

# $b$ -hadron production fractions, lifetimes and mixing parameters as of summer 2015

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## **Abstract**

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# Contents

<b>3</b>	<b><i>b</i>-hadron production fractions, lifetimes and mixing parameters</b>	<b>3</b>
3.1	<i>b</i> -hadron production fractions . . . . .	3
3.1.1	<i>b</i> -hadron production fractions in $\Upsilon(4S)$ decays . . . . .	3
3.1.2	<i>b</i> -hadron production fractions in $\Upsilon(5S)$ decays . . . . .	5
3.1.3	<i>b</i> -hadron production fractions at high energy . . . . .	8
3.2	<i>b</i> -hadron lifetimes . . . . .	13
3.2.1	Lifetime measurements, uncertainties and correlations . . . . .	14
3.2.2	Inclusive <i>b</i> -hadron lifetimes . . . . .	16
3.2.3	$B^0$ and $B^+$ lifetimes and their ratio . . . . .	17
3.2.4	$B_s^0$ lifetimes . . . . .	21
3.2.5	$B_c^+$ lifetime . . . . .	24
3.2.6	$\Lambda_b^0$ and <i>b</i> -baryon lifetimes . . . . .	24
3.2.7	Summary and comparison with theoretical predictions . . . . .	26
3.3	Neutral <i>B</i> -meson mixing . . . . .	28
3.3.1	$B^0$ mixing parameters $\Delta\Gamma_d$ and $\Delta m_d$ . . . . .	29
3.3.2	$B_s^0$ mixing parameters $\Delta\Gamma_s$ and $\Delta m_s$ . . . . .	33
3.3.3	<i>CP</i> violation in $B^0$ and $B_s^0$ mixing . . . . .	38
3.3.4	Mixing-induced <i>CP</i> violation in $B_s^0$ decays . . . . .	43
	<b>References</b>	<b>46</b>

### 3 $b$ -hadron production fractions, lifetimes and mixing parameters

Quantities such as  $b$ -hadron production fractions,  $b$ -hadron lifetimes, and neutral  $B$ -meson oscillation frequencies have been studied in the nineties at LEP and SLC ( $e^+e^-$  colliders at  $\sqrt{s} = m_Z$ ) as well as at the first version of the Tevatron ( $p\bar{p}$  collider at  $\sqrt{s} = 1.8$  TeV). Since then precise measurements of the  $B^0$  and  $B^+$  mesons have also been performed at the asymmetric  $B$  factories, KEKB and PEP-II ( $e^+e^-$  colliders at  $\sqrt{s} = m_{\Upsilon(4S)}$ ) while measurements related to the other  $b$  hadrons, in particular  $B_s^0$ ,  $B_c^+$  and  $\Lambda_b^0$ , have been performed at the upgraded Tevatron ( $\sqrt{s} = 1.96$  TeV) and are continuing at the LHC ( $pp$  collider at  $\sqrt{s} = 7$  and 8 TeV). In most cases, these basic quantities, although interesting by themselves, became necessary ingredients for the more complicated and refined analyses at the asymmetric  $B$  factories, the Tevatron and the LHC, in particular the time-dependent  $CP$  asymmetry measurements. It is therefore important that the best experimental values of these quantities continue to be kept up-to-date and improved.

In several cases, the averages presented in this chapter are needed and used as input for the results given in the subsequent chapters. Within this chapter, some averages need the knowledge of other averages in a circular way. This coupling, which appears through the  $b$ -hadron fractions whenever inclusive or semi-exclusive measurements have to be considered, has been reduced drastically in the past several years with increasingly precise exclusive measurements becoming available and dominating practically all averages.

In addition to  $b$ -hadron fractions, lifetimes and mixing parameters, this chapter also deals with the  $CP$ -violating phase  $\phi_s^{c\bar{c}s} \simeq -2\beta_s$ , which is the phase difference between the  $B_s^0$  mixing amplitude and the  $b \rightarrow c\bar{c}s$  decay amplitude, as well as the parameters of  $CP$  violation in the  $B$  mixing amplitudes. The angle  $\beta$ , which is the equivalent of  $\beta_s$  for the  $B^0$  system, is discussed in Chapter ??.

#### 3.1 $b$ -hadron production fractions

We consider here the relative fractions of the different  $b$ -hadron species found in an unbiased sample of weakly decaying  $b$  hadrons produced under some specific conditions. The knowledge of these fractions is useful to characterize the signal composition in inclusive  $b$ -hadron analyses, to predict the background composition in exclusive analyses, or to convert (relative) observed rates into (relative) branching fraction measurements. Many  $B$ -physics analyses need these fractions as input. We distinguish here the following three conditions:  $\Upsilon(4S)$  decays,  $\Upsilon(5S)$  decays, and high-energy collisions (including  $Z^0$  decays).

##### 3.1.1 $b$ -hadron production fractions in $\Upsilon(4S)$ decays

Only pairs of the two lightest (charged and neutral)  $B$  mesons can be produced in  $\Upsilon(4S)$  decays, and it is enough to determine the following branching fractions:

$$f^{+-} = \Gamma(\Upsilon(4S) \rightarrow B^+ B^-) / \Gamma_{\text{tot}}(\Upsilon(4S)), \quad (1)$$

$$f^{00} = \Gamma(\Upsilon(4S) \rightarrow B^0 \bar{B}^0) / \Gamma_{\text{tot}}(\Upsilon(4S)). \quad (2)$$

Table 1: Published measurements of the  $B^+/B^0$  production ratio in  $\Upsilon(4S)$  decays, together with their average (see text). Systematic uncertainties due to the imperfect knowledge of  $\tau(B^+)/\tau(B^0)$  are included. The latest *BABAR* result [1] supersedes the earlier *BABAR* measurements [2, 3].

Experiment and year	Ref.	Decay modes or method	Published value of $R^{+/-00} = f^{+/-}/f^{00}$	Assumed value of $\tau(B^+)/\tau(B^0)$
CLEO, 2001	[4]	$J/\psi K^{(*)}$	$1.04 \pm 0.07 \pm 0.04$	$1.066 \pm 0.024$
<i>BABAR</i> , 2002	[2]	$(c\bar{c})K^{(*)}$	$1.10 \pm 0.06 \pm 0.05$	$1.062 \pm 0.029$
CLEO, 2002	[5]	$D^*\ell\nu$	$1.058 \pm 0.084 \pm 0.136$	$1.074 \pm 0.028$
Belle, 2003	[6]	dilepton events	$1.01 \pm 0.03 \pm 0.09$	$1.083 \pm 0.017$
<i>BABAR</i> , 2004	[3]	$J/\psi K$	$1.006 \pm 0.036 \pm 0.031$	$1.083 \pm 0.017$
<i>BABAR</i> , 2005	[1]	$(c\bar{c})K^{(*)}$	$1.06 \pm 0.02 \pm 0.03$	$1.086 \pm 0.017$
Average			$1.059 \pm 0.027$ (tot)	$1.076 \pm 0.004$

In practice, most analyses measure their ratio

$$R^{+/-00} = f^{+/-}/f^{00} = \Gamma(\Upsilon(4S) \rightarrow B^+B^-)/\Gamma(\Upsilon(4S) \rightarrow B^0\bar{B}^0), \quad (3)$$

which is easier to access experimentally. Since an inclusive (but separate) reconstruction of  $B^+$  and  $B^0$  is difficult, specific exclusive decay modes,  $B^+ \rightarrow x^+$  and  $B^0 \rightarrow x^0$ , are usually considered to perform a measurement of  $R^{+/-00}$ , whenever they can be related by isospin symmetry (for example  $B^+ \rightarrow J/\psi K^+$  and  $B^0 \rightarrow J/\psi K^0$ ). Under the assumption that  $\Gamma(B^+ \rightarrow x^+) = \Gamma(B^0 \rightarrow x^0)$ , *i.e.* that isospin invariance holds in these  $B$  decays, the ratio of the number of reconstructed  $B^+ \rightarrow x^+$  and  $B^0 \rightarrow x^0$  mesons, after correcting for efficiency, is proportional to

$$\frac{f^{+/-} \mathcal{B}(B^+ \rightarrow x^+)}{f^{00} \mathcal{B}(B^0 \rightarrow x^0)} = \frac{f^{+/-} \Gamma(B^+ \rightarrow x^+) \tau(B^+)}{f^{00} \Gamma(B^0 \rightarrow x^0) \tau(B^0)} = \frac{f^{+/-} \tau(B^+)}{f^{00} \tau(B^0)}, \quad (4)$$

where  $\tau(B^+)$  and  $\tau(B^0)$  are the  $B^+$  and  $B^0$  lifetimes respectively. Hence the primary quantity measured in these analyses is  $R^{+/-00} \tau(B^+)/\tau(B^0)$ , and the extraction of  $R^{+/-00}$  with this method therefore requires the knowledge of the  $\tau(B^+)/\tau(B^0)$  lifetime ratio.

The published measurements of  $R^{+/-00}$  are listed in Table 1 together with the corresponding assumed values of  $\tau(B^+)/\tau(B^0)$ . All measurements are based on the above-mentioned method, except the one from Belle, which is a by-product of the  $B^0$  mixing frequency analysis using dilepton events (but note that it also assumes isospin invariance, namely  $\Gamma(B^+ \rightarrow \ell^+X) = \Gamma(B^0 \rightarrow \ell^+X)$ ). The latter is therefore treated in a slightly different manner in the following procedure used to combine these measurements:

- each published value of  $R^{+/-00}$  from CLEO and *BABAR* is first converted back to the original measurement of  $R^{+/-00} \tau(B^+)/\tau(B^0)$ , using the value of the lifetime ratio assumed in the corresponding analysis;
- a simple weighted average of these original measurements of  $R^{+/-00} \tau(B^+)/\tau(B^0)$  from CLEO and *BABAR* (which do not depend on the assumed value of the lifetime ratio) is then computed, assuming no statistical or systematic correlations between them;

- the weighted average of  $R^{+/-00} \tau(B^+)/\tau(B^0)$  is converted into a value of  $R^{+/-00}$ , using the latest average of the lifetime ratios,  $\tau(B^+)/\tau(B^0) = 1.076 \pm 0.004$  (see Sec. 3.2.3);
- the Belle measurement of  $R^{+/-00}$  is adjusted to the current values of  $\tau(B^0) = 1.520 \pm 0.004$  ps and  $\tau(B^+)/\tau(B^0) = 1.076 \pm 0.004$  (see Sec. 3.2.3), using the quoted systematic uncertainties due to these parameters;
- the combined value of  $R^{+/-00}$  from CLEO and *BABAR* is averaged with the adjusted value of  $R^{+/-00}$  from Belle, assuming a 100% correlation of the systematic uncertainty due to the limited knowledge on  $\tau(B^+)/\tau(B^0)$ ; no other correlation is considered.

The resulting global average,

$$R^{+/-00} = \frac{f^{+-}}{f^{00}} = 1.059 \pm 0.027, \quad (5)$$

is consistent with equal production of charged and neutral  $B$  mesons, although only at the  $2.2\sigma$  level.

On the other hand, the *BABAR* collaboration has performed a direct measurement of the  $f^{00}$  fraction using an original method, which neither relies on isospin symmetry nor requires the knowledge of  $\tau(B^+)/\tau(B^0)$ . Its analysis, based on a comparison between the number of events where a single  $B^0 \rightarrow D^{*-}\ell^+\nu$  decay could be reconstructed and the number of events where two such decays could be reconstructed, yields [7]

$$f^{00} = 0.487 \pm 0.010 \text{ (stat)} \pm 0.008 \text{ (syst)}. \quad (6)$$

The two results of Eqs. (5) and (6) are of very different natures and completely independent of each other. Their product is equal to  $f^{+-} = 0.516 \pm 0.019$ , while another combination of them gives  $f^{+-} + f^{00} = 1.003 \pm 0.029$ , compatible with unity. Assuming<sup>1</sup>  $f^{+-} + f^{00} = 1$ , also consistent with CLEO's observation that the fraction of  $\Upsilon(4S)$  decays to  $B\bar{B}$  pairs is larger than 0.96 at 95% CL [9], the results of Eqs. (5) and (6) can be averaged (first converting Eq. (5) into a value of  $f^{00} = 1/(R^{+/-00} + 1)$ ) to yield the following more precise estimates:

$$f^{00} = 0.486 \pm 0.006, \quad f^{+-} = 1 - f^{00} = 0.514 \pm 0.006, \quad \frac{f^{+-}}{f^{00}} = 1.058 \pm 0.024. \quad (7)$$

The latter ratio differs from one by  $2.4\sigma$ .

### 3.1.2 $b$ -hadron production fractions in $\Upsilon(5S)$ decays

Hadronic events produced in  $e^+e^-$  collisions at the  $\Upsilon(5S)$  (also known as  $\Upsilon(10860)$ ) energy can be classified into three categories: light-quark ( $u, d, s, c$ ) continuum events,  $b\bar{b}$  continuum events, and  $\Upsilon(5S)$  events. The latter two cannot be distinguished and will be called  $b\bar{b}$  events in the following. These  $b\bar{b}$  events, which also include  $b\bar{b}\gamma$  events because of possible initial-state radiation, can hadronize in different final states. We define  $f_{u,d}^{\Upsilon(5S)}$  as the fraction of  $b\bar{b}$

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<sup>1</sup>A few non- $B\bar{B}$  decay modes of the  $\Upsilon(4S)$  ( $\Upsilon(1S)\pi^+\pi^-$ ,  $\Upsilon(2S)\pi^+\pi^-$ ,  $\Upsilon(1S)\eta$ ) have been observed with branching fractions of the order of  $10^{-4}$  [8], corresponding to a partial width several times larger than that in the  $e^+e^-$  channel. However, this can still be neglected and the assumption  $f^{+-} + f^{00} = 1$  remains valid in the present context of the determination of  $f^{+-}$  and  $f^{00}$ .

Table 2: Published measurements of  $f_s^{\Upsilon(5S)}$ . All values have been obtained assuming  $f_{\mathcal{B}}^{\Upsilon(5S)} = 0$ . They are quoted as in the original publications, except for the most recent measurement which is quoted as  $1 - f_{u,d}^{\Upsilon(5S)}$ , with  $f_{u,d}^{\Upsilon(5S)}$  from Ref. [10]. The last line gives our average of  $f_s^{\Upsilon(5S)}$  assuming  $f_{\mathcal{B}}^{\Upsilon(5S)} = 0$ .

Experiment, year, dataset	Decay mode or method	Value of $f_s^{\Upsilon(5S)}$
CLEO, 2006, 0.42 fb <sup>-1</sup> [11]	$\Upsilon(5S) \rightarrow D_s X$	$0.168 \pm 0.026^{+0.067}_{-0.034}$
	$\Upsilon(5S) \rightarrow \phi X$	$0.246 \pm 0.029^{+0.110}_{-0.053}$
	$\Upsilon(5S) \rightarrow B\bar{B}X$	$0.411 \pm 0.100 \pm 0.092$
	CLEO average of above 3	$0.21^{+0.06}_{-0.03}$
Belle, 2006, 1.86 fb <sup>-1</sup> [12]	$\Upsilon(5S) \rightarrow D_s X$	$0.179 \pm 0.014 \pm 0.041$
	$\Upsilon(5S) \rightarrow D^0 X$	$0.181 \pm 0.036 \pm 0.075$
	Belle average of above 2	$0.180 \pm 0.013 \pm 0.032$
Belle, 2010, 23.6 fb <sup>-1</sup> [10]	$\Upsilon(5S) \rightarrow B\bar{B}X$	$0.263 \pm 0.032 \pm 0.051$
Average of all above after adjustments to inputs of Table 3		$0.215 \pm 0.031$

Table 3: External inputs on which the  $f_s^{\Upsilon(5S)}$  averages are based.

Branching fraction	Value	Explanation and reference
$\mathcal{B}(B \rightarrow D_s X) \times \mathcal{B}(D_s \rightarrow \phi\pi)$	$0.00374 \pm 0.00014$	derived from [13]
$\mathcal{B}(B_s^0 \rightarrow D_s X)$	$0.92 \pm 0.11$	model-dependent estimate [14]
$\mathcal{B}(D_s \rightarrow \phi\pi)$	$0.045 \pm 0.004$	[13]
$\mathcal{B}(B \rightarrow D^0 X) \times \mathcal{B}(D^0 \rightarrow K\pi)$	$0.0243 \pm 0.0011$	derived from [13]
$\mathcal{B}(B_s^0 \rightarrow D^0 X)$	$0.08 \pm 0.07$	model-dependent estimate [12, 14]
$\mathcal{B}(D^0 \rightarrow K\pi)$	$0.0387 \pm 0.0005$	[13]
$\mathcal{B}(B \rightarrow \phi X)$	$0.0343 \pm 0.0012$	world average [13, 11]
$\mathcal{B}(B_s^0 \rightarrow \phi X)$	$0.161 \pm 0.024$	model-dependent estimate [11]

events with a pair of non-strange bottom mesons ( $B\bar{B}, B\bar{B}^*, B^*\bar{B}, B^*\bar{B}^*, B\bar{B}\pi, B\bar{B}^*\pi, B^*\bar{B}\pi, B^*\bar{B}^*\pi$ , and  $B\bar{B}\pi\pi$  final states, where  $B$  denotes a  $B^0$  or  $B^+$  meson and  $\bar{B}$  denotes a  $\bar{B}^0$  or  $B^-$  meson),  $f_s^{\Upsilon(5S)}$  as the fraction of  $b\bar{b}$  events with a pair of strange bottom mesons ( $B_s^0\bar{B}_s^0, B_s^0\bar{B}_s^{*0}, B_s^{*0}\bar{B}_s^0$ , and  $B_s^{*0}\bar{B}_s^{*0}$  final states), and  $f_{\mathcal{B}}^{\Upsilon(5S)}$  as the fraction of  $b\bar{b}$  events without any bottom meson in the final state. Note that the excited bottom-meson states decay via  $B^* \rightarrow B\gamma$  and  $B_s^{*0} \rightarrow B_s^0\gamma$ . These fractions satisfy

$$f_{u,d}^{\Upsilon(5S)} + f_s^{\Upsilon(5S)} + f_{\mathcal{B}}^{\Upsilon(5S)} = 1. \quad (8)$$

The CLEO and Belle collaborations have published measurements of several inclusive  $\Upsilon(5S)$  branching fractions,  $\mathcal{B}(\Upsilon(5S) \rightarrow D_s X)$ ,  $\mathcal{B}(\Upsilon(5S) \rightarrow \phi X)$  and  $\mathcal{B}(\Upsilon(5S) \rightarrow D^0 X)$ , from which they extracted the model-dependent estimates of  $f_s^{\Upsilon(5S)}$  reported in Table 2.<sup>2</sup> This extraction

<sup>2</sup> It was realized just before finalizing this document that more recent results from Belle [15],  $f_s^{\Upsilon(5S)} = 0.172 \pm 0.030$ , have been overlooked. These results are not included in Table 2 nor in the averages presented here.

was performed under the implicit assumption  $f_{\mathcal{B}}^{\mathcal{Y}(5S)} = 0$ , using the relation

$$\frac{1}{2}\mathcal{B}(\mathcal{Y}(5S) \rightarrow D_s X) = f_s^{\mathcal{Y}(5S)} \times \mathcal{B}(B_s^0 \rightarrow D_s X) + \left(1 - f_s^{\mathcal{Y}(5S)} - f_{\mathcal{B}}^{\mathcal{Y}(5S)}\right) \times \mathcal{B}(B \rightarrow D_s X), \quad (9)$$

and similar relations for  $\mathcal{B}(\mathcal{Y}(5S) \rightarrow D^0 X)$  and  $\mathcal{B}(\mathcal{Y}(5S) \rightarrow \phi X)$ . In Table 2 we list also the values of  $f_s^{\mathcal{Y}(5S)}$  derived from measurements of  $f_{u,d}^{\mathcal{Y}(5S)} = \mathcal{B}(\mathcal{Y}(5S) \rightarrow B\bar{B}X)$  [11, 10], as well as our average value of  $f_s^{\mathcal{Y}(5S)}$ , all obtained under the assumption  $f_{\mathcal{B}}^{\mathcal{Y}(5S)} = 0$ .

However, the assumption  $f_{\mathcal{B}}^{\mathcal{Y}(5S)} = 0$  is known to be invalid since the observation of the following final states in  $e^+e^-$  collisions at the  $\mathcal{Y}(5S)$  energy:  $\mathcal{Y}(1S)\pi^+\pi^-$ ,  $\mathcal{Y}(2S)\pi^+\pi^-$ ,  $\mathcal{Y}(3S)\pi^+\pi^-$  and  $\mathcal{Y}(1S)K^+K^-$  [16, 17],  $h_b(1P)\pi^+\pi^-$  and  $h_b(2P)\pi^+\pi^-$  [18], and more recently  $\mathcal{Y}(1S)\pi^0\pi^0$ ,  $\mathcal{Y}(2S)\pi^0\pi^0$  and  $\mathcal{Y}(3S)\pi^0\pi^0$  [19]. The sum of the measurements of the corresponding visible cross-sections, adding also the contributions of the unmeasured  $\mathcal{Y}(1S)K^0\bar{K}^0$ ,  $h_b(1P)\pi^0\pi^0$  and  $h_b(2P)\pi^0\pi^0$  final states assuming isospin conservation, amounts to

$$\sigma^{\text{vis}}(e^+e^- \rightarrow (b\bar{b})hh) = 13.2 \pm 1.4 \text{ pb}, \quad \text{for } (b\bar{b}) = \mathcal{Y}(1S, 2S, 3S), h_b(1P, 2P) \text{ and } hh = \pi\pi, KK.$$

We divide this by the  $b\bar{b}$  production cross section,  $\sigma(e^+e^- \rightarrow b\bar{b}X) = 337 \pm 15 \text{ pb}$ , obtained as the average of the CLEO [14] and Belle [15]<sup>3</sup> measurements, to obtain

$$\mathcal{B}(\mathcal{Y}(5S) \rightarrow (b\bar{b})hh) = 0.039 \pm 0.004, \quad \text{for } (b\bar{b}) = \mathcal{Y}(1S, 2S, 3S), h_b(1P, 2P) \text{ and } hh = \pi\pi, KK,$$

which is to be considered as a lower bound for  $f_{\mathcal{B}}^{\mathcal{Y}(5S)}$ .

Following the method described in Ref. [20], we perform a  $\chi^2$  fit of the original measurements of the  $\mathcal{Y}(5S)$  branching fractions of Refs. [11, 12, 10], using the inputs of Table 3, the relations of Eqs. (8) and (9) and the one-sided Gaussian constraint  $f_{\mathcal{B}}^{\mathcal{Y}(5S)} \geq \mathcal{B}(\mathcal{Y}(5S) \rightarrow (b\bar{b})hh)$ , to simultaneously extract  $f_{u,d}^{\mathcal{Y}(5S)}$ ,  $f_s^{\mathcal{Y}(5S)}$  and  $f_{\mathcal{B}}^{\mathcal{Y}(5S)}$ . Taking all known correlations into account, the best fit values are

$$f_{u,d}^{\mathcal{Y}(5S)} = 0.761_{-0.042}^{+0.027}, \quad (10)$$

$$f_s^{\mathcal{Y}(5S)} = 0.200_{-0.031}^{+0.030}, \quad (11)$$

$$f_{\mathcal{B}}^{\mathcal{Y}(5S)} = 0.039_{-0.004}^{+0.050}, \quad (12)$$

where the strongly asymmetric uncertainty on  $f_{\mathcal{B}}^{\mathcal{Y}(5S)}$  is due to the one-sided constraint from the observed  $(b\bar{b})hh$  decays. These results, together with their correlation, imply

$$f_s^{\mathcal{Y}(5S)}/f_{u,d}^{\mathcal{Y}(5S)} = 0.263_{-0.044}^{+0.052}, \quad (13)$$

in fair agreement with the results of a *BABAR* analysis [21], performed as a function of centre-of-mass energy.<sup>4</sup>

The production of  $B_s^0$  mesons at the  $\mathcal{Y}(5S)$  is observed to be dominated by the  $B_s^{*0}\bar{B}_s^{*0}$  channel, with  $\sigma(e^+e^- \rightarrow B_s^{*0}\bar{B}_s^{*0})/\sigma(e^+e^- \rightarrow B_s^{*0}\bar{B}_s^{(*)0}) = (87.0 \pm 1.7)\%$  [22, 23]. The proportions of the various production channels for non-strange  $B$  mesons have also been measured [10].

<sup>3</sup> The results of Ref. [15] supersede the  $\sigma(e^+e^- \rightarrow b\bar{b}X)$  and  $f_s^{\mathcal{Y}(5S)}$  results of Ref. [12].

<sup>4</sup> This has not been included in the average, since no numerical value is given for  $f_s^{\mathcal{Y}(5S)}/f_{u,d}^{\mathcal{Y}(5S)}$  in Ref. [21].

### 3.1.3 $b$ -hadron production fractions at high energy

At high energy, all species of weakly decaying  $b$  hadrons may be produced, either directly or in strong and electromagnetic decays of excited  $b$  hadrons. It is often assumed that the fractions of these different species are the same in unbiased samples of high- $p_T$   $b$  jets originating from  $Z^0$  decays, from  $p\bar{p}$  collisions at the Tevatron, or from  $pp$  collisions at the LHC. This hypothesis is plausible under the condition that the square of the momentum transfer to the produced  $b$  quarks,  $Q^2$ , is large compared with the square of the hadronization energy scale,  $Q^2 \gg \Lambda_{\text{QCD}}^2$ . On the other hand, there is no strong argument that the fractions at different machines should be strictly equal, so this assumption should be checked experimentally. Although the available data is not sufficient at this time to perform a definitive check, it is expected that more refined analyses of the Tevatron Run II data and new analyses from LHC experiments may improve this situation and allow one to confirm or disprove this assumption with reasonable confidence. Meanwhile, the attitude adopted here is that these fractions are assumed to be equal at all high-energy colliders until demonstrated otherwise by experiment. Both CDF and LHCb report a  $p_T$  dependence for  $\Lambda_b^0$  production relative to  $B^+$  and  $B^0$ ; the number of  $\Lambda_b^0$  baryons observed at low  $p_T$  is enhanced with respect to that seen at LEP's higher  $p_T$ . Therefore we present three sets of complete averages: one set including only measurements performed at LEP, a second set including only measurements performed at the Tevatron, a third set including measurements performed at LEP, Tevatron and LHCb. The LHCb production fractions results by themselves are still incomplete, lacking measurements of the production of other weakly decaying heavy-flavour baryons,  $\Xi_b$  and  $\Omega_b$ , and a measurement of  $\bar{\chi}$  giving an extra constraint between  $f_d$  and  $f_s$ .

Contrary to what happens in the charm sector where the fractions of  $D^+$  and  $D^0$  are different, the relative amount of  $B^+$  and  $B^0$  is not affected by the electromagnetic decays of excited  $B^{*+}$  and  $B^{*0}$  states and strong decays of excited  $B^{**+}$  and  $B^{**0}$  states. Decays of the type  $B_s^{**0} \rightarrow B^{(*)}K$  also contribute to the  $B^+$  and  $B^0$  rates, but with the same magnitude if mass effects can be neglected. We therefore assume equal production of  $B^+$  and  $B^0$  mesons. We also neglect the production of weakly decaying states made of several heavy quarks (like  $B_c^+$  and doubly heavy baryons) which is known to be very small. Hence, for the purpose of determining the  $b$ -hadron fractions, we use the constraints

$$f_u = f_d \quad \text{and} \quad f_u + f_d + f_s + f_{\text{baryon}} = 1, \quad (14)$$

where  $f_u$ ,  $f_d$ ,  $f_s$  and  $f_{\text{baryon}}$  are the unbiased fractions of  $B^+$ ,  $B^0$ ,  $B_s^0$  and  $b$  baryons, respectively.

We note that there are many measurements of the production cross-sections of different species of  $b$  hadrons. In principle these could be included in a global fit to determine the production fractions. We do not perform such a fit at the current time, and instead average only the measured production fractions.

The LEP experiments have measured  $f_s \times \mathcal{B}(B_s^0 \rightarrow D_s^- \ell^+ \nu_\ell X)$  [24],  $\mathcal{B}(b \rightarrow \Lambda_b^0) \times \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \ell^- \bar{\nu}_\ell X)$  [25, 26] and  $\mathcal{B}(b \rightarrow \Xi_b^-) \times \mathcal{B}(\Xi_b^- \rightarrow \Xi^- \ell^- \bar{\nu}_\ell X)$  [27, 28]<sup>5</sup> from partially reconstructed final states including a lepton,  $f_{\text{baryon}}$  from protons identified in  $b$  events [30], and the production rate of charged  $b$  hadrons [31]. Ratios of  $b$ -hadron fractions have been measured at CDF using lepton+charm final states [32, 33, 34]<sup>6</sup>, double semileptonic decays with  $K^* \mu \mu$  and  $\phi \mu \mu$  final

<sup>5</sup> The DELPHI result of Ref. [28] is considered to supersede an older one [29].

<sup>6</sup> CDF updated their measurement of  $f_{\Lambda_b^0}/f_d$  [32] to account for a measured  $p_T$  dependence between exclusively reconstructed  $\Lambda_b^0$  and  $B^0$  [34].



states [35], and fully reconstructed  $B_s^0 \rightarrow J/\psi \phi$  decays [36]. Measurements of the production of other heavy flavour baryons at the Tevatron are included in the determination of  $f_{\text{baryon}}$  [37, 38, 39]<sup>7</sup> using the constraint

$$\begin{aligned} f_{\text{baryon}} &= f_{\Lambda_b^0} + f_{\Xi_b^0} + f_{\Xi_b^-} + f_{\Omega_b^-} \\ &= f_{\Lambda_b^0} \left( 1 + 2 \frac{f_{\Xi_b^-}}{f_{\Lambda_b^0}} + \frac{f_{\Omega_b^-}}{f_{\Lambda_b^0}} \right), \end{aligned} \quad (15)$$

where isospin invariance is assumed in the production of  $\Xi_b^0$  and  $\Xi_b^-$ . Other  $b$  baryons are expected to decay strongly or electromagnetically to those baryons listed. For the production measurements, both CDF and D0 reconstruct their  $b$  baryons exclusively to final states which include a  $J/\psi$  and a hyperon ( $\Lambda_b^0 \rightarrow J/\psi \Lambda$ ,  $\Xi_b^- \rightarrow J/\psi \Xi^-$  and  $\Omega_b^- \rightarrow J/\psi \Omega^-$ ). We assume that the partial decay width of a  $b$  baryon to a  $J/\psi$  and the corresponding hyperon is equal to the partial width of any other  $b$  baryon to a  $J/\psi$  and the corresponding hyperon. LHCb has also measured ratios of  $b$ -hadron fractions in charm+lepton final states [40] and in fully reconstructed hadronic two-body decays  $B^0 \rightarrow D^- \pi^+$ ,  $B_s^0 \rightarrow D_s^- \pi^+$  and  $\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-$  [41, 42].<sup>8</sup>

Both CDF and LHCb observe that the ratio  $f_{\Lambda_b^0}/f_d$  depends on the  $p_T$  of the charm+lepton system [34, 40].<sup>9</sup> CDF chose to correct an older result to account for the  $p_T$  dependence. In a second result, CDF binned their data in  $p_T$  of the charm+electron system [33]. The more recent LHCb measurement using hadronic decays [42] obtains the scale for  $R_{\Lambda_b^0} = f_{\Lambda_b^0}/f_d$  from their previous charm + lepton data [40]. The LHCb measurement using hadronic data also bins the same data in pseudorapidity ( $\eta$ ) and sees a linear dependence of  $R_{\Lambda_b^0}$ . Since  $\eta$  is not entirely independent of  $p_T$  it is impossible to tell at this time whether this dependence is just an artifact of the  $p_T$  dependence. Figure 1 shows the ratio  $R_{\Lambda_b^0}$  as a function of  $p_T$  for the  $b$  hadron, as measured by LHCb. LHCb fits their scaled hadronic data to obtain

$$R_{\Lambda_b^0} = (0.151 \pm 0.030) + \exp \left\{ -(0.57 \pm 0.11) - (0.095 \pm 0.016)[\text{GeV}/c]^{-1} \times p_T \right\}. \quad (16)$$

A value of  $R_{\Lambda_b^0}$  is also calculated for LEP and placed at the approximate  $p_T$  for the charm+lepton system, but this value does not participate in any fit.<sup>10</sup> Because the two LHCb results for  $R_{\Lambda_b^0}$  are not independent, we use only their semileptonic data for the averages. Note that the  $p_T$  dependence of  $R_{\Lambda_b^0}$  combined with the constraint from Eq. (14) implies a compensating  $p_T$  dependence in one or more of the production fractions,  $f_u$ ,  $f_d$ , or  $f_s$ .

Both CDF and LHCb have investigated the  $p_T$  dependence of  $f_s/f_d$  using fully reconstructed  $B_s^0$  and  $B^0$  decays. The CDF analysis reconstructed decays that include a  $J/\psi$  in the final state [36] and reports no significant  $p_T$  dependence on the ratio. However, their result is dominated by an 18% scale uncertainty from preliminary measurements of the branching ratios of the  $\mathcal{B}(B_s^0 \rightarrow J/\psi \phi)$  and  $\mathcal{B}(B^0 \rightarrow J/\psi K^*(892))$ . LHCb reported  $3\sigma$  evidence that the ratio

<sup>7</sup> D0 reports  $f_{\Omega_b^-}/f_{\Xi_b^-}$ . We use the CDF+D0 average of  $f_{\Xi_b^-}/f_{\Lambda_b^0}$  to obtain  $f_{\Omega_b^-}/f_{\Lambda_b^0}$  and then combine it with the CDF result.

<sup>8</sup> The results of Ref. [41] supersede those of Ref. [43].

<sup>9</sup> CDF compares the  $p_T$  distribution of fully reconstructed  $\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-$  with  $\bar{B}^0 \rightarrow D^+ \pi^-$ , which gives  $f_{\Lambda_b^0}/f_d$  up to a scale factor. LHCb compares the  $p_T$  in the charm+lepton system between  $\Lambda_b^0$  and  $B^0$  and  $B^+$ , giving  $R_{\Lambda_b^0}/2 = f_{\Lambda_b^0}/(f_u + f_d) = f_{\Lambda_b^0}/2f_d$ .

<sup>10</sup> The CDF semileptonic data would require significant corrections to obtain the  $p_T$  of the  $b$  hadron and be included on the same plot with the LHCb data. We do not have these corrections at this time.

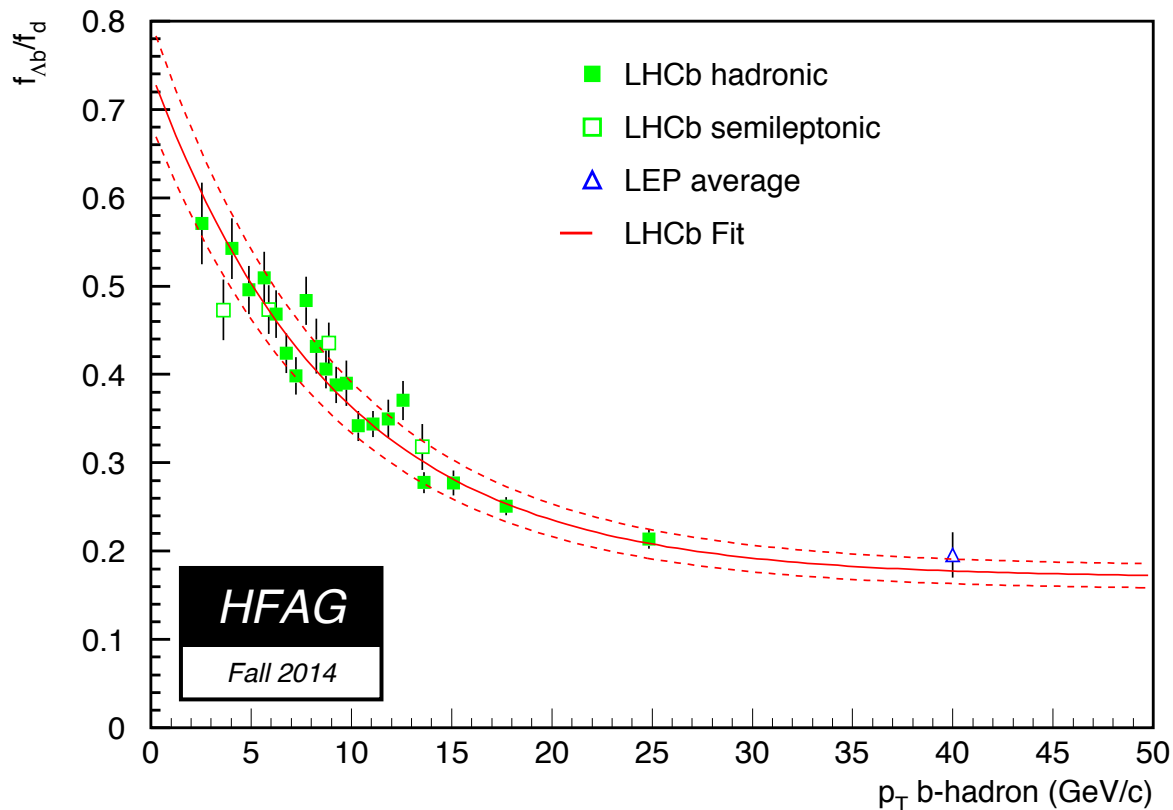


Figure 1: Ratio of production fractions  $f_{\Lambda_b^0}/f_d$  as a function of  $p_T$  of the  $b$  hadron from LHCb data for  $b$  hadrons decaying semileptonically [40] and fully reconstructed in hadronic decays [42]. The curve represents a fit to the LHCb hadronic data [42]. The computed LEP ratio is included at an approximate  $p_T$  in  $Z$  decays, but does not participate in any fit.

$f_s/f_d$  decreases with  $p_T$  using fully reconstructed  $B_s^0$  and  $B^0$  decays and theoretical predictions for branching ratios [41]<sup>8</sup>. Figure 2 shows the ratio  $R_s = f_s/f_u$  as a function of  $p_T$  measured by CDF and LHCb. Two fits are performed. The first fit, using a linear parameterization, yields  $R_s = (0.2760 \pm 0.0068) - (0.00191 \pm 0.00059)[\text{GeV}/c]^{-1} \times p_T$ . A second fit, using a simple exponential, yields  $R_s = \exp\{(-1.293 \pm 0.028) - (0.0077 \pm 0.0025)[\text{GeV}/c]^{-1} \times p_T\}$ . The two fits are nearly indistinguishable over the  $p_T$  range of the results, but the second gives a physical value for all  $p_T$ .  $R_s$  is also calculated for LEP and placed at the approximate  $p_T$  for the  $b$  hadron, though the LEP result doesn't participate in the fit. Our world average for  $R_s$  is also included in the figure for reference.

In order to combine or compare LHCb results with other experiments, the  $p_T$ -dependent  $f_{\Lambda_b^0}/(f_u + f_d)$  is weighted by the  $p_T$  spectrum.<sup>11</sup> Table 4 compares the  $p_T$ -weighted LHCb data

<sup>11</sup> In practice the LHCb data are given in 14 bins in  $p_T$  and  $\eta$  with a full covariance matrix [40]. The weighted average is calculated as  $D^T C^{-1} M / \sigma$ , where  $\sigma = D^T C^{-1} D$ ,  $M$  is a vector of measurements,  $C^{-1}$  is the inverse

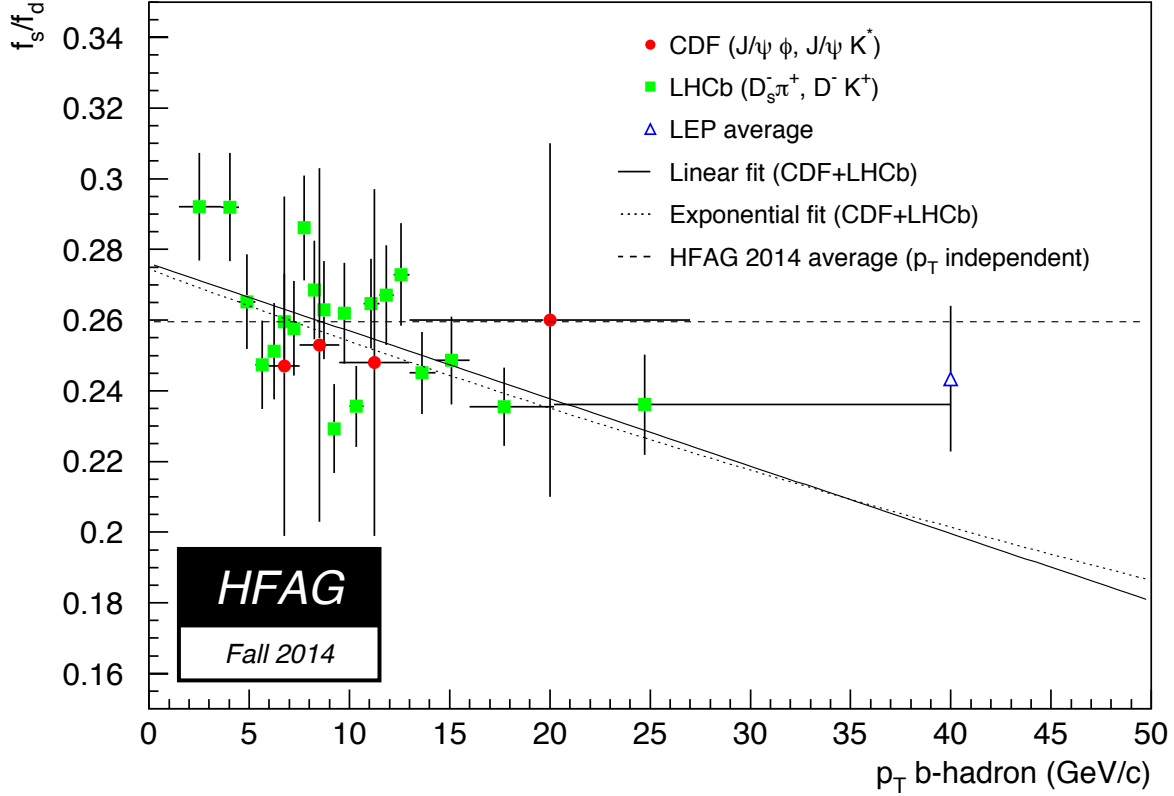


Figure 2: Ratio of production fractions  $f_s/f_d$  as a function of  $p_T$  of the reconstructed  $b$  hadrons for the CDF [36] and LHCb [41]<sup>8</sup> data. Note the suppressed zero for the vertical axis. The curves represent fits to the data: a linear fit (solid), and an exponential fit described in the text (dotted). The  $p_T$  independent value average of  $R_s$  (dashed) is shown for comparison. The computed LEP ratio is included at an approximate  $p_T$  in  $Z$  decays, but does not participate in any fit.

with comparable averages from CDF. The average CDF and LHCb data are in good agreement despite the  $b$  hadrons being produced in different kinematic regimes.

All these published results have been combined following the procedure and assumptions described in Ref. [44], to yield  $f_u = f_d = 0.405 \pm 0.006$ ,  $f_s = 0.104 \pm 0.005$  and  $f_{\text{baryon}} = 0.086 \pm 0.011$  under the constraints of Eq. (14). Repeating the combinations for LEP and the Tevatron, we obtain  $f_u = f_d = 0.413 \pm 0.008$ ,  $f_s = 0.089 \pm 0.013$  and  $f_{\text{baryon}} = 0.085 \pm 0.011$  when using the LEP data only, and  $f_u = f_d = 0.348 \pm 0.020$ ,  $f_s = 0.097 \pm 0.012$  and  $f_{\text{baryon}} = 0.207 \pm 0.046$  when using the Tevatron data only. As noted previously, the LHCb data are insufficient to determine a complete set of  $b$ -hadron production fractions. The world averages (LEP, Tevatron and LHCb) for the various fractions are presented here for comparison with previous averages.

<sup>8</sup>covariance matrix and  $D^T$  is the transpose of the design matrix (vector of 1's).

Table 4: Comparison of average production fraction ratios from CDF and LHCb. The kinematic regime of the charm+lepton system reconstructed in each experiment is also shown.

Quantity	CDF	LHCb
$f_s/(f_u + f_d)$	$0.149 \pm 0.022$	$0.131 \pm 0.009$
$f_{\Lambda_b^0}/(f_u + f_d)$	$0.212 \pm 0.058$	$0.223 \pm 0.022$
Average charm+lepton $p_T$	$\sim 13 \text{ GeV}/c$	$\sim 7 \text{ GeV}/c$
Pseudorapidity range	$-1 < \eta < 1$	$2 < \eta < 5$

Significant differences exist between the LEP and Tevatron fractions, therefore use of the world averages should be taken with some care. For these combinations other external inputs are used, *e.g.* the branching ratios of  $B$  mesons to final states with a  $D$ ,  $D^*$  or  $D^{**}$  in semileptonic decays, which are needed to evaluate the fraction of semileptonic  $B_s^0$  decays with a  $D_s^-$  in the final state.

Time-integrated mixing analyses performed with lepton pairs from  $b\bar{b}$  events produced at high-energy colliders measure the quantity

$$\bar{\chi} = f'_d \chi_d + f'_s \chi_s, \quad (17)$$

where  $f'_d$  and  $f'_s$  are the fractions of  $B^0$  and  $B_s^0$  hadrons in a sample of semileptonic  $b$ -hadron decays, and where  $\chi_d$  and  $\chi_s$  are the  $B^0$  and  $B_s^0$  time-integrated mixing probabilities. Assuming that all  $b$  hadrons have the same semileptonic decay width implies  $f'_i = f_i R_i$ , where  $R_i = \tau_i/\tau_b$  is the ratio of the lifetime  $\tau_i$  of species  $i$  to the average  $b$ -hadron lifetime  $\tau_b = \sum_i f_i \tau_i$ . Hence measurements of the mixing probabilities  $\bar{\chi}$ ,  $\chi_d$  and  $\chi_s$  can be used to improve our knowledge of  $f_u$ ,  $f_d$ ,  $f_s$  and  $f_{\text{baryon}}$ . In practice, the above relations yield another determination of  $f_s$  obtained from  $f_{\text{baryon}}$  and mixing information,

$$f_s = \frac{1}{R_s} \frac{(1+r)\bar{\chi} - (1 - f_{\text{baryon}} R_{\text{baryon}})\chi_d}{(1+r)\chi_s - \chi_d}, \quad (18)$$

where  $r = R_u/R_d = \tau(B^+)/\tau(B^0)$ .

The published measurements of  $\bar{\chi}$  performed by the LEP experiments have been combined by the LEP Electroweak Working Group to yield  $\bar{\chi} = 0.1259 \pm 0.0042$  [45]. This can be compared with the Tevatron average,  $\bar{\chi} = 0.147 \pm 0.011$ , obtained from D0 [46] and CDF [47].<sup>12</sup> The two averages deviate from each other by  $1.8\sigma$ ; this could be an indication that the production fractions of  $b$  hadrons at the  $Z$  peak or at the Tevatron are not the same. We choose to combine these two results in a simple weighted average, assuming no correlations, and, following the PDG prescription, we multiply the combined uncertainty by 1.8 to account for the discrepancy. Our world average is then  $\bar{\chi} = 0.1284 \pm 0.0069$ .

Introducing the  $\bar{\chi}$  average in Eq. (18), together with our world average  $\chi_d = 0.1856 \pm 0.0011$  (see Eq. (46) of Sec. 3.3.1), the assumption  $\chi_s = 1/2$  (justified by Eq. (55) in Sec. 3.3.2), the best knowledge of the lifetimes (see Sec. 3.2) and the estimate of  $f_{\text{baryon}}$  given above, yields  $f_s = 0.117 \pm 0.018$  (or  $f_s = 0.110 \pm 0.011$  using only LEP data, or  $f_s = 0.165 \pm 0.029$  using only Tevatron data), an estimate dominated by the mixing information. Taking into account

<sup>12</sup> The CDF result of Ref. [47] is from Run I data. A preliminary CDF measurement based on Run II data [48] is still unpublished and therefore no longer included in our averages.

Table 5: Time-integrated mixing probability  $\bar{\chi}$  (defined in Eq. (17)), and fractions of the different  $b$ -hadron species in an unbiased sample of weakly decaying  $b$  hadrons, obtained from both direct and mixing measurements. The correlation coefficients between the fractions are also given. The last column includes measurements performed at LEP, Tevatron and LHCb.

Quantity	$Z$ decays	Tevatron	LHCb [41] <sup>8</sup>	all
Mixing probability $\bar{\chi}$	$0.1259 \pm 0.0042$	$0.147 \pm 0.011$		$0.1284 \pm 0.0069$
$B^+$ or $B^0$ fraction $f_u = f_d$	$0.409 \pm 0.007$	$0.354 \pm 0.020$		$0.405 \pm 0.006$
$B_s^0$ fraction $f_s$	$0.101 \pm 0.008$	$0.106 \pm 0.011$		$0.105 \pm 0.005$
$b$ -baryon fraction $f_{\text{baryon}}$	$0.081 \pm 0.011$	$0.186 \pm 0.045$		$0.084 \pm 0.011$
$B_s^0/B^0$ ratio $f_s/f_d$	$0.248 \pm 0.023$	$0.300 \pm 0.030$	$0.256 \pm 0.020$	$0.260 \pm 0.014$
$\rho(f_s, f_u) = \rho(f_s, f_d)$	$-0.638$	$+0.281$		$-0.245$
$\rho(f_{\text{baryon}}, f_u) = \rho(f_{\text{baryon}}, f_d)$	$-0.816$	$-0.973$		$-0.894$
$\rho(f_{\text{baryon}}, f_s)$	$+0.075$	$-0.494$		$-0.216$

all known correlations (including that introduced by  $f_{\text{baryon}}$ ), this result is then combined with the set of fractions obtained from direct measurements (given above), to yield the improved estimates of Table 5, still under the constraints of Eq. (14). As can be seen, our knowledge on the mixing parameters substantially reduces the uncertainty on  $f_s$ . It should be noted that the results are correlated, as indicated in Table 5.

Although no recent measurements of the fractions have become available, the averages of Table 5 (and most notably the  $b$ -baryon fraction) have significantly improved in precision as compared to those given in our previous report [49]. This is mostly due to a new and precise model-independent measurement of the  $\Lambda_c^+ \rightarrow pK^-\pi^+$  branching fraction from Belle [50], which has been used to adjust the fractions obtained from direct measurements.

### 3.2 $b$ -hadron lifetimes

In the spectator model the decay of  $b$ -flavoured hadrons  $H_b$  is governed entirely by the flavour changing  $b \rightarrow Wq$  transition ( $q = c, u$ ). For this very reason, lifetimes of all  $b$ -flavoured hadrons are the same in the spectator approximation regardless of the (spectator) quark content of the  $H_b$ . In the early 1990's experiments became sophisticated enough to start seeing the differences of the lifetimes among various  $H_b$  species. The first theoretical calculations of the spectator quark effects on  $H_b$  lifetime emerged only few years earlier.

Currently, most such calculations are performed in the framework of the Heavy Quark Expansion, HQE. In the HQE, under certain assumptions (the most important of which is that of quark-hadron duality [51]), the decay rate of an  $H_b$  to an inclusive final state  $f$  is expressed as the sum of a series of expectation values of operators of increasing dimension, multiplied by the correspondingly higher powers of  $\Lambda_{\text{QCD}}/m_b$ :

$$\Gamma_{H_b \rightarrow f} = |\text{CKM}|^2 \sum_n c_n^{(f)} \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right)^n \langle H_b | O_n | H_b \rangle, \quad (19)$$

where  $|\text{CKM}|^2$  is the relevant combination of the CKM matrix elements. The coefficients  $c_n^{(f)}$  of this expansion, known as the Operator Product Expansion [52], can be calculated perturbatively. Hence, the HQE predicts  $\Gamma_{H_b \rightarrow f}$  in the form of an expansion in both  $\Lambda_{\text{QCD}}/m_b$  and

$\alpha_s(m_b)$ . The precision of current experiments makes it mandatory to go to the next-to-leading order in QCD, *i.e.* to include corrections of the order of  $\alpha_s(m_b)$  to the  $c_n^{(f)}$  terms. All non-perturbative physics is shifted into the expectation values  $\langle H_b | O_n | H_b \rangle$  of operators  $O_n$ . These can be calculated using lattice QCD or QCD sum rules, or can be related to other observables via the HQE [53]. One may reasonably expect that powers of  $\Lambda_{\text{QCD}}/m_b$  provide enough suppression that only the first few terms of the sum in Eq. (19) matter.

Theoretical predictions are usually made for the ratios of the lifetimes (with  $\tau(B^0)$  chosen as the common denominator) rather than for the individual lifetimes, for this allows several uncertainties to cancel. The precision of the current HQE calculations (see Refs. [54, 55, 56] for the latest updates) is in some instances already surpassed by the measurements, *e.g.* in the case of  $\tau(B^+)/\tau(B^0)$ . Also, HQE calculations are not assumption-free. More accurate predictions are a matter of progress in the evaluation of the non-perturbative hadronic matrix elements and verifying the assumptions that the calculations are based upon. However, the HQE, even in its present shape, draws a number of important conclusions, which are in agreement with experimental observations:

- The heavier the mass of the heavy quark, the smaller is the variation in the lifetimes among different hadrons containing this quark, which is to say that as  $m_b \rightarrow \infty$  we retrieve the spectator picture in which the lifetimes of all  $H_b$  states are the same. This is well illustrated by the fact that lifetimes are rather similar in the  $b$  sector, while they differ by large factors in the  $c$  sector ( $m_c < m_b$ ).
- The non-perturbative corrections arise only at the order of  $\Lambda_{\text{QCD}}^2/m_b^2$ , which translates into differences among  $H_b$  lifetimes of only a few percent.
- It is only the difference between meson and baryon lifetimes that appears at the  $\Lambda_{\text{QCD}}^2/m_b^2$  level. The splitting of the meson lifetimes occurs at the  $\Lambda_{\text{QCD}}^3/m_b^3$  level, yet it is enhanced by a phase space factor  $16\pi^2$  with respect to the leading free  $b$  decay.

To ensure that certain sources of systematic uncertainty cancel, lifetime analyses are sometimes designed to measure ratios of lifetimes. However, because of the differences in decay topologies, abundance (or lack thereof) of decays of a certain kind, *etc.*, measurements of the individual lifetimes are also common. In the following section we review the most common types of lifetime measurements. This discussion is followed by the presentation of the averaging of the various lifetime measurements, each with a brief description of its particularities.

### 3.2.1 Lifetime measurements, uncertainties and correlations

In most cases, the lifetime of an  $H_b$  state is estimated from a flight distance measurement and a  $\beta\gamma$  factor which is used to convert the geometrical distance into the proper decay time. Methods of accessing lifetime information can roughly be divided in the following five categories:

1. **Inclusive (flavour-blind) measurements.** These measurements are aimed at extracting the lifetime from a mixture of  $b$ -hadron decays, without distinguishing the decaying species. Often the knowledge of the mixture composition is limited, which makes these measurements experiment-specific. Also, these measurements have to rely on Monte Carlo simulation for estimating the  $\beta\gamma$  factor, because the decaying hadrons are not fully reconstructed. On the bright side, these are usually the largest statistics  $b$ -hadron lifetime

measurements that are accessible to a given experiment, and can, therefore, serve as an important performance benchmark.

2. **Measurements in semileptonic decays of a specific  $H_b$ .** The  $W$  boson from  $b \rightarrow Wc$  produces a  $\ell\nu_\ell$  pair ( $\ell = e, \mu$ ) in about 21% of the cases. The electron or muon from such decays provides a clean and efficient trigger signature. The  $c$  quark from the  $b \rightarrow Wc$  transition and the other quark(s) making up the decaying  $H_b$  combine into a charm hadron, which is reconstructed in one or more exclusive decay channels. Knowing what this charmed hadron is allows one to separate, at least statistically, different  $H_b$  species. The advantage of these measurements is in statistics, which is usually superior to the case of exclusively reconstructed  $H_b$  decays. Some of the main disadvantages are related to the difficulty of estimating the lepton+charm sample composition and to the Monte Carlo reliance for the momentum (and hence  $\beta\gamma$  factor) estimate.
3. **Measurements in exclusively reconstructed hadronic decays.** These have the advantage of complete reconstruction of the decaying  $H_b$  state, which allows one to infer the decaying species as well as to perform precise measurement of the  $\beta\gamma$  factor. Both lead to generally smaller systematic uncertainties than in the above two categories. The downsides are smaller branching ratios and larger combinatorial backgrounds, especially in  $H_b \rightarrow H_c\pi(\pi\pi)$  and multi-body  $H_c$  decays, or in a hadron collider environment with non-trivial underlying event. Decays of the type  $H_b \rightarrow J/\psi H_s$  are relatively clean and easy to trigger, due to the  $J/\psi \rightarrow \ell^+\ell^-$  signature, but their branching fraction is only about 1%.
4. **Measurements at asymmetric  $B$  factories.** In the  $\Upsilon(4S) \rightarrow B\bar{B}$  decay, the  $B$  mesons ( $B^+$  or  $B^0$ ) are essentially at rest in the  $\Upsilon(4S)$  frame. This makes direct lifetime measurements impossible in experiments at symmetric colliders producing  $\Upsilon(4S)$  at rest. At asymmetric  $B$  factories the  $\Upsilon(4S)$  meson is boosted resulting in  $B$  and  $\bar{B}$  moving nearly parallel to each other with the same boost. The lifetime is inferred from the distance  $\Delta z$  separating the  $B$  and  $\bar{B}$  decay vertices along the beam axis and from the  $\Upsilon(4S)$  boost known from the beam energies. This boost is equal to  $\beta\gamma \approx 0.55$  (0.43) in the *BABAR* (*Belle*) experiment, resulting in an average  $B$  decay length of approximately 250 (190)  $\mu\text{m}$ .  
 In order to determine the charge of the  $B$  mesons in each event, one of them is fully reconstructed in a semileptonic or hadronic decay mode. The other  $B$  is typically not fully reconstructed, only the position of its decay vertex is determined from the remaining tracks in the event. These measurements benefit from large statistics, but suffer from poor proper time resolution, comparable to the  $B$  lifetime itself. This resolution is dominated by the uncertainty on the decay vertices, which is typically 50 (100)  $\mu\text{m}$  for a fully (partially) reconstructed  $B$  meson. With very large future statistics, the resolution and purity could be improved (and hence the systematics reduced) by fully reconstructing both  $B$  mesons in the event.
5. **Direct measurement of lifetime ratios.** This method, initially applied in the measurement of  $\tau(B^+)/\tau(B^0)$ , is now also used for other  $b$ -hadron species at the LHC. The ratio of the lifetimes is extracted from the proper time dependence of the ratio of the observed yields of two different  $b$ -hadron species, both reconstructed in decay modes

with similar topologies. The advantage of this method is that subtle efficiency effects (partially) cancel in the ratio.

In some of the latest analyses, measurements of two (*e.g.*  $\tau(B^+)$  and  $\tau(B^+)/\tau(B^0)$ ) or three (*e.g.*  $\tau(B^+)$ ,  $\tau(B^+)/\tau(B^0)$ , and  $\Delta m_d$ ) quantities are combined. This introduces correlations among measurements. Another source of correlations among the measurements are the systematic effects, which could be common to an experiment or to an analysis technique across the experiments. When calculating the averages, such correlations are taken into account following the general procedure described in Ref. [57].

### 3.2.2 Inclusive $b$ -hadron lifetimes

The inclusive  $b$  hadron lifetime is defined as  $\tau_b = \sum_i f_i \tau_i$  where  $\tau_i$  are the individual species lifetimes and  $f_i$  are the fractions of the various species present in an unbiased sample of weakly decaying  $b$  hadrons produced at a high-energy collider.<sup>13</sup> This quantity is certainly less fundamental than the lifetimes of the individual species, the latter being much more useful in comparisons of the measurements with the theoretical predictions. Nonetheless, we perform the averaging of the inclusive lifetime measurements for completeness as well as for the reason that they might be of interest as “technical numbers.”

Table 6: Measurements of average  $b$ -hadron lifetimes.

Experiment	Method	Data set	$\tau_b$ (ps)	Ref.
ALEPH	Dipole	91	$1.511 \pm 0.022 \pm 0.078$	[58]
DELPHI	All track i.p. (2D)	91–92	$1.542 \pm 0.021 \pm 0.045$	[59] <sup>a</sup>
DELPHI	Sec. vtx	91–93	$1.582 \pm 0.011 \pm 0.027$	[60] <sup>a</sup>
DELPHI	Sec. vtx	94–95	$1.570 \pm 0.005 \pm 0.008$	[61]
L3	Sec. vtx + i.p.	91–94	$1.556 \pm 0.010 \pm 0.017$	[62] <sup>b</sup>
OPAL	Sec. vtx	91–94	$1.611 \pm 0.010 \pm 0.027$	[63]
SLD	Sec. vtx	93	$1.564 \pm 0.030 \pm 0.036$	[64]
Average set 1 ( $b$ vertex)			$1.572 \pm 0.009$	
ALEPH	Lepton i.p. (3D)	91–93	$1.533 \pm 0.013 \pm 0.022$	[65]
L3	Lepton i.p. (2D)	91–94	$1.544 \pm 0.016 \pm 0.021$	[62] <sup>b</sup>
OPAL	Lepton i.p. (2D)	90–91	$1.523 \pm 0.034 \pm 0.038$	[66]
Average set 2 ( $b \rightarrow \ell$ )			$1.537 \pm 0.020$	
CDF1	$J/\psi$ vtx	92–95	$1.533 \pm 0.015^{+0.035}_{-0.031}$	[67]
ATLAS	$J/\psi$ vtx	2010	$1.489 \pm 0.016 \pm 0.043$	[68] <sup>p</sup>
Average set 3 ( $b \rightarrow J/\psi$ )			$1.516 \pm 0.028$	

<sup>a</sup> The combined DELPHI result quoted in [60] is  $1.575 \pm 0.010 \pm 0.026$  ps.

<sup>b</sup> The combined L3 result quoted in [62] is  $1.549 \pm 0.009 \pm 0.015$  ps.

<sup>p</sup> Preliminary.

In practice, an unbiased measurement of the inclusive lifetime is difficult to achieve, because it would imply an efficiency which is guaranteed to be the same across species. So most of the

<sup>13</sup>In principle such a quantity could be slightly different in  $Z$  decays, at the Tevatron or at the LHC, in case the fractions of  $b$ -hadron species are not exactly the same; see the discussion in Sec. 3.1.3.



measurements are biased. In an attempt to group analyses which are expected to select the same mixture of  $b$  hadrons, the available results (given in Table 6) are divided into the following three sets:

1. measurements at LEP and SLD that accept any  $b$ -hadron decay, based on topological reconstruction (secondary vertex or track impact parameters);
2. measurements at LEP based on the identification of a lepton from a  $b$  decay; and
3. measurements at the Tevatron based on inclusive  $H_b \rightarrow J/\psi X$  reconstruction, where the  $J/\psi$  is fully reconstructed.

The measurements of the first set are generally considered as estimates of  $\tau_b$ , although the efficiency to reconstruct a secondary vertex most probably depends, in an analysis-specific way, on the number of tracks coming from the vertex, thereby depending on the type of the  $H_b$ . Even though these efficiency variations can in principle be accounted for using Monte Carlo simulations (which inevitably contain assumptions on branching fractions), the  $H_b$  mixture in that case can remain somewhat ill-defined and could be slightly different among analyses in this set.

On the contrary, the mixtures corresponding to the other two sets of measurements are better defined in the limit where the reconstruction and selection efficiency of a lepton or a  $J/\psi$  from an  $H_b$  does not depend on the decaying hadron type. These mixtures are given by the production fractions and the inclusive branching fractions for each  $H_b$  species to give a lepton or a  $J/\psi$ . In particular, under the assumption that all  $b$  hadrons have the same semileptonic decay width, the analyses of the second set should measure  $\tau(b \rightarrow \ell) = (\sum_i f_i \tau_i^3) / (\sum_i f_i \tau_i^2)$  which is necessarily larger than  $\tau_b$  if lifetime differences exist. Given the present knowledge on  $\tau_i$  and  $f_i$ ,  $\tau(b \rightarrow \ell) - \tau_b$  is expected to be of the order of 0.003 ps. On the other hand, the third set measuring  $\tau(b \rightarrow J/\psi)$  is expected to give an average smaller than  $\tau_b$  because of the  $B_c^+$  meson which has a significantly larger probability to decay to a  $J/\psi$  than other  $b$ -hadron species.

Measurements by SLC and LEP experiments are subject to a number of common systematic uncertainties, such as those due to (lack of knowledge of)  $b$  and  $c$  fragmentation,  $b$  and  $c$  decay models,  $\mathcal{B}(B \rightarrow \ell)$ ,  $\mathcal{B}(B \rightarrow c \rightarrow \ell)$ ,  $\mathcal{B}(c \rightarrow \ell)$ ,  $\tau_c$ , and  $H_b$  decay multiplicity. In the averaging, these systematic uncertainties are assumed to be 100% correlated. The averages for the sets defined above (also given in Table 6) are

$$\tau(b \text{ vertex}) = 1.572 \pm 0.009 \text{ ps}, \quad (20)$$

$$\tau(b \rightarrow \ell) = 1.537 \pm 0.020 \text{ ps}, \quad (21)$$

$$\tau(b \rightarrow J/\psi) = 1.516 \pm 0.028 \text{ ps}. \quad (22)$$

### 3.2.3 $B^0$ and $B^+$ lifetimes and their ratio

After a number of years of dominating these averages the LEP experiments yielded the scene to the asymmetric  $B$  factories and the Tevatron experiments. The  $B$  factories have been very successful in utilizing their potential – in only a few years of running, *BABAR* and, to a greater extent, *Belle*, have struck a balance between the statistical and the systematic uncertainties, with both being close to (or even better than) the impressive 1%. In the meanwhile, CDF and D0 have emerged as significant contributors to the field as the Tevatron Run II data flowed in.

Recently, the LHCb experiment reached a further step in precision, improving by a factor  $\sim 2$  over the previous best measurement.

At present time we are in an interesting position of having three sets of measurements (from LEP/SLC,  $B$  factories and the Tevatron) that originate from different environments, obtained using substantially different techniques and are precise enough for incisive comparison.

Table 7: Measurements of the  $B^0$  lifetime.

Experiment	Method	Data set	$\tau(B^0)$ (ps)	Ref.
ALEPH	$D^{(*)}\ell$	91–95	$1.518 \pm 0.053 \pm 0.034$	[69]
ALEPH	Exclusive	91–94	$1.25_{-0.13}^{+0.15} \pm 0.05$	[70]
ALEPH	Partial rec. $\pi^+\pi^-$	91–94	$1.49_{-0.15-0.06}^{+0.17+0.08}$	[70]
DELPHI	$D^{(*)}\ell$	91–93	$1.61_{-0.13}^{+0.14} \pm 0.08$	[71]
DELPHI	Charge sec. vtx	91–93	$1.63 \pm 0.14 \pm 0.13$	[72]
DELPHI	Inclusive $D^*\ell$	91–93	$1.532 \pm 0.041 \pm 0.040$	[73]
DELPHI	Charge sec. vtx	94–95	$1.531 \pm 0.021 \pm 0.031$	[61]
L3	Charge sec. vtx	94–95	$1.52 \pm 0.06 \pm 0.04$	[74]
OPAL	$D^{(*)}\ell$	91–93	$1.53 \pm 0.12 \pm 0.08$	[75]
OPAL	Charge sec. vtx	93–95	$1.523 \pm 0.057 \pm 0.053$	[76]
OPAL	Inclusive $D^*\ell$	91–00	$1.541 \pm 0.028 \pm 0.023$	[77]
SLD	Charge sec. vtx $\ell$	93–95	$1.56_{-0.13}^{+0.14} \pm 0.10$	[78] <sup>a</sup>
SLD	Charge sec. vtx	93–95	$1.66 \pm 0.08 \pm 0.08$	[78] <sup>a</sup>
CDF1	$D^{(*)}\ell$	92–95	$1.474 \pm 0.039_{-0.051}^{+0.052}$	[79]
CDF1	Excl. $J/\psi K^{*0}$	92–95	$1.497 \pm 0.073 \pm 0.032$	[80]
CDF2	Excl. $J/\psi K_S^0, J/\psi K^{*0}$	02–09	$1.507 \pm 0.010 \pm 0.008$	[81]
D0	Excl. $J/\psi K^{*0}$	03–07	$1.414 \pm 0.018 \pm 0.034$	[82]
D0	Excl. $J/\psi K_S^0$	02–11	$1.508 \pm 0.025 \pm 0.043$	[83]
D0	Inclusive $D^-\mu^+$	02–11	$1.534 \pm 0.019 \pm 0.021$	[84]
BABAR	Exclusive	99–00	$1.546 \pm 0.032 \pm 0.022$	[85]
BABAR	Inclusive $D^*\ell$	99–01	$1.529 \pm 0.012 \pm 0.029$	[86]
BABAR	Exclusive $D^*\ell$	99–02	$1.523_{-0.023}^{+0.024} \pm 0.022$	[87]
BABAR	Incl. $D^*\pi, D^*\rho$	99–01	$1.533 \pm 0.034 \pm 0.038$	[88]
BABAR	Inclusive $D^*\ell$	99–04	$1.504 \pm 0.013_{-0.013}^{+0.018}$	[89]
Belle	Exclusive	00–03	$1.534 \pm 0.008 \pm 0.010$	[90]
ATLAS	Excl. $J/\psi K^{*0}$	2010	$1.51 \pm 0.04 \pm 0.04$	[91] <sup>p</sup>
ATLAS	Excl. $J/\psi K_S^0$	2011	$1.509 \pm 0.012 \pm 0.018$	[92]
LHCb	Excl. $J/\psi K^{*0}$	2011	$1.524 \pm 0.006 \pm 0.004$	[93]
LHCb	Excl. $J/\psi K_S^0$	2011	$1.499 \pm 0.013 \pm 0.005$	[93]
LHCb	$K^+\pi^-$	2011	$1.524 \pm 0.011 \pm 0.004$	[94]
Average			$1.520 \pm 0.004$	

<sup>a</sup> The combined SLD result quoted in [78] is  $1.64 \pm 0.08 \pm 0.08$  ps.

<sup>p</sup> Preliminary.

The averaging of  $\tau(B^+)$ ,  $\tau(B^0)$  and  $\tau(B^+)/\tau(B^0)$  measurements is summarized<sup>14</sup> in Tables 7,

<sup>14</sup>We do not include the old unpublished measurements of Refs. [97, 98].

Table 8: Measurements of the  $B^+$  lifetime.

Experiment	Method	Data set	$\tau(B^+)$ (ps)	Ref.
ALEPH	$D^{(*)}\ell$	91–95	$1.648 \pm 0.049 \pm 0.035$	[69]
ALEPH	Exclusive	91–94	$1.58_{-0.18-0.03}^{+0.21+0.04}$	[70]
DELPHI	$D^{(*)}\ell$	91–93	$1.61 \pm 0.16 \pm 0.12$	[71] <sup>a</sup>
DELPHI	Charge sec. vtx	91–93	$1.72 \pm 0.08 \pm 0.06$	[72] <sup>a</sup>
DELPHI	Charge sec. vtx	94–95	$1.624 \pm 0.014 \pm 0.018$	[61]
L3	Charge sec. vtx	94–95	$1.66 \pm 0.06 \pm 0.03$	[74]
OPAL	$D^{(*)}\ell$	91–93	$1.52 \pm 0.14 \pm 0.09$	[75]
OPAL	Charge sec. vtx	93–95	$1.643 \pm 0.037 \pm 0.025$	[76]
SLD	Charge sec. vtx $\ell$	93–95	$1.61_{-0.12}^{+0.13} \pm 0.07$	[78] <sup>b</sup>
SLD	Charge sec. vtx	93–95	$1.67 \pm 0.07 \pm 0.06$	[78] <sup>b</sup>
CDF1	$D^{(*)}\ell$	92–95	$1.637 \pm 0.058_{-0.043}^{+0.045}$	[79]
CDF1	Excl. $J/\psi K$	92–95	$1.636 \pm 0.058 \pm 0.025$	[80]
CDF2	Excl. $J/\psi K$	02–09	$1.639 \pm 0.009 \pm 0.009$	[81]
CDF2	Excl. $D^0\pi$	02–06	$1.663 \pm 0.023 \pm 0.015$	[95]
BABAR	Exclusive	99–00	$1.673 \pm 0.032 \pm 0.023$	[85]
Belle	Exclusive	00–03	$1.635 \pm 0.011 \pm 0.011$	[90]
LHCb	Excl. $J/\psi K$	2011	$1.637 \pm 0.004 \pm 0.003$	[93]
Average			$1.638 \pm 0.004$	

<sup>a</sup> The combined DELPHI result quoted in [72] is  $1.70 \pm 0.09$  ps.

<sup>b</sup> The combined SLD result quoted in [78] is  $1.66 \pm 0.06 \pm 0.05$  ps.

Table 9: Measurements of the ratio  $\tau(B^+)/\tau(B^0)$ .

Experiment	Method	Data set	Ratio $\tau(B^+)/\tau(B^0)$	Ref.
ALEPH	$D^{(*)}\ell$	91–95	$1.085 \pm 0.059 \pm 0.018$	[69]
ALEPH	Exclusive	91–94	$1.27^{+0.23+0.03}_{-0.19-0.02}$	[70]
DELPHI	$D^{(*)}\ell$	91–93	$1.00^{+0.17}_{-0.15} \pm 0.10$	[71]
DELPHI	Charge sec. vtx	91–93	$1.06^{+0.13}_{-0.11} \pm 0.10$	[72]
DELPHI	Charge sec. vtx	94–95	$1.060 \pm 0.021 \pm 0.024$	[61]
L3	Charge sec. vtx	94–95	$1.09 \pm 0.07 \pm 0.03$	[74]
OPAL	$D^{(*)}\ell$	91–93	$0.99 \pm 0.14^{+0.05}_{-0.04}$	[75]
OPAL	Charge sec. vtx	93–95	$1.079 \pm 0.064 \pm 0.041$	[76]
SLD	Charge sec. vtx $\ell$	93–95	$1.03^{+0.16}_{-0.14} \pm 0.09$	[78] <sup>a</sup>
SLD	Charge sec. vtx	93–95	$1.01^{+0.09}_{-0.08} \pm 0.05$	[78] <sup>a</sup>
CDF1	$D^{(*)}\ell$	92–95	$1.110 \pm 0.056^{+0.033}_{-0.030}$	[79]
CDF1	Excl. $J/\psi K$	92–95	$1.093 \pm 0.066 \pm 0.028$	[80]
CDF2	Excl. $J/\psi K^{(*)}$	02–09	$1.088 \pm 0.009 \pm 0.004$	[81]
D0	$D^{*+}\mu D^0\mu$ ratio	02–04	$1.080 \pm 0.016 \pm 0.014$	[96]
BABAR	Exclusive	99–00	$1.082 \pm 0.026 \pm 0.012$	[85]
Belle	Exclusive	00–03	$1.066 \pm 0.008 \pm 0.008$	[90]
LHCb	Excl. $J/\psi K^{(*)}$	2011	$1.074 \pm 0.005 \pm 0.003$	[93]
Average			$1.076 \pm 0.004$	

<sup>a</sup> The combined SLD result quoted in [78] is  $1.01 \pm 0.07 \pm 0.06$ .

8, and 9. For  $\tau(B^+)/\tau(B^0)$  we average only the measurements of this quantity provided by experiments rather than using all available knowledge, which would have included, for example,  $\tau(B^+)$  and  $\tau(B^0)$  measurements which did not contribute to any of the ratio measurements.

The following sources of correlated (within experiment/machine) systematic uncertainties have been considered:

- for SLC/LEP measurements –  $D^{**}$  branching ratio uncertainties [44], momentum estimation of  $b$  mesons from  $Z^0$  decays ( $b$ -quark fragmentation parameter  $\langle X_E \rangle = 0.702 \pm 0.008$  [44]),  $B_s^0$  and  $b$ -baryon lifetimes (see Secs. 3.2.4 and 3.2.6), and  $b$ -hadron fractions at high energy (see Table 5);
- for BABAR measurements – alignment,  $z$  scale, PEP-II boost, sample composition (where applicable);
- for D0 and CDF Run II measurements – alignment (separately within each experiment).

The resultant averages are:

$$\tau(B^0) = 1.520 \pm 0.004 \text{ ps}, \quad (23)$$

$$\tau(B^+) = 1.638 \pm 0.004 \text{ ps}, \quad (24)$$

$$\tau(B^+)/\tau(B^0) = 1.076 \pm 0.004. \quad (25)$$

### 3.2.4 $B_s^0$ lifetimes

Like neutral kaons, neutral  $B$  mesons contain short- and long-lived components, since the light (L) and heavy (H) eigenstates,  $B_L$  and  $B_H$ , differ not only in their masses, but also in their total decay widths, with a decay width difference defined as  $\Delta\Gamma = \Gamma_L - \Gamma_H$ . Neglecting  $CP$  violation in  $B - \bar{B}$  mixing, which is expected to be very small [99, 100] (see also Sec. 3.3.3), the mass eigenstates are also  $CP$  eigenstates, with the light  $B_L$  state being  $CP$ -even and the heavy  $B_H$  state being  $CP$ -odd. While the decay width difference  $\Delta\Gamma_d$  can be neglected in the  $B^0$  system, the  $B_s^0$  system exhibits a significant value of  $\Delta\Gamma_s$ : the sign of  $\Delta\Gamma_s$  is known to be positive [101], *i.e.* the heavy eigenstate lives longer than the light eigenstate. Specific measurements of  $\Delta\Gamma_s$  and  $\Gamma_s = (\Gamma_L + \Gamma_H)/2$  are explained and averaged in Sec. 3.3.2, but the results for  $1/\Gamma_L$ ,  $1/\Gamma_H$  and the mean  $B_s^0$  lifetime, defined as  $\tau(B_s^0) = 1/\Gamma_s$ , are also quoted at the end of this section.

Many  $B_s^0$  lifetime analyses, in particular the early ones performed before the non-zero value of  $\Delta\Gamma_s$  was firmly established, ignore  $\Delta\Gamma_s$  and fit the proper time distribution of a sample of  $B_s^0$  candidates reconstructed in a certain final state  $f$  with a model assuming a single exponential function for the signal. We denote such effective lifetime measurements [102] as  $\tau_{\text{single}}(B_s^0 \rightarrow f)$ ; their true values may lie *a priori* anywhere between  $1/\Gamma_L = 1/(\Gamma_s + \Delta\Gamma_s/2)$  and  $1/\Gamma_H = 1/(\Gamma_s - \Delta\Gamma_s/2)$ , depending on the proportion of  $B_L$  and  $B_H$  in the final state  $f$ . More recent determinations of effective lifetimes may be interpreted as measurements of the relative composition of  $B_L$  and  $B_H$  decaying to the final state  $f$ . Table 10 summarizes the effective lifetime measurements.

Averaging measurements of  $\tau_{\text{single}}(B_s^0 \rightarrow f)$  over several final states  $f$  will yield a result corresponding to an ill-defined observable when the proportions of  $B_L$  and  $B_H$  differ. Therefore, the effective  $B_s^0$  lifetime measurements are broken down into several categories and averaged separately.

Table 10: Measurements of the effective  $B_s^0$  lifetimes obtained from single exponential fits.

Experiment	Final state $f$	Data set		$\tau_{\text{single}}(B_s^0 \rightarrow f)$ (ps)	Ref.
ALEPH	$D_s^- \ell^+$	flavour-specific	91–95	$1.54_{-0.13}^{+0.14} \pm 0.04$	[103]
CDF1	$D_s^- \ell^+$	flavour-specific	92–96	$1.36 \pm 0.09_{-0.05}^{+0.06}$	[104]
DELPHI	$D_s^- \ell^+$	flavour-specific	91–95	$1.42_{-0.13}^{+0.14} \pm 0.03$	[105]
OPAL	$D_s^- \ell^+$	flavour-specific	90–95	$1.50_{-0.15}^{+0.16} \pm 0.04$	[106]
D0	$D_s^- \mu^+ X$	flavour-specific	Run II	$10.4 \text{ fb}^{-1}$	$1.479 \pm 0.010 \pm 0.021$ [84]
CDF2	$D_s^- \pi^+(X)$	flavour-specific	02–06	$1.3 \text{ fb}^{-1}$	$1.518 \pm 0.041 \pm 0.027$ [107]
LHCb	$D_s^- D^+$	flavour-specific	11–12	$3 \text{ fb}^{-1}$	$1.52 \pm 0.15 \pm 0.01$ [108]
LHCb	$D_s^- \pi^+$	flavour-specific	11	$1 \text{ fb}^{-1}$	$1.535 \pm 0.015 \pm 0.014$ [109]
Average of above 8 flavour-specific lifetime measurements				$1.511 \pm 0.014$	
LHCb	$\pi^+ K^-$	flavour-specific	11	$1.0 \text{ fb}^{-1}$	$1.60 \pm 0.06 \pm 0.01$ [94]
ALEPH	$D_s h$	ill-defined	91–95	$1.47 \pm 0.14 \pm 0.08$	[110]
DELPHI	$D_s h$	ill-defined	91–95	$1.53_{-0.15}^{+0.16} \pm 0.07$	[111]
OPAL	$D_s$ incl.	ill-defined	90–95	$1.72_{-0.19-0.17}^{+0.20+0.18}$	[112]
CDF1	$J/\psi \phi$	$CP$ even+odd	92–95	$1.34_{-0.19}^{+0.23} \pm 0.05$	[67]
D0	$J/\psi \phi$	$CP$ even+odd	02–04	$1.444_{-0.090}^{+0.098} \pm 0.02$	[113]
ATLAS	$J/\psi \phi$	$CP$ even+odd	10	$40 \text{ pb}^{-1}$	$1.41 \pm 0.08 \pm 0.05$ [91] <sup>p</sup>
LHCb	$J/\psi \phi$	$CP$ even+odd	11	$1 \text{ fb}^{-1}$	$1.480 \pm 0.011 \pm 0.005$ [93]
Average of above 4 $J/\psi \phi$ lifetime measurements				$1.478 \pm 0.012$	
ALEPH	$D_s^{(*)+} D_s^{(*)-}$	mostly $CP$ even	91–95	$1.27 \pm 0.33 \pm 0.08$	[114]
LHCb	$K^+ K^-$	$CP$ -even	10	$0.037 \text{ fb}^{-1}$	$1.440 \pm 0.096 \pm 0.009$ [115]
LHCb	$K^+ K^-$	$CP$ -even	11	$1.0 \text{ fb}^{-1}$	$1.407 \pm 0.016 \pm 0.007$ [94]
Average of above 2 $K^+ K^-$ lifetime measurements				$1.408 \pm 0.017$	
LHCb	$D_s^+ D_s^-$	$CP$ -even	11–12	$3 \text{ fb}^{-1}$	$1.379 \pm 0.026 \pm 0.017$ [108]
Average of above 1 measurement of $1/\Gamma_L$				$1.379 \pm 0.031$	
LHCb	$J/\psi K_S^0$	$CP$ -odd	11	$1.0 \text{ fb}^{-1}$	$1.75 \pm 0.12 \pm 0.07$ [116]
CDF2	$J/\psi f_0(980)$	$CP$ -odd	02–08	$3.8 \text{ fb}^{-1}$	$1.70_{-0.11}^{+0.12} \pm 0.03$ [117]
LHCb	$J/\psi \pi^+ \pi^-$	$CP$ -odd	11	$1.0 \text{ fb}^{-1}$	$1.652 \pm 0.024 \pm 0.024$ [118]
Average of above 2 measurements of $1/\Gamma_H$				$1.656 \pm 0.033$	

<sup>p</sup> Preliminary.

- **Decays to a flavour-specific final state** without  $CP$  violation in the decay amplitude, such as  $B_s^0 \rightarrow D_s^- \ell^+ \nu$  or  $B_s^0 \rightarrow D_s^- \pi^+$ , have equal fractions of  $B_L$  and  $B_H$  at time zero.<sup>15</sup> If the resulting superposition of two exponential distributions is fitted with a single exponential function, one obtains a measure of the so-called *flavour-specific lifetime* [119]:

$$\tau_{\text{single}}(B_s^0 \rightarrow \text{flavour specific}) = \frac{1}{\Gamma_s} \frac{1 + \left(\frac{\Delta\Gamma_s}{2\Gamma_s}\right)^2}{1 - \left(\frac{\Delta\Gamma_s}{2\Gamma_s}\right)^2}. \quad (26)$$

The average of all flavour-specific  $B_s^0$  lifetime measurements<sup>16</sup> is

$$\tau_{\text{single}}(B_s^0 \rightarrow \text{flavour specific}) = 1.511 \pm 0.014 \text{ ps}. \quad (27)$$

This average does not include an effective lifetime measurement of  $B_s^0 \rightarrow \pi^+ K^-$  decays [94].

- **$B_s^0 \rightarrow D_s^\mp X$  decays** include flavour-specific decays but also decays with an unknown mixture of light and heavy components. Measurements performed with such inclusive states are no longer used in averages.
- **$B_s^0 \rightarrow J/\psi \phi$  decays** contain a well-measured mixture of  $CP$ -even and  $CP$ -odd states. There are no known correlations between the existing  $B_s^0 \rightarrow J/\psi \phi$  effective lifetime measurements; these are combined into the average<sup>17</sup>  $\tau_{\text{single}}(B_s^0 \rightarrow J/\psi \phi) = 1.478 \pm 0.012$  ps. A caveat is that different experimental acceptances may lead to different admixtures of the  $CP$ -even and  $CP$ -odd states, and simple fits to a single exponential may result in inherently different values of  $\tau_{\text{single}}(B_s^0 \rightarrow J/\psi \phi)$ . Analyses that separate the  $CP$ -even and  $CP$ -odd components in this decay through a full angular study, outlined in Sec. 3.3.2, provide directly precise measurements of  $1/\Gamma_s$  and  $\Delta\Gamma_s$  (see Table 22).
- **Decays to  $CP$  eigenstates** have also been measured, in the  $CP$ -even modes  $B_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-}$  by ALEPH [114],  $B_s^0 \rightarrow K^+ K^-$  by LHCb [115, 94]<sup>18</sup> and  $B_s^0 \rightarrow D_s^+ D_s^-$  by LHCb [108], as well as in the  $CP$ -odd modes  $B_s^0 \rightarrow J/\psi f_0(980)$  by CDF [117],  $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$  by LHCb [118] and  $B_s^0 \rightarrow J/\psi K_S^0$  by LHCb [116]. If these decays are dominated by a single weak phase and if  $CP$  violation can be neglected, then  $\tau_{\text{single}}(B_s^0 \rightarrow CP\text{-even}) = 1/\Gamma_L$  and  $\tau_{\text{single}}(B_s^0 \rightarrow CP\text{-odd}) = 1/\Gamma_H$  (see Eqs. (50) and (51) for approximate relations in presence of  $CP$  violation in the mixing). However, not all these modes can be considered as pure  $CP$  eigenstates: a small  $CP$ -odd component is most probably present in  $B_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-}$  decays. Furthermore the decays  $B_s^0 \rightarrow K^+ K^-$  and  $B_s^0 \rightarrow J/\psi K_S^0$  may suffer from  $CP$  violation due to interfering tree and loop amplitudes. The averages for the effective lifetimes obtained for decays to pure  $CP$ -even ( $D_s^+ D_s^-$ ) and  $CP$ -odd ( $J/\psi f_0(980)$ ),

<sup>15</sup>The assumption that such decays are flavour-specific is valid to an excellent approximation in the SM. However, there are few experimental tests of it.

<sup>16</sup>An old unpublished measurement [120] is not included.

<sup>17</sup>An old unpublished measurement [121] is not included.

<sup>18</sup>An old unpublished measurement of the  $B_s^0 \rightarrow K^+ K^-$  effective lifetime by CDF [122] is no longer considered.

$J/\psi \pi^+ \pi^-$ ) final states, where  $CP$  conservation can be assumed, are

$$\tau_{\text{single}}(B_s^0 \rightarrow CP\text{-even}) = 1.379 \pm 0.031 \text{ ps}, \quad (28)$$

$$\tau_{\text{single}}(B_s^0 \rightarrow CP\text{-odd}) = 1.656 \pm 0.033 \text{ ps}. \quad (29)$$

As described in Sec. 3.3.2, the effective lifetime averages of Eqs. (27), (28), and (29) are used as ingredients to improve the determination of  $1/\Gamma_s$  and  $\Delta\Gamma_s$  obtained from the full angular analyses of  $B_s^0 \rightarrow J/\psi \phi$  and  $B_s^0 \rightarrow J/\psi K^+ K^-$  decays. The resulting world averages for the  $B_s^0$  lifetimes are

$$\tau(B_{sL}) = \frac{1}{\Gamma_L} = \frac{1}{\Gamma_s + \Delta\Gamma_s/2} = 1.417 \pm 0.006 \text{ ps}, \quad (30)$$

$$\tau(B_{sH}) = \frac{1}{\Gamma_H} = \frac{1}{\Gamma_s - \Delta\Gamma_s/2} = 1.604 \pm 0.010 \text{ ps}, \quad (31)$$

$$\tau(B_s^0) = \frac{1}{\Gamma_s} = \frac{2}{\Gamma_L + \Gamma_H} = 1.505 \pm 0.004 \text{ ps}. \quad (32)$$

### 3.2.5 $B_c^+$ lifetime

Early measurements of the  $B_c^+$  meson lifetime, from CDF [123, 124, 125] and D0 [126], use the semileptonic decay mode  $B_c^+ \rightarrow J/\psi \ell^+ \nu$  and are based on a simultaneous fit to the mass and lifetime using the vertex formed with the leptons from the decay of the  $J/\psi$  and the third lepton. Correction factors to estimate the boost due to the missing neutrino are used. Correlated systematic errors include the impact of the uncertainty of the  $B_c^+$   $p_T$  spectrum on the correction factors, the level of feed-down from  $\psi(2S)$  decays, Monte Carlo modeling of the decay model varying from phase space to the ISGW model, and mass variations. With more statistics, CDF2 was able to perform the first  $B_c^+$  lifetime based on fully reconstructed  $B_c^+ \rightarrow J/\psi \pi^+$  decays [127], which does not suffer from a missing neutrino. Recent measurements from LHCb, both with  $B_c^+ \rightarrow J/\psi \mu^+ \nu$  [128] and  $B_c^+ \rightarrow J/\psi \pi^+$  [129] decays, achieve the highest level of precision.

All the measurements<sup>19</sup> are summarized in Table 11 and the world average, dominated by the LHCb measurements, is determined to be

$$\tau(B_c^+) = 0.507 \pm 0.009 \text{ ps}. \quad (33)$$

### 3.2.6 $\Lambda_b^0$ and $b$ -baryon lifetimes

The first measurements of  $b$ -baryon lifetimes, performed at LEP, originate from two classes of partially reconstructed decays. In the first class, decays with an exclusively reconstructed  $\Lambda_c^+$  baryon and a lepton of opposite charge are used. These products are more likely to occur in the decay of  $\Lambda_b^0$  baryons. In the second class, more inclusive final states with a baryon ( $p$ ,  $\bar{p}$ ,  $\Lambda$ , or  $\bar{\Lambda}$ ) and a lepton have been used, and these final states can generally arise from any  $b$  baryon. With the large  $b$ -hadron samples available at the Tevatron and the LHC, the most precise measurements of  $b$  baryons now come from fully reconstructed exclusive decays.

<sup>19</sup>We do not list (nor include in the average) an unpublished result from CDF2 [124].



Table 11: Measurements of the  $B_c^+$  lifetime.

Experiment	Method	Data set		$\tau(B_c^+)$ (ps)	Ref.
CDF1	$J/\psi \ell$	92–95	0.11 fb $^{-1}$	$0.46_{-0.16}^{+0.18} \pm 0.03$	[123]
CDF2	$J/\psi e$	02–04	0.36 fb $^{-1}$	$0.463_{-0.065}^{+0.073} \pm 0.036$	[125]
D0	$J/\psi \mu$	02–06	1.3 fb $^{-1}$	$0.448_{-0.036}^{+0.038} \pm 0.032$	[126]
CDF2	$J/\psi \pi$		6.7 fb $^{-1}$	$0.452 \pm 0.048 \pm 0.027$	[127]
LHCb	$J/\psi \mu$	12	2 fb $^{-1}$	$0.509 \pm 0.008 \pm 0.012$	[128]
LHCb	$J/\psi \pi$	11–12	3 fb $^{-1}$	$0.5134 \pm 0.0110 \pm 0.0057$	[129]
Average				$0.507 \pm 0.009$	

The following sources of correlated systematic uncertainties have been considered: experimental time resolution within a given experiment,  $b$ -quark fragmentation distribution into weakly decaying  $b$  baryons,  $\Lambda_b^0$  polarization, decay model, and evaluation of the  $b$ -baryon purity in the selected event samples. In computing the averages the central values of the masses are scaled to  $M(\Lambda_b^0) = 5620 \pm 2$  MeV/ $c^2$  [130] and  $M(b\text{-baryon}) = 5670 \pm 100$  MeV/ $c^2$ .

For the semi-inclusive lifetime measurements, the meaning of the decay model systematic uncertainties and the correlation of these uncertainties between measurements are not always clear. Uncertainties related to the decay model are dominated by assumptions on the fraction of  $n$ -body semileptonic decays. To be conservative, it is assumed that these are 100% correlated whenever given as an error. DELPHI varies the fraction of 4-body decays from 0.0 to 0.3. In computing the average, the DELPHI result is corrected to a value of  $0.2 \pm 0.2$  for this fraction.

Furthermore, in computing the average, the semileptonic decay results from LEP are corrected for a polarization of  $-0.45_{-0.17}^{+0.19}$  [44] and a  $\Lambda_b^0$  fragmentation parameter  $\langle X_E \rangle = 0.70 \pm 0.03$  [131].

Inputs to the averages are given in Table 12. For the  $\Lambda_b^0$  lifetime average, we only include measurements obtained with inclusive  $\Lambda_c^\pm \ell^\mp$ , inclusive  $\Lambda \ell^- \ell^+$ , and fully exclusive final states. The CDF  $\Lambda_b^0 \rightarrow J/\psi \Lambda$  lifetime result [138] is larger than the world average computed excluding this result by  $2.5 \sigma$ . It is nonetheless combined with the rest without adjustment of input errors. The world average  $\Lambda_b^0$  lifetime is then

$$\tau(\Lambda_b^0) = 1.466 \pm 0.010 \text{ ps} . \quad (34)$$

It turns out that the average obtained using only measurements performed with semileptonic  $\Lambda_b^0$  decays ( $1.245_{-0.069}^{+0.071}$  ps) is significantly different from the one using only measurements performed with exclusively reconstructed  $\Lambda_b^0$  decays ( $1.470 \pm 0.010$  ps). The latter is much more precise (and less prone to systematic uncertainties) than the former. This discrepancy can only be attributed to a systematic experimental effect or to a statistical fluctuation.

For the strange  $b$  baryons, we no longer include measurements based on  $\Xi^\mp \ell^\mp$  [27, 28, 29] final states which consist of a mixture of  $\Xi_b^0$  and  $\Xi_b^-$  baryons. Instead we only average results obtained with fully exclusive modes, and obtain

$$\tau(\Xi_b^-) = 1.560 \pm 0.040 \text{ ps} , \quad (35)$$

$$\tau(\Xi_b^0) = 1.464 \pm 0.031 \text{ ps} , \quad (36)$$

$$\tau(\Omega_b^-) = 1.57_{-0.20}^{+0.23} \text{ ps} . \quad (37)$$

Table 12: Measurements of the  $b$ -baryon lifetimes.

Experiment	Method	Data set	Lifetime (ps)	Ref.
ALEPH	$\Lambda\ell$	91–95	$1.20 \pm 0.08 \pm 0.06$	[26]
DELPHI	$\Lambda\ell\pi$ vtx	91–94	$1.16 \pm 0.20 \pm 0.08$	[132] <sup>b</sup>
DELPHI	$\Lambda\mu$ i.p.	91–94	$1.10^{+0.19}_{-0.17} \pm 0.09$	[133] <sup>b</sup>
DELPHI	$p\ell$	91–94	$1.19 \pm 0.14 \pm 0.07$	[132] <sup>b</sup>
OPAL	$\Lambda\ell$ i.p.	90–94	$1.21^{+0.15}_{-0.13} \pm 0.10$	[134] <sup>c</sup>
OPAL	$\Lambda\ell$ vtx	90–94	$1.15 \pm 0.12 \pm 0.06$	[134] <sup>c</sup>
ALEPH	$\Lambda_c^+\ell$	91–95	$1.18^{+0.13}_{-0.12} \pm 0.03$	[26] <sup>a</sup>
ALEPH	$\Lambda\ell^-\ell^+$	91–95	$1.30^{+0.26}_{-0.21} \pm 0.04$	[26] <sup>a</sup>
DELPHI	$\Lambda_c^+\ell$	91–94	$1.11^{+0.19}_{-0.18} \pm 0.05$	[132] <sup>b</sup>
OPAL	$\Lambda_c^+\ell, \Lambda\ell^-\ell^+$	90–95	$1.29^{+0.24}_{-0.22} \pm 0.06$	[106]
CDF1	$\Lambda_c^+\ell$	91–95	$1.32 \pm 0.15 \pm 0.07$	[135]
D0	$\Lambda_c^+\mu$	02–06	$1.290^{+0.119+0.087}_{-0.110-0.091}$	[136]
Average of above 6 (semileptonic $\Lambda_b^0$ decays)			$1.245^{+0.071}_{-0.069}$	
CDF2	$\Lambda_c^+\pi$	02–06	$1.401 \pm 0.046 \pm 0.035$	[137]
CDF2	$J/\psi\Lambda$	02–11	$1.565 \pm 0.035 \pm 0.020$	[138]
D0	$J/\psi\Lambda$	02–11	$1.303 \pm 0.075 \pm 0.035$	[83]
ATLAS	$J/\psi\Lambda$	2011	$1.449 \pm 0.036 \pm 0.017$	[92]
CMS	$J/\psi\Lambda$	2011	$1.503 \pm 0.052 \pm 0.031$	[139]
LHCb	$J/\psi\Lambda$	2011	$1.415 \pm 0.027 \pm 0.006$	[93]
LHCb	$J/\psi pK$	11–12	$1.479 \pm 0.009 \pm 0.010$	[140]
Average of above 7 (fully reconstructed $\Lambda_b^0$ decays)			$1.470 \pm 0.010$	
Average of above 13: $\Lambda_b^0$ lifetime =			$1.466 \pm 0.010$	
ALEPH	$\Xi\ell$	90–95	$1.35^{+0.37+0.15}_{-0.28-0.17}$	[27]
DELPHI	$\Xi\ell$	91–93	$1.5^{+0.7}_{-0.4} \pm 0.3$	[29] <sup>d</sup>
DELPHI	$\Xi\ell$	92–95	$1.45^{+0.55}_{-0.43} \pm 0.13$	[28] <sup>d</sup>
CDF2	$J/\psi\Xi^-$	02–11	$1.32 \pm 0.14 \pm 0.02$	[138]
LHCb	$J/\psi\Xi^-$	11–12	$1.55^{+0.10}_{-0.09} \pm 0.03$	[141]
LHCb	$\Xi_c^0\pi^-$	11–12	$1.599 \pm 0.041 \pm 0.022$	[142]
Average of above 3: $\Xi_b^-$ lifetime =			$1.560 \pm 0.040$	
LHCb	$\Xi_c^+\pi^-$	11–12	$1.477 \pm 0.026 \pm 0.019$	[143]
Average of above 1: $\Xi_b^0$ lifetime =			$1.464 \pm 0.031$	
CDF2	$J/\psi\Omega^-$	02–11	$1.66^{+0.53}_{-0.40} \pm 0.02$	[138]
LHCb	$J/\psi\Omega^-$	11–12	$1.54^{+0.26}_{-0.21} \pm 0.05$	[141]
Average of above 2: $\Omega_b^-$ lifetime =			$1.57^{+0.23}_{-0.20}$	

<sup>a</sup> The combined ALEPH result quoted in [26] is  $1.21 \pm 0.11$  ps.<sup>b</sup> The combined DELPHI result quoted in [132] is  $1.14 \pm 0.08 \pm 0.04$  ps.<sup>c</sup> The combined OPAL result quoted in [134] is  $1.16 \pm 0.11 \pm 0.06$  ps.<sup>d</sup> The combined DELPHI result quoted in [28] is  $1.48^{+0.40}_{-0.31} \pm 0.12$  ps.

### 3.2.7 Summary and comparison with theoretical predictions

Averages of lifetimes of specific  $b$ -hadron species are collected in Table 13. As described in

Table 13: Summary of the lifetime averages for the different  $b$ -hadron species.

$b$ -hadron species	Measured lifetime
$B^+$	$1.638 \pm 0.004$ ps
$B^0$	$1.520 \pm 0.004$ ps
$B_s^0$	$1/\Gamma_s = 1.505 \pm 0.004$ ps
$B_{sL}$	$1/\Gamma_L = 1.417 \pm 0.006$ ps
$B_{sH}$	$1/\Gamma_H = 1.604 \pm 0.010$ ps
$B_c^+$	$0.507 \pm 0.009$ ps
$\Lambda_b^0$	$1.466 \pm 0.010$ ps
$\Xi_b^-$	$1.560 \pm 0.040$ ps
$\Xi_b^0$	$1.464 \pm 0.031$ ps
$\Omega_b^-$	$1.57^{+0.23}_{-0.20}$ ps

Table 14: Measured ratios of  $b$ -hadron lifetimes relative to the  $B^0$  lifetime and ranges predicted by theory [55, 56].

Lifetime ratio	Measured value	Predicted range
$\tau(B^+)/\tau(B^0)$	$1.076 \pm 0.004$	1.04 – 1.08
$\tau(B_s^0)/\tau(B^0)$	$0.990 \pm 0.004$	0.99 – 1.01
$\tau(\Lambda_b^0)/\tau(B^0)$	$0.965 \pm 0.007$	0.86 – 0.95

the introduction to Sec. 3.2, the HQE can be employed to explain the hierarchy of  $\tau(B_c^+) \ll \tau(\Lambda_b^0) < \tau(B_s^0) \approx \tau(B^0) < \tau(B^+)$ , and used to predict the ratios between lifetimes. Typical predictions are compared to the measured lifetime ratios in Table 14. The prediction of the ratio between the  $B^+$  and  $B^0$  lifetimes,  $1.06 \pm 0.02$  [55], is in good agreement with experiment.

The total widths of the  $B_s^0$  and  $B^0$  mesons are expected to be very close and differ by at most 1% [144, 56]. This prediction is consistent with the experimental ratio  $\tau(B_s^0)/\tau(B^0) = \Gamma_d/\Gamma_s$ , which is smaller than 1 by  $(1.0 \pm 0.4)\%$ .

The ratio  $\tau(\Lambda_b^0)/\tau(B^0)$  has particularly been the source of theoretical scrutiny since earlier calculations using the HQE [52, 145] predicted a value larger than 0.90, almost  $2\sigma$  above the world average at the time. Many predictions cluster around a most likely central value of 0.94 [146]. More recent calculations of this ratio that include higher-order effects predict a lower ratio between the  $\Lambda_b^0$  and  $B^0$  lifetimes [55, 56] and reduce this difference. References [55, 56] present probability density functions of their predictions with a variation of theoretical inputs, and the indicated ranges in Table 14 are the RMS of the distributions from the most probable values, and for  $\tau(\Lambda_b^0)/\tau(B^0)$ , also encompass the earlier theoretical predictions [52, 145, 146]. Note that in contrast to the  $B$  mesons, complete NLO QCD corrections and fully reliable lattice determinations of the matrix elements for  $\Lambda_b^0$  are not yet available. As already mentioned, the CDF measurement of the  $\Lambda_b^0$  lifetime in the exclusive decay mode  $J/\psi \Lambda$  [138] is significantly higher than the world average before inclusion, with a ratio to the  $\tau(B^0)$  world average of  $\tau(\Lambda_b^0)/\tau(B^0) = 1.012 \pm 0.031$ , resulting in continued interest in lifetimes of  $b$  baryons.

The lifetimes of the most abundant  $b$ -hadron species are now all known to sub-percent precision. Neglecting the contributions of the rarer species ( $B_c^+$  meson and  $b$  baryons other

than the  $\Lambda_b^0$ ), one can compute the average  $b$ -hadron lifetime from the individual lifetimes and production fractions as

$$\tau_b = \frac{f_d\tau(B^0)^2 + f_u\tau(B^+)^2 + 0.5f_s\tau(B_{sH})^2 + 0.5f_s\tau(B_{sL})^2 + f_{\text{baryon}}\tau(\Lambda_b^0)^2}{f_d\tau(B^0) + f_u\tau(B^+) + 0.5f_s\tau(B_{sH}) + 0.5f_s\tau(B_{sL}) + f_{\text{baryon}}\tau(\Lambda_b^0)}. \quad (38)$$

Using the lifetimes of Table 13 and the fractions in  $Z$  decays of Table 5, taking into account the correlations between the fractions (Table 5) as well as the correlation between  $\tau(B_{sH})$  and  $\tau(B_{sL})$  ( $-0.390$ ), one obtains

$$\tau_b(Z) = 1.566 \pm 0.003 \text{ ps}. \quad (39)$$

This is in very good agreement with (and three times more precise than) the average of Eq. (20) for the inclusive measurements performed at LEP.

### 3.3 Neutral $B$ -meson mixing

The  $B^0 - \bar{B}^0$  and  $B_s^0 - \bar{B}_s^0$  systems both exhibit the phenomenon of particle-antiparticle mixing. For each of them, there are two mass eigenstates which are linear combinations of the two flavour states,  $B$  and  $\bar{B}$ . The heaviest (lightest) of these mass states is denoted  $B_H$  ( $B_L$ ), with mass  $m_H$  ( $m_L$ ) and total decay width  $\Gamma_H$  ( $\Gamma_L$ ). We define

$$\Delta m = m_H - m_L, \quad x = \Delta m/\Gamma, \quad (40)$$

$$\Delta\Gamma = \Gamma_L - \Gamma_H, \quad y = \Delta\Gamma/(2\Gamma), \quad (41)$$

where  $\Gamma = (\Gamma_H + \Gamma_L)/2 = 1/\bar{\tau}(B)$  is the average decay width.  $\Delta m$  is positive by definition, and  $\Delta\Gamma$  is expected to be positive within the Standard Model.<sup>20</sup>

There are four different time-dependent probabilities describing the case of a neutral  $B$  meson produced as a flavour state and decaying without  $CP$  violation to a flavour-specific final state. If  $CPT$  is conserved (which will be assumed throughout), they can be written as

$$\begin{cases} \mathcal{P}(B \rightarrow B) &= \frac{e^{-\Gamma t}}{2} \left[ \cosh\left(\frac{\Delta\Gamma}{2}t\right) + \cos(\Delta mt) \right] \\ \mathcal{P}(B \rightarrow \bar{B}) &= \frac{e^{-\Gamma t}}{2} \left[ \cosh\left(\frac{\Delta\Gamma}{2}t\right) - \cos(\Delta mt) \right] \left| \frac{q}{p} \right|^2 \\ \mathcal{P}(\bar{B} \rightarrow B) &= \frac{e^{-\Gamma t}}{2} \left[ \cosh\left(\frac{\Delta\Gamma}{2}t\right) - \cos(\Delta mt) \right] \left| \frac{p}{q} \right|^2 \\ \mathcal{P}(\bar{B} \rightarrow \bar{B}) &= \frac{e^{-\Gamma t}}{2} \left[ \cosh\left(\frac{\Delta\Gamma}{2}t\right) + \cos(\Delta mt) \right] \end{cases}, \quad (42)$$

where  $t$  is the proper time of the system (*i.e.* the time interval between the production and the decay in the rest frame of the  $B$  meson). At the  $B$  factories, only the proper-time difference  $\Delta t$  between the decays of the two neutral  $B$  mesons from the  $\Upsilon(4S)$  can be determined, but, because the two  $B$  mesons evolve coherently (keeping opposite flavours as long as neither of them has decayed), the above formulae remain valid if  $t$  is replaced with  $\Delta t$  and the production flavour is replaced by the flavour at the time of the decay of the accompanying  $B$  meson in a flavour-specific state. As can be seen in the above expressions, the mixing probabilities depend

<sup>20</sup> For reasons of symmetry in Eqs. (40) and (41),  $\Delta\Gamma$  is sometimes defined with the opposite sign. The definition adopted here, *i.e.* Eq. (41), is the one used by most experimentalists and many phenomenologists in  $B$  physics.

on three mixing observables:  $\Delta m$ ,  $\Delta\Gamma$ , and  $|q/p|^2$ , which signals  $CP$  violation in the mixing if  $|q/p|^2 \neq 1$ .

In the next sections we review in turn the experimental knowledge on the  $B^0$  decay-width and mass differences, the  $B_s^0$  decay-width and mass differences,  $CP$  violation in  $B^0$  and  $B_s^0$  mixing, and mixing-induced  $CP$  violation in  $B_s^0$  decays.

### 3.3.1 $B^0$ mixing parameters $\Delta\Gamma_d$ and $\Delta m_d$

A large number of time-dependent  $B^0$ - $\bar{B}^0$  oscillation analyses have been performed since almost 20 years by the ALEPH, DELPHI, L3, OPAL, CDF, D0, *BABAR*, Belle and LHCb collaborations. The corresponding measurements of  $\Delta m_d$  are summarized in Table 15, where only the most recent results are listed (*i.e.* measurements superseded by more recent ones are omitted<sup>21</sup>). Although a variety of different techniques have been used, the individual  $\Delta m_d$  results obtained at different colliders have remarkably similar precision. The systematic uncertainties are comparable to the statistical uncertainties; they are often dominated by sample composition, mistag probability, or  $b$ -hadron lifetime contributions. Before being combined, the measurements are adjusted on the basis of a common set of input values, including the averages of the  $b$ -hadron fractions and lifetimes given in this report (see Secs. 3.1 and 3.2). Some measurements are statistically correlated. Systematic correlations arise both from common physics sources (fractions, lifetimes, branching ratios of  $b$  hadrons), and from purely experimental or algorithmic effects (efficiency, resolution, flavour tagging, background description). Combining all published measurements listed in Table 15 and accounting for all identified correlations as described in Ref. [44] yields  $\Delta m_d = 0.5055 \pm 0.0016 \pm 0.0012$  ps<sup>-1</sup>.

On the other hand, ARGUS and CLEO have published measurements of the time-integrated mixing probability  $\chi_d$  [168, 169, 170], which average to  $\chi_d = 0.182 \pm 0.015$ . Following Ref. [170], the decay width difference  $\Delta\Gamma_d$  could in principle be extracted from the measured value of  $\Gamma_d = 1/\tau(B^0)$  and the above averages for  $\Delta m_d$  and  $\chi_d$  (provided that  $\Delta\Gamma_d$  has a negligible impact on the  $\Delta m_d$  and  $\tau(B^0)$  analyses that have assumed  $\Delta\Gamma_d = 0$ ), using the relation

$$\chi_d = \frac{x_d^2 + y_d^2}{2(x_d^2 + 1)} \quad \text{with} \quad x_d = \frac{\Delta m_d}{\Gamma_d} \quad \text{and} \quad y_d = \frac{\Delta\Gamma_d}{2\Gamma_d}. \quad (43)$$

However, direct time-dependent studies provide much stronger constraints:  $|\Delta\Gamma_d|/\Gamma_d < 18\%$  at 95% CL from DELPHI [149],  $-6.8\% < \text{sign}(\text{Re}\lambda_{CP})\Delta\Gamma_d/\Gamma_d < 8.4\%$  at 90% CL from *BABAR* [171], and  $\text{sign}(\text{Re}\lambda_{CP})\Delta\Gamma_d/\Gamma_d = (1.7 \pm 1.8 \pm 1.1)\%$  [172] from Belle, where  $\lambda_{CP} = (q/p)_d(\bar{A}_{CP}/A_{CP})$  is defined for a  $CP$ -even final state (the sensitivity to the overall sign of  $\text{sign}(\text{Re}\lambda_{CP})\Delta\Gamma_d/\Gamma_d$  comes from the use of  $B^0$  decays to  $CP$  final states). In addition, the D0 collaboration has recently extracted a value of  $\Delta\Gamma_d/\Gamma_d = (0.50 \pm 1.38)\%$  [173] from their measurements of the same-sign dimuon charge asymmetry, under the interpretation that the observed asymmetries are due to  $CP$  violation in neutral  $B$ -meson mixing and interference. More recently LHCb has obtained  $\Delta\Gamma_d/\Gamma_d = (-0.044 \pm 0.025 \pm 0.011)\%$  [93] by comparing measurements of the  $B^0 \rightarrow J/\psi K^{*0}$  and  $B^0 \rightarrow J/\psi K_S^0$  decays, following the method of Ref. [174]. Assuming  $\text{Re}\lambda_{CP} > 0$ , as expected from the global fits of the Unitarity Triangle within the Standard Model [175], a combination of these five results (after adjusting the DELPHI and

<sup>21</sup> Two old unpublished CDF2 measurements [166, 167] are also omitted from our averages, Table 15 and Fig. 3.

Table 15: Time-dependent measurements included in the  $\Delta m_d$  average. The results obtained from multi-dimensional fits involving also the  $B^0$  (and  $B^+$ ) lifetimes as free parameter(s) [87, 89, 90] have been converted into one-dimensional measurements of  $\Delta m_d$ . All the measurements have then been adjusted to a common set of physics parameters before being combined.

Experiment and Ref.	Method		$\Delta m_d$ in $\text{ps}^{-1}$			$\Delta m_d$ in $\text{ps}^{-1}$		
	rec.	tag	before adjustment			after adjustment		
ALEPH [147]	$\ell$	$Q_{\text{jet}}$	0.404	$\pm 0.045$	$\pm 0.027$			
ALEPH [147]	$\ell$	$\ell$	0.452	$\pm 0.039$	$\pm 0.044$			
ALEPH [147]	above two combined		0.422	$\pm 0.032$	$\pm 0.026$	0.440	$\pm 0.032$	$^{+0.020}_{-0.019}$
ALEPH [147]	$D^*$	$\ell, Q_{\text{jet}}$	0.482	$\pm 0.044$	$\pm 0.024$	0.482	$\pm 0.044$	$\pm 0.024$
DELPHI [148]	$\ell$	$Q_{\text{jet}}$	0.493	$\pm 0.042$	$\pm 0.027$	0.498	$\pm 0.042$	$\pm 0.024$
DELPHI [148]	$\pi^* \ell$	$Q_{\text{jet}}$	0.499	$\pm 0.053$	$\pm 0.015$	0.500	$\pm 0.053$	$\pm 0.015$
DELPHI [148]	$\ell$	$\ell$	0.480	$\pm 0.040$	$\pm 0.051$	0.493	$\pm 0.040$	$^{+0.042}_{-0.040}$
DELPHI [148]	$D^*$	$Q_{\text{jet}}$	0.523	$\pm 0.072$	$\pm 0.043$	0.518	$\pm 0.072$	$\pm 0.043$
DELPHI [149]	vtx	comb	0.531	$\pm 0.025$	$\pm 0.007$	0.525	$\pm 0.025$	$\pm 0.006$
L3 [150]	$\ell$	$\ell$	0.458	$\pm 0.046$	$\pm 0.032$	0.466	$\pm 0.046$	$\pm 0.028$
L3 [150]	$\ell$	$Q_{\text{jet}}$	0.427	$\pm 0.044$	$\pm 0.044$	0.438	$\pm 0.044$	$\pm 0.042$
L3 [150]	$\ell$	$\ell(\text{IP})$	0.462	$\pm 0.063$	$\pm 0.053$	0.468	$\pm 0.063$	$\pm 0.044$
OPAL [151]	$\ell$	$\ell$	0.430	$\pm 0.043$	$^{+0.028}_{-0.030}$	0.465	$\pm 0.043$	$^{+0.017}_{-0.016}$
OPAL [152]	$\ell$	$Q_{\text{jet}}$	0.444	$\pm 0.029$	$^{+0.020}_{-0.017}$	0.481	$\pm 0.029$	$\pm 0.013$
OPAL [153]	$D^* \ell$	$Q_{\text{jet}}$	0.539	$\pm 0.060$	$\pm 0.024$	0.544	$\pm 0.060$	$\pm 0.023$
OPAL [153]	$D^*$	$\ell$	0.567	$\pm 0.089$	$^{+0.029}_{-0.023}$	0.572	$\pm 0.089$	$^{+0.028}_{-0.022}$
OPAL [77]	$\pi^* \ell$	$Q_{\text{jet}}$	0.497	$\pm 0.024$	$\pm 0.025$	0.496	$\pm 0.024$	$\pm 0.025$
CDF1 [154]	$D \ell$	SST	0.471	$^{+0.078}_{-0.068}$	$^{+0.033}_{-0.034}$	0.470	$^{+0.078}_{-0.068}$	$^{+0.033}_{-0.034}$
CDF1 [155]	$\mu$	$\mu$	0.503	$\pm 0.064$	$\pm 0.071$	0.514	$\pm 0.064$	$^{+0.070}_{-0.069}$
CDF1 [156]	$\ell$	$\ell, Q_{\text{jet}}$	0.500	$\pm 0.052$	$\pm 0.043$	0.545	$\pm 0.052$	$\pm 0.036$
CDF1 [157]	$D^* \ell$	$\ell$	0.516	$\pm 0.099$	$^{+0.029}_{-0.035}$	0.523	$\pm 0.099$	$^{+0.028}_{-0.035}$
D0 [158]	$D^{(*)} \mu$	OST	0.506	$\pm 0.020$	$\pm 0.016$	0.506	$\pm 0.020$	$\pm 0.016$
BABAR [159]	$B^0$	$\ell, K, \text{NN}$	0.516	$\pm 0.016$	$\pm 0.010$	0.521	$\pm 0.016$	$\pm 0.008$
BABAR [160]	$\ell$	$\ell$	0.493	$\pm 0.012$	$\pm 0.009$	0.487	$\pm 0.012$	$\pm 0.006$
BABAR [89]	$D^* \ell \nu(\text{part})$	$\ell$	0.511	$\pm 0.007$	$\pm 0.007$	0.513	$\pm 0.007$	$\pm 0.007$
BABAR [87]	$D^* \ell \nu$	$\ell, K, \text{NN}$	0.492	$\pm 0.018$	$\pm 0.014$	0.493	$\pm 0.018$	$\pm 0.013$
Belle [161]	$D^* \pi(\text{part})$	$\ell$	0.509	$\pm 0.017$	$\pm 0.020$	0.513	$\pm 0.017$	$\pm 0.019$
Belle [6]	$\ell$	$\ell$	0.503	$\pm 0.008$	$\pm 0.010$	0.506	$\pm 0.008$	$\pm 0.008$
Belle [90]	$B^0, D^* \ell \nu$	comb	0.511	$\pm 0.005$	$\pm 0.006$	0.513	$\pm 0.005$	$\pm 0.006$
LHCb [162]	$B^0$	OST	0.499	$\pm 0.032$	$\pm 0.003$	0.499	$\pm 0.032$	$\pm 0.003$
LHCb [163]	$B^0$	OST, SST	0.5156	$\pm 0.0051$	$\pm 0.0033$	0.5156	$\pm 0.0051$	$\pm 0.0033$
LHCb [164]	$D \mu$	OST, SST	0.503	$\pm 0.011$	$\pm 0.013$	0.503	$\pm 0.011$	$\pm 0.013$
LHCb [165]	$D^{(*)} \mu$	OST	0.5036	$\pm 0.0020$	$\pm 0.0013$	0.5036	$\pm 0.0020$	$\pm 0.0013$
World average (all above measurements included):						0.5055 $\pm 0.0016 \pm 0.0012$		
– ALEPH, DELPHI, L3, OPAL and CDF1 only:						0.496 $\pm 0.010$ $\pm 0.009$		
– BABAR and Belle only:						0.509 $\pm 0.003$ $\pm 0.003$		
– LHCb only:						0.5051 $\pm 0.0018 \pm 0.0013$		

Table 16: Simultaneous measurements of  $\Delta m_d$  and  $\tau(B^0)$ , and their average. The Belle analysis also measures  $\tau(B^+)$  at the same time, but it is converted here into a two-dimensional measurement of  $\Delta m_d$  and  $\tau(B^0)$ , for an assumed value of  $\tau(B^+)$ . The first quoted error on each measurement is statistical and the second is systematic; in the case of adjusted measurements, the latter includes a contribution obtained from the variation of  $\tau(B^+)$  or  $\tau(B^+)/\tau(B^0)$  in the indicated range. Units are  $\text{ps}^{-1}$  for  $\Delta m_d$  and ps for lifetimes. The three different values of  $\rho(\Delta m_d, \tau(B^0))$  correspond to the statistical, systematic and total correlation coefficients between the adjusted measurements of  $\Delta m_d$  and  $\tau(B^0)$ .

Exp. & Ref.	Measured $\Delta m_d$	Measured $\tau(B^0)$	Measured $\tau(B^+)$	Assumed $\tau(B^+)$
<i>BABAR</i> [87]	$0.492 \pm 0.018 \pm 0.013$	$1.523 \pm 0.024 \pm 0.022$	—	$(1.083 \pm 0.017)\tau(B^0)$
<i>BABAR</i> [89]	$0.511 \pm 0.007 \begin{smallmatrix} +0.007 \\ -0.006 \end{smallmatrix}$	$1.504 \pm 0.013 \begin{smallmatrix} +0.018 \\ -0.013 \end{smallmatrix}$	—	$1.671 \pm 0.018$
Belle [90]	$0.511 \pm 0.005 \pm 0.006$	$1.534 \pm 0.008 \pm 0.010$	$1.635 \pm 0.011 \pm 0.011$	—
	Adjusted $\Delta m_d$	Adjusted $\tau(B^0)$	$\rho(\Delta m_d, B^0)$	Assumed $\tau(B^+)$
<i>BABAR</i> [87]	$0.492 \pm 0.018 \pm 0.013$	$1.523 \pm 0.024 \pm 0.022$	$-0.22 \ +0.71 \ +0.16$	$(1.076 \pm 0.004)\tau(B^0)$
<i>BABAR</i> [89]	$0.512 \pm 0.007 \pm 0.007$	$1.506 \pm 0.013 \pm 0.018$	$+0.01 \ -0.85 \ -0.48$	$1.638 \pm 0.004$
Belle [90]	$0.511 \pm 0.005 \pm 0.006$	$1.535 \pm 0.008 \pm 0.011$	$-0.27 \ -0.14 \ -0.19$	$1.638 \pm 0.004$
Average	$0.509 \pm 0.004 \pm 0.004$	$1.527 \pm 0.006 \pm 0.008$	$-0.19 \ -0.25 \ -0.23$	$1.638 \pm 0.004$

*BABAR* results to  $1/\Gamma_d = \tau(B^0) = 1.520 \pm 0.004$  ps) yields

$$\Delta\Gamma_d/\Gamma_d = 0.001 \pm 0.010. \quad (44)$$

Assuming  $\Delta\Gamma_d = 0$  and using  $1/\Gamma_d = \tau(B^0) = 1.520 \pm 0.004$  ps, the  $\Delta m_d$  and  $\chi_d$  results are combined through Eq. (43) to yield the world average

$$\Delta m_d = 0.5055 \pm 0.0020 \text{ ps}^{-1}, \quad (45)$$

or, equivalently,

$$x_d = 0.768 \pm 0.004 \quad \text{and} \quad \chi_d = 0.1856 \pm 0.0011. \quad (46)$$

Figure 3 compares the  $\Delta m_d$  values obtained by the different experiments.

The  $B^0$  mixing averages given in Eqs. (45) and (46) and the  $b$ -hadron fractions of Table 5 have been obtained in a fully consistent way, taking into account the fact that the fractions are computed using the  $\chi_d$  value of Eq. (46) and that many individual measurements of  $\Delta m_d$  at high energy depend on the assumed values for the  $b$ -hadron fractions. Furthermore, this set of averages is consistent with the lifetime averages of Sec. 3.2.

It should be noted that the most recent (and precise) analyses at the asymmetric  $B$  factories measure  $\Delta m_d$  as a result of a multi-dimensional fit. Two *BABAR* analyses [87,89], based on fully and partially reconstructed  $B^0 \rightarrow D^* \ell \nu$  decays respectively, extract simultaneously  $\Delta m_d$  and  $\tau(B^0)$  while the latest Belle analysis [90], based on fully reconstructed hadronic  $B^0$  decays and  $B^0 \rightarrow D^* \ell \nu$  decays, extracts simultaneously  $\Delta m_d$ ,  $\tau(B^0)$  and  $\tau(B^+)$ . The measurements of  $\Delta m_d$  and  $\tau(B^0)$  of these three analyses are displayed in Table 16 and in Fig. 4. Their two-dimensional average, taking into account all statistical and systematic correlations, and expressed at  $\tau(B^+) = 1.638 \pm 0.004$  ps, is

$$\left. \begin{array}{l} \Delta m_d = 0.509 \pm 0.006 \text{ ps}^{-1} \\ \tau(B^0) = 1.527 \pm 0.010 \text{ ps} \end{array} \right\} \text{ with a total correlation of } -0.23. \quad (47)$$

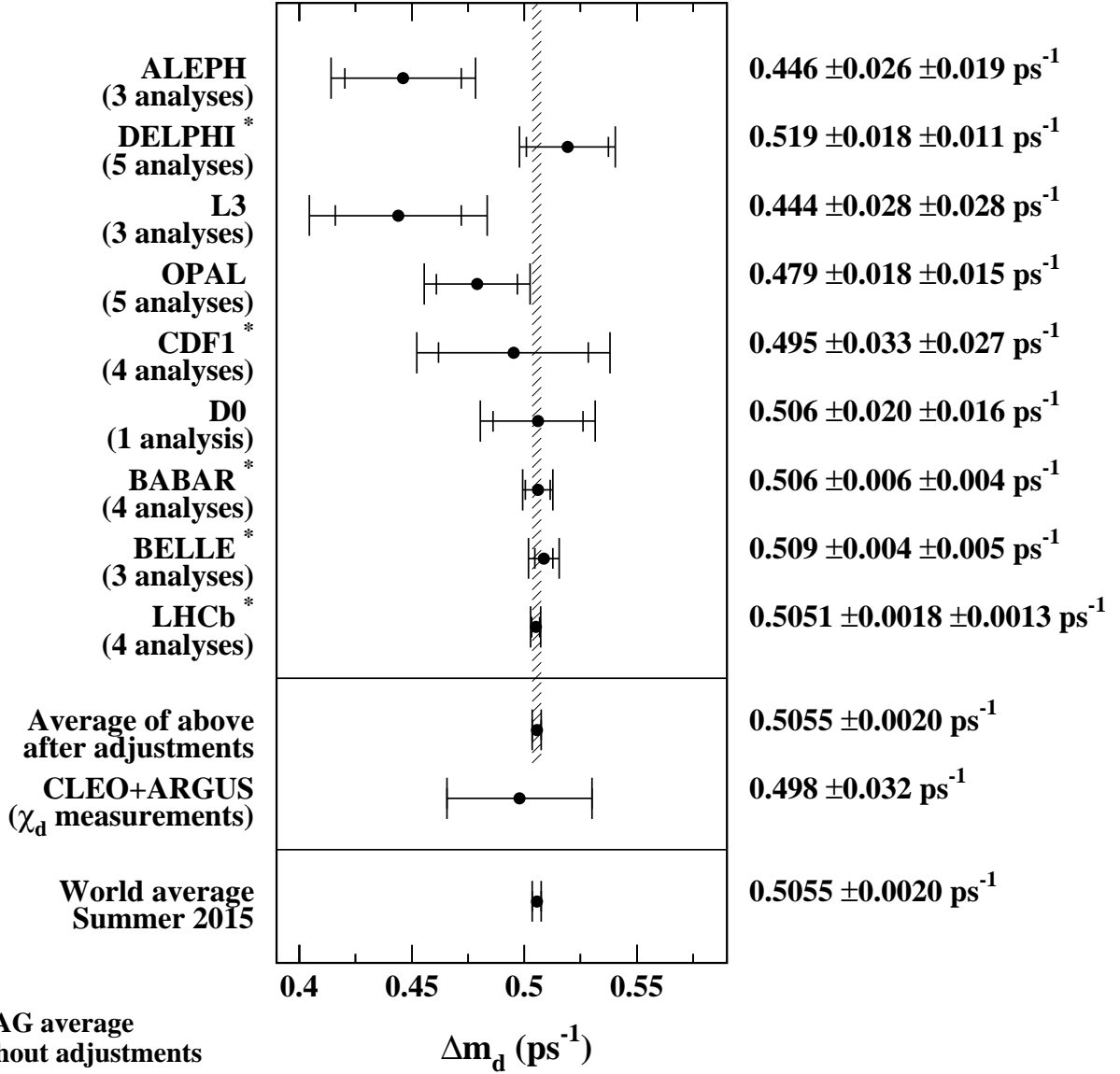


Figure 3: The  $B^0-\bar{B}^0$  oscillation frequency  $\Delta m_d$  as measured by the different experiments. The averages quoted for ALEPH, L3 and OPAL are taken from the original publications, while the ones for DELPHI, CDF, *BABAR*, Belle and LHCb have been computed from the individual results listed in Table 15 without performing any adjustments. The time-integrated measurements of  $\chi_d$  from the symmetric  $B$  factory experiments ARGUS and CLEO have been converted to a  $\Delta m_d$  value using  $\tau(B^0) = 1.520 \pm 0.004$  ps. The two global averages have been obtained after adjustments of all the individual  $\Delta m_d$  results of Table 15 (see text).



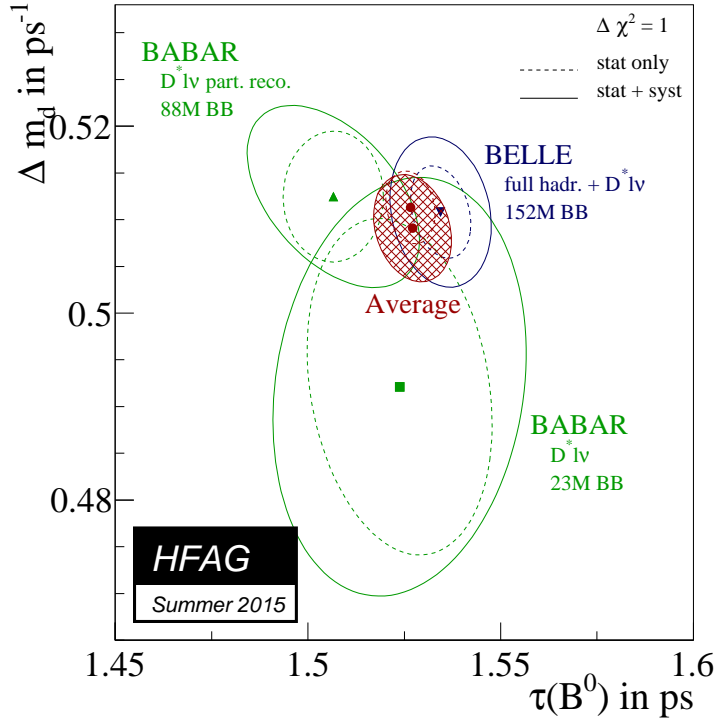


Figure 4: Simultaneous measurements of  $\Delta m_d$  and  $\tau(B^0)$  [87, 89, 90], after adjustment to a common set of parameters (see text). Statistical and total uncertainties are represented as dashed and solid contours respectively. The average of the three measurements is indicated by a hatched ellipse.

### 3.3.2 $B_s^0$ mixing parameters $\Delta\Gamma_s$ and $\Delta m_s$

Definitions and an introduction to  $\Delta\Gamma_s$  have been given in Sec. 3.2.4. Neglecting  $CP$  violation, the mass eigenstates are also  $CP$  eigenstates, with the short-lived state being  $CP$ -even and the long-lived state being  $CP$ -odd.

The best sensitivity to  $\Delta\Gamma_s$  is currently achieved by the recent time-dependent measurements of the  $B_s^0 \rightarrow J/\psi \phi$  (or more generally  $B_s^0 \rightarrow J/\psi K^+ K^-$ ) decay rates performed at CDF [176], D0 [177], ATLAS [178, 179], CMS [180, 181] and LHCb [182], where the  $CP$ -even and  $CP$ -odd amplitudes are statistically separated through a full angular analysis (see last two columns of Table 22). With the exception of the first CMS analysis [180], these studies use both untagged and tagged  $B_s^0$  candidates and are optimized for the measurement of the  $CP$ -violating phase  $\phi_s^{c\bar{c}s}$ , defined later in Sec. 3.3.4. The LHCb collaboration analyzed the  $B_s^0 \rightarrow J/\psi K^+ K^-$  decay, considering that the  $K^+ K^-$  system can be in a  $P$ -wave or  $S$ -wave state, and measured the dependence of the strong phase difference between the  $P$ -wave and  $S$ -wave amplitudes as a function of the  $K^+ K^-$  invariant mass [101]. This allowed, for the first time, the unambiguous determination of the sign of  $\Delta\Gamma_s$ , which was found to be positive at the  $4.7\sigma$  level. The following averages present only the  $\Delta\Gamma_s > 0$  solutions.

The available results [176, 177, 178, 179, 180, 181, 182] are shown in Table 17. They are combined, taking into account, in each analysis, the correlation between  $\Delta\Gamma_s$  and  $\Gamma_s$ . The results, displayed as the red contours labelled “ $B_s^0 \rightarrow J/\psi KK$  measurements” in the plots of

Table 17: Measurements of  $\Delta\Gamma_s$  and  $\Gamma_s$  using  $B_s^0 \rightarrow J/\psi\phi$  and  $B_s^0 \rightarrow J/\psi K^+K^-$  decays. Only the solution with  $\Delta\Gamma_s > 0$  is shown, since the two-fold ambiguity has been resolved in Ref. [101]. The first error is due to statistics, the second one to systematics. The last line gives our average.

Exp.	Mode	Dataset	$\Delta\Gamma_s$ (ps <sup>-1</sup> )	$\Gamma_s$ (ps <sup>-1</sup> )	Ref.
CDF	$J/\psi\phi$	9.6 fb <sup>-1</sup>	+0.068 ± 0.026 ± 0.009	0.654 ± 0.008 ± 0.004	[176]
D0	$J/\psi\phi$	8.0 fb <sup>-1</sup>	+0.163 <sup>+0.065</sup> <sub>-0.064</sub>	0.693 <sup>+0.018</sup> <sub>-0.017</sub>	[177]
ATLAS	$J/\psi\phi$	4.9 fb <sup>-1</sup>	+0.053 ± 0.021 ± 0.010	0.677 ± 0.007 ± 0.004	[178]
ATLAS	$J/\psi\phi$	14.3 fb <sup>-1</sup>	+0.096 ± 0.013 ± 0.007	0.??? ± 0.??? ± 0.???	[179] <sup>p</sup>
ATLAS	above 2 combined		+0.082 ± 0.011 ± 0.007	0.677 ± 0.003 ± 0.003	[179] <sup>p</sup>
CMS	$J/\psi\phi$	5.0 fb <sup>-1</sup>	+0.048 ± 0.024 ± 0.003	0.655 ± 0.008 ± 0.003	[180] <sup>p</sup>
CMS	$J/\psi\phi$	20 fb <sup>-1</sup>	+0.095 ± 0.013 ± 0.007	0.6704 ± 0.0043 ± 0.0051	[181] <sup>p</sup>
LHCb	$J/\psi K^+K^-$	3.0 fb <sup>-1</sup>	+0.0805 ± 0.0091 ± 0.0033	0.6603 ± 0.0027 ± 0.0015	[182]
All combined			+0.079 ± 0.006	0.6648 ± 0.0022	

<sup>p</sup> Preliminary.

Table 18: Averages of  $\Delta\Gamma_s$ ,  $\Gamma_s$  and related quantities, obtained from  $B_s^0 \rightarrow J/\psi\phi$  and  $B_s^0 \rightarrow J/\psi K^+K^-$  alone (first column), adding the constraints from the effective lifetimes measured in pure  $CP$  modes  $B_s^0 \rightarrow D_s^+D_s^-$  and  $B_s^0 \rightarrow J/\psi f_0(980)$ ,  $J/\psi\pi^+\pi^-$  (second column), and adding the constraint from the effective lifetime measured in flavour-specific modes  $B_s^0 \rightarrow D_s^-\ell^+\nu X$ ,  $D_s^-\pi^+$ ,  $D_s^-D^+$  (third column, recommended world averages).

	$B_s^0 \rightarrow J/\psi K^+K^-$ modes only (see Table 17)	$B_s^0 \rightarrow J/\psi K^+K^-$ modes + pure $CP$ modes	$B_s^0 \rightarrow J/\psi K^+K^-$ modes + pure $CP$ modes + flavour-specific modes
$\Gamma_s$	0.6648 ± 0.0022 ps <sup>-1</sup>	0.6641 ± 0.0021 ps <sup>-1</sup>	0.6645 ± 0.0020 ps <sup>-1</sup>
$1/\Gamma_s$	1.504 ± 0.005 ps	1.506 ± 0.005 ps	1.505 ± 0.004 ps
$1/\Gamma_L$	1.420 ± 0.006 ps	1.417 ± 0.006 ps	1.417 ± 0.006 ps
$1/\Gamma_H$	1.599 ± 0.011 ps	1.606 ± 0.011 ps	1.604 ± 0.010 ps
$\Delta\Gamma_s$	+0.079 ± 0.006 ps <sup>-1</sup>	+0.083 ± 0.006 ps <sup>-1</sup>	+0.082 ± 0.006 ps <sup>-1</sup>
$\Delta\Gamma_s/\Gamma_s$	+0.118 ± 0.010	+0.125 ± 0.009	+0.124 ± 0.009
$\rho(\Gamma_s, \Delta\Gamma_s)$	-0.326	-0.297	-0.243

Fig. 5, are given in the first column of numbers of Table 18.

An alternative approach, which is directly sensitive to first order in  $\Delta\Gamma_s/\Gamma_s$ , is to determine the effective lifetime of untagged  $B_s^0$  candidates decaying to pure  $CP$  eigenstates; we use here measurements with  $B_s^0 \rightarrow D_s^+D_s^-$  [108],  $B_s^0 \rightarrow J/\psi f_0(980)$  [117] and  $B_s^0 \rightarrow J/\psi\pi^+\pi^-$  [118] decays. The precise extraction of  $1/\Gamma_s$  and  $\Delta\Gamma_s$  from such measurements, discussed in detail in Ref. [102], requires additional information in the form of theoretical assumptions or external inputs on weak phases and hadronic parameters. If  $f$  designates a final state in which both  $B_s^0$  and  $\bar{B}_s^0$  can decay, the ratio of the effective  $B_s^0$  lifetime decaying to  $f$  relative to the mean  $B_s^0$

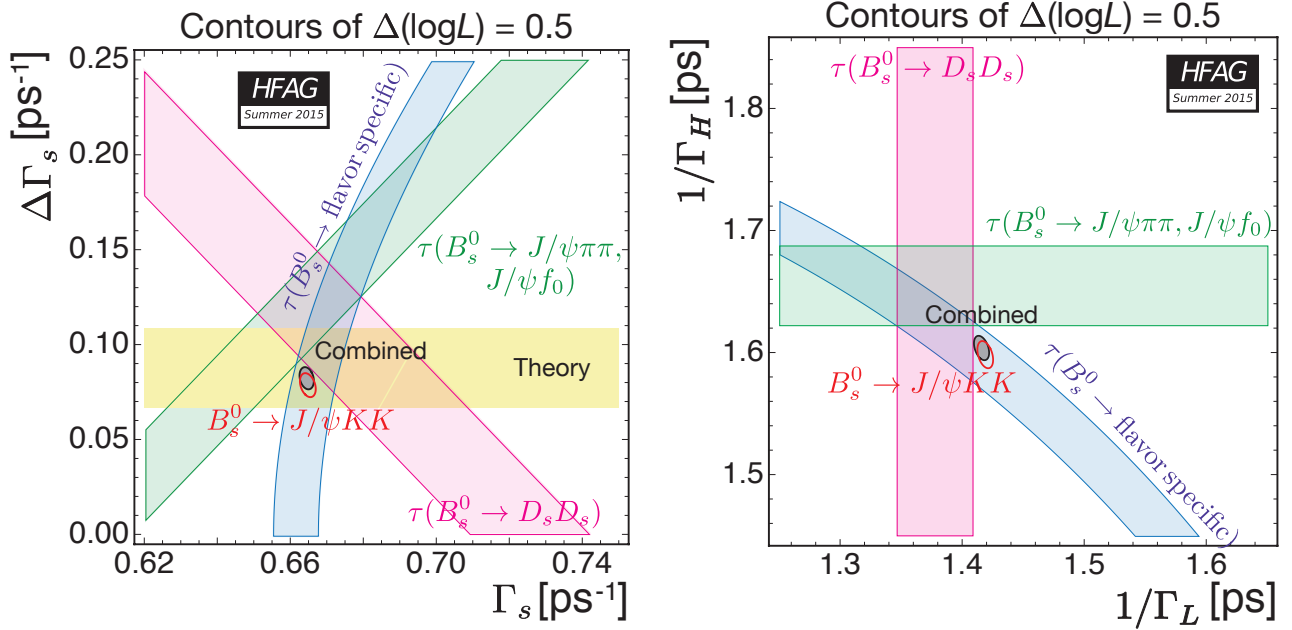


Figure 5: Contours of  $\Delta \ln L = 0.5$  (39% CL for the enclosed 2D regions, 68% CL for the bands) shown in the  $(\Gamma_s, \Delta\Gamma_s)$  plane on the left and in the  $(1/\Gamma_L, 1/\Gamma_H)$  plane on the right. The average of all the  $B_s^0 \rightarrow J/\psi \phi$  and  $B_s^0 \rightarrow J/\psi K^+ K^-$  results is shown as the red contour, and the constraints given by the effective lifetime measurements of  $B_s^0$  to flavour-specific, pure  $CP$ -odd and pure  $CP$ -even final states are shown as the blue, green and purple bands, respectively. The average taking all constraints into account is shown as the gray-filled contour. The yellow band is a theory prediction  $\Delta\Gamma_s = 0.087 \pm 0.021 \text{ ps}^{-1}$  [99] that assumes no new physics in  $B_s^0$  mixing.

lifetime is [102]<sup>22</sup>

$$\frac{\tau_{\text{single}}(B_s^0 \rightarrow f)}{\tau(B_s^0)} = \frac{1}{1 - y_s^2} \left[ \frac{1 - 2A_f^{\Delta\Gamma} y_s + y_s^2}{1 - A_f^{\Delta\Gamma} y_s} \right], \quad (48)$$

where

$$A_f^{\Delta\Gamma} = -\frac{2\text{Re}(\lambda_f)}{1 + |\lambda_f|^2}. \quad (49)$$

To include the measurements of the effective  $B_s^0 \rightarrow D_s^+ D_s^-$  ( $CP$ -even),  $B_s^0 \rightarrow J/\psi f_0(980)$  ( $CP$ -odd) and  $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$  ( $CP$ -odd) lifetimes as constraints in the  $\Delta\Gamma_s$  fit,<sup>23</sup> we neglect sub-leading penguin contributions and possible direct  $CP$  violation. Explicitly, in Eq. (49), we set  $A_{CP\text{-even}}^{\Delta\Gamma} = \cos \phi_s^{c\bar{c}s}$  and  $A_{CP\text{-odd}}^{\Delta\Gamma} = -\cos \phi_s^{c\bar{c}s}$ . Given the small value of  $\phi_s^{c\bar{c}s}$ , we have, to first

<sup>22</sup> The definition of  $A_f^{\Delta\Gamma}$  given in Eq. (49) has the sign opposite to that given in Ref. [102].

<sup>23</sup> The effective lifetimes measured in  $B_s^0 \rightarrow K^+ K^-$  (mostly  $CP$ -even) and  $B_s^0 \rightarrow J/\psi K_S^0$  (mostly  $CP$ -odd) are not used because we can not quantify the penguin contributions in those modes.

order in  $y_s$ :

$$\tau_{\text{single}}(B_s^0 \rightarrow CP\text{-even}) \approx \frac{1}{\Gamma_L} \left( 1 + \frac{(\phi_s^{c\bar{c}s})^2 y_s}{2} \right), \quad (50)$$

$$\tau_{\text{single}}(B_s^0 \rightarrow CP\text{-odd}) \approx \frac{1}{\Gamma_H} \left( 1 - \frac{(\phi_s^{c\bar{c}s})^2 y_s}{2} \right). \quad (51)$$

The numerical inputs are taken from Eqs. (28) and (29) and the resulting averages, combined with the  $B_s^0 \rightarrow J/\psi K^+ K^-$  information, are indicated in the second column of numbers of Table 18. These averages assume  $\phi_s^{c\bar{c}s} = 0$ , which is compatible with the  $\phi_s^{c\bar{c}s}$  average presented in Sec. 3.3.4.

Information on  $\Delta\Gamma_s$  can also be obtained from the study of the proper time distribution of untagged samples of flavour-specific  $B_s^0$  decays [119], where the flavour (*i.e.*  $B_s^0$  or  $\bar{B}_s^0$ ) at the time of decay can be determined by the decay products. In such decays, *e.g.* semileptonic  $B_s^0$  decays, there is an equal mix of the heavy and light mass eigenstates at time zero. The proper time distribution is then a superposition of two exponential functions with decay constants  $\Gamma_{L,H} = \Gamma_s \pm \Delta\Gamma_s/2$ . This provides sensitivity to both  $1/\Gamma_s$  and  $(\Delta\Gamma_s/\Gamma_s)^2$ . Ignoring  $\Delta\Gamma_s$  and fitting for a single exponential leads to an estimate of  $\Gamma_s$  with a relative bias proportional to  $(\Delta\Gamma_s/\Gamma_s)^2$ , as shown in Eq. (26). Including the constraint from the world-average flavour-specific  $B_s^0$  lifetime, given in Eq. (27), leads to the results shown in the last column of Table 18. These world averages are displayed as the gray contours labelled ‘‘Combined’’ in the plots of Fig. 5. They correspond to the lifetime averages  $1/\Gamma_s = 1.505 \pm 0.004$  ps,  $1/\Gamma_L = 1.417 \pm 0.006$  ps,  $1/\Gamma_H = 1.604 \pm 0.010$  ps, and to the decay-width difference

$$\Delta\Gamma_s = +0.082 \pm 0.006 \text{ ps}^{-1} \quad \text{and} \quad \Delta\Gamma_s/\Gamma_s = +0.124 \pm 0.009, \quad (52)$$

which is in good agreement with the Standard Model prediction  $\Delta\Gamma_s = 0.087 \pm 0.021 \text{ ps}^{-1}$  [99].

Independent estimates of  $\Delta\Gamma_s/\Gamma_s$  obtained from measurements of the  $B_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-}$  branching fraction [114, 183, 184, 185]<sup>24</sup> have not been used, since they are based on the questionable [99] assumption that these decays account for all  $CP$ -even final states. The results of early lifetime analyses attempting to measure  $\Delta\Gamma_s/\Gamma_s$  [74, 105, 111, 67] have not been used either.

The strength of  $B_s^0$  mixing is known to be large since more than 20 years. Indeed the time-integrated measurements of  $\bar{\chi}$  (see Sec. 3.1.3), when compared to our knowledge of  $\chi_d$  and the  $b$ -hadron fractions, indicated that  $\chi_s$  should be close to its maximal possible value of  $1/2$ . Many searches of the time dependence of this mixing were performed by ALEPH [187], CDF (Run I) [188], DELPHI [105, 111, 149, 189], OPAL [190, 191] and SLD [192, 193, 194], but did not have enough statistical power and proper time resolution to resolve the small period of the  $B_s^0$  oscillations.

$B_s^0$  oscillations have been observed for the first time in 2006 by the CDF collaboration [195], based on samples of flavour-tagged hadronic and semileptonic  $B_s^0$  decays (in flavour-specific final states), partially or fully reconstructed in  $1 \text{ fb}^{-1}$  of data collected during Tevatron’s Run II. This was shortly followed by independent evidence obtained by the D0 collaboration with  $2.4 \text{ fb}^{-1}$  of data [196]. More recently the LHCb collaboration obtained the most precise results using fully reconstructed  $B_s^0 \rightarrow D_s^- \pi^+$  and  $B_s^0 \rightarrow D_s^- \pi^+ \pi^- \pi^+$  decays at the LHC [197, 198]. LHCb has also observed  $B_s^0$  oscillations with  $B_s^0 \rightarrow J/\psi K^+ K^-$  decays [182] and with semileptonic  $B_s^0 \rightarrow D_s^- \mu^+ X$  decays [164]. The measurements of  $\Delta m_s$  are summarized in Table 19.

<sup>24</sup> The result of Ref. [184] supersedes that of Ref. [186].

Table 19: Measurements of  $\Delta m_s$ .

Experiment	Method	Data set		$\Delta m_s$ (ps <sup>-1</sup> )	Ref.
CDF2	$D_s^{(*)-} \ell^+ \nu$ , $D_s^{(*)-} \pi^+$ , $D_s^- \rho^+$		1 fb <sup>-1</sup>	17.77 ± 0.10 ± 0.07	[195]
D0	$D_s^- \ell^+ X$ , $D_s^- \pi^+ X$		2.4 fb <sup>-1</sup>	18.53 ± 0.93 ± 0.30	[196] <sup>u</sup>
LHCb	$D_s^- \pi^+$ , $D_s^- \pi^+ \pi^- \pi^+$	2010	0.034 fb <sup>-1</sup>	17.63 ± 0.11 ± 0.02	[197]
LHCb	$D_s^- \mu^+ X$	2011	1.0 fb <sup>-1</sup>	17.93 ± 0.22 ± 0.15	[164]
LHCb	$D_s^- \pi^+$	2011	1.0 fb <sup>-1</sup>	17.768 ± 0.023 ± 0.006	[198]
LHCb	$J/\psi K^+ K^-$	2011–2012	3.0 fb <sup>-1</sup>	17.711 <sup>+0.055</sup> <sub>-0.057</sub> ± 0.011	[182]
Average of CDF and LHCb measurements				17.757 ± 0.020 ± 0.007	

<sup>u</sup> Unpublished.

A simple average of the CDF and LHCb results<sup>25</sup>, taking into account the correlated systematic uncertainties between the three LHCb measurements, yields

$$\Delta m_s = 17.757 \pm 0.020 \pm 0.007 \text{ ps}^{-1} = 17.757 \pm 0.021 \text{ ps}^{-1} \quad (53)$$

and is illustrated in Figure 6. Multiplying this result with the mean  $B_s^0$  lifetime of Eq. (32),  $1/\Gamma_s = 1.505 \pm 0.004$  ps, yields

$$x_s = \frac{\Delta m_s}{\Gamma_s} = 26.72 \pm 0.09. \quad (54)$$

With  $2y_s = \Delta\Gamma_s/\Gamma_s = +0.124 \pm 0.009$  (see Eq. (52)) and under the assumption of no  $CP$  violation in  $B_s^0$  mixing, this corresponds to

$$\chi_s = \frac{x_s^2 + y_s^2}{2(x_s^2 + 1)} = 0.499304 \pm 0.000005. \quad (55)$$

The ratio of the  $B^0$  and  $B_s^0$  oscillation frequencies, obtained from Eqs. (45) and (53),

$$\frac{\Delta m_d}{\Delta m_s} = 0.02847 \pm 0.00012, \quad (56)$$

can be used to extract the following ratio of CKM matrix elements,

$$\left| \frac{V_{td}}{V_{ts}} \right| = \xi \sqrt{\frac{\Delta m_d m(B_s^0)}{\Delta m_s m(B^0)}} = 0.2157 \pm 0.0004 \pm 0.0107, \quad (57)$$

where the first quoted error is from experimental uncertainties (with the masses  $m(B_s^0)$  and  $m(B^0)$  taken from Ref. [13]), and where the second quoted error is from theoretical uncertainties in the estimation of the SU(3) flavour-symmetry breaking factor  $\xi = 1.268 \pm 0.063$ , obtained from unquenched lattice QCD calculations [199]. Note that Eq. (57) assumes that  $\Delta m_s$  and  $\Delta m_d$  only receive Standard Model contributions. An alternative approach would be to take  $V_{td}/V_{ts}$  from global fits to predict  $\Delta m_d/\Delta m_s$ , and then compare the prediction with the measurement of Eq. (56) to set limits on new physics effects.

<sup>25</sup> We do not include the old unpublished D0 [196] result in the average.

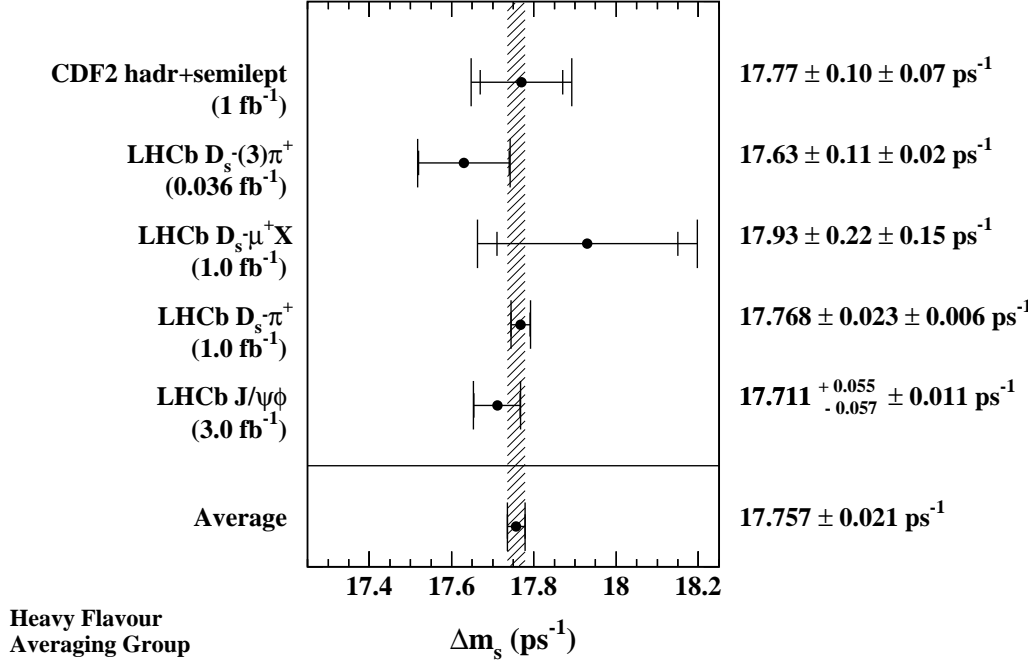


Figure 6: Published measurements of  $\Delta m_s$ , together with their average.

### 3.3.3 $CP$ violation in $B^0$ and $B_s^0$ mixing

Evidence for  $CP$  violation in  $B^0$  mixing has been searched for, both with flavour-specific and inclusive  $B^0$  decays, in samples where the initial flavour state is tagged. In the case of semileptonic (or other flavour-specific) decays, where the final state tag is also available, the following asymmetry

$$\mathcal{A}_{\text{SL}}^d = \frac{N(\overline{B}^0(t) \rightarrow \ell^+ \nu_\ell X) - N(B^0(t) \rightarrow \ell^- \bar{\nu}_\ell X)}{N(\overline{B}^0(t) \rightarrow \ell^+ \nu_\ell X) + N(B^0(t) \rightarrow \ell^- \bar{\nu}_\ell X)} = \frac{|p/q|_d^2 - |q/p|_d^2}{|p/q|_d^2 + |q/p|_d^2} \quad (58)$$

has been measured, either in time-integrated analyses at CLEO [170,200], *BABAR* [201], CDF [202]<sup>26</sup> and D0 [173,204], or in time-dependent analyses at OPAL [152], ALEPH [205], *BABAR* [171,206,207] and Belle [208]. Note that the asymmetry of time-dependent decay rates in Eq. (58) is time-independent. In the inclusive case, also investigated and published at ALEPH [205] and OPAL [76], no final state tag is used, and the asymmetry [209]

$$\frac{N(B^0(t) \rightarrow \text{all}) - N(\overline{B}^0(t) \rightarrow \text{all})}{N(B^0(t) \rightarrow \text{all}) + N(\overline{B}^0(t) \rightarrow \text{all})} \simeq \mathcal{A}_{\text{SL}}^d \left[ \frac{\Delta m_d}{2\Gamma_d} \sin(\Delta m_d t) - \sin^2 \left( \frac{\Delta m_d t}{2} \right) \right] \quad (59)$$

must be measured as a function of the proper time to extract information on  $CP$  violation.

<sup>26</sup> We do not include the unpublished measurement of Ref. [203] in our average.

Table 20: Measurements<sup>27</sup> of  $CP$  violation in  $B^0$  mixing and their average in terms of both  $\mathcal{A}_{\text{SL}}^d$  and  $|q/p|_d$ . The individual results are listed as quoted in the original publications, or converted<sup>31</sup> to an  $\mathcal{A}_{\text{SL}}^d$  value. When two errors are quoted, the first one is statistical and the second one systematic. The ALEPH and OPAL results assume no  $CP$  violation in  $B_s^0$  mixing.

Exp. & Ref.	Method	Measured $\mathcal{A}_{\text{SL}}^d$	Measured $ q/p _d$
CLEO [170]	partial hadronic rec.	+0.017 ±0.070 ±0.014	
CLEO [200]	dileptons	+0.013 ±0.050 ±0.005	
CLEO [200]	average of above two	+0.014 ±0.041 ±0.006	
BABAR [171]	full hadronic rec.		1.029 ±0.013 ±0.011
BABAR [206]	part. rec. $D^* X \ell \nu$	+0.0006 ±0.0017 <sup>+0.0038</sup> <sub>-0.0032</sub>	0.99971 ±0.00084 ±0.00175
BABAR [201]	dileptons	-0.0039 ±0.0035 ±0.0019	
Belle [208]	dileptons	-0.0011 ±0.0079 ±0.0085	1.0005 ±0.0040 ±0.0043
Average of above 6 $B$ factory results		-0.0019 ± 0.0027 (tot)	1.0009 ± 0.0013 (tot)
D0 [204]	$B^0 \rightarrow D^{(*)-} \mu + X$	+0.0068 ±0.0045 ±0.0014	
LHCb [210]	$B^0 \rightarrow D^{(*)-} \mu + X$	-0.0002 ±0.0019 ±0.0030	
Average of above 8 pure $B^0$ results		+0.0001 ± 0.0020 (tot)	1.0000 ± 0.0010 (tot)
D0 [173]	dimuons	-0.0062 ± 0.0043 (tot)	
Average of above 9 direct measurements		-0.0010 ± 0.0018 (tot)	1.0005 ± 0.0009 (tot)
OPAL [152]	leptons	+0.008 ±0.028 ±0.012	
OPAL [76]	inclusive (Eq. (59))	+0.005 ±0.055 ±0.013	
ALEPH [205]	leptons	-0.037 ±0.032 ±0.007	
ALEPH [205]	inclusive (Eq. (59))	+0.016 ±0.034 ±0.009	
ALEPH [205]	average of above two	-0.013 ± 0.026 (tot)	
Average of above 14 results		-0.0010 ± 0.0018 (tot)	1.0005 ± 0.0009 (tot)
Best fit value from 2D combination of $\mathcal{A}_{\text{SL}}^d$ and $\mathcal{A}_{\text{SL}}^s$ results (see Eq. (64))		-0.0015 ± 0.0017 (tot)	1.0007 ± 0.0009 (tot)

On the other hand, LHCb has studied the time-dependence of the charge asymmetry of  $B^0 \rightarrow D^{(*)-} \mu^+ \nu_\mu X$  decays without tagging the initial state [210], which would be equal to

$$\frac{N(D^{(*)-} \mu^+ \nu_\mu X) - N(D^{(*)+} \mu^- \bar{\nu}_\mu X)}{N(D^{(*)-} \mu^+ \nu_\mu X) + N(D^{(*)+} \mu^- \bar{\nu}_\mu X)} = \mathcal{A}_{\text{SL}}^d [1 - \cos(\Delta m_d t)] \quad (60)$$

in absence of detection and production asymmetries.

Table 20 summarizes the different measurements: in all cases asymmetries compatible with zero have been found, with a precision limited by the available statistics.

A simple average of all measurements performed at  $B$  factories [170, 200, 171, 206, 201, 208] yields  $\mathcal{A}_{\text{SL}}^d = -0.0019 \pm 0.0027$ ; adding also the D0 [204] and LHCb [210] measurements obtained with reconstructed semileptonic  $B^0$  decays yields

$$\mathcal{A}_{\text{SL}}^d = +0.0001 \pm 0.0020 \quad \iff \quad |q/p|_d = 1.0000 \pm 0.0010, \quad (61)$$

where the relation between  $\mathcal{A}_{\text{SL}}^d$  and  $|q/p|_d$  is given in Eq. (58). As discussed in more detail later in this section, the latest dimuon D0 analysis [173] separates the  $B^0$  and  $B_s^0$  contributions by exploiting the dependence on the muon impact parameter cut; combining the  $\mathcal{A}_{\text{SL}}^d$  result quoted by D0 with the above  $B^0$  average of Eq. (61) yields  $\mathcal{A}_{\text{SL}}^d = -0.0010 \pm 0.0018$ .

All the other  $B^0$  analyses performed at high energy, either at LEP or at the Tevatron, did not separate the contributions from the  $B^0$  and  $B_s^0$  mesons. Under the assumption of no  $CP$  violation in  $B_s^0$  mixing, a number of these analyses [46, 152, 205, 76] quote a measurement of  $\mathcal{A}_{\text{SL}}^d$  or  $|q/p|_d$  for the  $B^0$  meson. Including also these results<sup>27</sup> in the previous average under the assumption  $\mathcal{A}_{\text{SL}}^s = 0$  leads to  $\mathcal{A}_{\text{SL}}^d = -0.0010 \pm 0.0018$  (*i.e.* no change). The latter assumption makes sense within the Standard Model, since  $\mathcal{A}_{\text{SL}}^s$  is predicted to be much smaller than  $\mathcal{A}_{\text{SL}}^d$  [99], but may not be suitable in the presence of new physics.

The following constraints on a combination of  $\mathcal{A}_{\text{SL}}^d$  and  $\mathcal{A}_{\text{SL}}^s$  (or equivalently  $|q/p|_d$  and  $|q/p|_s$ ) have been obtained by the Tevatron experiments, using inclusive semileptonic decays of  $b$  hadrons:

$$\frac{1}{4} (f'_d \chi_d \mathcal{A}_{\text{SL}}^d + f'_s \chi_s \mathcal{A}_{\text{SL}}^s) = +0.0015 \pm 0.0038(\text{stat}) \pm 0.0020(\text{syst}) \quad \text{CDF1 [202]}, \quad (62)$$

$$\mathcal{A}_{\text{SL}}^b = \frac{f'_d \chi_d \mathcal{A}_{\text{SL}}^d + f'_s \chi_s \mathcal{A}_{\text{SL}}^s}{f'_d \chi_d + f'_s \chi_s} = -0.00496 \pm 0.00153(\text{stat}) \pm 0.00072(\text{syst}) \quad \text{D0 [173]}, \quad (63)$$

where the fractions  $f'$  have been defined in Eq. (17). While the imprecise CDF1 result is compatible with no  $CP$  violation<sup>28</sup>, the D0 result of Eq. (63), obtained by measuring the charge asymmetry of like-sign dimuons, differs by 2.8 standard deviations from the Standard Model expectation of  $\mathcal{A}_{\text{SL}}^b(\text{SM}) = (-2.3 \pm 0.4) \times 10^{-4}$  [173, 99]. With a more sophisticated analysis in bins of the muon impact parameters, D0 conclude that the overall deviation of the their measurements from the SM is at the level of  $3.6\sigma$ .

Using the average  $\mathcal{A}_{\text{SL}}^d = +0.0001 \pm 0.0020$  of Eq. (61), obtained from pure  $B^0$  measurements, the two results of Eqs. (62) and (63) are turned<sup>29</sup> into the measurements of  $\mathcal{A}_{\text{SL}}^s$  displayed in the top part of Fig. 7. Taking into account the uncertainties on the  $b$ -hadron fractions and mixing parameters, the value derived from the D0 analysis does not show evidence of  $CP$  violation in the  $B_s^0$  system. In addition, the third and fourth lines of Fig. 7 show direct determination of  $\mathcal{A}_{\text{SL}}^s$  obtained by D0 [211] and LHCb [212] by measuring the time-integrated charge asymmetry of untagged  $B_s^0 \rightarrow D_s \mu X$  decays. The four results of Fig. 7 are combined to yield  $\mathcal{A}_{\text{SL}}^s = -0.0083 \pm 0.0027(\text{stat}) \pm 0.0021(\text{syst}) = -0.0083 \pm 0.0034$  or, equivalently through Eq. (58),  $|q/p|_s = 1.0042 \pm 0.0014(\text{stat}) \pm 0.0010(\text{syst}) = 1.0042 \pm 0.0017$ . The quoted systematic errors include experimental systematics as well as the correlated dependence on external parameters.

As mentioned above, the D0 like-sign dimuon analysis investigates the dependence of the charge asymmetry as a function of the muon impact parameters. Interpreting the observed asymmetries in terms of  $CP$  violation in  $B$ -meson mixing and interference, and using the mixing parameters and the world  $b$ -hadron fractions of Ref. [49], the D0 collaboration extracts [173] values for  $\mathcal{A}_{\text{SL}}^d$  and  $\mathcal{A}_{\text{SL}}^s$  and their correlation coefficient<sup>30</sup>, as shown in Table 21. However, the individual contributions to the total quoted errors from this analysis and from the external inputs are not given, so the adjustment of these results to different or more recent values of

<sup>27</sup> A low-statistics result published by CDF using the Run I data [202] and an unpublished result by CDF using Run II data [203] are not included in our averages, nor in Table 20.

<sup>28</sup> A measurement from CDF2,  $\mathcal{A}_{\text{SL}}^b = +0.0080 \pm 0.0090(\text{stat}) \pm 0.0068(\text{syst})$  [203], more precise than the D0 measurement, is also compatible with no  $CP$  violation, but since it is unpublished since 2007 we no longer include it in our averages, nor in Fig. 7.

<sup>29</sup> For simplicity, we set  $f'_q = f_q$ .

<sup>30</sup> They also extract at the same time a value for  $\Delta\Gamma_d/\Gamma_d$  (see Sec. 3.3.1).



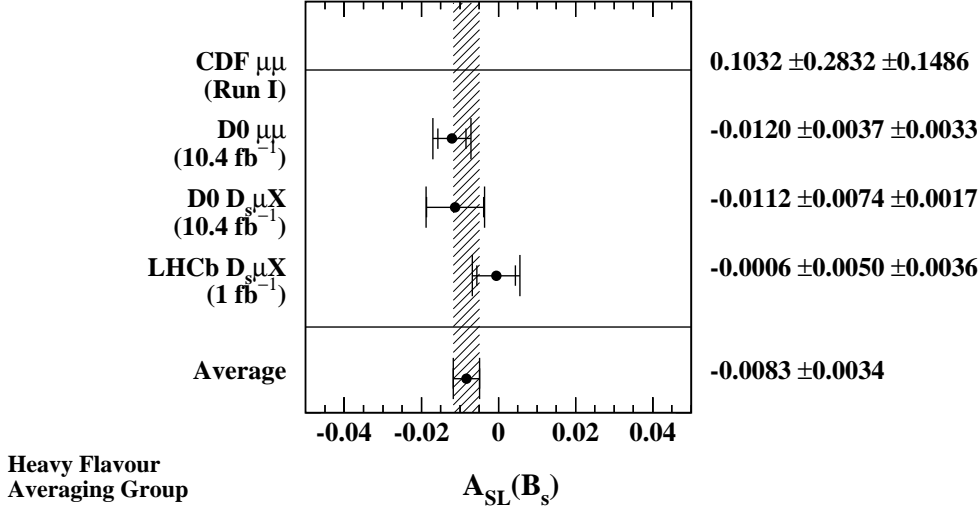


Figure 7: Measurements of  $\mathcal{A}_{\text{SL}}^s$ , derived from CDF [202],<sup>28</sup> D0 [173, 211] and LHCb [212] analyses, adjusted to the pure  $B^0$  average of  $\mathcal{A}_{\text{SL}}^d$ . The combined value of  $\mathcal{A}_{\text{SL}}^s$  is also shown.

Table 21: Direct measurements of  $CP$  violation in  $B_s^0$  and  $B^0$  mixing, together with their two-dimensional average. Only total errors are quoted.

Exp. & Ref.	Method	Measured $\mathcal{A}_{\text{SL}}^s$	Measured $\mathcal{A}_{\text{SL}}^d$	$\rho(\mathcal{A}_{\text{SL}}^s, \mathcal{A}_{\text{SL}}^d)$
D0 [211]	$B_s^0 \rightarrow D_s \mu X$	$-0.0112 \pm 0.0076$		
LHCb [212]	$B_s^0 \rightarrow D_s \mu X$	$-0.0006 \pm 0.0062$		
Average of above $B_s^0$ results		$-0.0048 \pm 0.0048$		
Average of $B^0$ results (Eq. (61))			$+0.0001 \pm 0.0020$	
D0 [173]	dimuons	$-0.0082 \pm 0.0099$	$-0.0062 \pm 0.0043$	$-0.61$
Average of all above		$-0.0075 \pm 0.0041$	$-0.0015 \pm 0.0017$	$-0.158$

the external inputs cannot (easily) be done. Using a two-dimensional fit, these values are combined with the pure  $B^0$  average of Eq. (61) and with the results from the  $B_s^0 \rightarrow D_s \mu X$  analyses [211, 212], assumed to be independent and also shown in Table 21. The result, shown graphically in Fig. 8, is

$$\mathcal{A}_{\text{SL}}^d = -0.0015 \pm 0.0017 \iff |q/p|_d = 1.0007 \pm 0.0009, \quad (64)$$

$$\mathcal{A}_{\text{SL}}^s = -0.0075 \pm 0.0041 \iff |q/p|_s = 1.0038 \pm 0.0021, \quad (65)$$

$$\rho(\mathcal{A}_{\text{SL}}^d, \mathcal{A}_{\text{SL}}^s) = -0.158. \quad (66)$$

The average of Fig. 7 ignores the impact parameter study of D0. The average of Eq. (65) ignores the CDF1 result, which has a very large uncertainty anyway. We choose the results of Eqs. (64), (65), and (66) as our final averages,<sup>31</sup> since they incorporate better the available

<sup>31</sup> Early analyses and (perhaps hence) the PDG use the complex parameter  $\epsilon_B = (p - q)/(p + q)$ ; if  $CP$

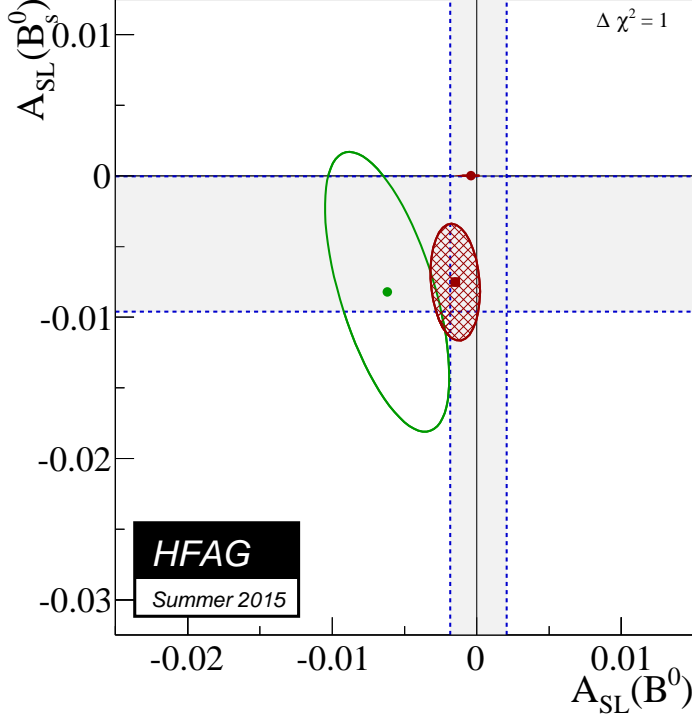


Figure 8: Direct measurements of  $\mathcal{A}_{\text{SL}}^s$  and  $\mathcal{A}_{\text{SL}}^d$  listed in Table 21 ( $B^0$  average as the vertical band,  $B_s^0$  average as the horizontal band, D0 dimuon result as the green ellipse), together with their two-dimensional average (red hatched ellipse). The red point close to  $(0, 0)$  is the Standard Model prediction of Ref. [99] with error bars multiplied by 10. The prediction and the experimental average deviate from each other by  $1.5\sigma$ .

published data.

The above averages show no evidence of  $CP$  violation in  $B^0$  and  $B_s^0$  mixing. They deviate by  $1.5\sigma$  from the very small predictions of the Standard Model,  $\mathcal{A}_{\text{SL}}^{d\text{ SM}} = -(4.1 \pm 0.6) \times 10^{-4}$  and  $\mathcal{A}_{\text{SL}}^{s\text{ SM}} = +(1.9 \pm 0.3) \times 10^{-5}$  [99]. Given the current size of the experimental uncertainties, there is still significant room for a possible new physics contribution, especially in the  $B_s^0$  system. In this respect, the deviation of the D0 dimuon asymmetry [173] from expectation has generated a lot of excitement, however recent results from D0 and LHCb have not yet settled the issue, and more experimental data (especially from LHCb) is awaited eagerly.

At the more fundamental level,  $CP$  violation in  $B_s^0$  mixing<sup>32</sup> is caused by the weak phase difference

$$\phi_{12} = \arg[-M_{12}/\Gamma_{12}], \quad (67)$$

where  $M_{12}$  and  $\Gamma_{12}$  are the off-diagonal elements of the mass and decay matrices of the  $B_s^0 - \bar{B}_s^0$

violation in the mixing is small,  $\mathcal{A}_{\text{SL}}^d \cong 4\text{Re}(\epsilon_B)/(1 + |\epsilon_B|^2)$  and the averages of Eqs. (61) and (64) correspond to  $\text{Re}(\epsilon_B)/(1 + |\epsilon_B|^2) = +0.0000 \pm 0.0005$  and  $-0.0004 \pm 0.0004$ , respectively.

<sup>32</sup> Of course, a similar formalism exists for the  $B^0$  system; for simplicity we omit here the subscript  $s$  for  $\phi_{12}$ ,  $M_{12}$  and  $\Gamma_{12}$ .

system. This is related to the observed decay-width difference through the relation

$$\Delta\Gamma_s = 2|\Gamma_{12}| \cos \phi_{12} + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right), \quad (68)$$

where quadratic (or higher-order) terms in the small quantity  $|\Gamma_{12}/M_{12}| \sim \mathcal{O}(m_b^2/m_t^2)$  can be neglected. The SM prediction for this phase is tiny [99],

$$\phi_{12}^{\text{SM}} = 0.0038 \pm 0.0010; \quad (69)$$

however, new physics in  $B_s^0$  mixing could change this observed phase to

$$\phi_{12} = \phi_{12}^{\text{SM}} + \phi_{12}^{\text{NP}}. \quad (70)$$

The  $B_s^0$  semileptonic asymmetry can be expressed as [213]

$$\mathcal{A}_{\text{SL}}^s = \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right) = \frac{\Delta\Gamma_s}{\Delta m_s} \tan \phi_{12} + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right). \quad (71)$$

Using this relation, the current knowledge of  $\mathcal{A}_{\text{SL}}^s$ ,  $\Delta\Gamma_s$  and  $\Delta m_s$ , given in Eqs. (65), (52), and (53) respectively, yield an experimental determination of  $\phi_{12}$ ,

$$\tan \phi_{12} = \mathcal{A}_{\text{SL}}^s \frac{\Delta m_s}{\Delta\Gamma_s} = -1.6 \pm 0.9, \quad (72)$$

which represents only a very weak constraint at present.

### 3.3.4 Mixing-induced $CP$ violation in $B_s^0$ decays

$CP$  violation induced by  $B_s^0 - \bar{B}_s^0$  mixing has been a field of very active study and fast experimental progress in the past couple of years. The main observable is the  $CP$ -violating phase  $\phi_s^{c\bar{c}s}$ , defined as the weak phase difference between the  $B_s^0 - \bar{B}_s^0$  mixing amplitude and the  $b \rightarrow c\bar{c}s$  decay amplitude.

The golden mode for such studies is  $B_s^0 \rightarrow J/\psi \phi$ , followed by  $J/\psi \rightarrow \mu^+ \mu^-$  and  $\phi \rightarrow K^+ K^-$ , for which a full angular analysis of the decay products is performed to separate statistically the  $CP$ -even and  $CP$ -odd contributions in the final state. As already mentioned in Sec. 3.3.2, CDF [176], D0 [177], ATLAS [178, 179], CMS [181] and LHCb [182] have used both untagged and tagged  $B_s^0 \rightarrow J/\psi \phi$  (and  $B_s^0 \rightarrow J/\psi K^+ K^-$ ) events for the measurement of  $\phi_s^{c\bar{c}s}$ . LHCb [214] has used  $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$  events, analyzed with a full amplitude model including several  $\pi^+ \pi^-$  resonances (*e.g.*  $f_0(980)$ ), although the  $J/\psi \pi^+ \pi^-$  final state had already been shown to be almost  $CP$  pure with a  $CP$ -odd fraction larger than 0.977 at 95% CL [215]. In addition, LHCb has used the  $B_s^0 \rightarrow D_s^+ D_s^-$  channel [216] to measure  $\phi_s^{c\bar{c}s}$ .

All CDF, D0, ATLAS and CMS analyses provide two mirror solutions related by the transformation  $(\Delta\Gamma_s, \phi_s^{c\bar{c}s}) \rightarrow (-\Delta\Gamma_s, \pi - \phi_s^{c\bar{c}s})$ . However, the LHCb analysis of  $B_s^0 \rightarrow J/\psi K^+ K^-$  resolves this ambiguity and rules out the solution with negative  $\Delta\Gamma_s$  [101], a result in agreement with the Standard Model expectation. Therefore, in what follows, we only consider the solution with  $\Delta\Gamma_s > 0$ .

Table 22: Direct experimental measurements of  $\phi_s^{c\bar{c}s}$ ,  $\Delta\Gamma_s$  and  $\Gamma_s$  using  $B_s^0 \rightarrow J/\psi\phi$ ,  $J/\psi K^+K^-$ ,  $J/\psi\pi^+\pi^-$  and  $D_s^+D_s^-$  decays. Only the solution with  $\Delta\Gamma_s > 0$  is shown, since the two-fold ambiguity has been resolved in Ref. [101]. The first error is due to statistics, the second one to systematics. The last line gives our average.

Exp.	Mode	Dataset	$\phi_s^{c\bar{c}s}$	$\Delta\Gamma_s$ (ps <sup>-1</sup> )	Ref.
CDF	$J/\psi\phi$	9.6 fb <sup>-1</sup>	$[-0.60, +0.12]$ , 68% CL	$+0.068 \pm 0.026 \pm 0.009$	[176]
D0	$J/\psi\phi$	8.0 fb <sup>-1</sup>	$-0.55^{+0.38}_{-0.36}$	$+0.163^{+0.065}_{-0.064}$	[177]
ATLAS	$J/\psi\phi$	4.9 fb <sup>-1</sup>	$+0.12 \pm 0.25 \pm 0.05$	$+0.053 \pm 0.021 \pm 0.010$	[178]
ATLAS	$J/\psi\phi$	14.3 fb <sup>-1</sup>	$-0.119 \pm 0.088 \pm 0.036$	$+0.096 \pm 0.013 \pm 0.007$	[179] <sup>p</sup>
ATLAS	above 2 combined		$-0.094 \pm 0.083 \pm 0.033$	$+0.082 \pm 0.011 \pm 0.007$	[179] <sup>p</sup>
CMS	$J/\psi\phi$	20 fb <sup>-1</sup>	$-0.075 \pm 0.097 \pm 0.031$	$+0.095 \pm 0.013 \pm 0.007$	[181]
LHCb	$J/\psi K^+K^-$	3.0 fb <sup>-1</sup>	$-0.058 \pm 0.049 \pm 0.006$	$+0.0805 \pm 0.0091 \pm 0.0033$	[182]
LHCb	$J/\psi\pi^+\pi^-$	3.0 fb <sup>-1</sup>	$+0.070 \pm 0.068 \pm 0.008$	—	[214]
LHCb	above 2 combined		$-0.010 \pm 0.039(\text{tot})$	—	[182]
LHCb	$D_s^+D_s^-$	3.0 fb <sup>-1</sup>	$+0.02 \pm 0.17 \pm 0.02$	—	[216]
All combined			$-0.034 \pm 0.033$	$+0.084 \pm 0.007$	

<sup>p</sup> Preliminary.

We perform a combination of the CDF [176], D0 [177], ATLAS [178, 179], CMS [181] and LHCb [182, 214] results summarized in Table 22. This is done by adding the two-dimensional log profile-likelihood scans of  $\Delta\Gamma_s$  and  $\phi_s^{c\bar{c}s}$  from the four  $B_s^0 \rightarrow J/\psi\phi$  ( $B_s^0 \rightarrow J/\psi K^+K^-$ ) analyses and a one-dimensional log profile-likelihood of  $\phi_s^{c\bar{c}s}$  from the  $B_s^0 \rightarrow J/\psi\pi^+\pi^-$  and  $B_s^0 \rightarrow D_s^+D_s^-$  analyses; the combined likelihood is then maximized with respect to  $\Delta\Gamma_s$  and  $\phi_s^{c\bar{c}s}$ .

In the  $B_s^0 \rightarrow J/\psi\phi$  and  $B_s^0 \rightarrow J/\psi K^+K^-$  analyses,  $\phi_s^{c\bar{c}s}$  and  $\Delta\Gamma_s$  come from a simultaneous fit that determines also the  $B_s^0$  lifetime, the polarisation amplitudes and strong phases. While the correlation between  $\phi_s^{c\bar{c}s}$  and all other parameters is small, the correlations between  $\Delta\Gamma_s$  and the polarisation amplitudes are sizeable. However, since the various experiments use different conventions for the amplitudes and phases, a full combination including all correlations is not performed. Instead, our average only takes into account the correlation between  $\phi_s^{c\bar{c}s}$  and  $\Delta\Gamma_s$ .

In the recent LHCb  $B_s^0 \rightarrow J/\psi K^+K^-$  analysis [182], the  $\phi_s^{c\bar{c}s}$  values are measured for the first time for each polarization of the final state. Since those values are compatible within each other, we still use the unique value of  $\phi_s^{c\bar{c}s}$  for our world average, corresponding to the one measured by the other-than-LHCb analyses. In the same analysis, the statistical correlation coefficient between  $\phi_s^{c\bar{c}s}$  and  $|\lambda|$  (which signals  $CP$  violation in the decay if different from unity) is measured to be very small ( $-0.02$ ). We neglect this correlation in our average. Furthermore, the statistical correlation coefficient between  $\phi_s^{c\bar{c}s}$  and  $\Delta\Gamma_s$  is measured to be small ( $-0.08$ ). When averaging LHCb results of  $B_s^0 \rightarrow J/\psi K^+K^-$ ,  $B_s^0 \rightarrow J/\psi\pi^+\pi^-$  and  $B_s^0 \rightarrow D_s^+D_s^-$ , we neglect this correlation coefficient (putting it to zero). Given the increasing experimental precision, we have also stopped using the two-dimensional  $\Delta\Gamma_s - \phi_s^{c\bar{c}s}$  histograms provided by the CDF and D0 collaborations: we are now approximating those with two-dimensional Gaussian likelihoods.

We obtain the individual and combined contours shown in Fig. 9. Maximizing the likelihood,

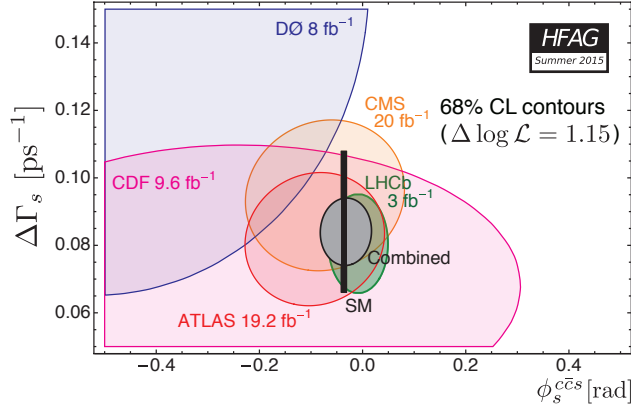


Figure 9: 68% CL regions in  $B_s^0$  width difference  $\Delta\Gamma_s$  and weak phase  $\phi_s^{c\bar{c}s}$  obtained from individual and combined CDF [176], D0 [177], ATLAS [178,179], CMS [181] and LHCb [182,214] likelihoods of  $B_s^0 \rightarrow J/\psi\phi$ ,  $B_s^0 \rightarrow J/\psi K^+K^-$  and  $B_s^0 \rightarrow J/\psi\pi^+\pi^-$ . The expectation within the Standard Model [175,99] is shown as the black rectangle.

we find, as summarized in Table 22:

$$\Delta\Gamma_s = +0.084 \pm 0.007 \text{ ps}^{-1}, \quad (73)$$

$$\phi_s^{c\bar{c}s} = -0.034 \pm 0.033. \quad (74)$$

The above  $\Delta\Gamma_s$  average is consistent, but highly correlated with the average of Eq. (52). Our final recommended average for  $\Delta\Gamma_s$  is the one of Eq. (52), which includes all available information on  $\Delta\Gamma_s$ .

In the Standard Model and ignoring sub-leading penguin contributions,  $\phi_s^{c\bar{c}s}$  is expected to be equal to  $-2\beta_s$ , where  $\beta_s = \arg[-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)]$  is a phase analogous to the angle  $\beta$  of the usual CKM unitarity triangle (aside from a sign change). An indirect determination via global fits to experimental data gives [175]

$$(\phi_s^{c\bar{c}s})^{\text{SM}} = -2\beta_s = -0.0363_{-0.0014}^{+0.0012}. \quad (75)$$

The average value of  $\phi_s^{c\bar{c}s}$  from Eq. (74) is consistent with this Standard Model expectation.

New physics could contribute to  $\phi_s^{c\bar{c}s}$ . Assuming that new physics only enters in  $M_{12}$  (rather than in  $\Gamma_{12}$ ), one can write [99]

$$\phi_s^{c\bar{c}s} = -2\beta_s + \phi_{12}^{\text{NP}}, \quad (76)$$

where the new physics phase  $\phi_{12}^{\text{NP}}$  is the same as that appearing in Eq. (70). In this case

$$\phi_{12} = \phi_{12}^{\text{SM}} + 2\beta_s + \phi_s^{c\bar{c}s} = 0.007 \pm 0.033, \quad (77)$$

where the numerical estimation was performed with the values of Eqs. (69), (75), and (74). This can serve as a reference value to which the measurement of Eq. (72) can be compared.

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