Semileptonic B Decays

Contents

1 Semileptonic B Decays

The charge of the working group "Semileptonic B Decay Parameters" consists in

- the determination of the best values of the inclusive and exclusive semileptonic B decay branching ratios from the combined data, both for Cabibbo-favored and Cabibbo-suppressed decays, fully taking into account correlations between the experiments;
- determining the best possible values of $F(1)|V_{cb}|$ from exclusive semileptonic B decays and of $|V_{cb}|$ from the inclusive semileptonic B decay rate;
- a detailed understanding of the correlated theoretical errors and the encouragement of a uniform and consistent error estimation among the active experiments.

Currently, this program is implemented to varying degrees. The work of the "LEP Heavy Flavor Steering Group" has resulted in well-understood averages for Cabibbo-favored exclusive and inclusive semileptonic decays (see, *e.g.*, Ref. [1]).

The situation is different for Cabibbo-suppressed semileptonic B decays. In this edition of the HFAG updates, we present an average of inclusive determinations of $|V_{ub}|$. Work is in progress to achieve the same for exclusive decays (see Ref. [2] for an alternative best estimate of $|V_{ub}|$).

In the following a detailed description of all parameters and published results relevant for the determination of the combined results is provided. The description is based on the information available on the web-page at

`http://www.slac.stanford.edu/xorg/hfag/semi/winter04/winter04.shtml`

1.1 Introduction

In the combination of the published results, the central values and errors are rescaled to a common set of input parameters, summarized in Table 1 and provided in the file `common.param` (accessible from the web-page). All measurements with a dependency on any of these parameters are rescaled to the central values given in Table 1, and their error is recalculated based on the error provided in the column "Excursion". The detailed dependency for each measurement is contained in files (provided by the experiments) accessible from the web-page.

Table 1: Common input parameters for the combination of semileptonic B decays. Most of the parameters are taken from Ref. [3]. This table is encoded in the file `common.param`.

Parameter	Assumed Value	Excursion	Description
rb	21.646	± 0.065	R_b
bdst	1.27	± 0.021	$\mathcal{B}(\overline{B} \to D^* \tau \overline{\nu})$
bdsd	1.62	± 0.040	$\mathcal{B}(\overline{B} \to D^* D)$
bdst2	0.65	± 0.013	$\mathcal{B}(b \to D^* \tau)$ (OPAL incl)
bdsd2	4.2	± 1.5	$\mathcal{B}(b \to D^* D)$ (OPAL incl)
bdsd3	0.87	$^{+0.23}_{-0.19}$	$\mathcal{B}(b \to D^* D)$ (DELPHI incl)
xe	0.702	± 0.008	B fragmentation: $\langle E_B \rangle / E_{beam}$
bdsi	17.3	± 2.0	$\mathcal{B}(b \to D^{*+}$ incl)
cdsi	22.6	± 1.4	$\mathcal{B}(c \to D^{*+}$ incl)
tb0	1.536	± 0.014	$\tau(B^0)$
tbplus	1.671	± 0.018	$\tau(B^+)$
tbps	1.461	± 0.057	$\tau(B_s)$
fbd	39.7	± 1.0	B_d fraction at $\sqrt{s} = m_{Z^0}$
fbs	10.7	± 1.1	B_s fraction at $\sqrt{s} = m_{Z^0}$
fbar	9.85	± 1.7	Baryon fraction at $\sqrt{s} = m_{Z^0}$
dst	67.7	± 0.5	$\mathcal{B}(D^{*+} \to D\pi^+)$
dkpp	9.10	± 0.7	$\mathcal{B}(D^+ \to K - \pi^+\pi^+)$
dkp	3.85	± 0.09	$\mathcal{B}(D^0 \to K - \pi^+)$
dkpzp	13.1	± 1.6	$\mathcal{B}(D^0 \to K - \pi^+\pi^0)$
dkppp	7.46	± 0.31	$\mathcal{B}(D^0 \to K - \pi^+\pi^+\pi^-)$
dkzpp	2.99	± 0.18	$\mathcal{B}(D^0 \to K^0\pi^+\pi^-)$
dkln	7.0	± 0.4	$\mathcal{B}(D^0 \to K^-\ell^+\nu)$
dkk	4.3	± 0.2	$\mathcal{B}(D^0 \to K^-K^+)$
dkx	1.1	± 0.025	$K^-\pi^+X$ rates
dkox	0.42	± 0.05	$\mathcal{B}(D^0 \to K^0 X)$
dnlx	6.87	± 0.28	$\mathcal{B}(D^0 \to X \ell \overline{\nu})$
dkpcl	61.2	± 2.9	$\mathcal{B}(D^{*0} \to D^0\pi^0)$
dssR	0.64	± 0.11	$\mathcal{B}(b \to D^{**}\ell\overline{\nu}) \times \mathcal{B}D^{**} \to D^{*+}X$
fb0	0.489	± 0.012	$f_0 = \mathcal{B}(\Upsilon(4S) \to B^0\overline{B}^0)$
chi	0.091	± 0.003	$\chi = \chi_d f_0$

1.2 Exclusive Cabibbo-favored Decays

Aspects of the phenomenology of exclusive Cabibbo-favored B decays and their use in the determination of $|V_{cb}|$ in the context of Heavy Quark Effective Theory (HQET) are described in many places, *e.g.*, in Ref. [1] and will not be repeated here.

Averages are provided for both the branching ratios $\mathcal{B}(\overline{B} \to D^* \ell \overline{\nu})$ plus $\mathcal{B}(\overline{B} \to D \ell \overline{\nu})$ and the CKM matrix element $|V_{cb}|$ multiplied by the form factor at zero recoil of the decay $\overline{B} \to D^* \ell \overline{\nu}$ and $\overline{B} \to D \ell \overline{\nu}$, respectively.

1.2.1 $\overline{B} \to D^* \ell \overline{\nu}$

The measurements included in the average are scaled to a consistent set of input parameters and their errors, and therefore some of the (older) measurements are subject to considerable adjustments:

- The published results by ALEPH [4] and BELLE [7] as quoted in the abstracts of their respective papers are based on a linear parametrization of the form factor and have been redetermined. In order to obtain a model-independent error for the form factor uncertainties, the errors of both analyses have also been recomputed to be based on the CLEO measurement instead of the published error based on QCD sum rules.
- Updates in the branching fractions of D mesons and the production of D^{**} mesons in B decays have lead to increased rates for most of the measurements.
- The average B^0 lifetime has changed considerably since the ALEPH measurement with the much higher precision available at the B-factories.
- The production of B^0 meson at $\sqrt{s} = m_{Z^0}$ has a direct impact on all LEP measurements. Adjusting results on the $\Upsilon(4S)$ for the branching fraction of $\mathcal{B}(\Upsilon(4S) \to B^0\overline{B}^0) \equiv f_{00}$ yields an increase compared to the assumption of $f_{00} = 0.5$.
- Many input parameters are now better known—this decreases some of the systematic errors of the rescaled results with respect to the original publication.

The largest correlated errors are the fraction of B^0 mesons (fbd and fb0, respectively), the form factors $R_1(1)$ and $R_2(1)$ at zero recoil, B^0 meson lifetime, branching fractions of D mesons, and the details of D^{**} modeling.

At LEP, the measurements have been done both with “inclusive” analyses based on a partial reconstruction of the $\overline{B} \to D^* \ell \overline{\nu}$ decay and a full reconstruction of the exclusive decay. The average branching ratio $\mathcal{B}(\overline{B} \to D^*\ell\overline{\nu})$ is determined in a one-dimensional fit from the measurements provided in Table 2. The statistical correlation between the results from the same experiment (DELPHI and OPAL) is taken into account. The $\chi^2/dof = 17.3/7$ is below the level where the PDG starts to introduce scale factors. It is dominated by the two measurements by CLEO and *BABAR*, respectively.

Table 2: Average branching ratio $\mathcal{B}(\overline{B} \to D^*\ell\overline{\nu})$ and individual results.

Experiment	$\mathcal{B}(\overline{B} \to D^*\ell\overline{\nu})[\%]$ (rescaled)	$\mathcal{B}(\overline{B} \to D^*\ell\overline{\nu})[\%]$ (published)
ALEPH (excl) [4]	$5.74 \pm 0.26_{stat} \pm 0.36_{syst}$	$5.53 \pm 0.26_{stat} \pm 0.52_{syst}$
OPAL (excl) [5]	$5.48 \pm 0.19_{stat} \pm 0.50_{syst}$	$5.11 \pm 0.19_{stat} \pm 0.49_{syst}$
OPAL (incl) [5]	$6.19 \pm 0.27_{stat} \pm 0.58_{syst}$	$5.92 \pm 0.27_{stat} \pm 0.68_{syst}$
DELPHI (incl) [6]	$5.03 \pm 0.13_{stat} \pm 0.36_{syst}$	$4.70 \pm 0.13_{stat}\ {}^{+0.36}_{-0.31}\ {}_{syst}$
BELLE (excl) [7]	$4.70 \pm 0.23_{stat} \pm 0.43_{syst}$	$4.59 \pm 0.23_{stat} \pm 0.49_{syst}$
CLEO (excl) [8]	$6.14 \pm 0.19_{stat} \pm 0.39_{syst}$	$6.09 \pm 0.19_{stat} \pm 0.40_{syst}$
DELPHI (excl) [9]	$5.77 \pm 0.22_{stat} \pm 0.57_{syst}$	$5.90 \pm 0.22_{stat} \pm 0.50_{syst}$
BABAR (excl) [10]	$4.60 \pm 0.03_{stat} \pm 0.27_{syst}$	$4.68 \pm 0.03_{stat} \pm 0.29_{syst}$
Average	$\mathbf{5.13 \pm 0.19}$	$\boldsymbol{\chi^2/dof = 17.2/7}$

The average for $F(1)|V_{cb}|$ is determined by the two-dimensional combination the results provided in Table 3. This allows to fully incorporate the correlation between $F(1)|V_{cb}|$ and ρ^2.

For a determination of $|V_{cb}|$, the form factor at zero recoil $F(1)$ needs to be computed. A possible choice is $F(1) = 0.91 \pm 0.04_{theo}$ [11], resulting in

$$|V_{cb}| = 40.1 \pm 0.9_{exp} \pm 1.8_{theo}$$

The $\chi^2/dof = 29.7/14$ is below the level where the PDG starts to introduce scale factors. It is dominated by the two measurements by CLEO and *BABAR*, respectively.

1.2.2 $\overline{B} \to D\ell\overline{\nu}$

The average branching ratio $\mathcal{B}(\overline{B} \to D\ell\overline{\nu})$ is determined by the combination the results provided in Table 4. The error sources here are the same as discussed in section 1.2.1, but generally at a higher level due to larger background levels and less stringent kinematic constraints.

The average for $G(1)|V_{cb}|$ is determined by the two-dimensional combination the results provided in Table 5.

Table 3: Average of $F(1)|V_{cb}|$ determined in the decay $\overline{B} \to D^* \ell \overline{\nu}$ and individual results. The fit for the average has $\chi^2/dof = 29.7/14$

Experiment	$F(1)\|V_{cb}\|[10^{-3}]$ (rescaled) $F(1)\|V_{cb}\|[10^{-3}]$ (published)	ρ^2 (rescaled) ρ^2 (published)
ALEPH (excl) [4]	$33.6 \pm 2.1_{stat} \pm 1.6_{syst}$ $32.0 \pm 2.1_{stat} \pm 2.0_{syst}$	$0.75 \pm 0.25_{stat} \pm 0.37_{syst}$ $0.37 \pm 0.26_{stat} \pm 0.14_{syst}$
OPAL (incl) [5]	$38.5 \pm 1.2_{stat} \pm 2.4_{syst}$ $37.5 \pm 1.2_{stat} \pm 2.6_{syst}$	$1.25 \pm 0.14_{stat} \pm 0.39_{syst}$ $1.12 \pm 0.14_{stat} \pm 0.29_{syst}$
OPAL (excl) [5]	$39.1 \pm 1.6_{stat} \pm 2.3_{syst}$ $36.8 \pm 1.6_{stat} \pm 2.0_{syst}$	$1.48 \pm 0.21_{stat} \pm 0.27_{syst}$ $1.31 \pm 0.21_{stat} \pm 0.16_{syst}$
DELPHI (incl) [6]	$36.9 \pm 1.4_{stat} \pm 2.5_{syst}$ $35.5 \pm 1.4_{stat} \, {}^{+2.3}_{-2.4}{}_{syst}$	$1.50 \pm 0.14_{stat} \pm 0.37_{syst}$ $1.34 \pm 0.14_{stat} \, {}^{+0.24}_{-0.22}{}_{syst}$
BELLE (excl) [7]	$36.3 \pm 1.9_{stat} \pm 1.9_{syst}$ $35.8 \pm 1.9_{stat} \pm 1.9_{syst}$	$1.45 \pm 0.16_{stat} \pm 0.20_{syst}$ $1.45 \pm 0.16_{stat} \pm 0.20_{syst}$
CLEO (excl) [8]	$43.3 \pm 1.3_{stat} \pm 1.8_{syst}$ $43.1 \pm 1.3_{stat} \pm 1.8_{syst}$	$1.61 \pm 0.09_{stat} \pm 0.21_{syst}$ $1.61 \pm 0.09_{stat} \pm 0.21_{syst}$
DELPHI (excl) [9]	$38.7 \pm 1.8_{stat} \pm 2.2_{syst}$ $39.2 \pm 1.8_{stat} \pm 2.3_{syst}$	$1.32 \pm 0.15_{stat} \pm 0.33_{syst}$ $1.32 \pm 0.15_{stat} \pm 0.33_{syst}$
BABAR (excl) [10]	$33.9 \pm 0.2_{stat} \pm 1.1_{syst}$ $34.0 \pm 0.2_{stat} \pm 1.3_{syst}$	$1.23 \pm 0.02_{stat} \pm 0.27_{syst}$ $1.23 \pm 0.02_{stat} \pm 0.28_{syst}$
Average	$\mathbf{36.5 \pm 0.8}$	$\mathbf{1.47 \pm 0.13}$

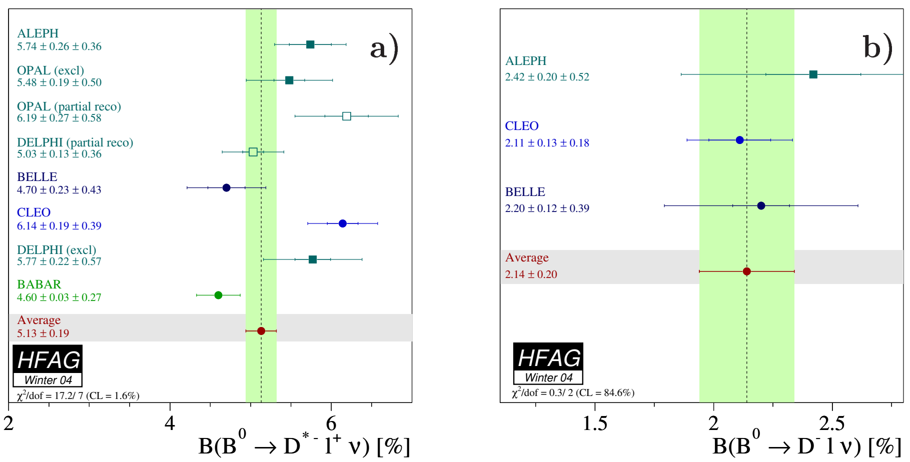

Figure 1: Average branching ratio of exclusive semileptonic B decays. (a) $\overline{B} \to D^* \ell \overline{\nu}$ and (b) $\overline{B} \to D \ell \overline{\nu}$.

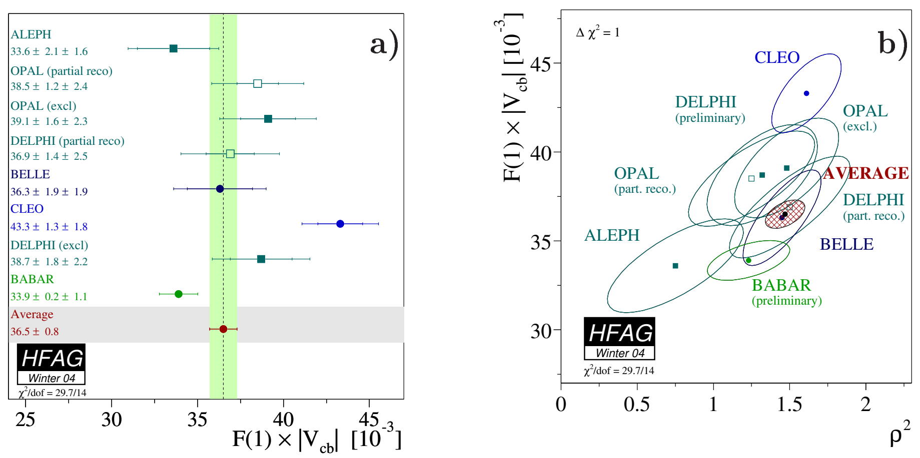

Figure 2: (a) Illustration of the average $F(1)|V_{cb}|$ and rescaled measurements of exclusive $\overline{B} \to D^* \ell \overline{\nu}$ decays determined in a two-dimensional fit. (b) Illustration of $F(1)|V_{cb}|$ vs. ρ^2. The error ellipses correspond to $\Delta\chi^2 = 1$.

Table 4: Average of the branching ratio $\mathcal{B}(\overline{B} \to D\ell\overline{\nu})$ and individual results.

Experiment	$\mathcal{B}(\overline{B} \to D\ell\overline{\nu})[\%]$ (resc.)	$\mathcal{B}(\overline{B} \to D\ell\overline{\nu})[\%]$ (publ.)
ALEPH [4]	$2.42 \pm 0.20_{stat} \pm 0.50_{syst}$	$2.35 \pm 0.20_{stat} \pm 0.44_{syst}$
CLEO [12]	$2.11 \pm 0.13_{stat} \pm 0.17_{syst}$	$2.20 \pm 0.16_{stat} \pm 0.19_{syst}$
BELLE [13]	$2.21 \pm 0.12_{stat} \pm 0.39_{syst}$	$2.13 \pm 0.12_{stat} \pm 0.39_{syst}$
Average	$\mathbf{2.14 \pm 0.20}$	$\boldsymbol{\chi^2/dof = 0.3/2}$

For a determination of $|V_{cb}|$, the form factor at zero recoil $G(1)$ needs to be computed. A possible choice is $G(1) = 1.04 \pm 0.06_{theo}$ [11], resulting in

$$|V_{cb}| = 40.2 \pm 3.6_{exp} \pm 2.3_{theo}$$

Table 5: Average of $G(1)|V_{cb}|$ determined in the decay $\overline{B} \to D\ell\overline{\nu}$ and individual results. The fit for the average has $\chi^2/dof = 0.3/4$.

Experiment	$G(1)\|V_{cb}\|[10^{-3}]$ (rescaled) $G(1)\|V_{cb}\|[10^{-3}]$ (published)	ρ^2 (rescaled) ρ^2 (published)
ALEPH [4]	$39.2 \pm 10.0_{stat} \pm 6.4_{syst}$ $31.1 \pm 9.9_{stat} \pm 8.6_{syst}$	$0.97 \pm 0.98_{stat} \pm 0.38_{syst}$ $0.20 \pm 0.98_{stat} \pm 0.50_{syst}$
CLEO [12]	$44.4 \pm 5.8_{stat} \pm 3.5_{syst}$ $44.8 \pm 0.61_{stat} \pm 0.37_{syst}$	$1.27 \pm 0.25_{stat} \pm 0.14_{syst}$ $1.27 \pm 0.25_{stat} \pm 0.14_{syst}$
BELLE [13]	$41.7 \pm 4.4_{stat} \pm 5.2_{syst}$ $41.1 \pm 4.4_{stat} \pm 5.1_{syst}$	$1.12 \pm 0.22_{stat} \pm 0.14_{syst}$ $1.12 \pm 0.22_{stat} \pm 0.14_{syst}$
Average	$\mathbf{41.8 \pm 3.7}$	$\mathbf{1.15 \pm 0.16}$

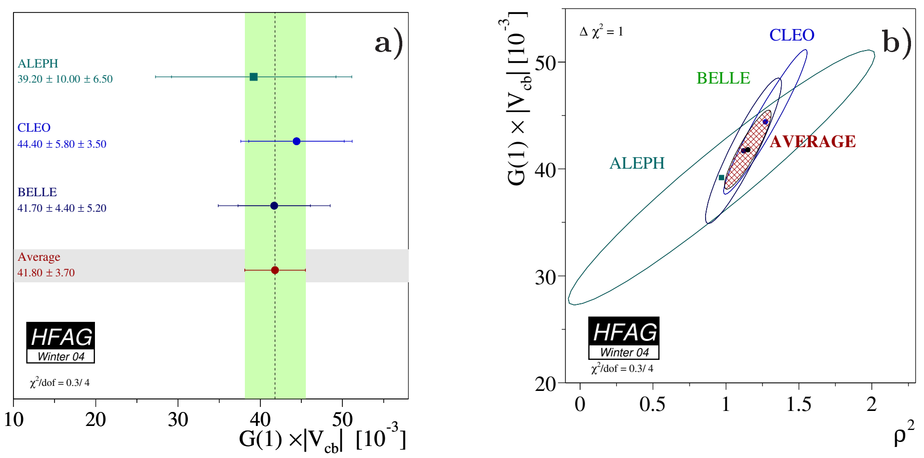

Figure 3: (a) Illustration of the average $G(1)|V_{cb}|$ and rescaled measurements of exclusive $\overline{B} \to D\ell\overline{\nu}$ decays determined in a two-dimensional fit. (b) Illustration of $F(1)|V_{cb}|$ vs. ρ^2. The error ellipses correspond to $\Delta\chi^2 = 1$.

1.3 Inclusive Cabibbo-favored Decays

Aspects of the theory and phenomenology of inclusive Cabibbo-favored B decays and their use in the determination of $|V_{cb}|$ in the context of the Heavy Quark Expansion (HQE), an Operator Product Expansion based on HQET, are described in many places, *e.g.*, in Ref. [1] and references therein, and will not be repeated here.

Averages are provided for the total semileptonic branching ratio $\mathcal{B}(\overline{B} \to X\ell\overline{\nu})$. In the future, it is foreseen to also provide averages for the partial semileptonic branching fraction $\mathcal{B}(\overline{B} \to X\ell\overline{\nu}; E_\ell > 0.6\,\mathrm{GeV})$.

1.3.1 Total Semileptonic Branching Fraction

The average for the total branching ratio $\mathcal{B}(\overline{B} \to X\ell\overline{\nu})$ is determined by the combination the results provided in Table 6. In this average, the extrapolation of the measured rate to the total decay rate is performed by each experiment, usually with a fit of several components to the experimental spectrum. The measurements and the average are illustrated in Figure **??**. It is foreseen to also provide an average of the partial branching fraction in the future. The extrapolation to the full spectrum is not necessary a priori, as the HQE allows the direct determination of $|V_{cb}|$ from a partial rate.

Table 6: Average of the total semileptonic branching fractions $\mathcal{B}(\overline{B} \to X\ell\overline{\nu})$ determined in tagged measurements on the $\Upsilon(4S)$.

Experiment		$\mathcal{B}_{tot}(\overline{B} \to X\ell\overline{\nu})[\%]$ (resc.)	$\mathcal{B}_{tot}(\overline{B} \to X\ell\overline{\nu})[\%]$ (publ.)
ARGUS (ℓ-tag)	[14]	$9.75 \pm 0.50 \pm 0.60$	$9.70 \pm 0.50 \pm 0.60$
BABAR(B_{reco}-tag)	[15]	$10.40 \pm 0.50 \pm 0.46$	$10.40 \pm 0.50 \pm 0.46$
BELLE (ℓ-tag)	[16]	$10.96 \pm 0.12 \pm 0.50$	$10.90 \pm 0.12 \pm 0.49$
BABAR(e-tag)	[17]	$10.91 \pm 0.18 \pm 0.29$	$10.87 \pm 0.18 \pm 0.30$
BELLE (B_{reco}-tag)	[18]	$11.19 \pm 0.20 \pm 0.29$	$11.19 \pm 0.20 \pm 0.31$
CLEO (ℓ-tag)	[19]	$10.91 \pm 0.08 \pm 0.30$	$10.88 \pm 0.08 \pm 0.33$
Average		$\mathbf{10.90 \pm 0.23}$	$\boldsymbol{\chi^2/dof = 5.0/5}$

1.4 Exclusive Cabibbo-suppressed Decays

Here we list results on exclusive determinations of $|V_{ub}|$. An average of (these) exclusive $b \to u\ell\overline{\nu}$ results is envisioned for the future. The measurements are separated into two classes: a first one averaging over the entire Q^2 range (shown in Table 7), and a second class, where the decay rate is measured differentially in (few) bins of Q^2 (shown in Table 8).

Table 7: Summary of exclusive determinations of $\mathcal{B}(\overline{B} \to X\ell\overline{\nu})$ and $|V_{ub}|$ using the entire Q^2 range. The errors quoted correspond to statistical, experimental systematic and theoretical systematic, respectively.

Experiment	Mode	$\mathcal{B}[10^{-4}]$	$\lvert V_{ub}\rvert$ $[10^{-3}]$ (resc.)
CLEO [21]	$B^0 \to \rho^-\ell^+\nu$	$2.69 \pm 0.41\ ^{+0.35}_{-0.40} \pm 0.50$	$3.24 \pm 0.25\ ^{+0.21}_{-0.24} \pm 0.58$
BABAR [22]	$B^0 \to \rho^-\ell^+\nu$	$3.29 \pm 0.42 \pm 0.47 \pm 0.60$	$3.59 \pm 0.23 \pm 0.26 \pm 0.66$
BELLE [23]	$B^0 \to \omega\ell^+\nu$	$1.3 \pm 0.4 \pm 0.2 \pm 0.3$	$3.1 \pm 0.2 \pm 0.2 \pm 0.6$

Table 8: Summary of exclusive determinations of $\mathcal{B}(\overline{B} \to X\ell\overline{\nu})$ and $|V_{ub}|$ binned in Q^2. The errors quoted correspond to statistical, experimental systematic, theoretical systematic, and signal form-factor shaoe, respectively.

Experiment	Mode	$\mathcal{B}[10^{-4}]$	$\lvert V_{ub}\rvert$ $[10^{-3}]$ (resc.)
CLEO [24]	$B^0 \to \pi^-\ell^+\nu$	$1.33 \pm 0.18 \pm 0.11 \pm 0.01 \pm 0.07$	$3.24 \pm 0.22 \pm 0.13\ ^{+0.55}_{-0.39} \pm 0.09$
CLEO [24]	$B^0 \to \rho^-\ell^+\nu$	$2.17 \pm 0.34\ ^{+0.47}_{-0.54} \pm 0.41 \pm 0.01$	$3.00 \pm 0.21\ ^{+0.29}_{-0.35}\ ^{+0.49}_{-0.38} \pm 0.28$

1.5 Inclusive Cabibbo-suppressed Decays

A recent discussion of the theoretical issues and errors is provided, *e.g.*, in Refs. [25] and [26]. An independent estimate of the size of the theoretical uncertainties can be found in Ref. [2]. The following description is focusing on a technical description of the relevant details of the averaging procedure.

1.5.1 Determination of $|V_{ub}|$

Inclusive determinations of $|V_{ub}|$ based on charmless semileptonic B decays have been presented by all four LEP experiments [27] and, more recently, by the experiments at the $\Upsilon(4S)$. Here, we combine measurements of the *BABAR*, BELLE, and CLEO collaborations as they present measurements in well-defined phase-space regions and are based on the same theoretical description of the decay in terms of an OPE. Due to the large background from Cabibbo-favored $\overline{B} \to X_c \ell \overline{\nu}$ decays, the measurements are sensitive to $B \to X_u \ell \overline{\nu}$ decays only in restricted regions of phase-space. The convergence of the OPE is affected by these restrictions and complicate the extrapolation to the full rate and thus the determination of $|V_{ub}|$.

In the theoretical description, the motion of the b quark inside the B meson is described with a "shape function", usually dependent on two non-perturbative parameters. These parameters can be determined from a measurement of the photon energy spectrum in the rare decay $b \to s\gamma$. This decay has a smaller branching fraction than the decay $B \to X_u \ell \overline{\nu}$, which limits the precision possible in this approach. Recently, there has been theoretical interest to use moments measured in semileptonic $\overline{B} \to X_c \ell \overline{\nu}$ decays for the determination of the shape function parameters.

In the average $|V_{ub}|$ presented here, all measurements are scaled to a common set of the shape function parameters and their uncertainties. Both are taken from the photon energy spectrum in $B \to s\gamma$ decays as measured by the CLEO collaboration [28, 29]. The (large) errors of the shape function parameters are dominated by statistical uncertainties in the measurement of the photon energy spectrum.

For the average, the systematic error is divided into the categories shown in table 9. In the averaging procedure, the errors belonging to a specific category are taken as 100% correlated between the different measurements. The numerical values of the error for each measurement is given in a spreadsheet [30].

In table 9, the largest and most critical uncertainties are the extrapolation "fu" and the additional errors not provided by the experiments:

- fu: This error estimates the uncertainty in the extrapolation from the observed phase-space to the full phase-space. For this, all measurements

Table 9: Errors in the average of $|V_{ub}|$. The numerical values for all measurements can be found in a spreadsheet [30]. The entries in the "Name" column correspond to the names in the spreadsheet.

Name	Explanation
Uncorr	**Quadratic sum of statistical and experimental errors**
Statistical	Statistical
Exp	Experimental detector systematics (uncorrelated)
Corr	**Quadratic sum of correlated errors**
B2C(bg)	Modeling of $\overline{B} \to X_c \ell \overline{\nu}$ (Branching fractions, form factors, etc)
B2U(eff)	Modeling of $B \to X_u \ell \overline{\nu}$ (Branching fractions, ε_{vis}, $s\overline{s}$ popping, etc)
BF→Vub	Theoretical error $\Gamma \to |V_{ub}|$ (m_b, α_s)
fu(extrapol)	Extrapolation from visible range
Additional errors not provided by the experiments	
SSF	Subleading shape functions (power corrections)
WA	Weak annihilation
QHD	(Local) Quark-Hadron Duality

use the triple-differential calculation of Ref. [31]. The shape function is parametrized with the "exponential form". All measurements are rescaled to correspond to the shape function parameters determined from the photon energy spectrum measurement of CLEO [28]. The extrapolation errors for each measurement are determined by the uncertainty of the shape function parameters [29], largely dominated by statistics. Recent theoretical work [32] may allow the extraction of the shape function parameters from the high-precision measurements of moments in semi-leptonic $\overline{B} \to X_c \ell \overline{\nu}$ decays.

- SSF: Uncertainties due to power corrections are summarized into the term labeled "subleading shape function" effects. They have been calculated for the endpoint measurements [35]; the corrections are used as estimate for the error (without scaling the central values). These corrections strongly depend on the fraction of phase-space covered in a measurement. They are much larger for the *BABAR* and Belle endpoint analyses with the higher lepton energy threshold. We use the uncertainty for the case $E_\ell > 2.2$GeV as estimate for the m_X analysis of *BABAR*. This is roughly consistent with theoretical expectations [33]. For the (m_X, Q^2) results, the error is expected to be significantly smaller; here we use half of the m_X error. The SSF errors are summarized in table 10.

- WA: The error for weak annihilation estimates the uncertainty from parametrically enhanced nonperturbative effects predominantly expected at high-Q^2. The estimates in Ref. [34] are used for the (m_X, Q^2) analysis and scaled to the other phase-space regions as shown in table 10. Because of the concentration at high-Q^2, the effects of weak annihilation are diluted as more phase-space is covered.

- QHD: The error due to the violation of quark-hadron duality is estimated for the (m_X, Q^2) analysis in Ref. [2]. It is found to be substantially smaller that the uncertainties from weak annihilation and power corrections. Therefore, we neglect it here.

Table 10: Scaling of theoretical errors in different regions of phase-space. The m_X analysis requires for the hadronic system in $B \to X_u \ell \overline{\nu}$ decays $m_X < 1.55\,\mathrm{GeV}/c^2$, the (m_X, Q^2) analysis selects the phase-space $m_X < 1.7\,\mathrm{GeV}/c^2, Q^2 > 8\,\mathrm{GeV}^2$.

Component	m_X	(m_X, Q^2)	$E_\ell > 2.2$GeV	$E_\ell > 2.3$GeV
SSF	4%	2%	4%	8%
WA	2%	4%	6%	8%

The average of $|V_{ub}|$ is based on the measurements summarized in Table 11 and amounts to (see figure 4)

$$|V_{ub}| = (4.57 \pm 0.61) \times 10^{-3}.$$

The error of this average contains a relative contribution of 5.6% for the translation of $\Gamma(b \to u\ell\overline{\nu})$ to $|V_{ub}|$ and has been derived in Ref. [3]. This error contains contributions from the perturbative expansion and the uncertainty of the b quark mass ($\Delta m_b = 90\,\mathrm{MeV}/c^2$ is used here).

The current extrapolation error on $|V_{ub}|$ is based on the approach of Ref. [31] for the shape function effect, where the shape function parameter errors are based only on the photon energy spectrum. Recent reports [32, 41] studied the radiative corrections to the shape function and the prescription of Ref. [31] must be modified. In particular Bosch *et al.* [32] showed that the sensitivity of the extrapolation to the shape function is reduced for the m_X and (m_X, Q^2) analyses, but becomes larger for the endpoint analyses. Furthermore, it may become possible to substantially improve the precision of the shape function parameters (and hence reduce the extrapolation errors) when using the moments measurements in semileptonic B decays.

1.5.2 Discussion

In the future, improvements to this average will be studied and implemented. The following is a starting point, not an exhaustive list, for possible improvements.

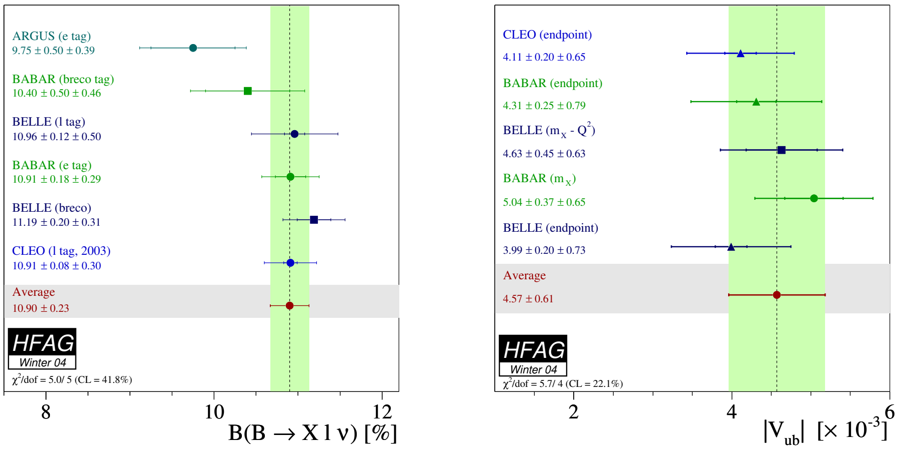

Figure 4: (a) Average $\mathcal{B}(\overline{B} \to X\ell\overline{\nu})$ from inclusive decays. (b) Average $|V_{ub}|$ from inclusive charmless semileptonic B decays.

- The paper by de Fazio and Neubert [31] with a shape function parametrization provides a triple differential decay rate for $B \to X_u\ell\overline{\nu}$ decays that is used in all measurements over the entire phase-space. Recent theoretical work seems to indicate that it can be improved even at its nominal order ($1/m_B$ and α_s) [32, 41].
- Every experiment should provide the result in an unfolded rate so that theoretical improvements can be applied in the future. The idea is to separate the "unfolding" of a rate or distribution from the "extrapolation" (the former is experimental, the latter theoretical).
- All three experiments have a "hybrid" MC model for signal decays. The differential spectra should be compared. Its applicability should be understood (probably good for unfolding, but questionable for extrapolation).
- There should be agreement on one specific recipe to evaluate the common errors. This can include items like
  - $\overline{B} \to X_c\ell\overline{\nu}$ modeling:
    - Variation of inclusive and exclusive branching fractions for B and D decays (full list and range)
    - Variation of form factors describing $\overline{B} \to X_c\ell\overline{\nu}$ decays
  - $B \to X_u\ell\overline{\nu}$ modeling and theoretical error
    - Variation of exclusive branching fraction (full list and range)
    - Comparison with inclusive-only signal model and comparison with exclusive-only signal model (?)

Table 11: Average of $|V_{ub}|$ determined in inclusive measurements on the $\Upsilon(4S)$. The errors quoted for the measurements correspond to uncorrelated and correlated, respectively.

Experiment	Method ([GeV$^{(2)}$])	$\|V_{ub}\|$ [10^{-3}] (resc.)	$\|V_{ub}\|$ [10^{-3}] (publ.)
CLEO [36]	Endpoint, $p_\ell > 2.2$	$4.11 \pm 0.20 \pm 0.65$	$4.08 \pm 0.20 \pm 0.62$
BABAR [37]	Endpoint, $p_e > 2.3$	$4.31 \pm 0.25 \pm 0.79$	$4.43 \pm 0.26 \pm 0.67$
BABAR [38]	$m_X < 1.55$	$5.04 \pm 0.37 \pm 0.65$	$4.62 \pm 0.35 \pm 0.51$
BELLE [39]	$(m_X < 1.70, Q^2 > 8)$	$4.63 \pm 0.45 \pm 0.63$	$4.66 \pm 0.45 \pm 0.61$
BELLE [40]	Endpoint, $p_\ell > 2.3$	$3.99 \pm 0.20 \pm 0.73$	$3.99 \pm 0.20 \pm 0.61$
Average		$\mathbf{4.57 \pm 0.61}$	$\boldsymbol{\chi^2/dof = 7.6/5}$

- $s\bar{s}$ popping (if applicable in the analysis)
- Central values of shape function parameters
- Variation of shape function parameters (this is linked to recent theoretical work)

A close collaboration between the experiments will be necessary to obtain an even more consistent treatment of errors not only for the most critical ones (as shown here), but also for the remaining smaller errors.

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