HALO FORMATION IN 3-D BUNCHES WITH SELF-CONSISTENT STATIONARY DISTRIBUTIONS

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Abstract

We have constructed, analytically and numerically, a new class of self-consistent 6-D phase space stationary distributions. Stationary distributions allow us to study the halo development mechanism without being obscured by beam redistribution and its effect on halo formation. The beam is then mismatched longitudinally and/or transversely, and we explore the formation of longitudinal and transverse halos in 3-D axisymmetric beam bunches. Of particular importance is the result that, due to the coupling between longitudinal and transverse motion, a longitudinal or transverse halo is observed for a mismatch less than 10% if the mismatch in the other plane is large.

1 INTRODUCTION

The need for high current in a variety of new accelerator applications has focused a great deal of attention on understanding the phenomenon of halo formation in ion beams, which can cause excessive radioactivation of the accelerator. Starting in about 1991, a variety of two-dimensional (2-D) simulation studies [1, 2] have led to the conclusion that halos are formed when a beam is mismatched to a focusing channel, exciting some sort of collective oscillation(s) of the beam which are in resonance with the non-linear oscillation of individual ions.

Most of the simulations studies start with rms matched beams which are *not* stationary solutions of the Vlasov equation (See for example [3].). As a result, the initial beam undergoes some sort of redistribution in phase space, masking the possible development of halos. Our effort has been devoted to populating a stationary distribution in phase space, in the hope that the halo development mechanism can be studied without being obscured by the "relaxation" of the beam in phase space. We have particularly studied initial distributions which are stationary by virtue of being a function only of the Hamiltonian [4, 5].

It is clear that a realistic treatment of halo formation must take into account 3-D beam bunches and 6-D phase space distributions. Recently, Barnard and Lund [6] performed numerical studies with a 3-D beam bunch using the particle-core model, drawing attention to the existence and importance of a longitudinal halo for a spheroidal bunch. However, all studies based on the particle-core model do not address the question of whether halo formation is influenced by the density redistribution which follows for a non-stationary beam, even if it is rms matched [3]. In fact, halo formation in 2-D due to the redistribution process in rms matched beams was shown, for example, by Okamoto [7] and Jameson [8]. We therefore continue our effort to study the halo development mechanism in 3-D beam bunches in the absence of the redistribution process. Such an approach allows us to study the fundamental mechanism of halo formation associated with the beam mismatch. To accomplish this we have constructed, analytically and numerically, a new class of stationary 6-D phase space distributions for a spheroidal beam bunch. Our present analysis assumes smoothed linear external transverse and longitudinal restoring force gradients, k_z, k_y, k_x. In general, the distribution can be chosen to have an approximately ellipsoidal boundary. However, for simplicity, we treat the azimuthally symmetric case $(k_x = k_y)$ for which the beam bunch is approximately spheroidal. This is the focus of the present investigation. More details can be found in [9].

2 STATIONARY 6-D PHASE SPACE DISTRIBUTION

2.1 Distribution and charge density

We take for the azimuthally symmetric 6-D phase space distribution

$$f(\boldsymbol{x}, \boldsymbol{p}) = N(H_0 - H)^{-1/2} \tag{1}$$

where

$$H = k_x r^2/2 + k_z z^2/2 + e\Phi_{sc}(\boldsymbol{x}) + mv^2/2. \tag{2}$$

Here $\boldsymbol{p} = m\boldsymbol{v}$, $r^2 = x^2 + y^2$, and k_x, k_z are the smoothed transverse and longitudinal restoring force gradients. The quantity $\Phi_{sc}(\boldsymbol{x})$ is the electrostatic potential due to the space charge of the bunch. The distribution is normalized such that

$$\int d\boldsymbol{x} \int d\boldsymbol{p} f(\boldsymbol{x}, \boldsymbol{p}) = 1. \tag{3}$$

The charge distribution corresponding to Eq. (1) is

$$\begin{aligned} \rho(\boldsymbol{x}) &= Q \int d\boldsymbol{p} f(\boldsymbol{x}, \boldsymbol{p}) \\ &= NQm^3 \int d\boldsymbol{v} \left[G(\boldsymbol{x}) - \frac{mv^2}{2} \right]^{-1/2}, \end{aligned} \tag{4}$$

where

$$G(\boldsymbol{x}) \equiv H_0 - \frac{k_x r^2}{2} - \frac{k_z z^2}{2} - e\Phi_{sc}(\boldsymbol{x}). \tag{5}$$

Performing the integral over $d\boldsymbol{v} \equiv v^2 dv d\Omega_v$ in Eq. (4) leads to

$$\rho(\boldsymbol{x}) = QG(\boldsymbol{x}) / \int d\boldsymbol{x} G(\boldsymbol{x}), \tag{6}$$

where the normalization constant satisfies

$$2\sqrt{2}\pi^2 N m^{3/2} \int d\boldsymbol{x} G(\boldsymbol{x}) = 1. \tag{7}$$

From Eq. (5) and Poisson's equation, we write

$$\nabla^2 G(\boldsymbol{x}) = -k_s - e\nabla^2 \Phi_{sc} = -k_s + (e/\epsilon_0)\rho(\boldsymbol{x}), \tag{8}$$

where

$$k_s = 2k_x + k_z. \tag{9}$$

Using Eq. (6), we obtain the partial differential equation for $G(\boldsymbol{x})$

$$\nabla^2 G(\boldsymbol{x}) = -k_s + \kappa^2 G(\boldsymbol{x}), \tag{10}$$

where

$$\kappa^2 = (eQ/\epsilon_0) / \int d\boldsymbol{x} G(\boldsymbol{x}). \tag{11}$$

The solution of Eq. (10) for an axisymmetric, spheroidal shaped bunch can most easily be written in the spherical coordinates R, θ for which

$$z = R\cos\theta \ , \ r = R\sin\theta, \tag{12}$$

as

$$G(\boldsymbol{x}) = (k_s/\kappa^2) g(\boldsymbol{x}) \tag{13}$$

where

$$g(\boldsymbol{x}) = 1 + \sum_{\ell=0}^{\infty} \alpha_\ell P_{2\ell}(\cos\theta) i_{2\ell}(\kappa R). \tag{14}$$

Here $P_{2\ell}(\cos\theta)$ are the even (fore-aft symmetric) Legendre polynomials and $i_{2\ell}(\kappa R)$ are the spherical Bessel functions (regular at $\kappa R = 0$) of imaginary argument.

Since $g(\boldsymbol{x})$ is proportional to the charge density, the edge of the bunch is defined as the border $g(\boldsymbol{x}) = 0$, closest to the origin. We therefore choose the α_ℓ's so that the surface of the bunch reproduces, as closely as possible, the ellipsoidal surface.

We also note that $m\langle \dot{x}^2 \rangle = m\langle \dot{y}^2 \rangle = m\langle \dot{z}^2 \rangle = m\langle v^2 \rangle/3$ because H depends only on v^2 and $\boldsymbol{x}$. Thus our choice of a stationary distribution of the form $f(H)$ *automatically* corresponds to equipartition (equal average kinetic energy in the three spatial directions).

2.2 *Numerical implementation*

We have developed a 3-D particle-in-cell (PIC) code HALO3D to test the analytic model described above, and to explore halo formation [9]. The single-particle equations of motion are integrated using a symplectic, split-operator technique [10]. The space charge calculation uses area weighting ("Cloud-in-Cell") and implements open boundary conditions with the Hockney convolution algorithm [11]. The code runs on parallel computers, and in particular, the space charge calculation has been optimized for parallel platforms using the Ferrell-Bertschinger method [12]. Some details about the code can be found in [13]. Details about effects of non-linear RF fields can be found in [14].

We initially populate the 6-D phase space according to Eq. (1), and then mismatch the x, y, z coordinates by factors $\mu_x = \mu_y = 1 + \delta a/a$, $\mu_z = 1 + \delta c/c$ and the corresponding momenta by $1/\mu_x = 1/\mu_y$, $1/\mu_z$, with a, c being the minor and major semiaxes of our spheroidal bunch, respectively.

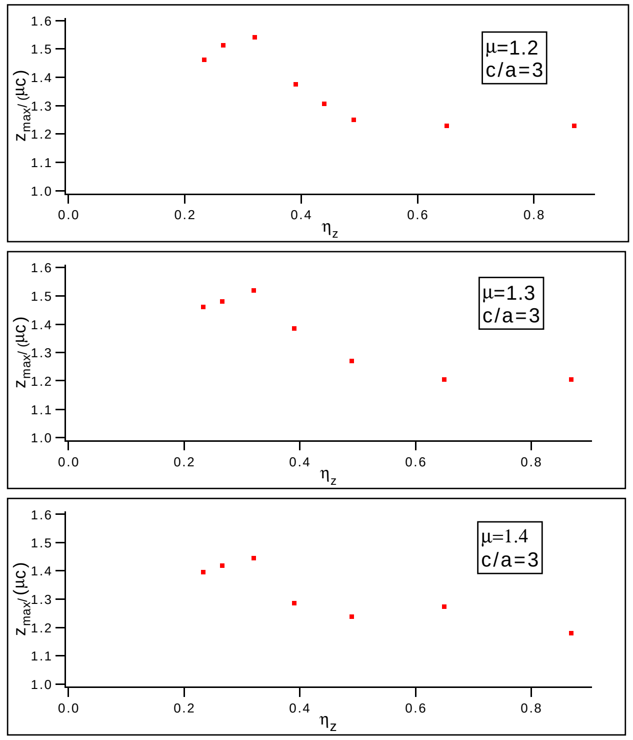

Figure 1: Longitudinal halo extent for different mismatches.

3 ORBIT SIMULATIONS

3.1 *Longitudinal halo*

Due to the fact that longitudinal tune depression is always less than the transverse one for elongated equipartitioned bunches the longitudinal halo is our primary focus. An important quantity is the ratio of the halo radius to that of the matched distribution. We performed a systematic study for different c/a and mismatch factors in the range of interest [15], by looking at the halo extent at the time when the beam comes to a roughly saturated state after the development of a halo. Our new result is the dependence of the halo extent on tune depression shown in Fig. 1 for $c/a = 3$ and mismatch parameters $\mu = 1.2, 1.3, 1.4$ being the same in all directions x, y, z. One sees a significant increase in halo extent for severe tune depressions. In addition the halo extent clearly depends on the mismatch parameter. The

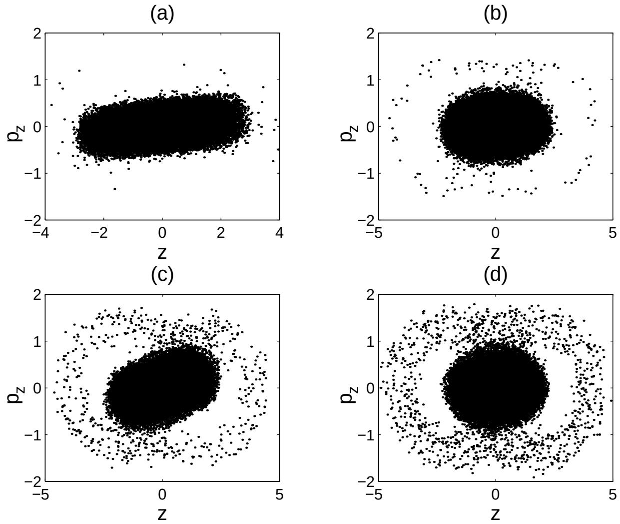

Figure 2: Dependence of halo intensity on the mismatch for $c/a = 3$, $\eta_x = 0.65$, $\eta_z = 0.49$ (with 32,768 particles plotted). a) $\mu = 1.1$. b) $\mu = 1.2$. c) $\mu = 1.3$. d) $\mu = 1.4$.

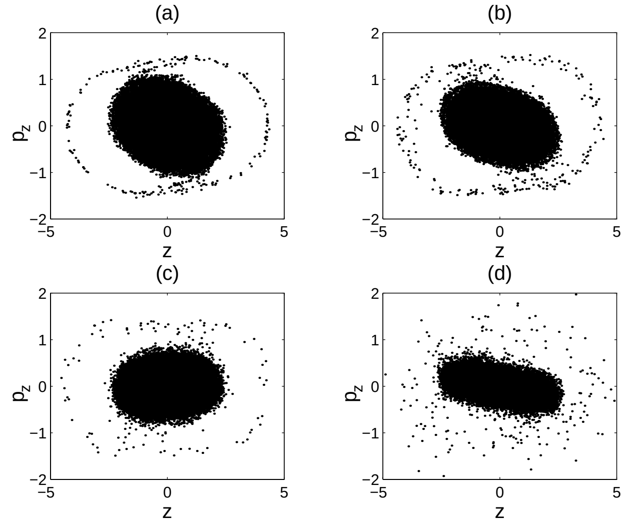

Figure 3: Dependence of halo intensity on tune depression for $c/a = 3$, $\mu = 1.2$. a) $\eta_z = 0.87$. b) $\eta_z = 0.65$. c) $\eta_z = 0.49$. d) $\eta_z = 0.32$.

approximately linear dependence of the halo extent on the mismatch factor μ indicates that a serious effort should be made to match the beam to the channel as accurately as possible. Similar investigation for other c/a can be found in [9].

Simulation results show that the halo intensity (roughly defined as the fraction of particles outside the core in phase space) depends primarily on the mismatch. Figure 2 presents the phase space diagram (with only 32,768 particles plotted) after the halos have stabilized, for $\eta_x = 0.65$, $\eta_z = 0.49$ ($c/a = 3$) with several mismatches $\mu = 1.1, 1.2, 1.3, 1.4$. Severe mismatches lead to several percent of the particles in the halo, which is clearly outside acceptable limits. In Fig. 3 we present the phase space diagram for different tune depressions $\eta_z = 0.87, 0.65, 0.49, 0.32$ with $\mu = 1.2$ ($c/a = 3$) for which the fraction of particles in the halo is about 0.5 %. No significant dependence of halo intensity on the tune depression is seen. However, for tune depression $\eta_z \leq 0.4$ the clear peanut diagram in the longitudinal phase space now has a chaotic behavior.

One more important feature is how fast the halo develops. We first make the observation that for comparable mismatches the longitudinal halo develops much faster than the transverse halo when the mismatches and/or tune depressions are not severe. Such behavior simply occurs because for fixed charge we have $\eta_z < \eta_x$ for elongated equipartitioned bunches. For severe mismatches and/or tune depressions both the longitudinal and transverse halos develop very quickly. A typical picture is shown in Fig. 4.

Of particular interest is the clear dependence on tune depression. Specifically, for more severe tune depression the halo starts to develop earlier as can be seen in Fig. 5 where the development of the halo is shown for $c/a = 3$, $\mu = 1.2$ and different tune depressions.

Another important characteristic of the longitudinal halo is its dependence on the mismatch when there is no mismatch in the radial direction. The number of particles in the halo drops dramatically with μ_z. In fact, we see no halo for $\mu_z < 1.2$ (< 20% longitudinal mismatch). Note that the situation changes when the effect of coupling is significant.

3.2 *Transverse halo*

The transverse halo closely duplicates all the features observed for non-linear stationary distributions in 2-D simulations [16]. The agreement between 2-D and 3-D simulations is very good. The only two significant differences seen are related to the rate of halo development. In the present 3-D simulations there is a clear dependence on the tune depression which was not the case in the corresponding 2-D simulations [16]. The second difference is that the transverse halo in the 3-D simulations develops significantly faster than in 2-D for comparable mismatches and tune depressions. More details can be found in [9].

3.3 *Coupling effects*

In performing 3-D simulations we encounter halo formation in a beam bunch, where we clearly see coupling between the longitudinal and transverse motion. It was already noted [9] that due to the coupling between r and z, a transverse or longitudinal halo is observed even for a very small mismatch (less than 10%) as long as there is a significant mismatch in the other plane. Further numerical investigation of this question showed that the effect of coupling becomes extremely important for nearly spherical bunches ($c/a \leq 2$) which is typical of the parameter range of interest for the APT design [15]. For example, for the short bunch with $c/a = 2$, with only a longitudinal initial mismatch ($\mu_z = 1.5$, $\mu_x = \mu_y = 1.0$), one finds particles

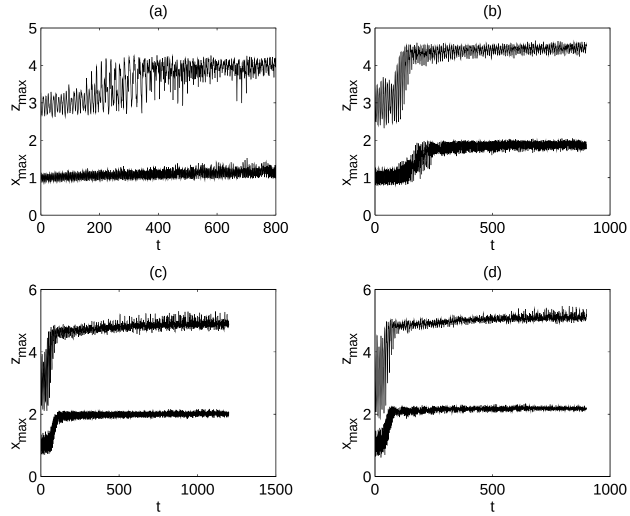

Figure 4: Halo development for comparable mismatches ($c/a = 3$, $\eta_x = 0.65$, $\eta_z = 0.49$). a) $\mu = 1.1$. b) $\mu = 1.2$. c) $\mu = 1.3$. d) $\mu = 1.4$.

at large amplitude in both the longitudinal and transverse directions, as can be seen in Fig. 6 for the 6-D stationary distribution. Of course, the intensity of particles in the transverse halo is much smaller than it is when there is in addition a transverse initial mismatch. (In our example in Fig. 6, we have 0.05 percent of the particles in the transverse halo with zero transverse mismatch compared with several percent in the longitudinal halo.) A similar effect due to coupling was seen for the non-stationary distributions [17].

4 SUMMARY

Most of the previous studies were concerned with halos in long beams. In the current work we address the question of halo formation in a beam bunch which is of particular interest for the Accelerator Production of Tritium project where relatively short bunches are proposed [15].

We have constructed, analytically and numerically, a new class of 6-D phase space stationary distributions for an azimuthally symmetric beam bunch of arbitrary charge in the shape of a prolate spheroid [9]. The stationary distribution allows us to study the halo development mechanism in 3-D beam bunches where no phase space redistribution takes place. Our choice of parameters automatically assures equipartition. In our calculations the beam remains equipartitioned through the channel. We therefore study the halo development in 3-D beams which are in thermal equilibrium, without the redistribution introduced by any equipartition process which may take place. Such an approach gives us an excellent chance to investigate the major mechanism of halo formation associated purely with the beam mismatch.

We then use a PIC code with smoothed linear external focusing forces, in which the initial stationary distribution

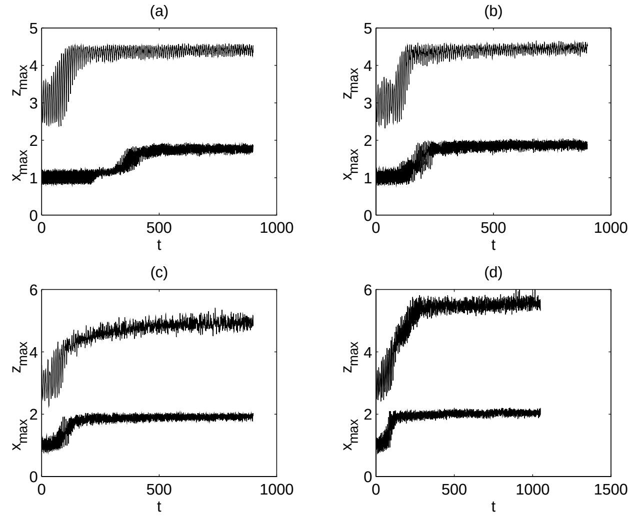

Figure 5: Dependence of the rate of halo development on tune depressions for $c/a = 3$, $\mu = 1.2$. a) $\eta_x = 0.79$, $\eta_z = 0.65$. b) $\eta_x = 0.65$, $\eta_z = 0.49$. c) $\eta_x = 0.53$, $\eta_z = 0.39$. d) $\eta_x = 0.45$, $\eta_z = 0.32$.

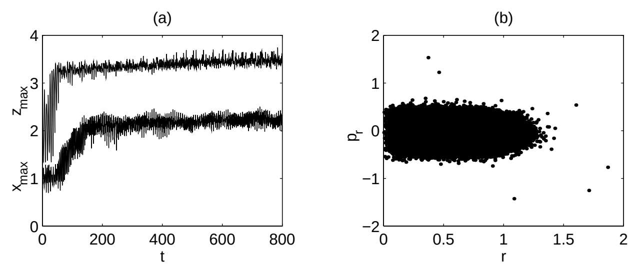

Figure 6: Coupling effect for the 6-D stationary distribution with zero transverse mismatch $\mu_x = \mu_y = 1.0$, $\mu_z = 1.5$ ($c/a = 2$, $\eta_x = 0.55$, $\eta_z = 0.45$) a) maximum x and z b) $r - p_r$ diagram at $t = 800$ for particles with the angular momentum $|L_z| < 0.1$ (with 25,000 particles plotted).

is mismatched in both the transverse and longitudinal directions, and find that both transverse and longitudinal halos can develop, depending on the choice of tune depressions and mismatches. We also found that the effect of coupling between the r and z planes is very important in the halo development mechanism and can lead to serious consequencies, especially as the bunch shape becomes more spherical.

Our main conclusion is that the longitudinal halo is of great importance because it develops earlier than the transverse halo for elongated bunches with comparable longitudinal and transverse mismatches, and because it occurs even for mismatches of order 10%. In addition, the control of the longitudinal halo could be challenging if the phase width of a beam bunch in the RF bucket cannot be made sufficiently small.

After we established the parameters which lead to halo formation in 3-D beam bunches for the self-consistent 6-D phase space stationary distribution [9], we explored rms matched distributions which are *not* self-consistent, to determine the extent to which the relatively rapid redistribution of the 6-D phase space contributes to the formation of halos [17, 18].

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