)(p^- - k^- - \frac{f_2 - \imath\epsilon}{p^+ - k^+}) \\
[3] &= (p'^+ - k^+)(p'^- - k^- - \frac{f_3 - \imath\epsilon}{p'^+ - k^+}) \\
[4] &= (p^+ - k^+)(p^- - k^- - \frac{f_4 - \imath\epsilon}{p^+ - k^+}) \\
[5] &= (p'^+ - k^+)(p'^- - k^- - \frac{f_5 - \imath\epsilon}{p'^+ - k^+}) \\
[6] &= (p^+ - k^+)(p^- - k^- - \frac{f_6 - \imath\epsilon}{p'^+ - k^+}) \\
[7] &= (p'^+ - k^+)(p'^- - k^- - \frac{f_7 - \imath\epsilon}{p'^+ - k^+})
\end{aligned}
$$

- Where the functions $f_i$ are given by:

$$
\begin{aligned}
f_1 &= k_\perp^2 + m_q^2 \\
f_2 &= (k-p)_\perp^2 + m_q^2 \\
f_3 &= (k-p')_\perp^2 + m_q^2 \\
f_4 &= (k-p)_\perp^2 + m_R^2 \; . \\
f_5 &= (k-p')_\perp^2 + m_R^2 \\
f_6 &= (k-p)_\perp^2 + m_R^2 \; . \\
f_7 &= (k-p')_\perp^2 + m_R^2
\end{aligned}
$$

- Pair Terms

$$J_{xx}^{+\,(Pair)} = \lim_{\delta^+\to 0}\int d^3k \frac{Tr[J_{xx}^{+\,(Bad)}]}{[1][2][4][5][6][7]}\mathcal{O}_{prop}$$

$$J_{zx}^{+\,(Pair)} = \lim_{\delta^+\to 0}\int d^3k \frac{Tr[J_{zx}^{+\,(Bad)}]}{[1][2][4][5][6][7]}\mathcal{O}_{prop}$$

$$J_{zz}^{+\,(Pair)} = \lim_{\delta^+\to 0}\int d^3k \frac{Tr[J_{zz}^{+\,(Bad)}]}{[1][2][4][5][6][7]}\mathcal{O}_{prop}$$

- $\mathcal{O}_{prop} = \frac{m_\rho^2}{4(p^\mu \;\; k^\mu + m_q \;\; m_\rho)(p`^\mu \;\; k^\mu + m_q \;\; m_\rho)}$

$d^3k \equiv \frac{d^2k_\perp dk^+}{2(2\pi)^3}$

- Limit: $\delta^+ \longrightarrow 0$

$$\lim_{\delta^+\to 0} \int d^3k \frac{Tr_{iJ}^{+A}}{[1][2][4][5][6][7]}\frac{k^{-3}}{2}\mathcal{O}_{prop} \sim \; O(\delta^+2)$$

$$\lim_{\delta^+\to 0} \int d^3k \frac{Tr_{iJ}^{+B}}{[1][2][4][5][6][7]}k^{-2}\mathcal{O}_{prop} \sim \; O(\delta^+)$$

$$\lim_{\delta^+\to 0} \int d^3k \frac{Tr_{iJ}^{+B}}{[1][2][4][5][6][7]}k^{-}\mathcal{O}_{prop} \sim \; O(\delta^+2)$$

- I. Grach and L. Kondratyku Prescription: $I_{00}^{+}$

$$
\begin{aligned}
G_0^{GK\ (Pair)} &= \frac{1}{3}[J_{xx}^{+\ (Bad)} + \eta J_{zz}^{+\ (Bad)}] = \\
&\frac{1}{3}[-k^- q_x(\sqrt{\eta}\sqrt{1+\eta})Tr_{ij}^B + \\
&(-\eta k^- k^+ Tr_{ij}^B)]\ = 0 \\
G_1^{GK\ (Pair)} &= [-J_{zz}^{+\ (Bad)} - \frac{J_{zx}^{+\ (Bad)}}{\sqrt{\eta}}]\ = 0 \\
G_2^{GK\ (Pair)} &= \frac{\sqrt{2}}{3}[J_{xx}^{+\ (Bad)} + \eta J_{zz}^{+\ (Bad)}]\ = 0
\end{aligned}
$$

No Pair Terms Contribution !!!

- Vertex $\Gamma(q^\mu, \gamma^\nu)$
- Trace

$$Tr_{ji} = Tr[(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\not{\epsilon}^\mu(\not{k} + m)]\epsilon_j^\mu(2k_\mu - p'_\mu)$$

- Light-Front Trace

$$Tr_{ji} = Tr[(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\not{\epsilon}_\perp\gamma^+]\frac{k^-}{2}\epsilon_j^\mu(2k_\mu - p'_\mu)$$

- Bad Terms (Bad) / Pair Terms

$$\begin{aligned}
Tr_{xx}^{Bad} &= Tr[(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\gamma^-\gamma^+]\frac{k^-2}{2} \\
Tr_{yy}^{Bad} &= 0 \\
Tr_{zz}^{Bad} &= Tr[(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\gamma^-\gamma^+]\frac{k^-2}{2} \\
Tr_{zz}^{Bad} &= Tr[(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\gamma^-\gamma^+]\frac{k^-2}{2}\sqrt{\eta}
\end{aligned}$$

- By Definition

$$Tr_{ij}^C = Tr[(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\gamma^-\gamma^+]$$

$$\begin{aligned}
Tr_{xx}^{Bad} &= Tr_{ij}^C\ \frac{-k^-2\eta}{2} \\
Tr_{zz}^{Bad} &= Tr_{ij}^C\ \frac{k^-2}{2} \\
Tr_{zz}^{Bad} &= Tr_{ij}^C\ \frac{k^-2\sqrt{\eta}}{2}
\end{aligned}$$

- Pair Terms

- Interval $p^+ < k^+ < p'^+$ where: $p'^+ = p^+ + \delta^+$

$$J^{+\ (Par)}_{xx} = \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J^{+\ Par}_{xx}]}{[1][2][4][5][6][7]} \times \frac{m_\rho}{2(p'^\mu\ k^\mu + m_q\ m_\rho)} \to 0$$

$$J^{+\ (Par)}_{zx} = \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J^{+\ Par}_{zx}]}{[1][2][4][5][6][7]} \times \frac{m_\rho}{2(p'^\mu\ k^\mu + m_q\ m_\rho)} \to 0$$

$$J^{+\ (Par)}_{zz} = \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J^{+\ (Par)}_{zz}]}{[1][2][4][5][6][7]} \times \frac{m_\rho}{2(p'^\mu\ k^\mu + m_q\ m_\rho)} \to 0$$

- Vertex $\Gamma(\gamma^\mu, q^\nu)$

No Pair Terms Contributions !!!

- Vertex $\Gamma(\gamma^\mu, \gamma^\nu)$

$$Tr_{ji} \;=\; Tr[\not\epsilon_f^\alpha(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\not\epsilon_i^\alpha(\not k + m)]$$

- Bad Terms (Bad) $\propto (k^-)$

$$Tr_{ji}^{Bad} = \frac{k^-}{2}\, Tr[\not\epsilon_f^\alpha(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\not\epsilon^\alpha\gamma^+]$$

$$\begin{aligned}
Tr_{xx}^{Bad} &= \frac{-k^-\;\eta}{8}\, Tr[\gamma^-(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\not\epsilon_i^\alpha\gamma^-\gamma^+] \\
Tr_{yy}^{Bad} &= k^-(k^+ - p^+)^2 = 0 \\
Tr_{zz}^{Bad} &= \frac{k^-}{8}\, Tr[\gamma^-(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\not\epsilon_i^\alpha\gamma^-\gamma^+] \\
Tr_{zx}^{Bad} &= \frac{-k^-\sqrt{\eta}}{8}\, Tr[\gamma^-(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\not\epsilon_i^\alpha\gamma^-\gamma^+]
\end{aligned}$$

- Fact: $\rightarrow k^{-(m+1)}\ (p^+ - k^+)^n$

No Pair Terms Contribution if $m < n$

- Simplification: is VIP

$$\begin{aligned}
Tr_{xx}^{Bad} &= -\eta\, Tr_{zz}^{Bad} \\
Tr_{zx}^{Bad} &= -\sqrt{\eta}\, Tr_{zz}^{Bad}
\end{aligned}$$

- Pair Terms

$$J_{xx}^{+\,(Pair)} = \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J_{xx}^{+\,(Bad)}]}{[1][2][4][5][6][7]} \neq 0$$

$$J_{zx}^{+\,(Pair)} = \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J_{zx}^{+(Bad)}]}{[1][2][4][5][6][7]} \neq 0$$

$$J_{zz}^{+\,(Pair)} = \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J_{zz}^{+\,(Bad)}]}{[1][2][4][5][6][7]} \neq 0$$

- Pair Term Contribution !!!

- Grach and Kondratyku : Elimination $I_{00}^{+}$

$$G_0^{GK} = \frac{1}{3}[J_{xx}^{+} + 2J_{yy}^{+} - \eta J_{yy}^{+} + \eta J_{zz}^{+}]$$

$$G_1^{GK} = J_{yy}^{+} - J_{zz}^{+} - \frac{J_{zx}^{+}}{\sqrt{\eta}}$$

$$G_2^{GK} = \frac{\sqrt{2}}{3}[J_{xx}^{+} + J_{yy}^{+}(-1-\eta) + \eta J_{zz}^{+}]$$

- Pair Terms Combination

$$G_0^{GK\ (Pair)} = \frac{1}{3}[J_{xx}^{+\ (Bad)} + \eta J_{zz}^{+\ (Bad)}] = \frac{1}{3}[-\eta J_{zz}^{+\ (Bad)} + \eta J_{zz}^{+\ (Bad)}] = 0$$

$$G_1^{GK\ (Pair)} = -J_{zz}^{+\ (Bad)} - \frac{J_{zx}^{+\ (Bad)}}{\sqrt{\eta}} = -J_{zz}^{+\ (Bad)} + \sqrt{\eta}\frac{J_{zz}^{+\ (Bad)}}{\sqrt{\eta}} = 0$$

$$G_2^{GK\ (Pair)} = \frac{\sqrt{2}}{3}[J_{xx}^{+\ (Bad)} + \eta J_{zz}^{+\ (Bad)}] = \frac{\sqrt{2}}{3}[-\eta J_{zz}^{+\ (Bad)} + \eta J_{zz}^{+\ (Bad)}] = 0$$

- Final Result:

No Pair Terms Contribution!!

**Ref.** De Melo and T. Frederico

Braz. J. Phys. Vol.34, 3A, (2004) 881

B.L.G. Bakker and C.R. Ji

Phy. Rev. D65 (2002) 116001

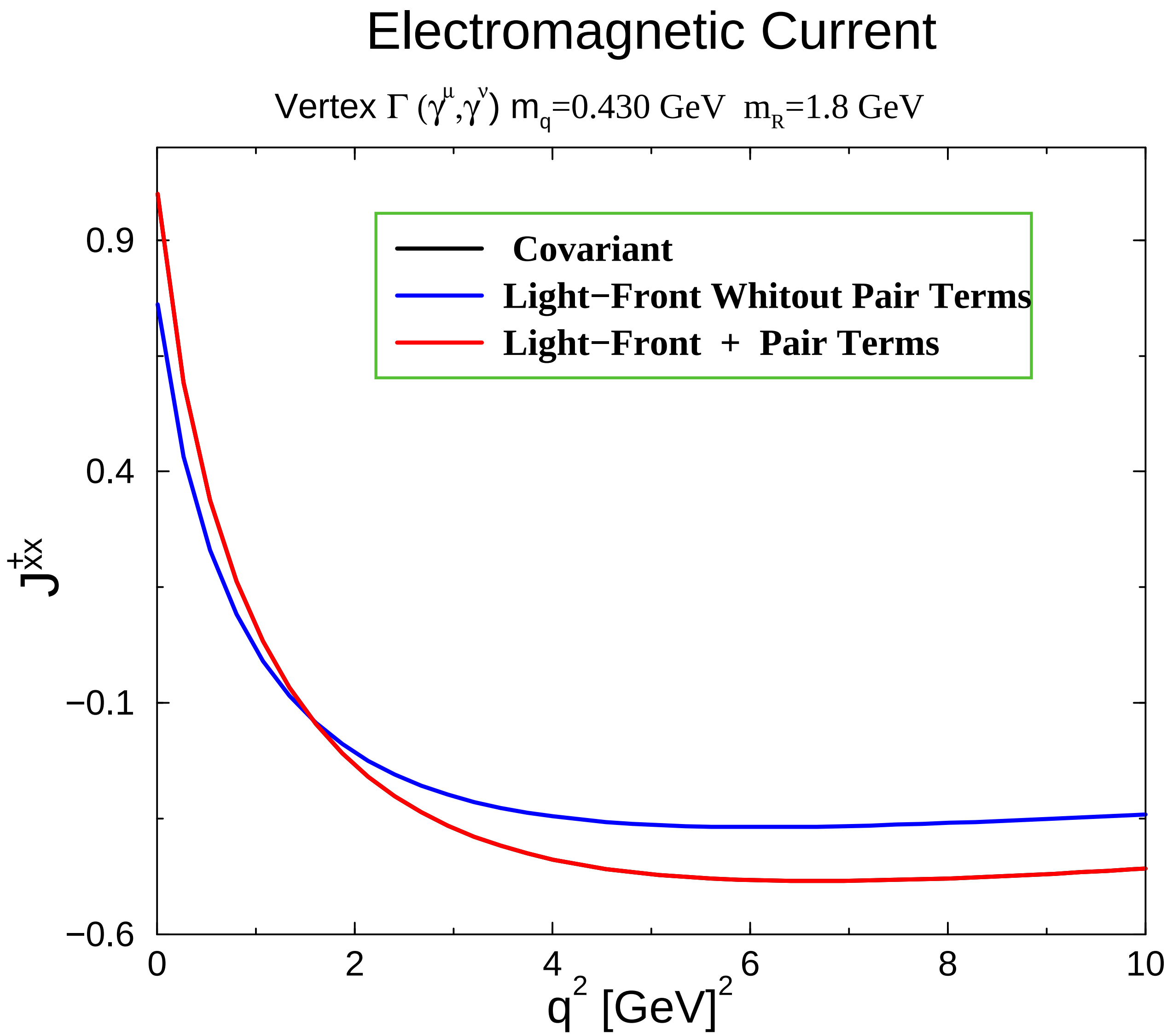


Fig. 7: Spin-1 Electromagetic Current

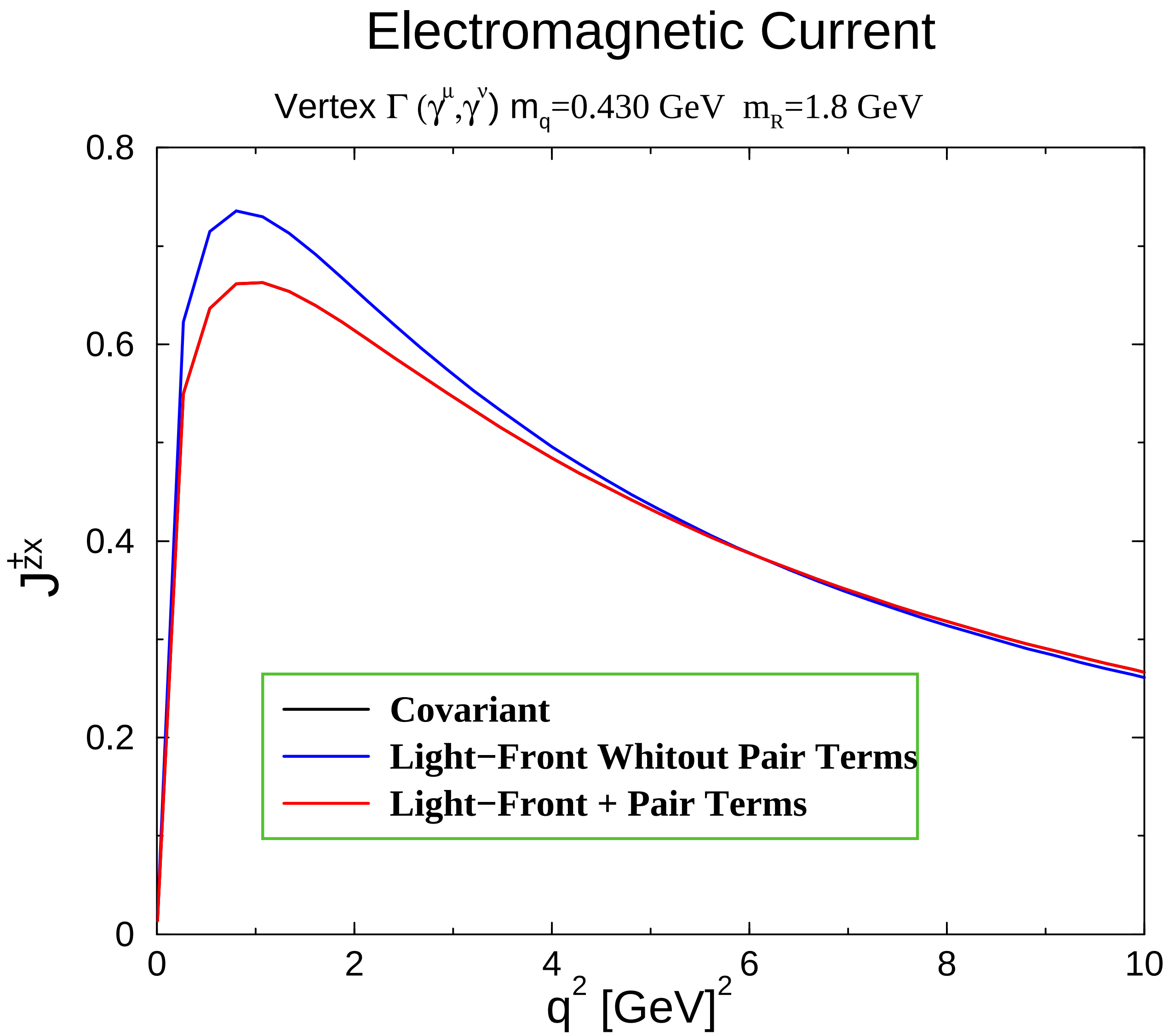


Fig. 8: Spin-1 Electromagetic Current

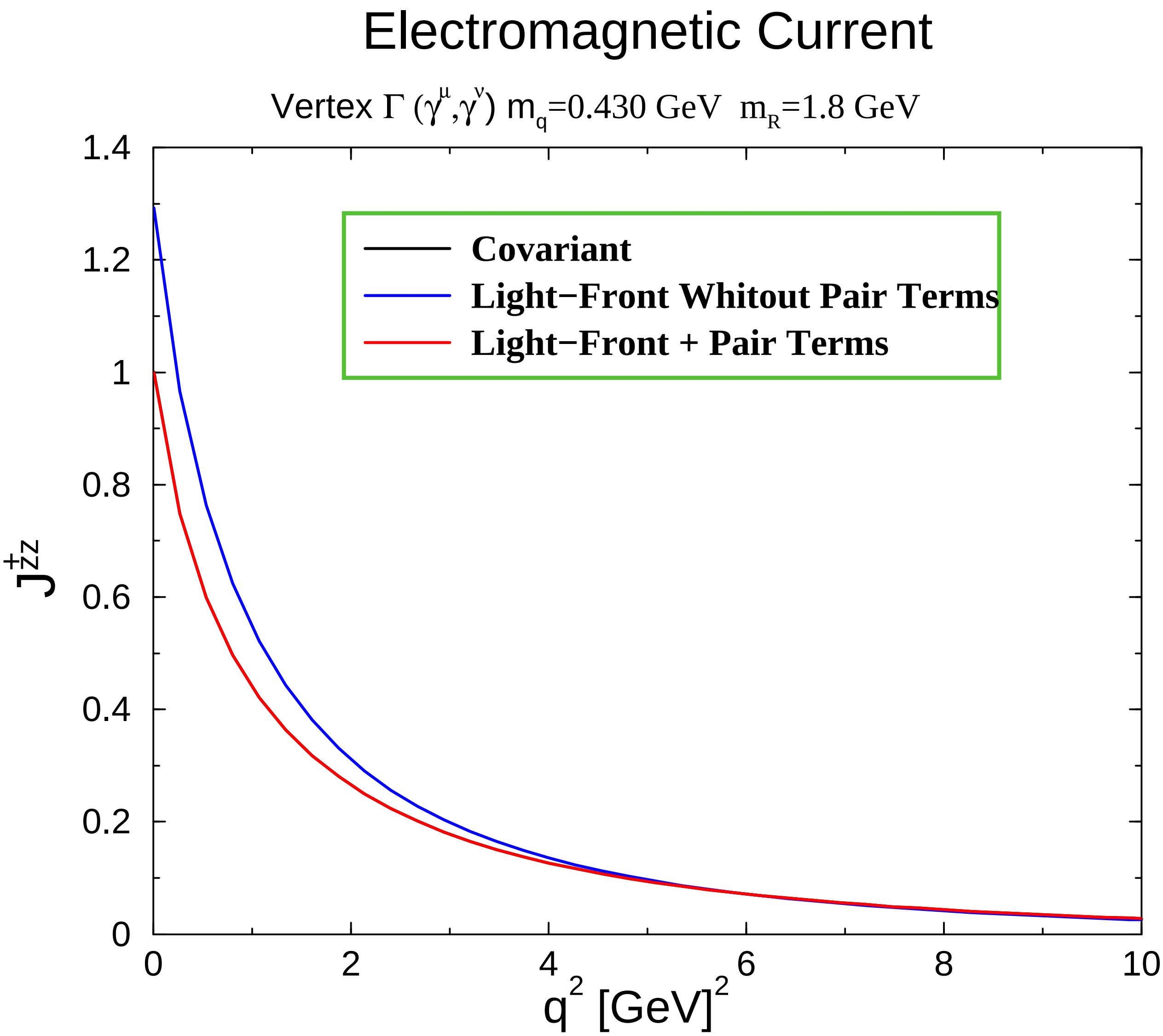


Fig. 9: Spin-1 Electromagetic Current

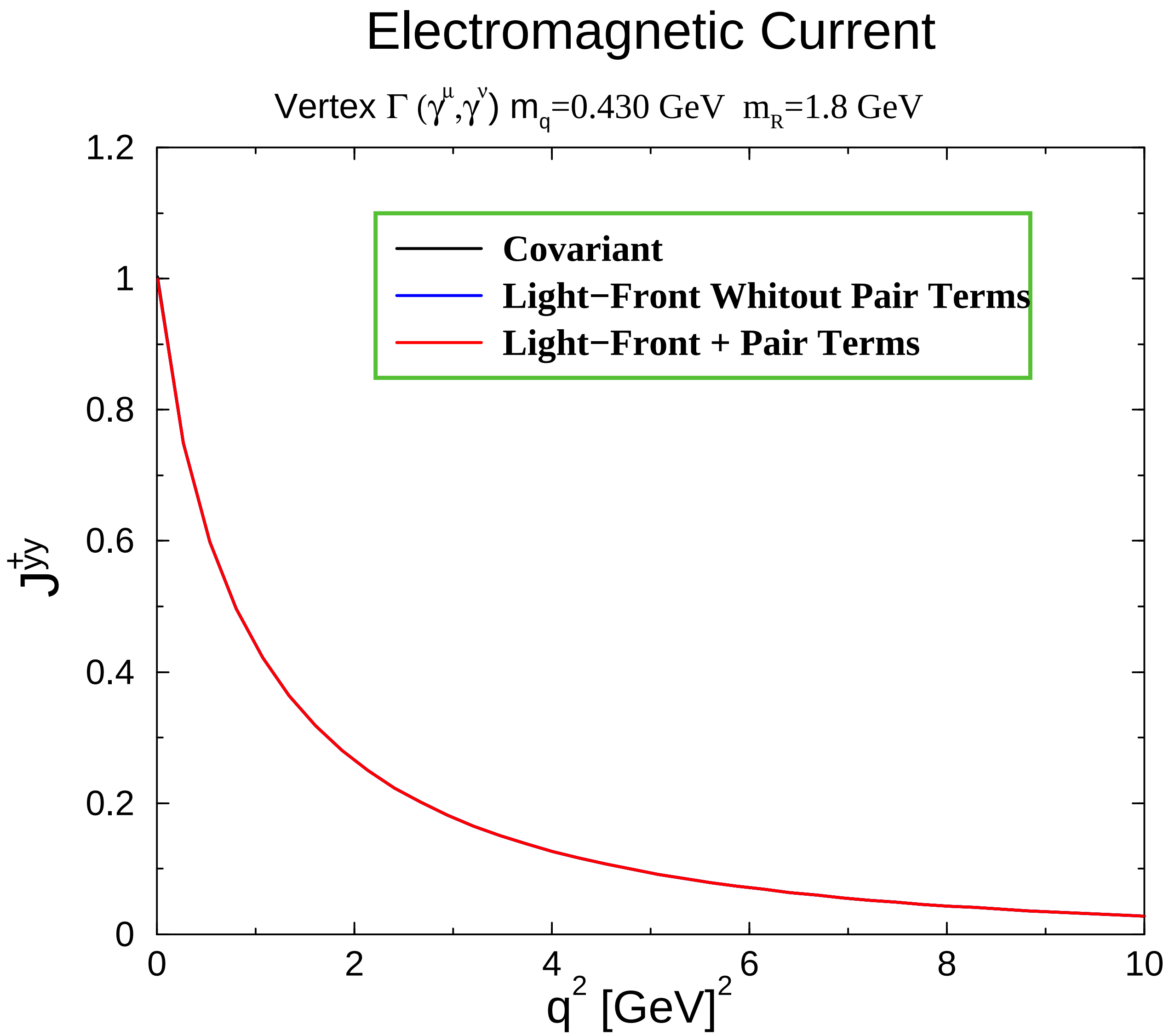


Fig. 10: Spin-1 Electromagetic Current

# Conclusions

- Light-Front $\Longrightarrow \begin{cases} \textit{Bound States} \\ \textit{Covariance} \end{cases}$
- Rotational Invariance Broken $\Longrightarrow k^-$ Problematic
- Terms $\begin{cases} - \textit{Good} \\ - \textit{Bad} \end{cases}$
- Electromagnetic Current: " + " , "- " and " $\perp$ "
- Pair Terms Contribution: $\Longrightarrow J^+$ and $J^-$
- Particles $\begin{cases} - Bosons \\ - Pseudoscalar \\ - Vector \end{cases}$
- Pairs Terms Contribution $\Longrightarrow$ Full Covariance Restorate

$J^+$ is not free of the Pair Terms Contribution !!!