# Towards $V_{ub}$: A Study of the $b$ Quark Shape Function using the Belle $B \to X_s\gamma$ Photon Spectrum

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## 1 Introduction

In this note we detail the extraction of the shape function parameters, $m_b$ and $\mu_\pi^2$ from the Belle measured $B \to X_s\gamma$ photon spectrum.

## 2 The Belle Raw $B \to X_s\gamma$ Photon Spectrum

Belle has made a measurement of the $B \to X_s\gamma$ photon spectrum using the Belle data set[1]. Belle measures signal yields for photon energies between 1.5 and 2.8 GeV. For the first time ever significant yields were extracted for the region 1.8-2.0 GeV. The analysis used a 140/15 fb$^{-1}$ ON/OFF resonance data sample. The raw spectrum is plotted in figure 1.

## 3 Method

The photon spectrum derived from the $B \to X_s\gamma$ MC is fitted to the measured raw photon spectrum. From this fit the shape function parameters, $m_b$ and $\mu_\pi^2$ are obtained.

**Shape function forms** The three shape function ansatz are described in table 1 and plotted in figure 2. The plot previews the results by using parameter values corresponding to the best fits - Exponential $m_b = 4.60$ GeV/$c^2$ & $\mu_\pi^2 = 0.47$ GeV$^2$/$c^2$ , Gaussian $m_b = 4.64$ GeV/$c^2$ & $\mu_\pi^2 = 0.36$ GeV$^2$/$c^2$ and Roman $m_b = 4.60$ GeV/$c^2$ & $\mu_\pi^2 = 0.45$ GeV$^2$/$c^2$ . Notice there are only slight differences between the curves.

**Raw MC spectra** It is not feasible to generate signal MC with all the variations of shape function form and parameter ranges. Instead we make use of a large inclusive $B \to X_s\gamma$ MC sample wherein the $M_{X_s}$ spectrum is

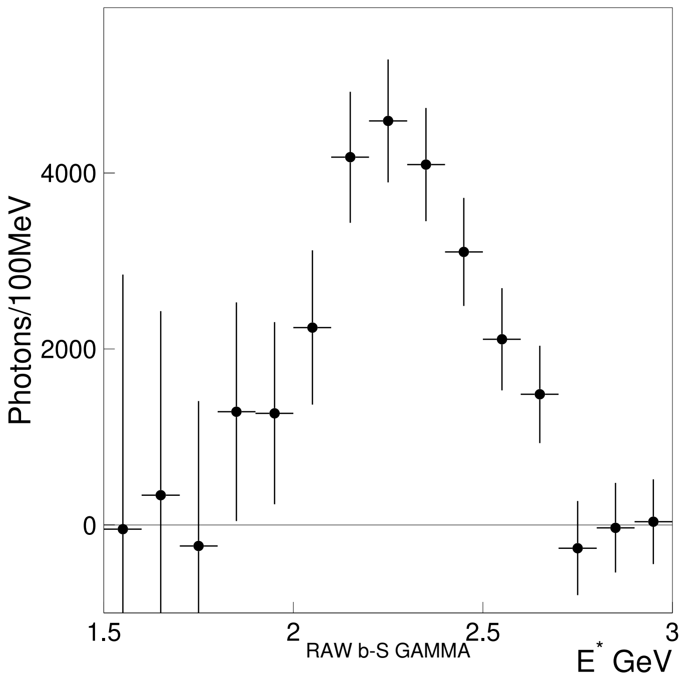


Figure 1: Raw $B \to X_s\gamma$ photon energy spectrum

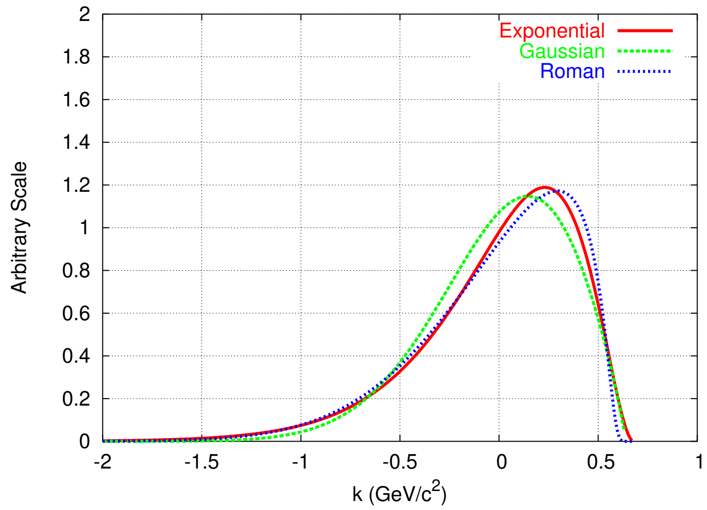


Figure 2: Shape function curves

| Shape Function | Form |
|---|---|
| Exponential | $F(k_+;a) = N(1-x)^a e^{(1+a)x}$ |
| Gaussian | $F(k_+;c) = N(1-x)^c e^{-b(1-x)^2}$<br>$b = \Gamma(\frac{c+2}{2})/\Gamma(\frac{c+1}{2})$ |
| Roman | $F(k_+;\rho) = N\frac{1}{\sqrt{\pi}}\frac{\bar{\Lambda}}{p_F}\exp\big\{-\frac{1}{4}\big(\frac{p_F}{\bar{\Lambda}}\frac{\rho}{1-x} - \frac{\bar{\Lambda}}{p_F}(1-x)\big)^2\big\}$<br>$\frac{\bar{\Lambda}}{p_F} = \frac{\rho}{\sqrt{\pi}}e^{\rho/2}K_1(\rho/2)$ |
| | $x = \frac{k_+}{\bar{\Lambda}}$<br>$-m_b \leq k_+ \leq \bar{\Lambda}$<br>$a, c, \rho, N$ are chosen<br>to satisfy<br>$A_0 = 1$ $A_1 = 0$ $A_2 = \mu_\pi^2/3$<br>$A_n = \int_{-m_b}^{\bar{\Lambda}} F(k_+)dk_+$ |

Table 1: Shape functions

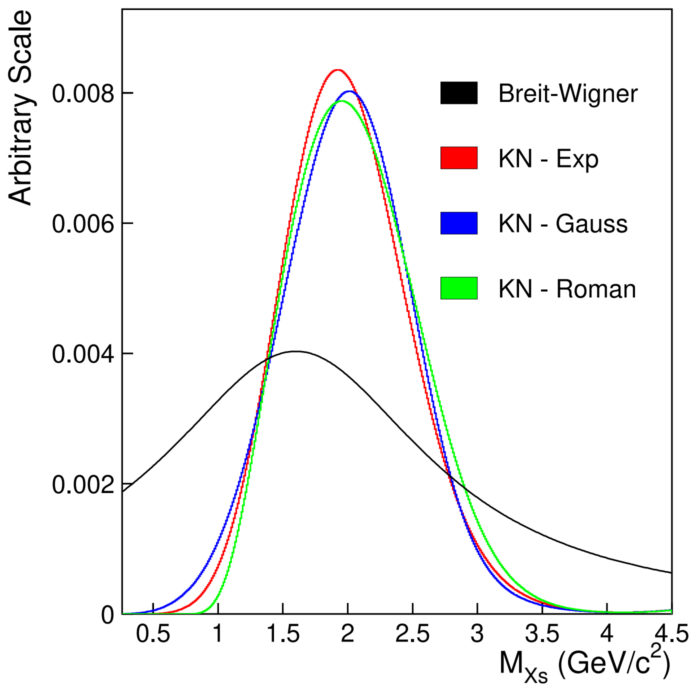


Figure 3: $M_{X_s}$ spectra

generated according to a Breit-Wigner form (mass $m = 1.6$ GeV/$c^2$ and width $\Gamma = 2.5$ GeV). The MC is reweighted in $M_{X_s}$ to agree with spectra calculated by Kagan and Neubert[2]. Generated MC and Kagan-Neubert (KN) calculated $M_{X_s}$ spectra are displayed in figure 3.

MC spectra are obtained after the application of the Belle $B \to X_s\gamma$ analysis cuts. Literally thousands of spectra are produced, spanning three shape function forms; Exponential, Gaussian and Roman, with $m_b = 4.35 - 4.75$ GeV/$c^2$ and $\mu_\pi^2 = 0.10 - 1.50$ GeV$^2$/$c^2$.

**Fits** Chi-squared fits of the MC photon spectra to the raw data spectrum are performed. The normalisation parameter is floated in the fit. For each shape function form we associate the best fit parameters to the spectra with the minimum fit chi-squared, $\chi^2_{\rm min}$, and measure the $1\sigma$ ellipse defined as $\chi^2(m_b, \mu_\pi^2) = \chi^2_{\rm min} + 1$.

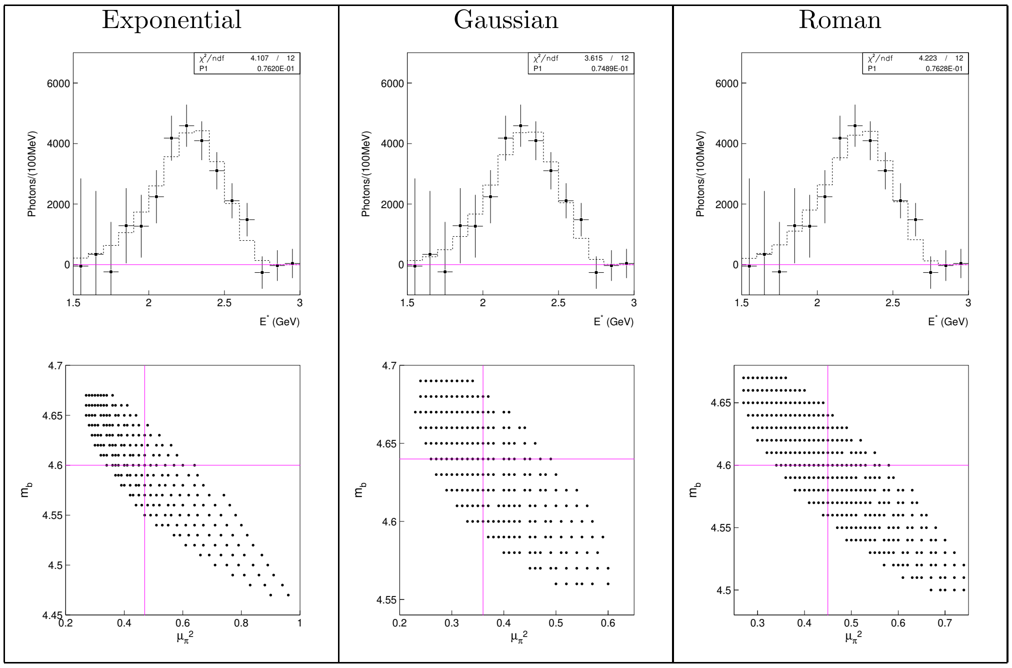


Figure 4: The best fit results

# 4 Results

The best fit and one sigma ellipse results of fits to the raw data spectrum are displayed in figure 4 and table 2 for each shape function form. We find the best fit for parameters $m_b$ and $\mu_\pi^2$ are consistent across all three shape function forms. The value for $m_b$ confirms the Belle result for the first moment of the corrected spectrum $< E_\gamma >= 2.292 \pm 0.026 \pm 0.034$ GeV since $2 < E_\gamma > \approx m_b = 4.58$. This also explains the discrepancy with the CLEO data as they measure $< E_\gamma >= 2.346 \pm 0.032 \pm 0.011$ GeV.

| Belle | | | | | | |
|---|---|---|---|---|---|---|
| Shape Function | Central $c$ | | Extreme1 $e_1$ | | Extreme2 $e_2$ | |
| | $m_b$ | $\mu_\pi^2$ | $m_b$ | $\mu_\pi^2$ | $m_b$ | $\mu_\pi^2$ |
| Exponential | 4.60 | 0.47 | 4.67 | 0.27 | 4.47 | 0.96 |
| Gaussian | 4.64 | 0.36 | 4.69 | 0.23 | 4.56 | 0.60 |
| Roman | 4.60 | 0.45 | 4.67 | 0.27 | 4.50 | 0.74 |

Table 2: The best fit shape parameter values

## References


[1] P. Koppenburg *et al.* [Belle Collaboration], arXiv:hep-ex/0403004.

[2] A. L. Kagan and M. Neubert, Eur. Phys. J. C **7** (1999) 5 [arXiv:hep-ph/9805303].