THE BOX DIAGRAM IN YUKAWA THEORY

Ben Bakker

Department of Physics and Astronomy
Vrije Universiteit, Amsterdam, The Netherlands

I Motivation

II Yukawa model

III LF perturbation theory

IV Results

V Conclusions

This work was done in collaboration with Cheung Ji (NC State, Raleigh) and Jorn Boomsma (VU, Amsterdam).

I Motivation

Light-front perturbation theory shows *singularities* not present in the manifestly covariant approach.

Examples

1. Instantaneous propagators
   Tamed by the *blink* mechanism.
   Norbert Ligterink et al., Phys. Rev. D **52**, 5954 (1995)

2. Arc contributions
   Analyzed and tamed.
   B.L.G. Bakker, M.A. DeWitt, C.-R. Ji, and Yu. Mishchenko,
   Phys. Rev. D **72**, 076005 (2005)

3. Regularized amplitudes $A_{\rm cov}$ and $A_{\rm LF}$ may not be identical.
   Analyzed and found not physically relevant
   in the Standard Model.
   B.L.G. Bakker and C.-R. Ji, Phys. Rev. D **71**, 053005 (2005)

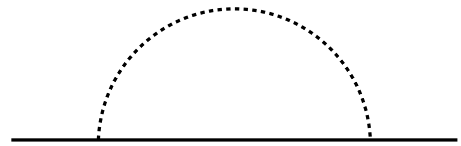
4. Covariant amplitude finite, light-front contributions diverge.
   Analyzed for Yukawa model, equivalence not checked
   Miranda van Iersel, Thesis (2004)

II Yukawa model

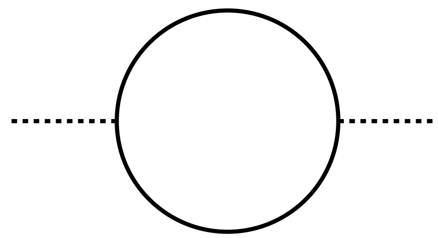
The model is the spin-0 – spin-1/2 Yukawa model.

Primitive divergent diagrams

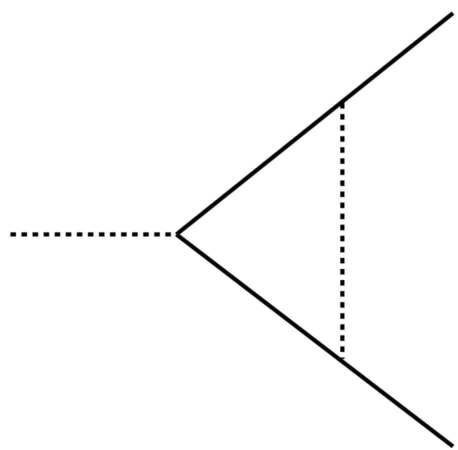
Self energy



Vacuum polarization



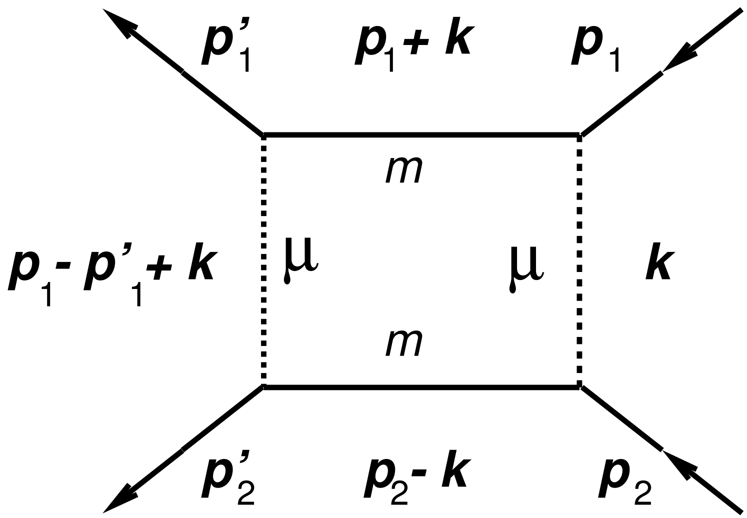
Vertex correction



The box diagram is finite.

Calculation of the covariant box

J. Boomsma, C.-R. Ji, and BLGB, 2006



The covariant box diagram with momenta and masses defined.

The scattering amplitude $\mathcal{T}$ is connected to the invariant amplitude $\mathcal{M}$ as follows

$$\mathcal{T} = \bar{u}(p_1', s_1')\bar{u}(p_2', s_2')\mathcal{M}u(p_1, s_1)u(p_2, s_2). \tag{1}$$

Following the usual procedure–Wick rotation, Feynman parameters, shift–one finds the expression for $\mathcal{M}$

$$\mathcal{M} = N\, 6 \int_T d\alpha_1 \dots d\alpha_4 \int \frac{d^4k}{(2\pi)^4} \tag{2}$$

$$\frac{[\gamma(1)\cdot(p_1+k)+m]\;[\gamma(2)\cdot(p_2-k)+m]}{\{\alpha_1(k^2-\mu^2)+\alpha_2[(k-p_2)^2-m^2]+\alpha_3[(k+p_1-p_1')^2-\mu^2]+\alpha_4[(k+p_1)^2-m^2]\}^4}.$$

The *on-shell* scattering amplitude can be expanded in terms of *four form factors* F_i multiplying the spin matrix.

$$\begin{aligned}
\mathcal{M} &= O_1F_1 + O_2F_2 + O_3F_3 + O_4F_4, \\
O_1 &= I(1)\otimes I(2), \\
O_2 &= [I(1)\otimes(p_1\cdot\Gamma(2)) + (p_2\cdot\Gamma(1))\otimes I(2)], \\
O_3 &= (p_2\cdot\Gamma(1))\otimes(p_1\cdot\Gamma(2)), \\
O_4 &= \Gamma(1)\cdot\Gamma(2),
\end{aligned} \tag{3}$$

The form factors can be expressed in terms of integrals over the Feynman parameters.

$$
\begin{aligned}
F_1 &= 2mM\left(2D_{\alpha_1} + D_{\alpha_2}\right) + M^2\left(2D_{\alpha_1} + D_{\alpha_2\alpha_4}\right) + 2m^2\left(D_{\alpha_1} + D_{\alpha_2}\right), \\
F_2 &= mD_{\alpha_2} + M\left(2D_{\alpha_1\alpha_2} + D_{\alpha_2\alpha_4}\right), \\
F_3 &= D_{\alpha_2\alpha_4}, \\
F_4 &= -D_2/4.
\end{aligned} \tag{4}
$$

These integrals are

$$
\begin{aligned}
D_{\alpha_i} &= \frac{i}{(4\pi)^2}\frac{1}{6}\int_T d\alpha \frac{\alpha_i}{M_{\rm cov}^4}, \\
D_{\alpha_i\alpha_j} &= \frac{i}{(4\pi)^2}\frac{1}{6}\int_T d\alpha \frac{\alpha_i\alpha_j}{M_{\rm cov}^4}, \\
D_2 &= \frac{-i}{(4\pi)^2}\frac{1}{3}\int_T d\alpha \frac{1}{M_{\rm cov}^2}.
\end{aligned} \tag{5}
$$

The spin-matrix elements $\mathcal{T}$ have the following symmetry

$$
\mathcal{T} = \begin{pmatrix} \mathcal{T}_{11} & \mathcal{T}_{12} & \mathcal{T}_{13} & \mathcal{T}_{14} \\ \mathcal{T}_{21} & \mathcal{T}_{22} & \mathcal{T}_{23} & \mathcal{T}_{24} \\ -\mathcal{T}_{24}^* & \mathcal{T}_{23}^* & \mathcal{T}_{22}^* & -\mathcal{T}_{21}^* \\ \mathcal{T}_{14}^* & -\mathcal{T}_{13}^* & -\mathcal{T}_{12}^* & \mathcal{T}_{11}^* \end{pmatrix}, \tag{6}
$$

where the numbering of the spin states is

$$
|1\rangle = |\uparrow\uparrow\rangle, \quad |2\rangle = |\uparrow\downarrow\rangle, \quad |3\rangle = |\downarrow\uparrow\rangle, \quad |4\rangle = |\downarrow\downarrow\rangle.
$$

For any given kinematics one can try to solve for the form factors

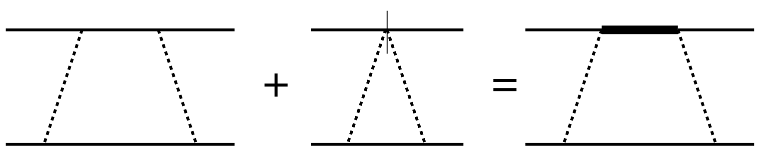
$$
\mathcal{T}_{ab} = \sum_i O^i_{ab} F_i. \tag{7}
$$

III Ligh-front perturbation theory

We perform the usual k^--integration to expand the covariant box in light-front amplitudes

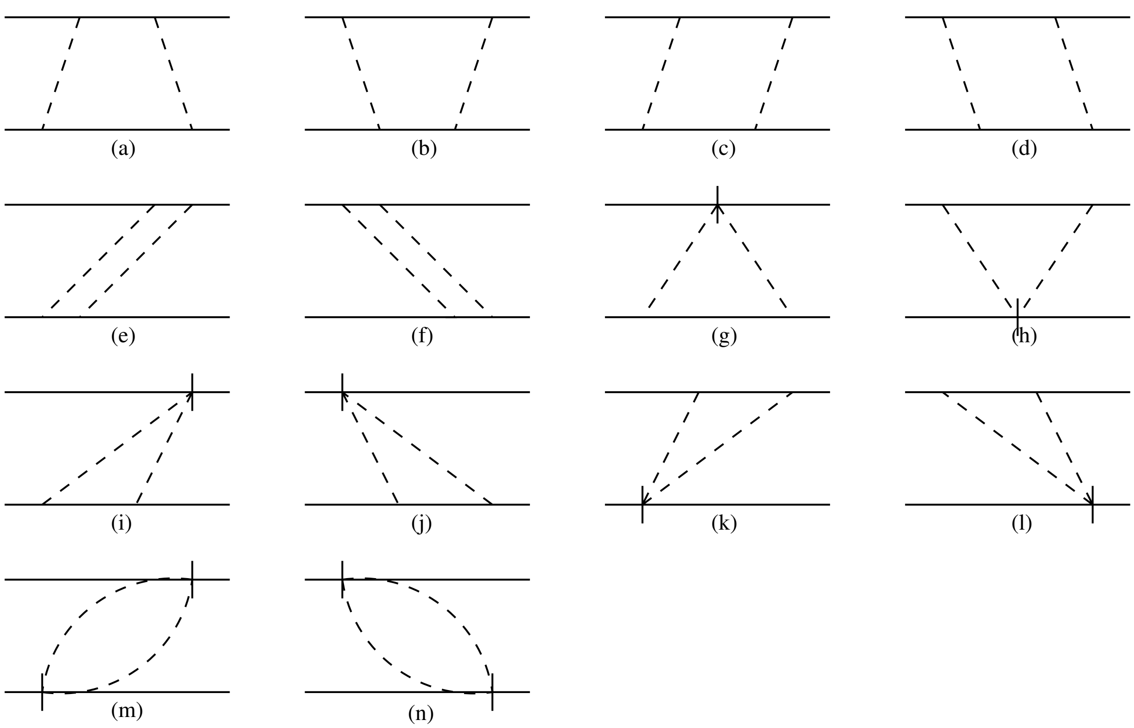
$$\mathcal{T} = \sum_D \mathcal{T}^D, \quad D = 1, \ldots 4. \tag{8}$$

We use the blink construction here to remove the cancelling singularities of the fermion propagators, e.g.



The LF amplitude with blinks is obtained adding the amplitude with an instantaneous fermion propagator to the amplitude with LF fermion propagators only.

The LF time ordered diagrams are



Adding diagrams (a) and (g) gives diagram 1, adding (b) and (h) gives diagram 1, the open diamond (c) is diagram 3, and adding diagrams (e), (i), (k), and (m) gives diagram 4, the stretched box.

Miranda van Iersel, Thesis, (2004)

All LF time ordered boxes are divergent. The divergences are due to the k^- dependence of the fermion propagators

The sum of the divergences vanishes.

If the LF time ordered boxes with instantaneous parts, $(g)\ldots(n)$, are dropped, the singularities do not cancel.

If the stretched box,(e), (f), and $(i)\ldots(n)$, is dropped, the singularities do not cancel either.

In order to calculate the LF amplitudes we need to *regularize*. We used two regularizations.

DR$_2$, dimensional regularization in the perpendicular momenta

Pauli-Villars for the boson.

We shall present our results as contributions to the invariant form factors from the LF amplitudes.

Light-front 'form factors'

LF 'form factors' are defined by relating LF amplitudes to LF 'form factors' as the full covariant amplitudes are related to the invariant form factors, viz

$$\begin{aligned} \mathcal{T}_{ab} &= \sum_i O^i_{ab}\, F_i \\ \mathcal{T} &= \sum_D \mathcal{T}^D, \\ \mathcal{T}^D_{ab} &= \sum_i O^i_{ab}\, F^D_i \end{aligned} \tag{9}$$

Depending on the kinematics these equations may not have a unique solution for $F_i^D, D = 1, \dots 4, i = 1, \dots 4$.

We define *valence* and *nonvalence* parts as follows

$$F_i^{\mathrm{val}} = F_i^1 + F_i^2 + F_i^4, \quad F_i^{\mathrm{nv}} = F_i^3. \tag{10}$$

The nonvalence part is the part that contains the four-particle Fock sector of the covariant box.

IV Results

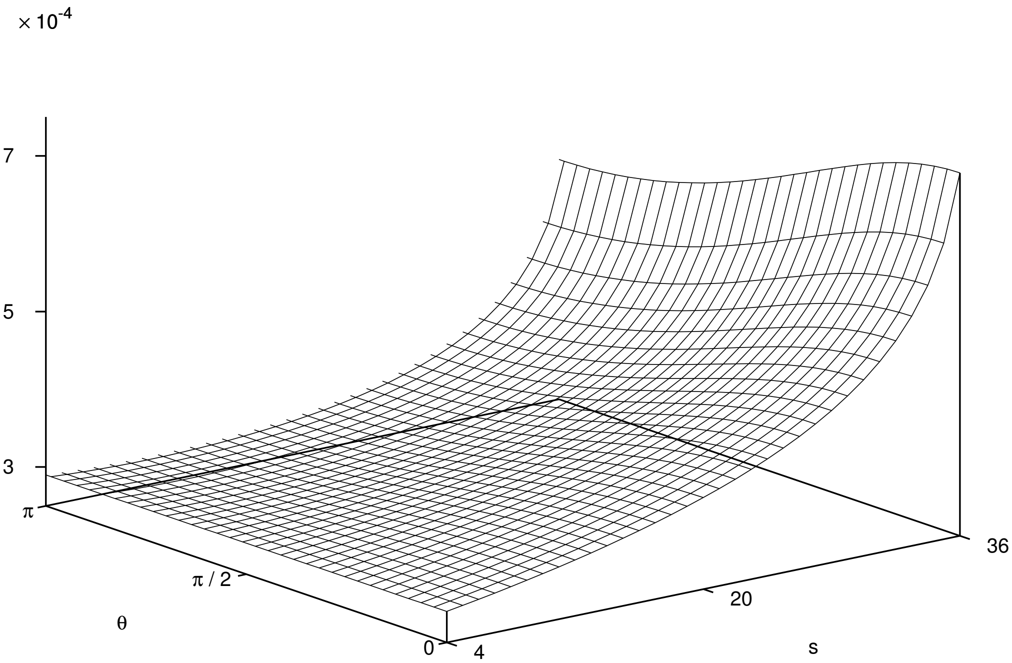
We have chosen the parameters of our model to mimick the bound-state situation. Model parameters: mass of the fermion inside the box $m = 1$, mass of the boson $\mu = 2.5$, mass of the fermions outside $M = 1$.

The threshold for real momenta of the external fermions is $s = 4M^2 = 4$, the threshold for inelasticitiy is $s = 4m^2 = 36$. we present results in this window. All form factors are plotted as functions of the the scattering angle in the CMS, θ and the Mandelstam variable s.

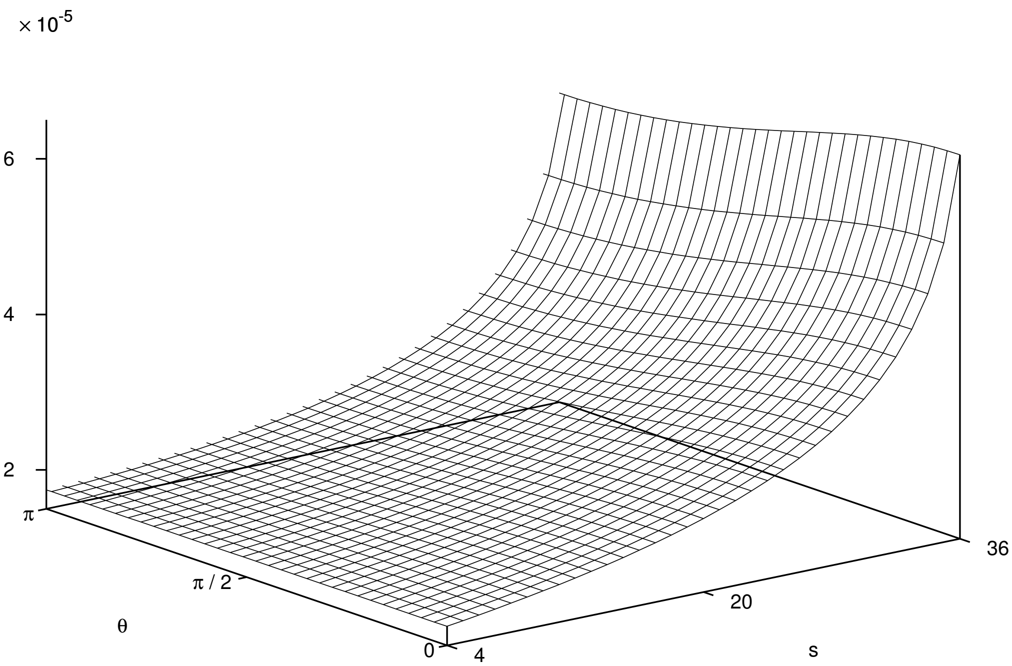
We show

1. The invariant form factors calculated directly from the invariant integrals $D_{\alpha_i \alpha_j}$.
2. For one form factor, F_1, we show the contribution of the four LF amplitudes.
3. The valence and nonvalence contributions to the form factors.
4. The valence and nonvalence contributions to the form factors for fixed $s = 20$, as a function of the scattering angle θ.
5. The valence and nonvalence contributions to the form factors for fixed $\theta = \pi/2$, as a function of s.

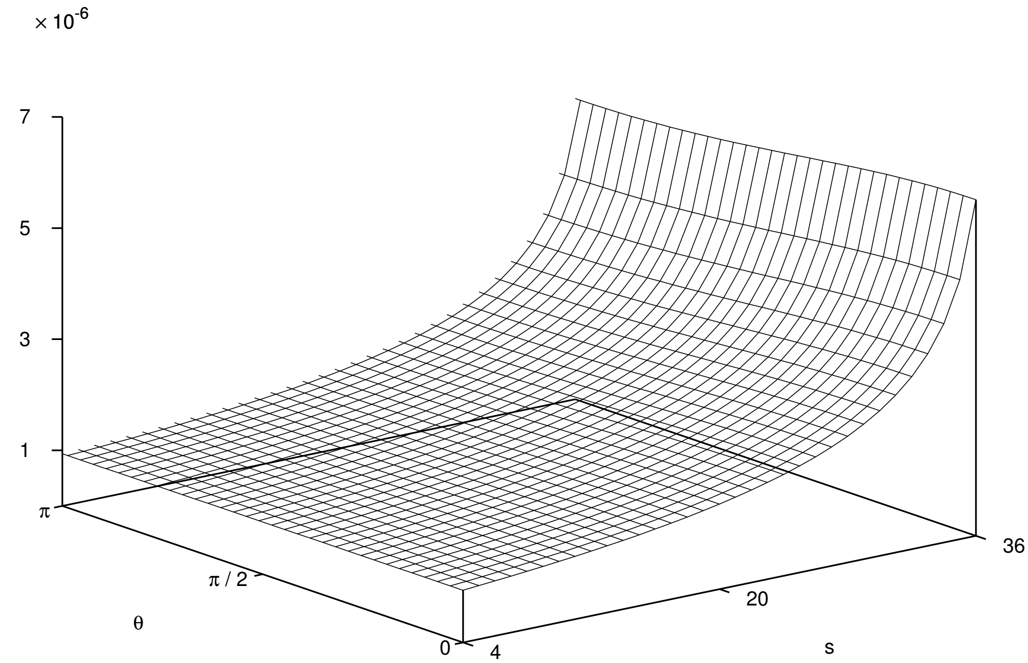
Form factor F_1



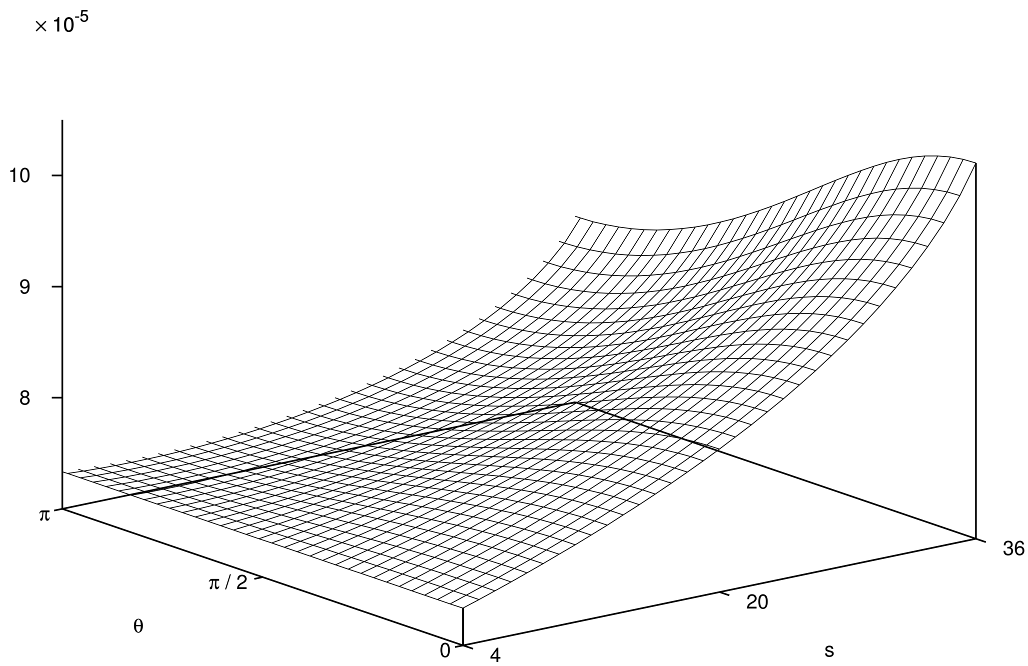
Form factor F_2



Form factor F_3



Form factor F_4



The logarithmic divergences in LF have the form

$$\mathcal{T}^D \propto \int \frac{d^2 k_\perp}{k_\perp^2}. \tag{11}$$

They show up in DR_2 as poles in ϵ, $S^D_{\mathrm{DR}_2} \times 1/\epsilon$, if we replace the integral by an integral in $2-2\epsilon$ dimensions.

In Pauli-Villars regularization the divergent terms have the form $S^D_{\mathrm{PV}} \times \log(\Lambda^2/\mu^2)$.

The coefficients of the divergent parts cancel out,

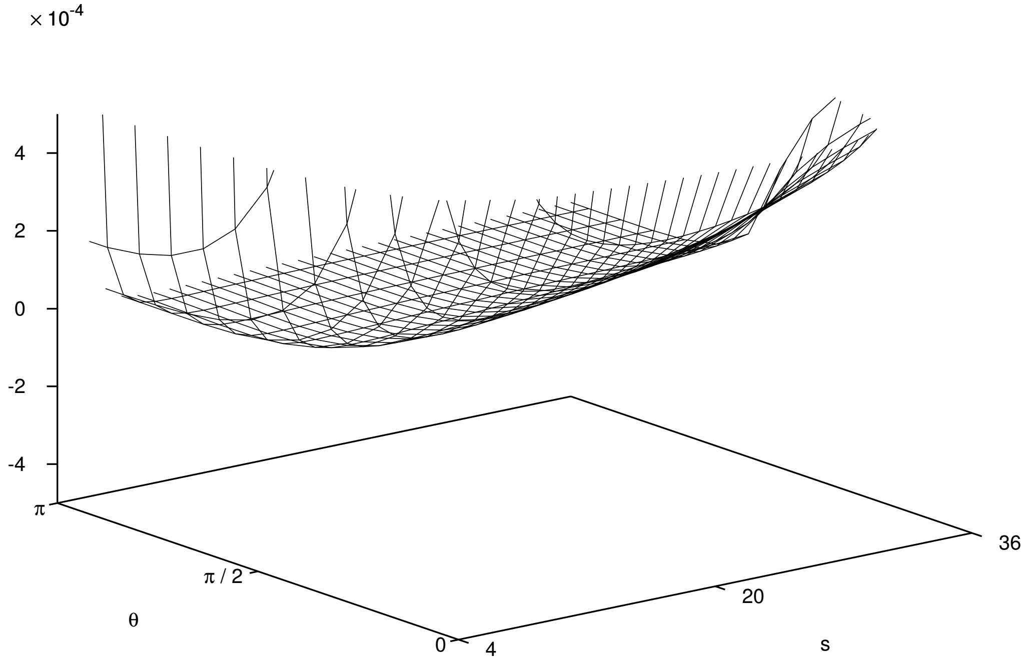
$$\sum_D S^D_{\mathrm{DR}_2} = 0, \sum_D S^D_{\mathrm{PV}} = 0.$$

The finite parts add up to the covariant amplitude.

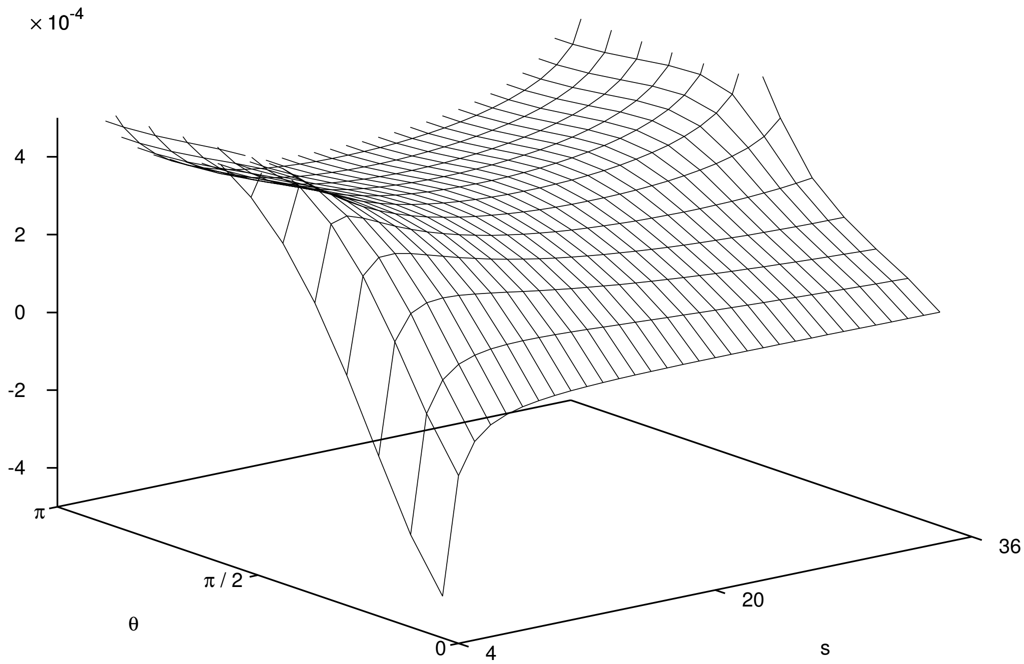
The finite parts are singular when the scattering angle θ of the momentum of the external fermions p goes to zero.

Depending on the kinematics the stretched box, diagram $D = 3$, has a *zero mode*, i.e., the integration interval shrinks to zero, while the integrand for $\mathcal{T}_3$ blows up proportional to one over the length of the integration interval.
To see the effect of negelcting the stretched box, which contains the four-body or nonvalence contribution, we show the valence and nonvalence contribution to the form factors.

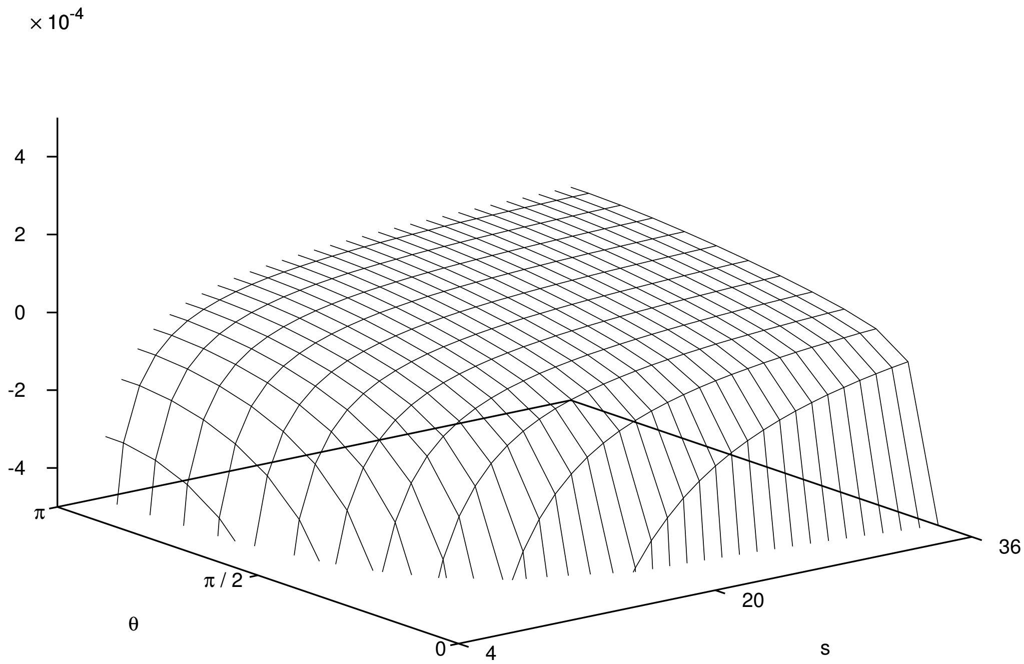
Contribution from LF diagram 1 to F_1



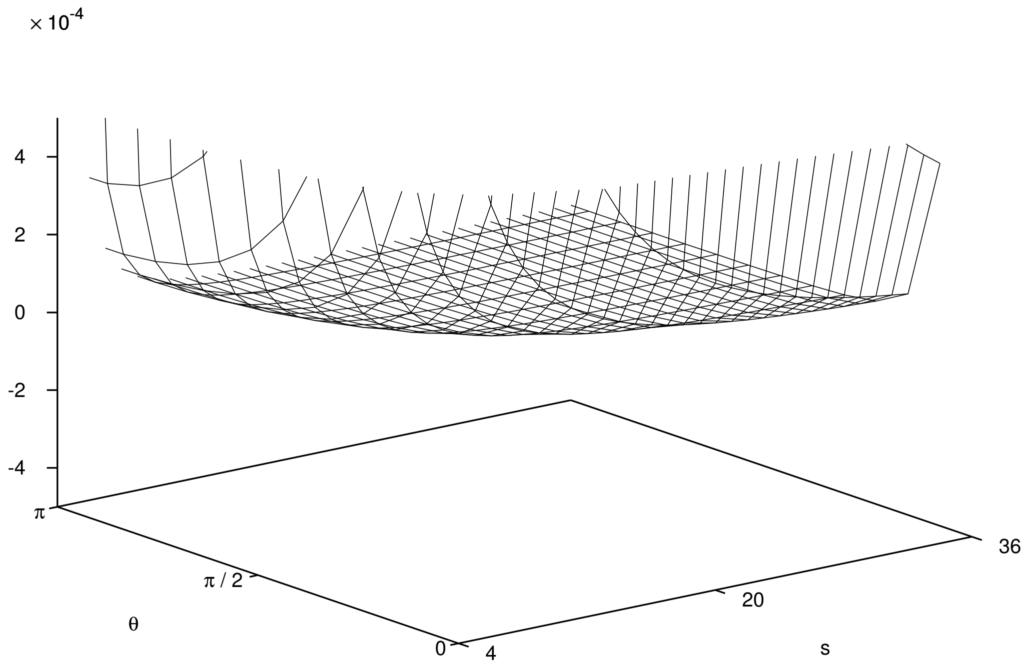
Contribution from LF diagram 2 to F_1



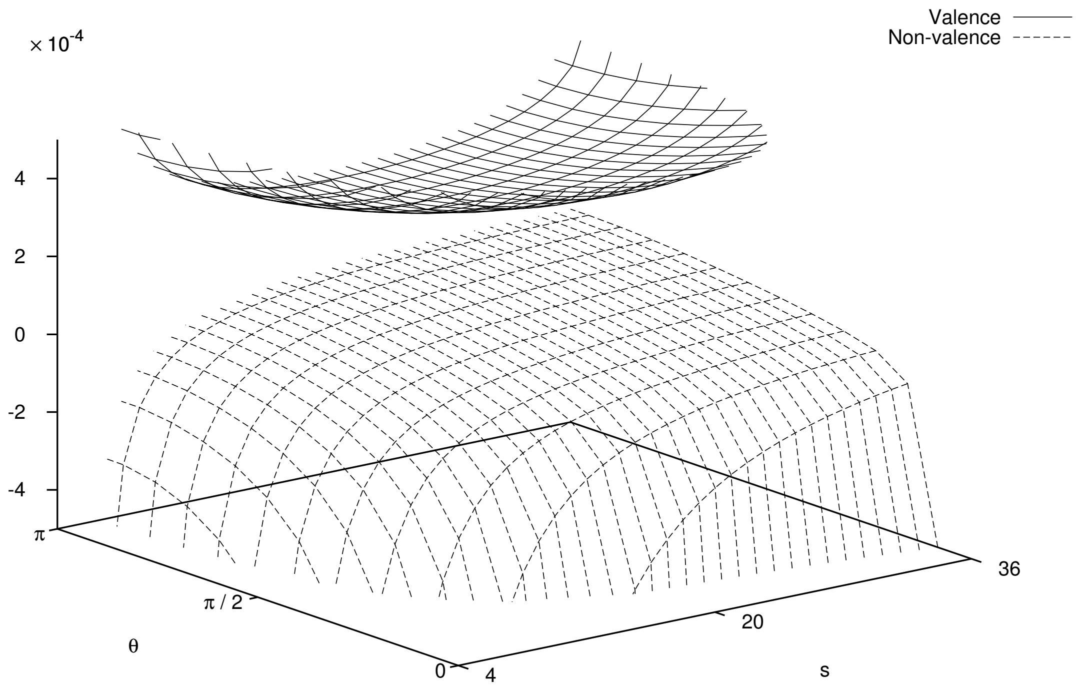
Contribution from LF diagram 3 to F_1



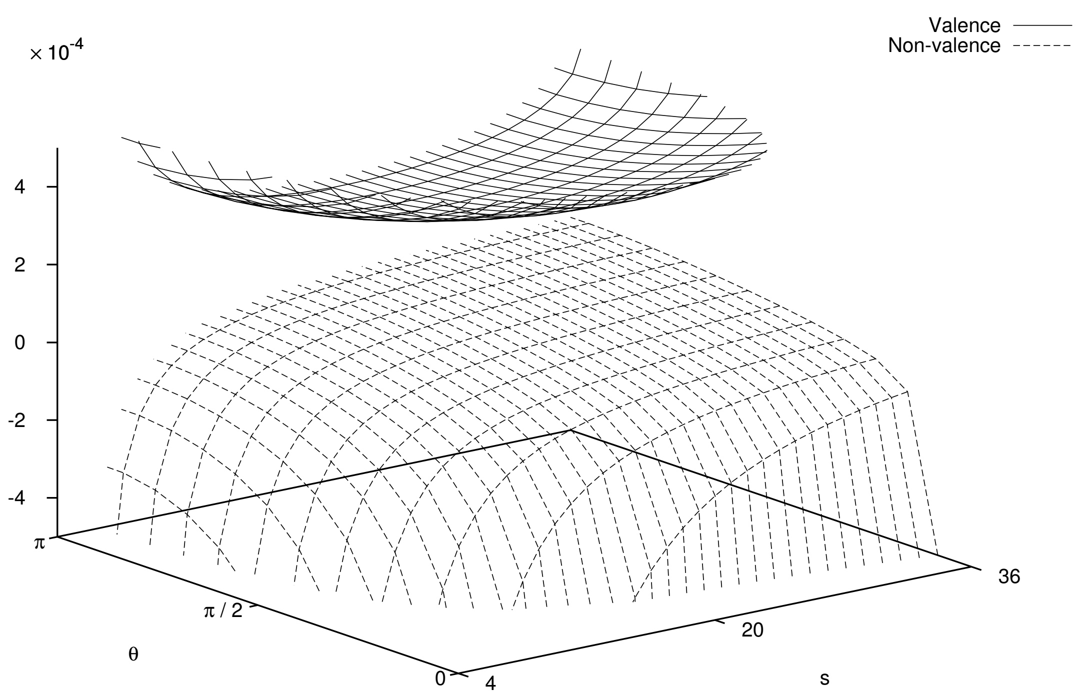
Contribution from LF diagram 4 to F_1



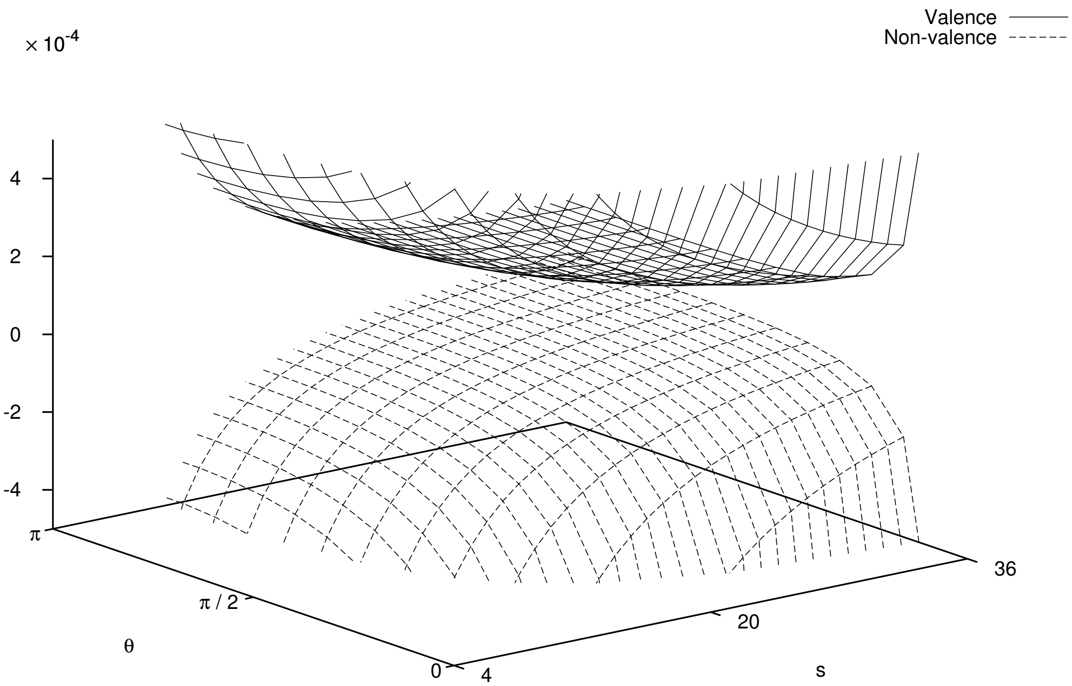
F_1


Valence
Non-valence
× 10⁻⁴
4
2
0
-2
-4
π
π / 2
θ
0
4
20
36
s

F_2

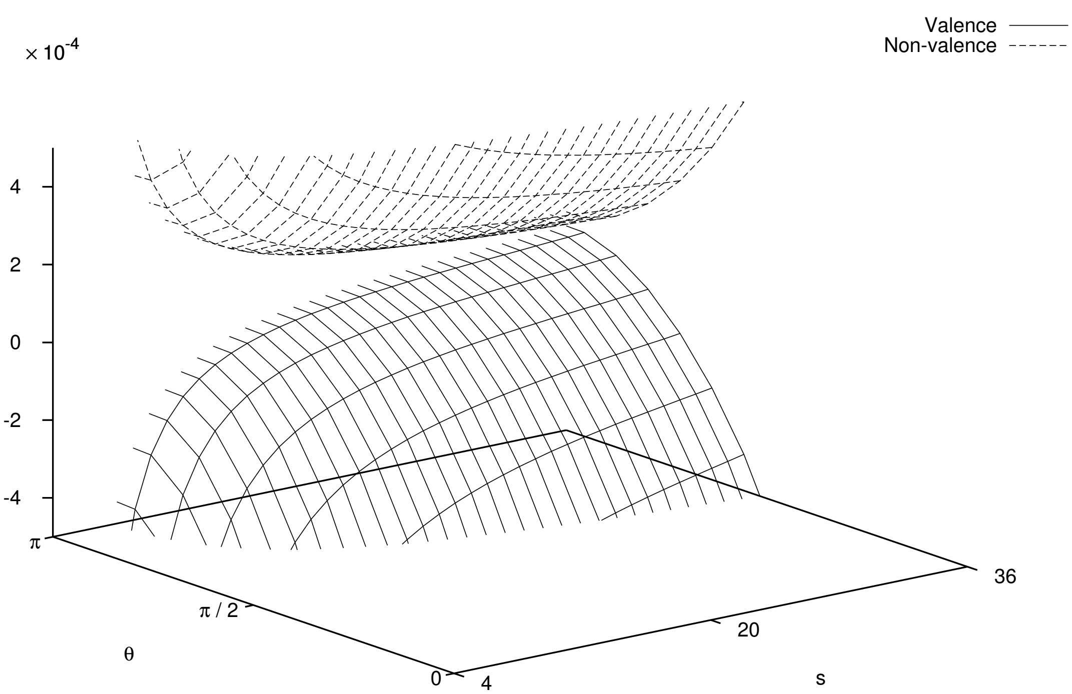

Valence
Non-valence
× 10⁻⁴
4
2
0
-2
-4
π
π / 2
θ
0
4
20
36
s

F_3

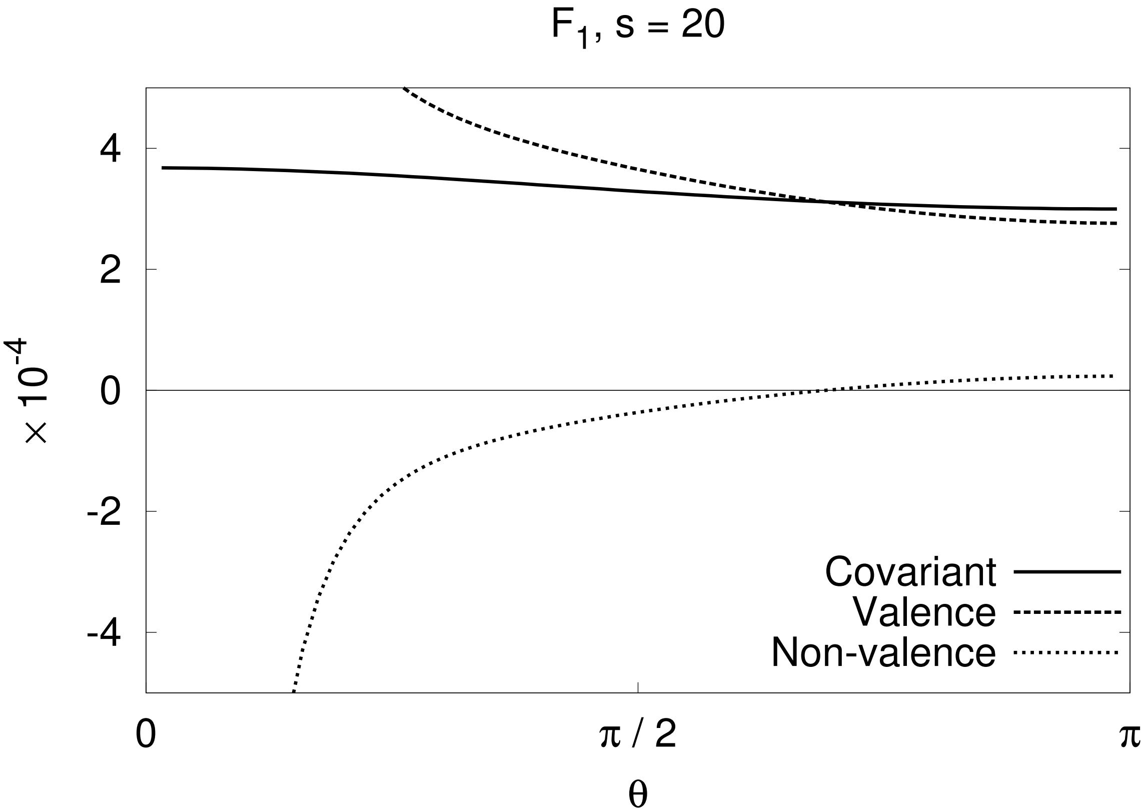


Valence
Non-valence
× 10^-4
4
2
0
-2
-4
π
π / 2
θ
0
4
20
36
s

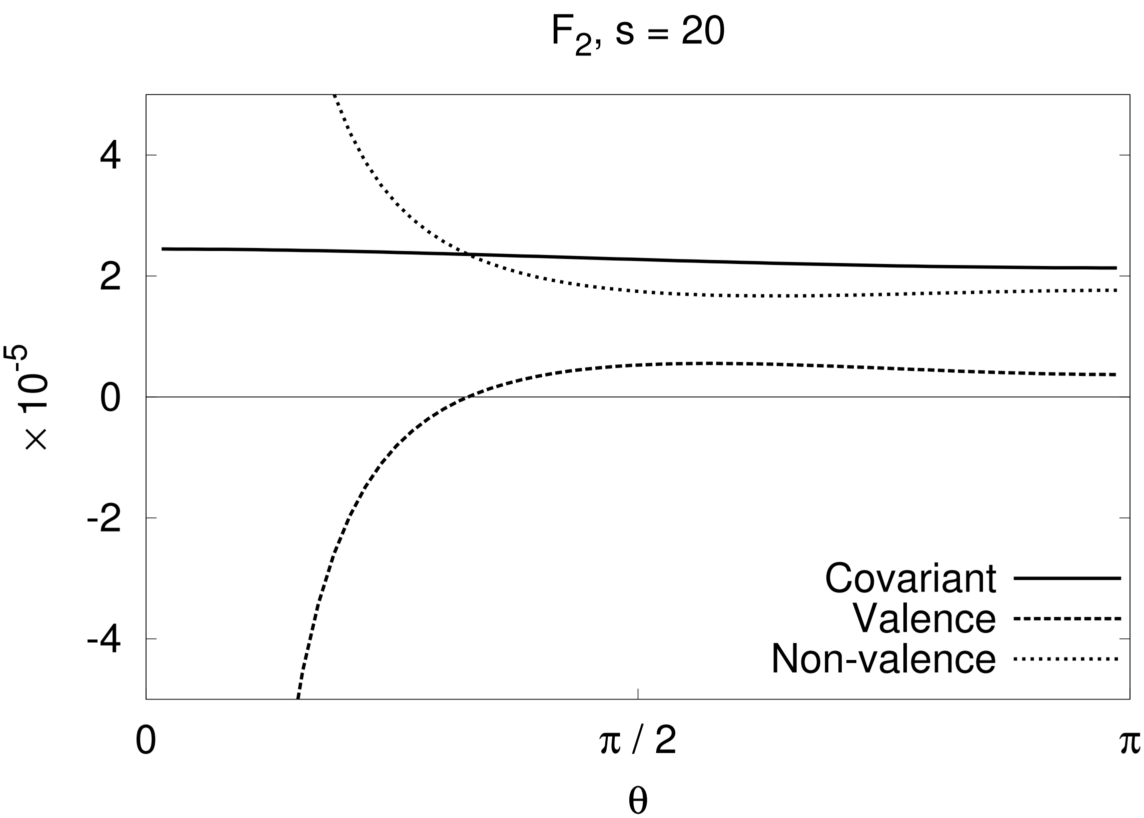
F_4



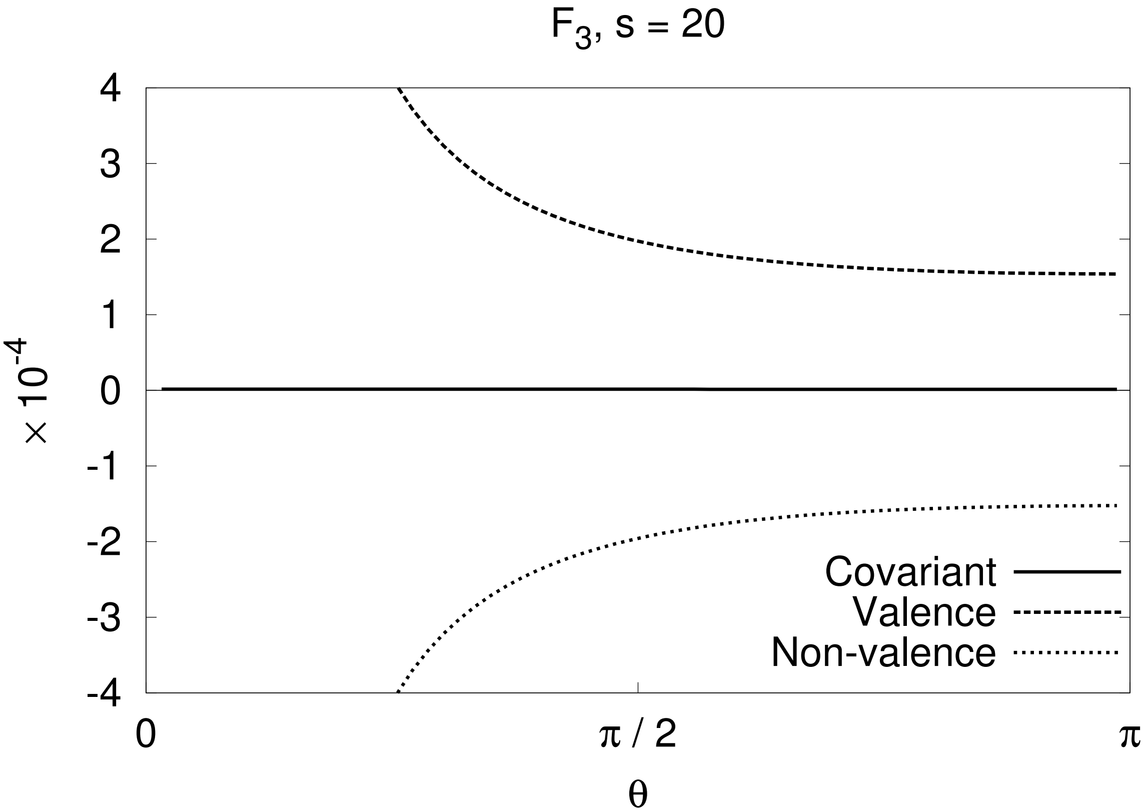
Valence
Non-valence
× 10^-4
4
2
0
-2
-4
π
π / 2
θ
0
4
20
36
s



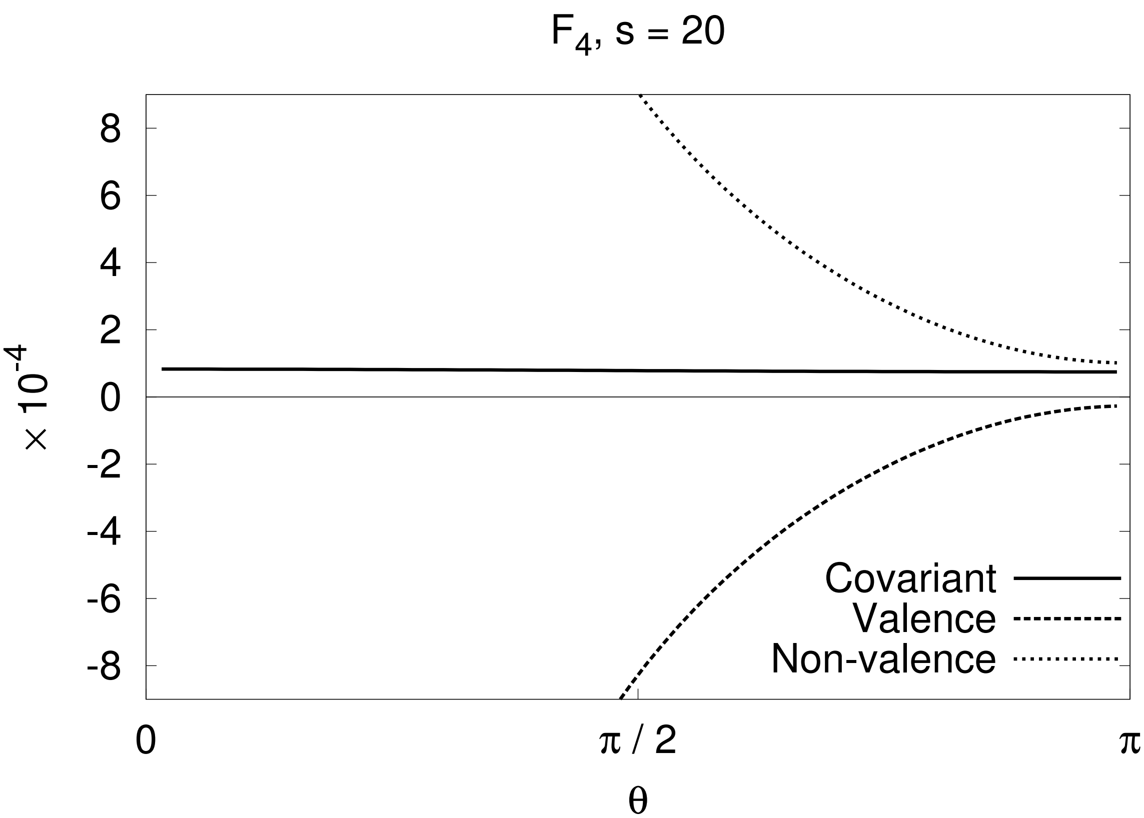
F1, s = 20
× 10-4
4
2
0
-2
-4
0
π / 2
π
θ
Covariant
Valence
Non-valence



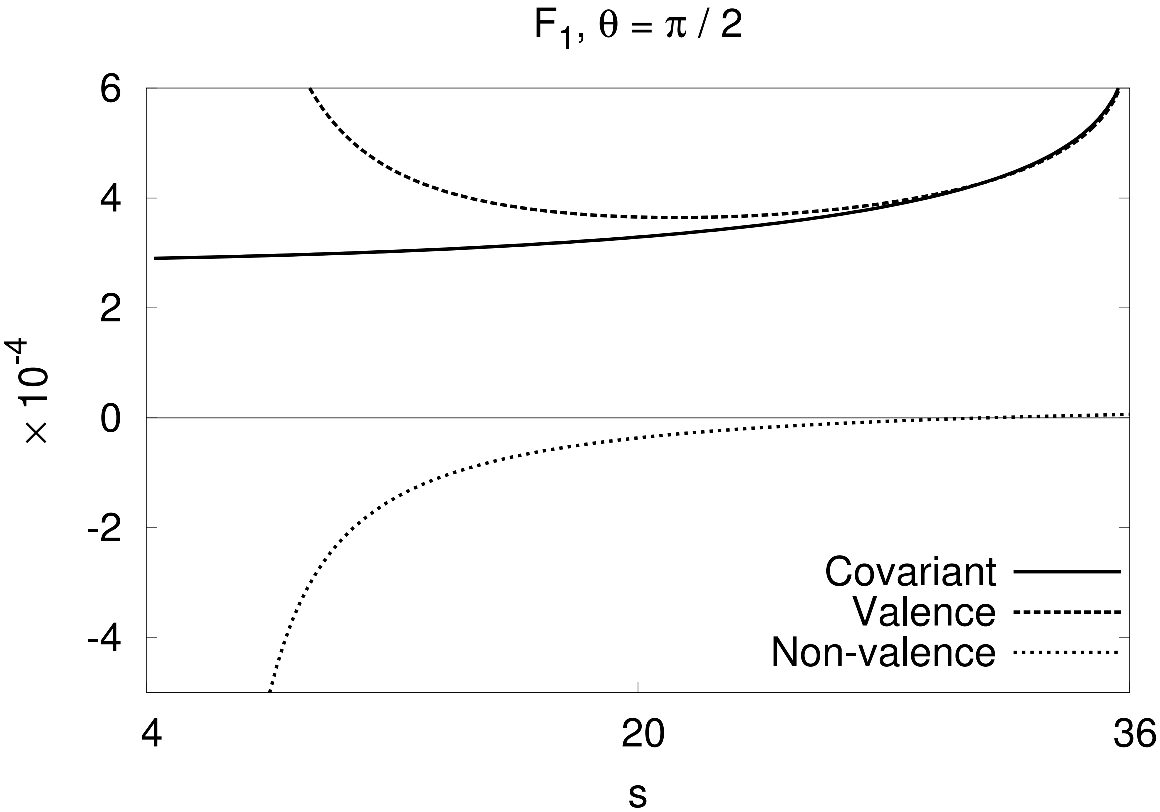
F2, s = 20
× 10-5
4
2
0
-2
-4
0
π / 2
π
θ
Covariant
Valence
Non-valence



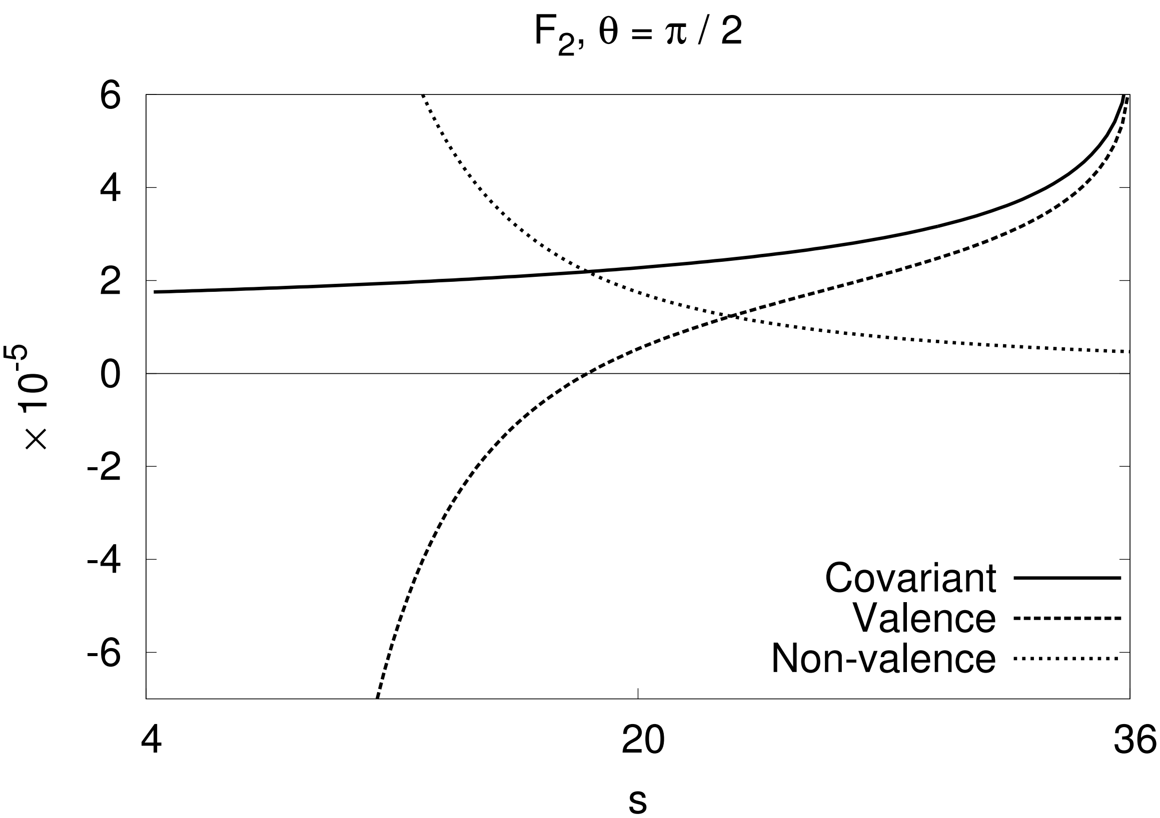
F3, s = 20
4
3
2
1
0
-1
-2
-3
-4
× 10-4
Covariant
Valence
Non-valence
0
π / 2
π
θ



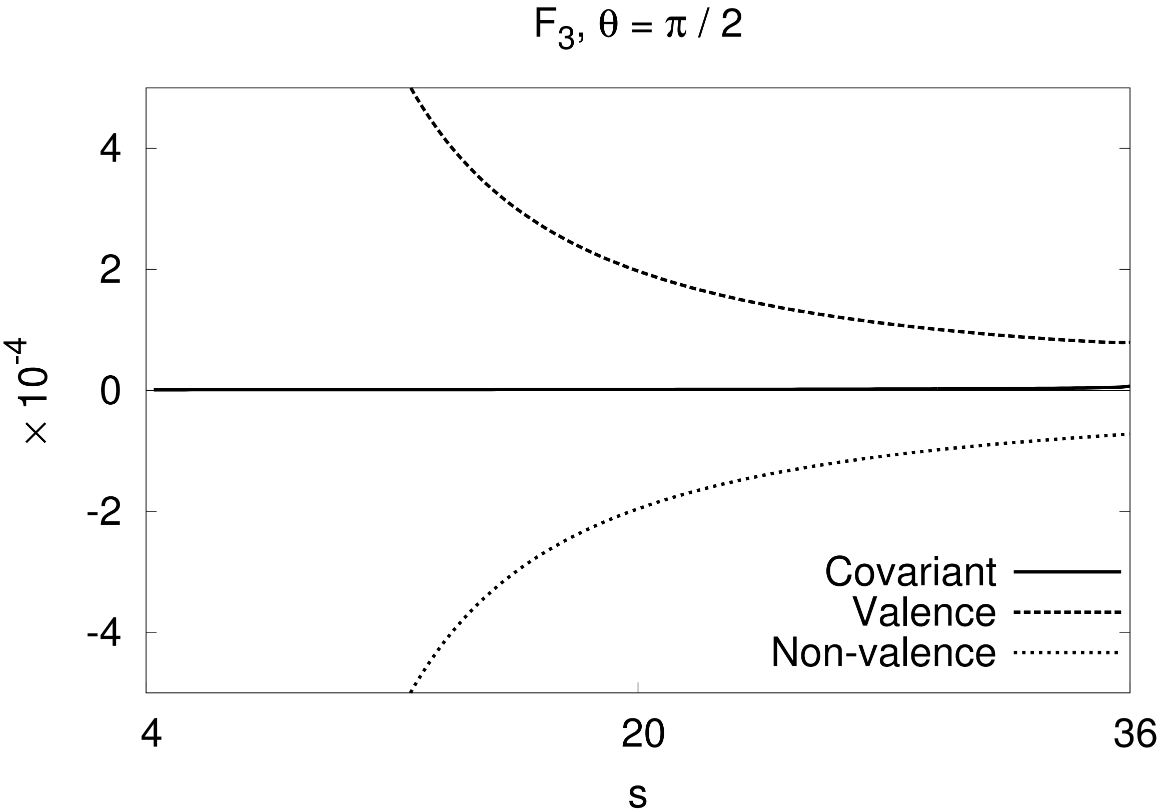
F4, s = 20
8
6
4
2
0
-2
-4
-6
-8
× 10-4
Covariant
Valence
Non-valence
0
π / 2
π
θ



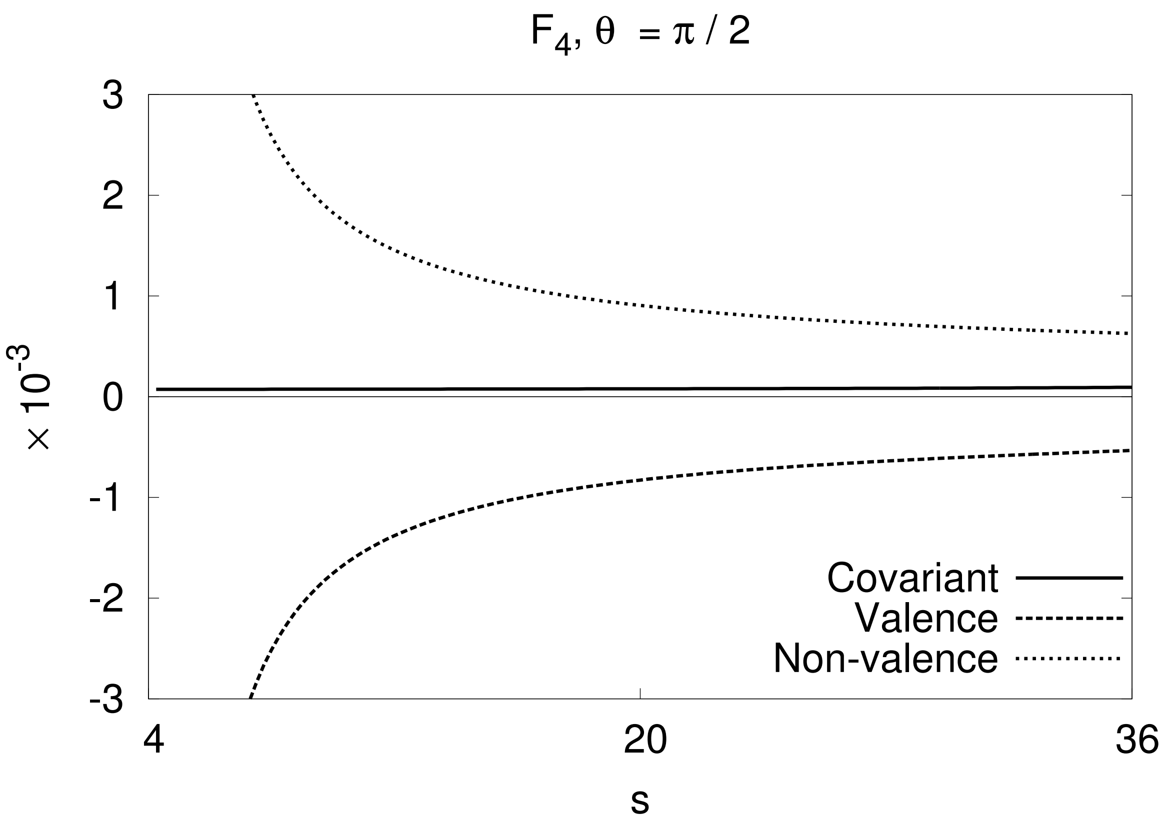
F1, θ = π / 2
6
4
2
0
-2
-4
× 10^-4
4
20
36
s
Covariant
Valence
Non-valence



F2, θ = π / 2
6
4
2
0
-2
-4
-6
× 10^-5
4
20
36
s
Covariant
Valence
Non-valence



F3, θ = π / 2
× 10-4
4
2
0
-2
-4
4
20
36
s
Covariant
Valence
Non-valence



F4, θ = π / 2
× 10-3
3
2
1
0
-1
-2
-3
4
20
36
s
Covariant
Valence
Non-valence

V Conclusions

1. The LF Yukawa model has singularities that must be tamed (Głazek et al., ...).

2. The singularities of the box diagram are due exclusively to the spin operator $\gamma^+(1) \otimes \gamma^+(2)$ occurring in the numerator of the integrand determining the box diagram.
   Any attempt at calculating bound states of fermions must remove this singularity.

3. Discarding higher Fock states gives amplitudes that have singularities in some kinematical regions, which hamper an accurate calculation of bound states.

4. The stretched box has a zero mode contribution.

5. *Conjecture* If a covariant amplitude is given by an unconditionally convergent integral, the LF contributions add up to this amplitude. If a covariant amplitude is given by a conditionally convergent integral, the LF summed contributions may differ by a finite amount from it.