Glue-Glue Scattering Amplitudes on the Light-Cone Worldsheet.

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Ref: D. C, J. Qiu, and C. B. Thorn, hep-th/0507280
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Introduction

• The generic worldsheet representation (proposed by K. Bardakci and C. B. Thorn, Nucl. Phys. **B626** (2002) 287, hep-th/0110301) applies directly only to the bare diagrams of the quantum field theory, and one must apply an explicit UV cutoff to give it concrete meaning.
To handle IR divergences p^+ is discretised.
$\longrightarrow$ break Lorentz invariance
Are the CTs **local**?

• A check of Lorentz invariance at one loop requires the complete evaluation of the amplitudes for a manifestly physical scattering process, e.g. on-shell glue-glue scattering. A complete light-cone gauge calculation of this process is, to our knowledge, unavailable.

♠ In this talk we present the helicity amplitudes with helicity nonconservation. Tree amplitudes for such processes vanish $\rightarrow$ one loop amplitudes are finite in both the infrared and ultraviolet.

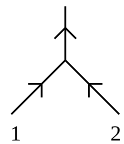
♠ UV regulator: The worldsheet friendly ultraviolet cutoff $\delta > 0$ is implemented by simply inserting a factor $e^{-\delta \sum_i \boldsymbol{q}_i^2}$ in the loop integrand, where $\boldsymbol{q}_i$ is the transverse dual momentum of the region bounded by loop i.

• IR regulator: define the worldsheet path integral on a lattice discretizing the + component of momenta
$p^+ = lm$, $l = 1, 2, \ldots$, where $p^\pm \equiv (p^0 \pm p^z)/\sqrt{2}$.

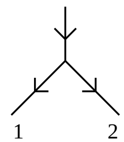
Feynman Rules for Light-cone gauge Yang-Mills

The non-vanishing three transverse gluon vertices:

(All momenta are incoming)



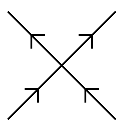
$$= \frac{2gp_3^+}{p_1^+ p_2^+}\left(p_1^+ p_2^\wedge - p_2^+ p_1^\wedge\right) = \frac{2gp_3^+}{p_1^+ p_2^+} K_{12}^\wedge$$



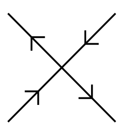
$$= \frac{2gp_3^+}{p_1^+ p_2^+}\left(p_1^+ p_2^\vee - p_2^+ p_1^\vee\right) = \frac{2gp_3^+}{p_1^+ p_2^+} K_{12}^\vee$$

Here, $p_j^\wedge = (p_j^x + ip_j^y)/\sqrt{2}$, $p_j^\vee = (p_j^x - ip_j^y)/\sqrt{2}$, and p_j^+ are the components of the momentum *entering* the diagram on leg j.

The quartic vertices in this helicity basis are given by



$$= -2g^2 \frac{p_1^+ p_3^+ + p_2^+ p_4^+}{(p_1^+ + p_4^+)^2}$$



$$= +2g^2 \left(\frac{p_1^+ p_2^+ + p_3^+ p_4^+}{(p_1^+ + p_4^+)^2} + \frac{p_1^+ p_4^+ + p_2^+ p_3^+}{(p_1^+ + p_2^+)^2} \right)$$

K Identities

$$K_{ij}^{\mu} \equiv p_i^+ p_j^\mu - p_j^+ p_i^\mu$$

By momentum conservation:

$$\sum_j K_{ij}^{\mu} = 0.$$

From the fact that K is an anti-symmetric product we have Bianchi-like identities

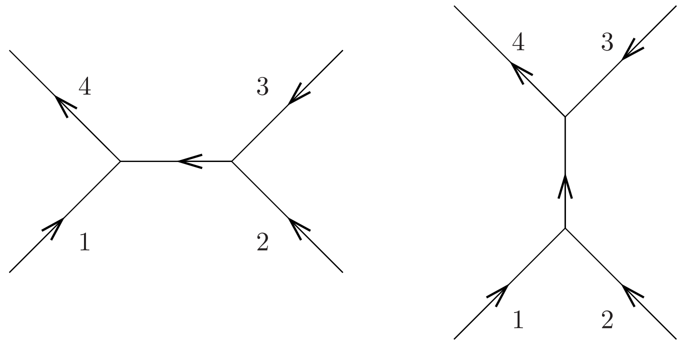
$$\begin{aligned} p_i^+ K_{jk}^{\mu} + p_k^+ K_{ij}^{\mu} + p_j^+ K_{ki}^{\mu} &= 0 \\ K_{li}^{\wedge} K_{jk}^{\wedge} + K_{lk}^{\wedge} K_{ij}^{\wedge} + K_{lj}^{\wedge} K_{ki}^{\wedge} &= 0 \end{aligned}$$

The most powerful type of identity:

$$\begin{aligned} \sum_j \frac{K_{ij}^{\wedge} K_{jk}^{\vee}}{p_j^+} &= -p_i^+ p_k^+ \sum_j \frac{p_j^2}{2p_j^+} \\ &= 0 \quad (\mathrm{on-shell}) \end{aligned}$$

Tree Amplitudes

$\wedge\ \wedge\ \wedge \vee$

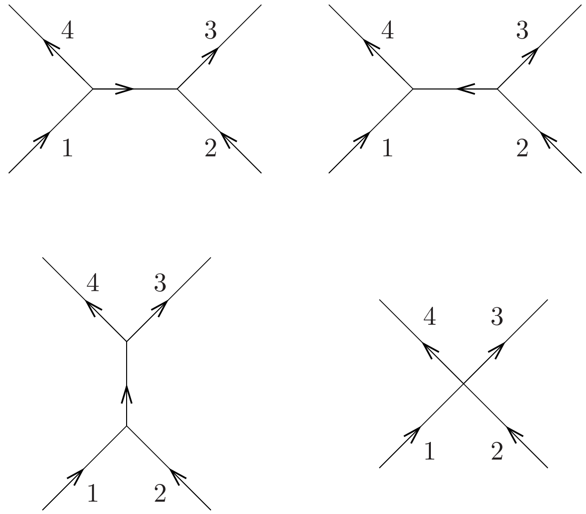


$$A_{tree}^{\wedge\wedge\wedge\vee} \;=\; -\frac{p_4^+}{p_1^+ p_2^+ p_3^+}\left[\frac{K_{32}^{\wedge} K_{14}^{\wedge}}{(p_2+p_3)^2} + \frac{K_{43}^{\wedge} K_{21}^{\wedge}}{(p_1+p_2)^2}\right]$$

With all four gluons off-shell

$$A_{tree}^{\wedge\wedge\wedge\vee} \;=\; -\frac{p_4^+ (K_{43}^{\wedge} K_{32}^{\wedge} p_1^2 + K_{14}^{\wedge} K_{43}^{\wedge} p_2^2 + K_{21}^{\wedge} K_{14}^{\wedge} p_3^2 + K_{32}^{\wedge} K_{21}^{\wedge} p_4^2)}{p_1^+ p_2^+ p_3^+ (p_1+p_2)^2 (p_2+p_3)^2}$$

$\wedge\wedge\vee\vee$

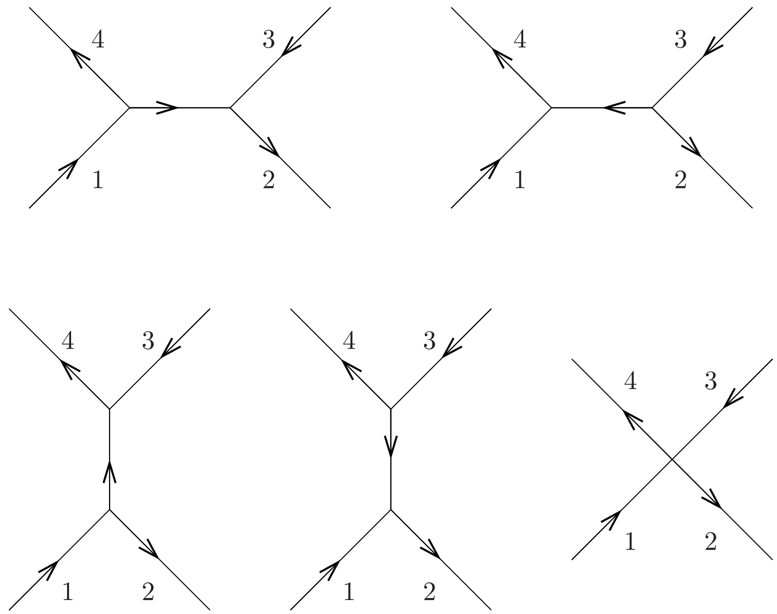


$$A^{\wedge\wedge\vee\vee}_{tree} = -\frac{1}{(p_1^+ + p_4^+)^2}\left[\frac{p_1^+ p_3^+}{p_2^+ p_4^+}\frac{K^{\vee}_{14}K^{\wedge}_{32}}{(p_1+p_4)^2} + \frac{p_2^+ p_4^+}{p_1^+ p_3^+}\frac{K^{\wedge}_{14}K^{\vee}_{32}}{(p_1+p_4)^2} + \frac{p_1^+ p_3^+ + p_2^+ p_4^+}{2}\right] - \frac{(p_1^+ + p_2^+)^2 K^{\wedge}_{21}K^{\vee}_{43}}{p_1^+ p_2^+ p_3^+ p_4^+ (p_1+p_2)^2}$$

$$A^{\wedge\wedge\vee\vee}_{tree} = \frac{p_3^+ p_4^+ K^{\wedge 4}_{12}}{2p_1^+ p_2^+ K^{\wedge}_{12}K^{\wedge}_{23}K^{\wedge}_{34}K^{\wedge}_{41}} \qquad \text{(On Shell)}$$

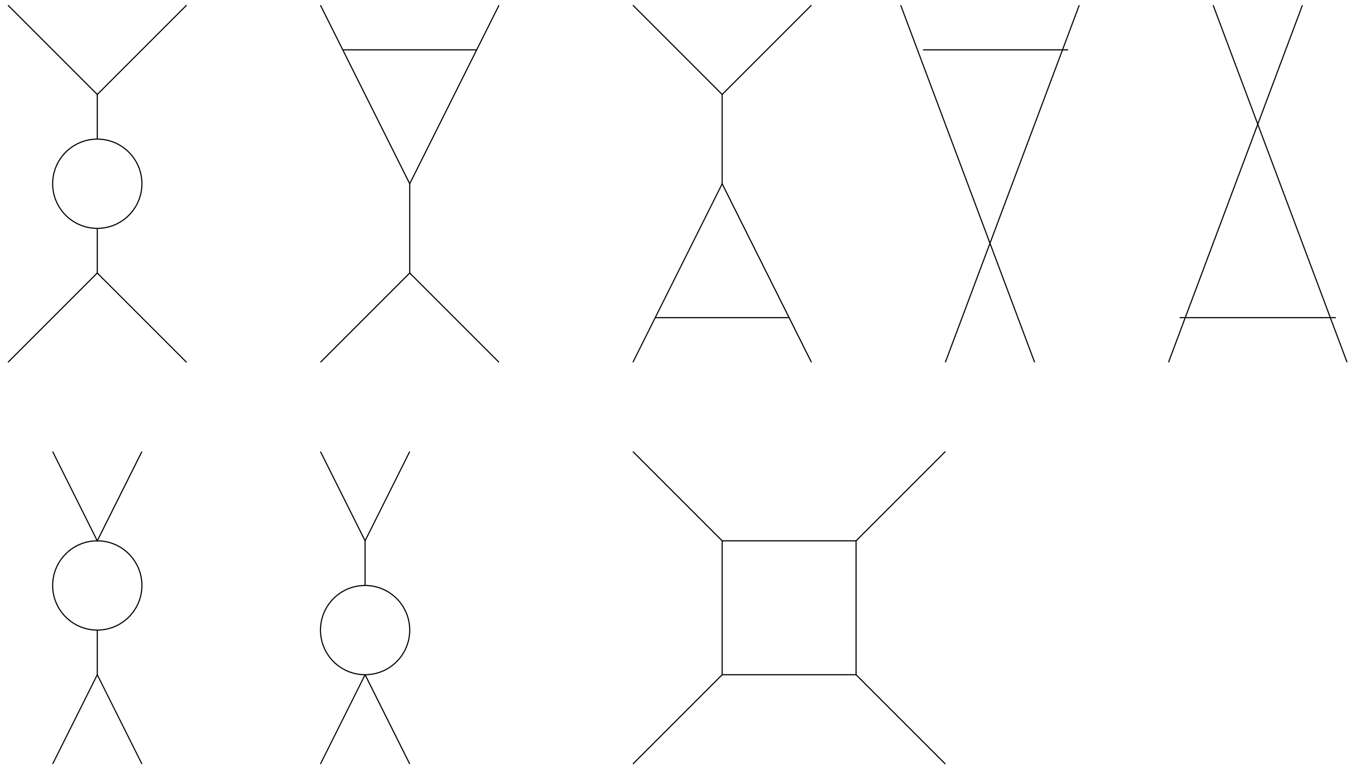
which is essentially the **Parke-Taylor**(PRL 56 (1986), 2459) form of the answer.

$\wedge\vee\wedge\vee$



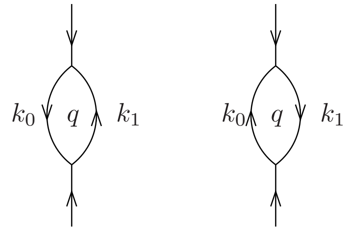
$$A_{tree}^{\wedge\vee\wedge\vee} = \frac{p_2^+ p_4^+ K_{13}^{\wedge 4}}{2 p_1^+ p_3^+ K_{12}^{\wedge} K_{23}^{\wedge} K_{34}^{\wedge} K_{41}^{\wedge}} \qquad (\mathrm{Parke-Taylor})$$

1 loop digrams for Glue-Glue Scattering



Gluon Self-Energy

like helicity component $\Pi^{\wedge\wedge}$:



$$\Pi^{\wedge\wedge} = \frac{g^2 N_c}{12\pi^2}[k_0^{\wedge 2} + k_1^{\wedge 2} + k_0^{\wedge} k_1^{\wedge}]$$

Lorentz invariance would imply that $\Pi^{\wedge\wedge} = 0$ on-shell.
We therefore must introduce a counterterm that exactly cancels this result:

$$\Pi^{\wedge\wedge}_{\mathrm{TOT}} \equiv \Pi^{\wedge\wedge} + \Pi^{\wedge\wedge}_{\mathrm{C.T.}} = 0.$$

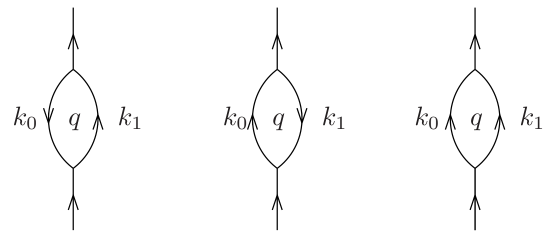
worldsheet description of the CT:

The dual momenta $\boldsymbol{k}_0, \boldsymbol{k}_1$ are the boundary values of the worldsheet field $\boldsymbol{q}(\sigma, \tau)$. We can write the contribution of the counterterm to the worldsheet path integral as

$$\begin{aligned} \frac{-T}{2p^+}\Pi^{\wedge\wedge}_{\mathrm{C.T.}} &= \frac{T}{2p^+}\frac{g^2 N_c}{12\pi^2}\left[\frac{3}{2}k_0^{\wedge 2} + \frac{3}{2}k_1^{\wedge 2} - \frac{1}{2}p^{\wedge 2}\right] \\ &= \frac{g^2 N_c}{16\pi^2}\int d\tau \frac{q^{\wedge 2}(0) + q^{\wedge 2}(p^+)}{p^+} - \frac{g^2 N_c}{48\pi^2}\int d\tau d\sigma \left(\frac{\partial q^{\wedge}}{\partial \sigma}\right)^2 \end{aligned}$$

The first term is a boundary term and the second is a bulk term.

Helicity conserving contributions $\Pi^{\wedge\vee}$



$$\begin{aligned}
\Pi^{\wedge\vee} &= \Pi^{\vee\wedge} = \\
&= -\frac{g^2 N_c}{4\pi^2} p^2 \sum_{q^+} \frac{1}{p^+} \left(\frac{q^+(p^+ - q^+)}{p^{+2}} + \frac{p^+ - q^+}{q^+} + \frac{q^+}{p^+ - q^+} \right) I(H\delta) \\
&\quad + \frac{g^2 N_c}{4\pi^2} \frac{1}{\delta} \sum_{q^+} \frac{1}{p^+} \left(1 + \frac{p^{+2}}{q^{+2}} + \frac{p^{+2}}{(p^+ - q^+)^2} \right) \\
H &\equiv x(1-x)p^2, \qquad x = 1 - \frac{q^+}{p^+} \\
I(H\delta) &\equiv \int_0^\infty \frac{e^{-H\delta u - \delta u (x\boldsymbol{k}_0 + (1-x)\boldsymbol{k}_1)^2/(1+u)} du}{1+u} \quad \widetilde{\delta \to 0} \quad -\gamma - \ln\{H\delta\}
\end{aligned}$$

The q^+ sums diverge when q^+ becomes continuous.

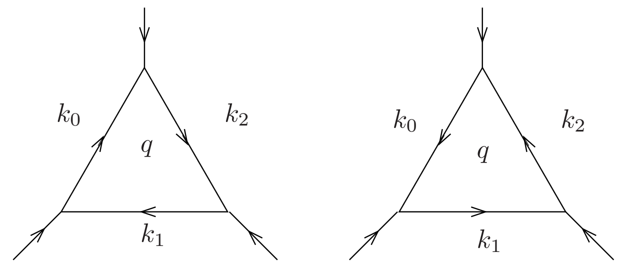
- spurious artifacts of the light-cone gauge. (nothing to do with the usual UV divergences) They must cancel in physical quantities without invoking renormalization or counterterms.

- The quadratic divergence $(1/\delta)$ is cancelled by readjusting the boundary energy and adding a gluon mass counter term.

Cubic Vertex Function

we present the final answers for the vertex corrections with two on-shell gluons.

1) triangle diagram with three like-helicities: $\Gamma_{\triangle}^{\wedge\wedge\wedge}$



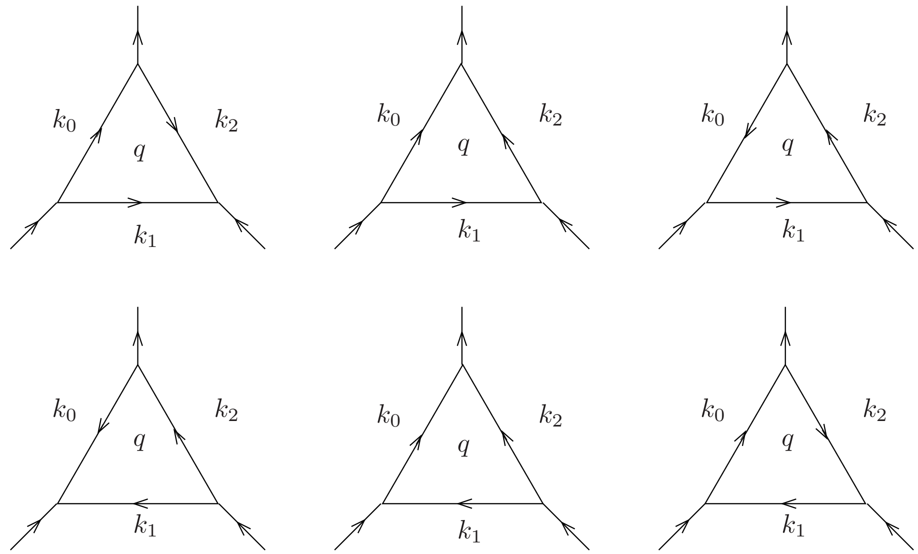
$$\Gamma_{\triangle}^{\wedge\wedge\wedge} = -\frac{(g\sqrt{N_c})^3}{6\pi^2}\frac{K^{\wedge 3}}{p_1^+ p_2^+ p_3^+ p_o^2}$$

$$\Gamma_{\triangle}^{\vee\vee\vee} = -\frac{(g\sqrt{N_c})^3}{6\pi^2}\frac{K^{\vee 3}}{p_1^+ p_2^+ p_3^+ p_o^2}$$

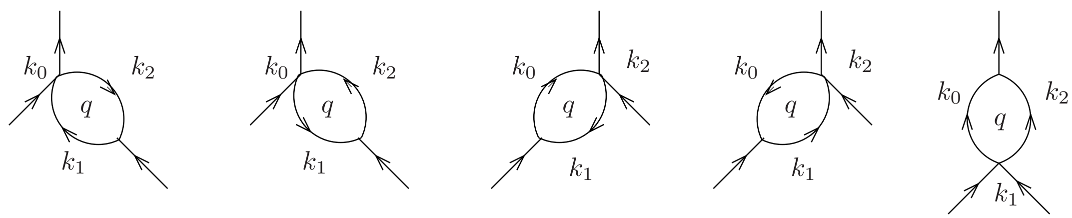
where p_o is the momentum of the off-shell gluon.

2) diagrams with two like-helicities: $\Gamma_{\triangle}^{\wedge\wedge\vee}$

Triangle diagrams:



Swordfish diagrams:



- Linear divergence in q^+ in both triangle and swordfish diagrams.
- when triangles and swordfishes are combined together, linear divergences cancel out leaving only the logrithmic div!

Combination of swordfish and triangle diagrams with two like-helicities and two legs on-shell gives

$$\Gamma_{1\ \mathrm{loop}} = -\frac{(g\sqrt{N_c})^3}{12\pi^2}\sum_i k_i - \frac{g^2 N_c}{8\pi^2}\Gamma_{\mathrm{tree}}\left(\frac{70}{9} - \frac{11}{3}\ln(\delta p_o^2 e^{\gamma}) + S\right)$$
$$+\alpha\frac{(g\sqrt{N_c})^3}{12\pi^2}\frac{K}{p_o^+}$$

the vectors k_i, K carry the polarization of the two like-helicity gluons

p_o is the four-momentum of the off-shell gluon

$\alpha = 1$ when the on-shell gluons have like-helicity

$\alpha = 0$ otherwise.

S is an infrared sensitive term, depends on the location of the off-shell gluon, but not on any of the gluon helicities.

The first term on the right of $\Gamma_{1\ \text{loop}}$ violates Lorentz invariance => must be canceled by a counterterm.

The required counterterm:

$$\Gamma^{\wedge\wedge\vee}_{\text{C.T.}} = +\frac{(g\sqrt{N_c})^3}{12\pi^2}(k_0^{\wedge} + k_1^{\wedge} + k_2^{\wedge})$$

(symmetric in the three legs)

Reduction of Box Diagrams

♣ Triangle diagrams easier to compute than box diagrams.

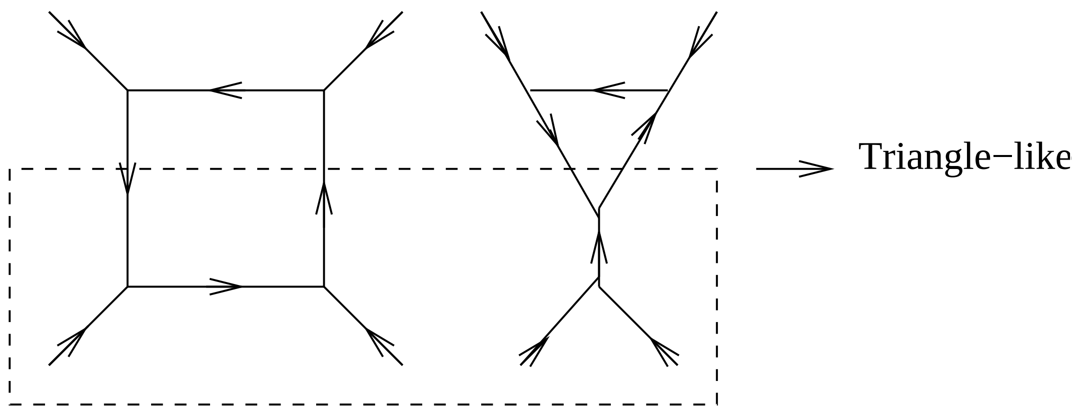
We use the off-shell expressions for tree amplitudes to reduce the box diagrams into triangle like diagrams.

• on-shell tree amplitudes: zero/simple.

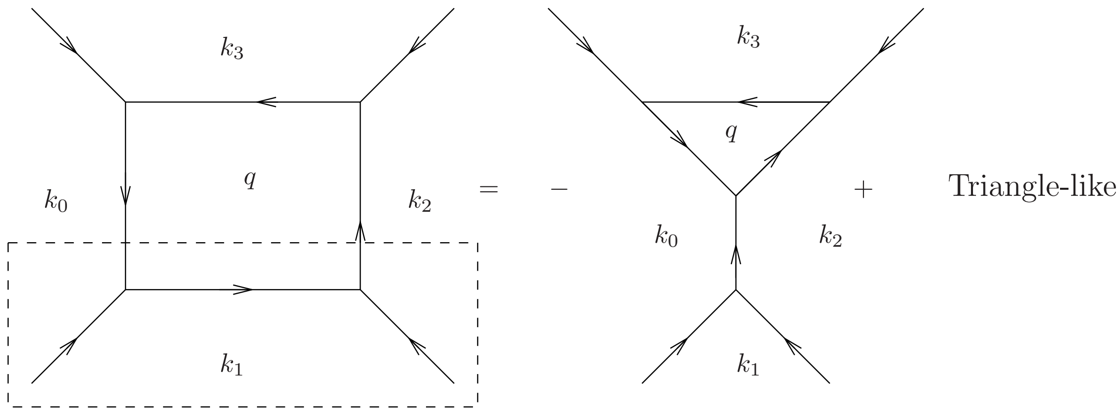
• off-shell tree amplitudes: on-shell + terms having atleast one factor of the virtuality p^2 of one of the off-shell legs

$$A_{tree}^{\wedge\wedge\wedge\vee} = -\frac{p_4^+(K_{43}^\wedge K_{32}^\wedge p_1^2 + K_{14}^\wedge K_{43}^\wedge p_2^2 + K_{21}^\wedge K_{14}^\wedge p_3^2 + K_{32}^\wedge K_{21}^\wedge p_4^2)}{p_1^+ p_2^+ p_3^+ (p_1+p_2)^2 (p_2+p_3)^2}$$

→ cancels a propagator reducing the box integral to a triangle like integral.


Triangle−like

$\wedge\wedge\wedge\wedge$ at one loop:



- The cubic vertex correction comes only due to the triangle diagrams with all legs having helicity $\wedge$, i.e., $\Gamma_\Delta^{\wedge\wedge\wedge}$.
- With the counter term included the self energy $\Pi^{\wedge\wedge} = 0$.

=> The physical one loop scattering amplitude contains no self energy insertions and is

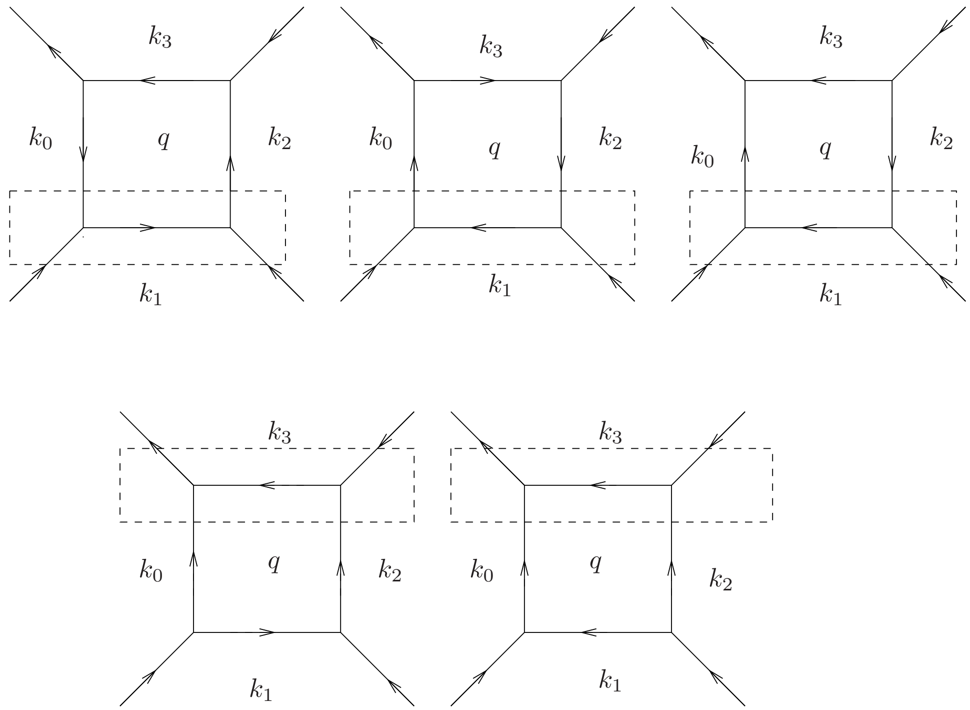
$$\Gamma^{\wedge\wedge\wedge\wedge} = \frac{g^4 N_c^2}{3\pi^2}\frac{1}{p_1^+ p_2^+ p_3^+ p_4^+}\left[\frac{K_{34}^\wedge {K_{12}^\wedge}^3}{p_{12}^4} + \frac{K_{23}^\wedge {K_{41}^\wedge}^3}{p_{14}^4} + \frac{K_{41}^\wedge {K_{23}^\wedge}^3}{p_{14}^4} + \frac{K_{12}^\wedge K_{41}^\wedge K_{23}^\wedge (K_{41}^\wedge + K_{23}^\wedge) + K_{12}^\wedge K_{34}^\wedge ({K_{41}^\wedge}^2 + {K_{23}^\wedge}^2)}{p_{12}^2 p_{14}^2}\right]$$

$$= -\frac{g_s^4 N_c^2}{48\pi^2}\frac{K_{21}^\wedge K_{43}^\wedge}{K_{21}^\vee K_{43}^\vee}$$

where $g_s = g\sqrt{2}$ is the conventional QCD coupling constant.

$\wedge \wedge \wedge \vee$ at one loop:

Box reduction

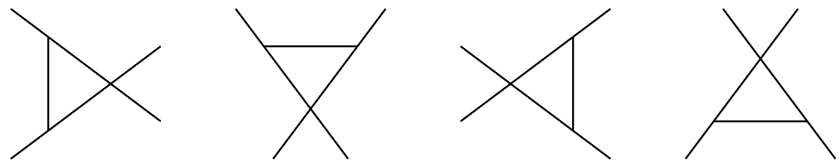


In the loop integrand, there are four distinct triangle denominator structures descended from the box denominator $(q_0^2 q_1^2 q_2^2 q_3^2)^{-1}$:
$(q_0^2 q_1^2 q_3^2)^{-1}$ $(q_1^2 q_2^2 q_3^2)^{-1}$ $(q_0^2 q_1^2 q_2^2)^{-1}$ and $(q_0^2 q_2^2 q_3^2)^{-1}$.
$(q_i = q - k_i)$

Divide the whole worldsheet into three patches $0 \leq q^+ \leq -p_4^+$, $-p_4^+ \leq q^+ \leq p_1^+$ and $p_1^+ \leq q^+ \leq p_{12}^+$. (Assuming $p_1^+,\ p_2^+ > 0$ and $p_1^+ + p_4^+ > 0$).

Due to kinematic constraints on q^+, different diagrams live in different regions on the worldsheet.

Linear divergences in q^+ are cancelled when combined with quartic triangles in each region of the worldsheet.



All the triangle-like and quartic triangle terms combine nicely to produce

$$\Gamma_{TL}^{\wedge\wedge\wedge\vee} = \frac{(2g)^4 N_c^2}{32\pi^2}\{B_0 + \frac{p_4^+}{p_1^+ p_2^+ p_3^+}\frac{K_{12}^\wedge K_{34}^\wedge}{p_{12}^2}[\frac{11}{3}\ln(\delta e^\gamma p_{12}^2) - \frac{11}{3}\ln(\delta e^\gamma p_{14}^2) - S^{q^+}(p_i^+)]\}.$$

$S^{q^+}(p_i^+)$ contains the terms with (IR sesitive) logarithms.

All the log terms coming from the box are exactly cancelled by the similar terms in cubic vertex correction and self-energy correction terms!

Terms left over are finite in both UV and IR.

The physical scattering amplitude

$$\Gamma^{\wedge\wedge\wedge\vee} = \Gamma_{TL}^{\wedge\wedge\wedge\vee} + \Gamma_{\Delta}^{\wedge\wedge\wedge\vee} + \Gamma_{SE}^{\wedge\wedge\wedge\vee}$$

$$\mathbf{\Gamma^{\wedge\wedge\wedge\vee} = -\frac{g_s^4 N_c^2}{96\pi^2}\frac{p_2^+ p_4^+ K_{13}^{\wedge 2}}{K_{43}^\wedge K_{32}^\vee K_{21}^\vee K_{14}^\wedge}(p_{12}^2 + p_{14}^2)}$$

Summary and Conclusions

- ✌ We have shown that in the one loop level gauge theory on the lightcone worldsheet survives the renormalization procedure.
  - ♣ We have enumerate the counter terms and
  - ♣ shown that they can be described locally on the worldsheet.
- ✌ In the way to do so we have presented a novel technique for calculation of scattering amplitudes in gauge theory.
- ✌ The on-shell glue-glue scattering in the tree level on the lightcone worldsheet reproduces *Parke-Taylor* amplitudes.

  One loop amplitudes for helicity violating scatterings agree with known results.