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COMBOS:
a program to **COM**bine results from **B OS**cillations analyses

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Abstract

This document presents a computer program, COMBOS, written to combine the results of various B oscillation analyses. The program has been designed to be able to compute averages of Δm_{d} measurements, as well as averages of $\mathrm{B_s}$ oscillation amplitudes. The results from the various analyses are read from files which have been defined with a very flexible format. At the present time, the program uses the combination algorithm which is a χ^2 fit taking into account statistical uncertainties, uncorrelated systematic uncertainties and correlated systematic uncertainties in a standard way. The program can easily be extended to include other combination algorithms and compare their results.

1 Introduction

COMBOS is a FORTRAN program relying only on the CERNLIB program library. It is developed for the LEP B oscillations working group to combine the LEP results on Δm_{d} and Δm_{s}. The input data to that program consists of the B oscillation results of the various analyses to be combined and instructions on how to perform the combination.

Sections 2 and 3 describe the input format for this program. Defining such a format is important to allow the different experiments to exchange data related to their B oscillation analyses (and their combinations) in a coherent and convenient fashion.

The COMBOS program itself is briefly described in Section 4. Note that this program is expected to evolve in function of the needs expressed by the LEP B oscillations working group.

2 Input format: analytical description

The description of the proposed format for the input file of a program to combine B oscillations analyses is given here in a complete and analytical way. The reader who doesn't want to start with all the boring details should jump directly to Section 3, where the same description is given in a more pragmatic way using examples, before coming back to this section.

2.1 Spaces, tabs and comments

Each physical input line is truncated to 256 characters, and therefore any characters appearing in columns 257 and up are ignored.

Space characters (hexadecimal ASCII code 20) and tab characters (hexadecimal ASCII code 9) found on input are treated as "blank" characters. Leading blank characters on an input line are ignored and a sequence of more than one blank character is treated as a single blank character. This implies that blank lines, i.e. physical lines of the input file which contain only blank characters, are ignored.

Lines in which the first non-blank character is the character "`*`" are also ignored (comment lines). The special characters "`#`", "`|`" and "`!`" can be used for in-line comments: if one of these characters appears in an input line, the rest of the line (including that special character) is ignored. This implies that comment lines can also start with "`#`", "`|`" or "`!`".

If necessary, the list of these comment characters can be modified with a line starting with the word "`COMMENTS`":

```
COMMENTS abcd
```

The word "`COMMENTS`" can be abbreviated up to 4 characters (i.e. "`COMM`") and can be given in uppercase, lowercase or mixed-case characters. It must be followed, on the same line, by a 4 letter word containing the new comment characters. In any subsequent input line, the first comment character "`a`" will be recognized only if it is the first non-bank character in the line, and the other 3 characters "`b`", "`c`" and "`d`" will be recognized anywhere in the line. Note that the 4 characters given on the "`COMMENTS`" line don't need to be different. The comments characters remain in effect until changed again with a new "`COMMENTS`" line. To go back to the default set of comment characters, use the line:

```
COMMENTS *#|!
```

A line starting with the word `EXIT` or `QUIT` will prevent the rest of the input file from being read by the program (see Section 2.2), and therefore all subsequent lines in the input file will be effectively treated as comments.

2.2 Input file nesting and control

If a line begins with the special word "`INCLUDE`", then the second word on that line is expected to be the name of an existing file:

```
INCLUDE file_name
```

The word "`INCLUDE`" can be abbreviated up to 4 characters (i.e. "`INCL`") and can be given in uppercase, lowercase or mixed-case characters. The file name however has to be given with the appropriate case when significant (for example on UNIX systems); it cannot contain a blank character and it is truncated to 80 characters. In such an `INCLUDE` line, the two special characters "`|`" and "`!`" loose their property of specifying the start of an in-line comment and can therefore be used inside file names; however, the rest of the `INCLUDE` line after the file name is ignored.

The file specified on an `INCLUDE` line will be opened and the next input lines will be read from that new input file; after the end of that file is reached, the next input lines are read from the original input file starting at the line which follows the `INCLUDE` line in the original input file. A file specified on an `INCLUDE` line can itself contain `INCLUDE` lines; up to 10 levels of nesting are available. When the end of the main input file is reached, the program stops.

However, if a line begins with the special word "`EXIT`", then the current input file is closed, and if a line begins with the special word "`QUIT`" then all input files are closed.

2.3 Tokens, keywords and logical lines

After removal of all the comments and expansion of the `INCLUDE` statements, each input line is broken into tokens separated by one or more blank characters. By definition a token cannot contain any of the following characters: " " (blank), "`|`", and "`!`". Each token is truncated to 16 characters and translated to upper case before being interpreted; this ensures case insensitivity. A "keyword" is defined as a token which is the first token on its physical input line and that matches one of the 16 strings listed in the first column of Table 1. A keyword can be abbreviated, but only if its abbreviated form is unambiguous and has at least 4 characters. The first token in the input file has to be a keyword.

Each keyword is expected to be followed (on the same physical line or on subsequent lines) by j tokens containing non-numerical (alphabetical) strings and then by k tokens containing numerical strings. The constraints on j and k depend on the keyword and are indicated in Table 1. A keyword and its following $j + k$ tokens form together a "logical line" Note that a logical line can span over several physical lines of the input file. If the first token after the end of a logical line is not a keyword, then a new logical line is started at this token, assuming the same keyword as in the previous logical line.

The order in which the keyword are allowed to appear in the input file and the meaning of each keyword and its associated logical line are given in the following sections.

2.4 The `OUTPUT` logical line

The `OUTPUT` keyword can appear anywhere in the input file. Its associated logical line has the following format:

```
OUTPUT lunlog lunerr lunout lunkum
* for example: OUTPUT 6 10 11
```

where `lunlog` is the logical unit for the normal "log file" output, `lunerr` is the logical unit where the warning and error messages issued by the program should be written, `lunout` is the logical unit where the output logical lines will be written (see Section 4), and `lunkum` is the logical unit for the KUMAC output. These four logical units are initially set to 6, 6, -1, and -1 respectively (a negative value indicates that the corresponding output is turned off), i.e. the program will only write messages on the predefined FORTRAN

Keyword	Constraint on number of non-numerical tokens j	Constraint on number of numerical tokens k
`OUTPUT`	$j = 0$	$1 \leq k \leq 4$
`BEGIN`	$j \geq 1$	$k \geq 0$
`COMBINE`	$1 \leq j \leq 3$	$k = 0$
`LUMP`	$j \geq 2$	$k = 0$
`MEASUREMENTS`	$1 \leq j \leq 4$	$k = 0$
`DISPLAY`	$1 \leq j \leq 4$	$k = 0$
`STEPS`	$j = 1$	$k \geq 0$
`PARAMETERS`	$j = 1$	$k = 3$
`SWITCH`	$j \geq 0$	$0 \leq k \leq 1$
`MINOS`	$j \geq 1$	$k = 0$
`SYNONYMS`	$j \geq 2$	$k = 0$
`CALL`	$j \geq 1$	$0 \leq k \leq 1$
`SUPERSEDE`	$1 \leq j \leq 3$	$k = 0$
`STAT_CORR_WITH`	$1 \leq j \leq 3$	$k = 1$
`DATA`	$j \geq 1$	$k =$ multiple of j
`END`	$j = 0$	$k = 0$

Table 1: List of recognised keywords. Each keyword is expected to be followed by j non-numerical (alphabetical) tokens, and then k numerical tokens.

logical unit 6 (standard output), and not write the output logical lines nor produce a KUMAC output. Each time an `OUTPUT` logical line is read from the input, the values of these four logical values will be changed to the integers `lunlog`, `lunerr`, `lunout` and `lunkum` specified after the keyword. A value of 0 (or a missing value) will not change the corresponding logical unit; a negative value will turn off the corresponding output (but the log file output cannot be turned off); a value of 5 or above 89 is not allowed (reserved logical units).

The "logical lines" output (written in the third logical unit specified on an `OUTPUT` logical line) consists of a set of logical lines that are in principle equivalent to the input to the COMBOS program. They show how the COMBOS program interpreted its input.

The "KUMAC" output (written in the third logical unit specified on an `OUTPUT` logical line) is a PAW macro that can be executed inside PAW. It contains all the information necessary to display graphically the COMBOS results. It is meant to be called from another PAW macro which actually produces the plot. A rather general and elaborate example of such a macro to execute the COMBOS KUMAC output is provided in file `combos.kumac`.

2.5 The `BEGIN` and `END` logical lines

Apart from the `OUTPUT` keyword described above, the input file has to be constructed with blocks of logical lines that are delimited by the `BEGIN` and `END` keywords. Such a block, starting with a `BEGIN` keyword and ending with an `END` keyword, defines an "analysis";

it can refer either to a single analysis (for example the ALEPH Δm_{s} analysis using $\mathrm{D_s}$ events), or to a group of analyses that have to be combined. Each logical line which appears inside such a BEGIN–END block applies to that analysis of group of analyses. In the case of a single analysis, the block has to provide all the information from the analysis, including the results of the measurement with all the contributions to the statistical and systematic uncertainties. In the case of a group of analyses, the block has to provide all the information which is needed to perform a combination of analyses, including the list of analyses to combine and the way the combination is to be done.

The BEGIN logical line has the following format:

```
BEGIN experiment_name method_name status reference ...
* for example: BEGIN ALEPH DS+HADRON/COMB PRELIMINARY WARSAW 1996
*              BEGIN ALEPH LEPTON/QJET PREPRINT CERN-PPE/96-102
*              BEGIN L3 LEPTON/LEPTON
*                    PUBLISHED Phys. Lett. B383 (1996) 487-498
```

where experiment_name, method_name and status are three non-numerical tokens which will together identify the analysis. In case of a single analysis, experiment_name is the name of the experiment, method_name is the name of the method used to study B oscillations, and status is an extra qualifier to indicate the status of the analysis. See Table 3 for a proposed list of possible values for these tokens to identify a single analysis. In the case of a group of analyses, these three tokens can be given any (non-numerical) convenient value. Any additional token(s) given on the same physical input line (reference), will be interpreted as a reference for the analysis. In any case, the first token (experiment_name) needs to be specified, and all the other ones will default to blank if omitted.

The END logical line has the following format:

```
END
```

and it will indicate the end of the "analysis" definition. In case of a group of analyses, it will be the signal that the program should proceed with the combination.

2.6 BEGIN–END block defining a single analysis

A single MEASUREMENTS logical line and one or more DATA logical lines are necessary in a BEGIN–END block defining a single analysis. Such a block can also include at most one STEPS logical line, one DISPLAY logical line, and any number of PARAMETERS, SUPERSEDE and STAT_CORR_WITH logical lines. No other logical line is allowed in such a block.

The MEASUREMENTS logical line has the following format:

```
MEASUREMENTS measured_quantity stat_uncert total_syst_uncert total_uncert
* for example: MEASUREMENTS AMPLITUDE STAT
```

where measured_quantity is the name of the quantity measured, stat_uncert is the name of the statistical uncertainty, and total_syst_uncert is the name of the total systematic uncertainty, and total_uncert is the name of the total (stat+syst) uncertainty. The names total_syst_uncert and total_uncert can be omitted, like shown in the example above.

The optional DISPLAY logical line has the same format as the MEASUREMENTS logical line; whereas the MEASUREMENTS logical line indicates the names of the measured quantity and its uncertainties to be used in combinations, the DISPLAY logical line indicates the names of the quantities to be displayed in plots, table, etc ..., in case these are different from the ones specified on the MEASUREMENTS logical line. For example,

```
MEASUREMENTS DMD  STAT  SYST
DISPLAY      DMD2 STAT2 SYST2
```

means that the quantities DMD, STAT and SYST have to be used in subsequent combinations, but that the quantities DMD2, STAT2 and SYST2 should be used instead when displaying the information on plots and summary tables. Note that the same name is allowed to appear both in a MEASUREMENTS and in a DISPLAY logical line. The special name SUPERSEDED appearing on the DISPLAY logical line, i.e.

```
DISPLAY SUPERSEDED
```

means that the quantities from the analyses superseded by the present analysis (as defined by the SUPERSEDE logical line described below) should be displayed instead of the quantities on the MEASUREMENTS logical line. The special word SUPERSEDED cannot be abbreviated. Nesting of "DISPLAY SUPERSEDED" logical lines is not allowed, i.e. superseded analyses cannot contain a "DISPLAY SUPERSEDED" logical line in their definition.

The optional STEPS logical line can have one of the following two formats:

```
STEPS step_quantity
* for example: STEPS DMS
```

or

```
STEPS step_quantity value_1 value_2 ... value_n
* for example: STEPS DMS 0.0 2.0 4.0 6.0 8.0 10.0 12.0 14.0 16.0 20.0
```

where step_quantity is the name of the quantity which is varied in steps to perform the measurements, and value_1, value_2, ..., and value_n are the values taken by the quantity step_quantity in each of the n steps. If these values are not specified on the STEPS logical lines, they have to be specified on at least one DATA logical line, as explained below. Note that these values can be specified both on the STEPS and on DATA logical lines, as long as they are the same.

Each optional PARAMETERS logical lines has one of the following formats:

```
PARAMETERS par_name par_value neg_excursion pos_excursion
PARAMETERS par_name par_value pos_excursion neg_excursion
* for example: PARAMETER TAUBS 1.551 +0.106 -0.106
```

where par_name is the name of a parameter (normally a physics constant), par_value is the value that was assumed for that parameter in the analysis, and pos_excursion (neg_excursion) is the positive (negative) excursion of that parameter used to estimate the associated systematic uncertainty. The excursion values have to be specified with their signs. See Table 4 for a proposed list of parameter names. A parameter name must normally correspond to the name of a systematic contribution. If this is not the case, then it must correspond to the name of a preparation routine; in that case, the three numerical values following the parameter name on the logical line are data with a special meaning defined for the corresponding preparation routine.

The DATA logical lines have the following general format:

```
DATA name_1         name_2         ... name_n
     value_1_step_1 value_2_step_1 ... value_n_step_1
     value_1_step_2 value_2_step_2 ... value_n_step_2
     ...
     value_1_step_m value_2_step_m ... value_n_step_m
*
* for example: DATA  DMS AMPLITUDE STAT+ SYST+ TAUBS+ RESOL+
*                                  STAT- SYST- TAUBS- RESOL-
*                    1.0  -0.345   +0.23 +0.04 +0.03  -0.02
*                                  -0.23 -0.04 -0.03  +0.02
*
*          or: DATA DMS 0 1 2 3 4 5 6 7 8 9 10
```

where n and m are any integer numbers above zero. Each of the n names (i.e. name_1, name_2, ..., name_n) can be any name in the following categories:

a) the name of the step variable as specified on the STEPS logical line;

b) the name of the measured quantity as specified on the MEASUREMENTS logical line;

c) the name of the "measured quantity to be displayed" as specified on the DISPLAY logical line;

d) the name of the statistical uncertainty, the total systematic uncertainty, or the total uncertainty as specified on the MEASUREMENTS logical line;

e) the name of the "statistical uncertainty to be displayed", the "total systematic uncertainty to be displayed", or the "total uncertainty to be displayed" as specified on the DISPLAY logical line;

f) one of the names listed in categories d) and e) above, followed immediately by the "+" (or the "−") character;

g) the name of a parameter as specified on a PARAMETERS logical line or the name of a systematic contribution not associated with a parameter (this later case corresponds to any name which does not match a name given on a STEPS, MEASUREMENTS, or PARAMETERS logical line);

h) one of the names listed in category g) above, followed immediately by the "+" (or the "−") character.

These names indicate the meaning of the numerical values that follow on the logical line. The number of numerical values must be a multiple of n. The first n values (i.e. value_1_step_1, value_2_step_1, ..., value_n_step_1) refer to the first measurement at the first value of the step variable (step 1), the next n values refer to the second measurement at the second value of the step variable (step 2), and the last `n` values refer to the last measurement at the last value of the step variable (step `m`). Each value has the following meaning according to the name to which it is associated, using the same category numbering as above:

a) the value of the step variable;

b) the central value of the measurement itself;

c) the central value of the "measurement to be displayed";

d) the statistical or total systematic uncertainty on the measurement;

e) the "statistical uncertainty to be displayed" or the "total systematic uncertainty to be displayed";

f) the positive (or negative) statistical or total systematic uncertainty, to be combined and/or displayed; the sign of that value has to be positive (or negative);

g) the change in the measured value (compared to the central value) obtained when the associated parameter or some other unspecified parameter is increased, i.e. set to its nominal value plus its positive excursion; the sign of this change is significant, a positive (negative) change implying that the measured value is increasing (decreasing) when the parameter value is increased; this change will be assumed to be symmetric, i.e. the change in the measured value obtained when the parameter is decreased will be assumed to be exactly opposite to the change obtained when the parameter is increased;

h) the change in the measured value (compared to the central value) obtained when the associated parameter or some other unspecified parameter is increased (or decreased), i.e. set to its nominal value plus its positive (or negative) excursion; the sign of this change is significant.

Several `DATA` logical lines can be provided. If a step variable has been declared, then the step values must be specified at least once on a `DATA` logical line, and all `DATA` logical lines must provide values for the same number of steps `m`. Note that the steps do not have to be ordered in increasing or decreasing values of the step variable and that the steps do not need to be of equal size. The central values, statistical uncertainties, total systematic uncertainties, as well as all the systematic contributions to the systematic uncertainties on the measurements all have to be specified on a `DATA` logical line, but only once.

The `SUPERSEDE` logical line has the following format:

```
SUPERSEDE experiment_name method_name status
* for example: SUPERSEDE ALEPH DS+HADRON/COMB PRELIMINARY
```

where `experiment_name`, `method_name` and `status` are specifying an analysis which is superseded by the current analysis. The superseded analysis does not have to be already defined in a previous `BEGIN`–`END` block.

The `STAT_CORR_WITH` logical line has the following format:

```
STAT_CORR_WITH experiment_name method_name status correlation
* for example: STAT_CORR_WITH DELPHI LEPTON/LEPTON PREPRINT 0.3
```

where `experiment_name`, `method_name` and `status` are specifying an analysis with which the current analysis is statistically correlated. The specified analysis does not have to be already defined in a previous `BEGIN`–`END` block. The numerical value `correlation` is the correlation coefficient (which must be between 0 and 1).

2.7 `BEGIN`–`END` block defining a group of analyses

At least one `COMBINE` logical line and at least one `CALL` logical line is necessary in a `BEGIN`–`END` block defining a group of analyses. A single `MEASUREMENTS` logical line is also required. Such a block can also include at most one `STEPS` logical line and any number of `PARAMETERS` and `LUMP` logical lines. The only other allowed keywords are the `SWITCH` and `SYNONYMS` keywords.

The `COMBINE` logical lines are used to indicate which analyses are included in the group; it has the following format:

```
COMBINE experiment_name method_name status
* for example: COMBINE OPAL * *
```

where `experiment_name`, `method_name` and `status` are specifying an analysis to be included in the group. This analysis must have been defined in a previous `BEGIN`–`END` block. Any of the three tokens following the `COMBINE` keyword can also be given as "`*`" and will match any string (wild-card). Note that the use of a wild-card is not possible in the first token of a physical line (because it would otherwise be interpreted as a comment).

A certain number of options exist, which define how the combination will be performed. These options act as "switches" which can have only two states: "ON" or "OFF". Each option (or switch) has a default value (or state) which can be changed by the means of a `SWITCH` logical line. The format of a `SWITCH` logical line is the following:

```
SWITCH name_1 name_2 ... name_n switch_value
* for example: SWITCH SYSTEMATICS OFF
```

where `name_1`, `name_2`, ...`name_n` are option names, and where `switch_value` is the new value to be assigned to these options. Allowed values for `switch_value` are `ON` (or 1), `OFF` (or 0), and `TOGGLE`, with obvious meanings. If `switch_value` is omitted, then the options specified on the logical line will be reset to their default values. The option names can be abbreviated. The list of options, their names, abbreviated forms and default values are given in Table 2. Their meaning will be explained below, in due time for each option. The wild-card "`*`", if used as an option name, specifies all options; for example the logical line

```
SWITCH *
```

will reset all options to their default values. Note that all switches are automatically reset to their default values for each new combined analysis.

The `LUMP` logical line is used to specify, for a given analysis included in the combination, which of its contributions to the systematic uncertainty have to be lumped before

Full option name	Shortest abbreviated form	Default value
ADJUSTMENTS	ADJU	ON
SYMM_ADJU	SYMM_ADJU	OFF
SYMM_COMB	SYMM_COMB	OFF
SYSTEMATICS	SYST	ON
SYST_CORR	SYST_CORR	ON
STAT_CORR	STAT_CORR	ON
INTERPOLATION	INTE	ON
EXTRAPOLATION	EXTR	OFF
QUOTE_STAT	QUOT	ON
LUMPS	LUMP	ON

Table 2: List of options for a combined analysis.

proceeding with the combination. Each `LUMP` logical line must appear immediately after a `COMBINE` logical line, or after another `LUMP` logical line. It will refer to the last analysis mentioned on the previous `COMBINE` logical line. It has the following format:

```
LUMP lump_name sys_name_1 sys_name_2 ... sys_name_n
* for example: LUMP BLIFETIMES TAUBU TAUBD TAUBS
```

where `lump_name` will be the name of the "lump", and `sys_name_1`, `sys_name_2`, ..., and `sys_name_n` are the names of contributions to the systematic uncertainty to be included in the lump. These contributions must have been defined in the `BEGIN`–`END` block describing the analysis to which the `LUMP` logical line refers; they may or may not be associated to parameters. A given contribution cannot be included in more than one lump. In the combination, lumps with the same name will be considered as fully correlated across the analyses. The lump definitions hold only for the combination in which they appear The lump name cannot exceed 10 characters. The effect of the `LUMP` logical lines can be cancelled by the presence of the following logical line:

```
SWITCH LUMPS OFF
```

The `MEASUREMENTS` logical line has the following format:

```
MEASUREMENTS measured_quantity stat_uncert total_syst_uncert total_uncert
* for example: MEASUREMENTS AMPLITUDE
```

where `measured_quantity` is the name of the quantity measured, `stat_uncert` is the name of the statistical uncertainty, and `total_syst_uncert` is the name of the total systematic uncertainty, and `total_uncert` is the name of the total (stat+syst) uncertainty. The names `stat_uncert`, `total_syst_uncert` and `total_uncert` can be omitted, like shown in the example above; if omitted, they will be given the default values `STAT`, `SYST`, and `TOTAL_ERROR` respectively.

The `STEPS` logical line has the same format as in the blocks defining single analyses. It is needed if the analyses in the group have it; in that case the name of the step variable must match the ones in the individual analyses. The step values are optional. The presence or absence of step values will affect the "step synchronisation" described in Section 4.1.

The `PARAMETERS` logical lines have the same format as in the blocks defining single analyses. They are needed if the analyses in the group have them; in that case the name of the parameters must match the ones in the individual analyses. However, a parameter can be given a value and excursions that are not the same as in the individual analyses. In that case, the individual measurements and the corresponding systematic uncertainties will be adjusted to match the new (common) value of the parameter (see Section 4.3). This automatic adjustment can be prevented if a `SWITCH` logical line is provided with the following format:

```
SWITCH ADJUSTMENTS OFF
```

Note also that parameters can be defined for a group of analyses which do not appear in any of the individual analyses in the group.

The `CALL` logical lines are used to indicate which routine(s) should be called to perform the combination of the analyses in the group ("combination routines"), which routine(s), if any, should be called before any combination is performed, and which routine(s), if any, should be called for final calculations after the combined results are available for all steps ("termination routines"). It has the following format:

```
CALL routine_1 routine_2 ... routine_n logical_unit
* for example: CALL CHI2_SYM_MIN 20
```

where `routine_1`, `routine_2`, ..., `routine_n` are the names of `n` routines (combination and/or termination routines) to be called to perform the combination, and where `logical_unit` is the logical unit where these routines have to write their output. The value `logical_unit` can be omitted and it that case the output of the combination routines will appear in the log file. A value of zero or less for `logical_unit` will suppress the printed output of the routines specified on the logical line; a value of 5 or above 89 is not allowed (reserved logical units). One would normally call only one single combination routine; the possibility of calling several combination routines exists in case one wants to compare the results given by different algorithms or techniques to perform the combination. If two routines need to be called but have to print their output on different logical units, then these two routines should be specified on separate `CALL` logical lines since only one logical unit is allowed per `CALL` logical line. For example:

```
CALL ROUTINE_A 20 ROUTINE_B 21
```

will assign logical unit 20 to routine `ROUTINE_A` and logical unit 21 to routine `ROUTINE_B` (remember that when two logical lines corresponding to the same keyword follow each other, it is not necessary to start a new line and repeat the keyword).

Synonyms can be defined with the following logical line:

```
SYNONYMS  name synonym_1 synonym_2 ... synonym_n
* for example: SYNONYMS RESOL RESOLUTION
```

where `synonym_1`, `synonym_2`, ..., `synonym_n` are `n` synonyms being defined for the name `name`. These definitions hold only for the combination in which they appear. If one of the strings appearing on the `SYNONYMS` logical line is the name of a parameter, then all other strings on that same logical line have to be names not associated with a parameter (i.e. two parameter names cannot be "synonymed"). In principle, recursivity (i.e. defining synonyms for synonyms) is possible. Note that lump names cannot be synonymed.

In the combination, the various analyses in the group will be considered to be statistically independent, unless specified otherwise with `STAT_CORR_WITH` logical lines. "Unlumped" systematic contributions which have the same name (or have names that have been defined as synonyms using a SYNONYMS logical line) are considered to be fully correlated and unlumped systematic contributions which have different names are considered to be fully uncorrelated. Lumps of systematic contributions are treated as correlated if the have the same name and uncorrelated if they have different names. If the statistical correlations have to be ignored, then a `SWITCH` logical line has to be provided with the following formats:

```
SWITCH STAT_CORR OFF
```

If the systematic correlations have to be ignored, then the following logical line will do:

```
SWITCH SYST_CORR OFF
```

If all correlations have to be ignored, then the following logical line will do:

```
SWITCH STAT_CORR SYST_CORR OFF
```

The possibility exists to ignore all systematics uncertainties in the combination; this can be done with:

```
SWITCH SYSTEMATICS OFF
```

Some combination routines can only handle symmetric uncertainties. If these routines are called and one or more analyses to combine have asymmetric errors, all errors will automatically be symmetrized before the combination is performed. The other combination routines use the asymmetric errors by default. If one wants to force the use of symmetric errors in all cases, the following logical line needs to be provided:

```
SWITCH SYMM_COMB ON
```

If the `SYMM_COMB` option has been turned on, then not only the uncertainties will be symmetrized, but also all parameter excursions.

A line like

```
MINOS name_1 name_2 ... name_n
MINOS ALL
```

causes the combination routines to calculate asymmetric (i.e. MINOS) errors for the specified parameters `name_1`, `name_2`, ..., `name_n`, or, if the name `ALL` is specified, for all parameters. Note that MINOS errors are calculated only for those combination methods which implement the necessary code to call routine `MINOS`. In this version the only combination method so set up is `CHI2_ASYM_MIN`.

```
BEGIN    ALEPH                      ! experiment name
         LEPTON/LEPTON              ! method name
         PREPRINT CERN-PPE/96-102  ! status and reference
PARAMETERS !    assumed_value  pos_excurs  neg_excurs
   taubd        1.564          +0.48       -0.48     ! Bd lifetime
   taubs        1.551          +0.106      -0.106    ! Bs lifetime
   dms          6.00           +35.0       -0.0      ! Bs oscillation frequency
   fbs          0.102          +0.016      -0.016    ! Bs fraction
   flamb        0.128          +0.039      -0.039    ! Lambda_b fraction
*  aleph_fcasc  0.070          +0.010      -0.010    ! fraction of b-->c-->l
*  aleph_resol  1.07           +0.070      -0.070    ! resolution
*  aleph_fbkg   0.024          +0.012      -0.012    ! fraction of background

MEASUREMENT dmd   statistical systematic
DATA        dmd   statistical systematic
            0.426 0.039       0.052
DATA
   taubd   taubs   flamb   fbs     aleph_fcasc  aleph_resol
   -0.014  -0.016  +0.014  -0.021  -0.033        -0.004
   dms+    dms-    aleph_fbkg  aleph_sysll
   +0.007  0.0000  -0.013      +0.018
END
```

Figure 1: Example of file containing the results of a $\Delta m_{\rm d}$ analysis.

3 Input format: description through examples

This section provides examples that are in principle enough to understand the basics of the proposed input format. An attempt is also made to establish a list of names (experiments, methods, parameters, ...) to be used consistently across analyses.

3.1 Input file describing the results of a particular analysis

It is proposed that each experiment provides, for each analysis, an ASCII file containing the results of the analysis as well as the parameters (physics constants) that were assumed. The file for the ALEPH $\Delta m_{\rm d}$ analysis using dilepton events could look like the example of Fig. 1, and a file with the result of an imaginary $\Delta m_{\rm s}$ analysis is given in Fig. 2.

The experiment name, method name and status (specified after the `BEGIN` keyword) should only take predefined values. The list of these values is given in Table 3. The method name has the form `selection/tag` where `selection` indicates what is "selected" in the hemisphere where the proper time of the neutral B meson candidate is measured (and often used as the tag of the decaying state of that candidate) and `tag` indicates what is used to tag the production state (with `COMB` indicating a combination of various tags). The list of method names can/will be extended as/if new analyses techniques are being developed. The status word should be `PRELIMINARY` for preliminary results shown in

```
BEGIN MY_EXPERIMENT NEW_TECHNIQUE VERY_PRELIMINARY
MEASUREMENTS AMPLITUDE STAT SYST
STEP DMS ! step variable
PARAMETERS FBS     0.102 +0.016 -0.016   TAUBS  1.500 +0.010 -0.010
DATA
DMS AMPLITUDE  STAT+  STAT-  SYST+  SYST-   FBS+   FBS- TAUBS+ TAUBS- MY_ETA+ MY_ETA-
 0.    -0.232 +0.021 -0.021 +0.002 -0.002 +0.001 -0.001 -0.001 +0.001  +0.001  -0.001
 3.    -0.422 +0.042 -0.042 +0.002 -0.002 +0.001 -0.001 -0.001 +0.001  +0.001  -0.001
 6.     0.032 +0.132 -0.132 +0.002 -0.002 +0.001 -0.001 -0.001 +0.001  +0.001  -0.001
 9.     0.734 +0.424 -0.424 +0.002 -0.002 +0.001 -0.001 -0.001 +0.001  +0.001  -0.001
12.    -0.430 +0.670 -0.670 +0.002 -0.002 +0.001 -0.001 -0.001 +0.001  +0.001  -0.001
END
```

Figure 2: Example of file containing the results of an imaginary Δm_{s} analysis.

Experiment names	Method names	Status names
ALEPH	LEPTON/LEPTON	PRELIMINARY
DELPHI	LEPTON/QJET	PREPRINT
L3	LEPTON/COMB	PUBLISHED
OPAL	DSTAR/COMB	
	DSTAR+LEPTON/COMB	
LEP	LEPTON+KAON/QJET	
	DS+LEPTON/QJET	
	DS+LEPTON/COMB	
	DS+HADRON/COMB	
	...	

Table 3: Tokens allowed after a BEGIN keyword to identify a particular B oscillations analysis at LEP.

conferences, `PREPRINT` for final results available as preprint and submitted to a journal, and `PUBLISHED` for results published in a journal.

The measured quantities are `DMD` for a $\Delta m_{\rm d}$ analysis or `AMPLITUDE` for a $\Delta m_{\rm s}$ analysis (or even `DMS` for a $\Delta m_{\rm s}$ analysis which can actually measure $\Delta m_{\rm s}$).

Table 4 gives a list of names that can appear after a `PARAMETERS` or `DATA` keyword, together with their meanings. This list can/should be extended as new parameters are used in B oscillations analyses. These parameters should have the same meaning in all analyses, because the associated systematic contributions will be considered as correlated between the various analyses (see Section 4.2). All other names of parameters or systematic uncertainties have to be chosen with care because, if two analyses have a systematic contribution with the same name (or with names that are synonyms), this contribution will be considered as correlated. The names of the parameters and the systematic uncertainties which will never be correlated across experiment should be prefixed with "`experiment_`" where `experiment` is the name of the experiment. For example, a systematic contribution due to resolution should be names `ALEPH_RESOL`, `OPAL_RESOL1`, `OPAL_RESOL2`, etc ..., to ensure that systematic uncertainties due to resolution will never be considered as correlated between various experiments. Of course, such systematic uncertainty could be correlated among various analyses in the same experiment and in this case a common name should be chosen.

3.2 Input file to drive the combination of several analyses

An example of file to use as input to the COMBOS program is given in Fig. 3.

4 The COMBOS program

This section describes the COMBOS program itself. This program consists of several parts:

- a set of routines to read and perform checks on the input data; the input data is expected to follow the format described in Section 2; if no command line arguments are provided when COMBOS is run, the input data is read from the predefined FORTRAN logical unit 5 ("standard input"); otherwise, the command line arguments are interpreted as file names and the input data is read from these files; however, each file specified on the command line will be processed independently;

- a master routine, which is called at the end of each `BEGIN`–`END` block defining a group of analyses to combine; this routine will synchronise the steps of the various analyses to combine (and interpolate or extrapolate between steps of the individual analyses if needed), determine which systematic contributions are correlated (see Section 4.2), adjust the results of these analyses according to the common parameter values and excursions (see Section 4.3), and symmetrize the uncertainties (see Section 4.5); after all the necessary checks are made, it will pack the data to be combined and call individual combination routines to do the real work;

- a set of combination routines, called by the master routine, and implementing each a different combination algorithm (see Section 4.7);

Parameter name	Meaning
Measured (Measurable) Quantities	
DMD	Δm_{d}
DMS	Δm_{s}
AMPLITUDE	$\mathrm{B_s}$ oscillation amplitude (at a given value of Δm_{s})
b-hadron Lifetimes	
TAUB	average b-hadron lifetime
TAUBD	$\mathrm{B_d}$ lifetime
TAUBU	$\mathrm{B_u}$ lifetime
TAUBS	$\mathrm{B_s}$ lifetime
TAULAMB	b-baryon lifetime
b-hadron Lifetime Related Quantities	
TAUBD/TAUB	Ratio $\mathrm{B_d}$ to average lifetime
TAUBU/TAUB	Ratio $\mathrm{B_u}$ to average lifetime
TAUBS/TAUB	Ratio $\mathrm{B_s}$ to average lifetime
TAULAMB/TAUB	Ratio b-baryon to average lifetime
TAUBU/TAUBD	Ratio $\mathrm{B_u}$ to $\mathrm{B_d}$ lifetime
TAUBS/TAUBD	Ratio $\mathrm{B_s}$ to $\mathrm{B_d}$ lifetime
TAULAMB/TAUBD	Ratio b-baryon to $\mathrm{B_d}$ lifetime
TAUBU-TAUBD	Difference $\mathrm{B_u}$ and $\mathrm{B_d}$ lifetimes
Source Fraction Related Quantities	
FBS	$f_{\mathrm{B_s}}$
FBD	$f_{\mathrm{B_d}} = f_{\mathrm{B_u}}$
FLAMB	$f_{\Lambda_{\mathrm{b}}}$
CHIBAR	$\overline{\chi}$
CHID	χ_{d}
CHID_UPSILON4S	χ_{d} from $\Upsilon(4S)$ measurements
CHIS	χ_{s}
FBS_FROM_BR	$f_{\mathrm{B_s}}$ derived from branching ratio measurements
CORR_FLAMB_FBS_	Correlation coefficient between FLAMB and FBS_FROM_BR
FCASC	Correlated part of error from $\mathrm{b} \to \mathrm{c} \to \ell$ decays
D^* Analysis Specific Quantities	
FBUBD	Fraction of $\mathrm{B_u}$ in ($\mathrm{B_d}$) sample
MBUBD	Mistag of $\mathrm{B_u}$ in ($\mathrm{B_d}$) sample
TAUD0	$\mathrm{D^0}$ lifetime
...	...

Table 4: List of names.

```
* Load results of individual analyses: output cards will be written on unit 21

OUTPUT 6 6 21
INCLUDE aleph_analyses.cards   ! include the file describing all the ALEPH results
INCLUDE delphi_analyses.cards  ! include the file describing all the DELPHI results
INCLUDE opal_analyses.cards    ! include the file describing all the OPAL results
INCLUDE l3_analyses.cards      ! include the file describing all the L3 results

OUTPUT 6 6 25                  ! from now on, output cards will be written on unit 25

* delta_m_d combinations
* ======================

BEGIN LEP DMD PRELIMINARY for next Winter Conferences
INCLUDE final_parameters.cards ! Read common values to be used for the parameters
COMBINE * * *                  ! Use all the DMD analyses in this combination
MEASUREMENT DMD
CALL CHI2_SYM                  ! Select an algorithm to perform the combination
END

* now repeat combination, but without including preliminary results
  BEGI    LEP DMD
  !INCL    old_parameters.cards   Old values not used anymore
  INCL    final_parameters.cards
  COMB    * * PUBLISHED * * PREPRINT
  MEAS    DMD
  CALL    CHI2_SYM
  END

* delta_m_s combinations (using amplitude method)
* ======================

BEGIN LEP AMPLITUDE_DS PRELIMINARY for next Winter Conference
INCLUDE final_parameters.cards
* Use only Ds based analyses
COMBINE   * DS+LEPTON/QJET *     * DS+LEPTON/COMB *     * DS+HADRON/COMB *
MEASUREMENT AMPLITUDE
CALL CHI2_SYM
END

BEGIN QUICK_TEST
INCLUDE final_parameters.cards
SWITCH ADJUSTMENTS OFF ! don't adjust results to the new parameter values and excursions
COMBINE DELPHI * *  ! use only DELPHI analyses in this combination
MEASUREMENT AMPLITUDE
CALL CHI2_SYM 6 NEW_COMBINATION 10
    ! ... because we want to compare the results of two combination algorithms
END
```

Figure 3: Example of input file to combine Δm_{d} and Δm_{s} results.

- an output routine to write (in an output file) both the input data and the results of the combinations, in a format compatible with the input format (i.e. that output file could be used as input in a subsequent run of the program and would give the same results).

4.1 Step synchronisation, interpolation and extrapolation

In case a `STEP` logical line is specified in the `BEGIN–END` block defining the group of analyses to combine, the master routine will decide at which values of the step variables the combination will be performed. The strategy depends on the presence or absence of step values on the `STEP` logical line.

If step values are specified on the `STEP` logical line, then the combination will be performed at each of these step values which is inside the range of step values of at least one of the analyses included in the group. Since the step values at which the combination will be done don't necessarily correspond to step values in the individual analyses, the data in each of these analyses will be interpolated (or extrapolated) between the data provided for the two nearest step values. The interpolation (or extrapolation) is linear and is applied (before the adjustment and lumping procedure described in Sections 4.3 and 4.4) to the central value of the measurements, to their statistical uncertainties and to all the contributions to their systematic uncertainties. The steps for which interpolation or extrapolation is required for at least one individual analysis are ignored (i.e. no combination performed) if the `INTERPOLATION` option is turned off in a `SWITCH` logical line, e.g.

```
SWITCH INTERPOLATION OFF
```

If no step values are specified on the `STEP` logical line, then the combination will be performed at each step value which appears at least once in any of the individual analyses included in the group.

Of course, if no step values are specified on the `STEP` logical line and the `INTERPOLATION` option is turned off, then the combination will be performed at each step value which appears in common to all the analyses included in the group.

In case there is a need to extrapolate, the behaviour of the program depends on the value of the `EXTRAPOLATION` option. If that option is turned on (in a `SWITCH` logical line), then extrapolation occurs. If that option is off (default), then no extrapolation is performed and the statistical uncertainty of the corresponding measurement is set to a very large positive value such as to completely deweight the measurement in the combination.

4.2 Determining the correlations

The master routine decides on the correlation of the various uncertainties in the following way. Statistical uncertainties are always treated as uncorrelated if the `STAT_CORR` option was turned off by a `SWITCH` logical line. Otherwise, statistical uncertainties are treated as uncorrelated, unless specified otherwise with `STAT_CORR_WITH` logical lines. If the `SYST_CORR` option was turned off by a `SWITCH` logical line, then all systematic uncertainties are assumed to be uncorrelated. Otherwise, unlumped systematic uncertainties with the same name (or with names that have been defined as synonyms) are treated as fully (i.e $+100\%$ or -100%) correlated and other unlumped systematic uncertainties, with different names that have not been synonymed, are treated as uncorrelated. Lumps of systematic uncertainties are treated as correlated (uncorrelated) if they have same (different) names.

Note that a parameter does not need to be associated to a systematic contribution for it to be considered as correlated in different analyses. However, for technical reasons, if a parameter is associated to a systematic contribution, that contribution will be considered as "correlated" even if it appears only in one of the analyses to be combined (in that case it is correlated just with itself). Similarly, a lump with a unique name will also be treated as correlated just with itself.

Note also that the total systematic uncertainties are not used in the combination.

4.3 Adjusting the results of individual analyses

The master routine decides on the corrections to apply to the results of each analysis in a group of analyses to combine. These corrections are then applied on the data before possible "lumping" of systematic contributions, and in any case before the combination is performed. A new set of corrections is determined for each group of analyses to combine. If the `ADJUSTMENTS` option was turned off by a `SWITCH` logical line, then no corrections will be applied. Otherwise, the results of each analysis will receive a correction for each parameter used in the analysis.

The correction associated to a given parameter is determined from the comparison of the value α and excursions $\Delta\alpha^{\pm}$ used in the analysis for that parameter with the value α_c and excursions $\Delta\alpha_c^{\pm}$ to be used in the combination.

The correction consists of an adjustment of the central value of the measurement, x, and an adjustment of the positive and negative systematic uncertainties due to that parameter, σ^+ and σ^-. Note that $\sigma^{\pm}$ is defined as the change obtained in the central value of the measurement when the value of the parameter is changed from α to $\alpha \pm |\Delta\alpha^{\pm}|$, and that both $\sigma^{\pm} > 0$ and $\sigma^{\pm} < 0$ are possible.

By default, i.e. if option `SYMM_ADJU` is off, the adjusted central value, x_{adj}, is computed as:

$$x_{\mathrm{adj}} = \begin{cases} x + \left|\frac{\alpha_c-\alpha}{\Delta\alpha^+}\right| \sigma^+ & \text{if } \alpha_c \geq \alpha \\ x + \left|\frac{\alpha_c-\alpha}{\Delta\alpha^-}\right| \sigma^- & \text{if } \alpha_c < \alpha \end{cases} \tag{1}$$

and the adjusted systematic uncertainties, $\sigma_{\mathrm{adj}}^{\pm}$ are computed as:

$$\sigma_{\mathrm{adj}}^{\pm} = \left|\frac{\Delta\alpha_c^{\pm}}{\Delta\alpha^{\pm}}\right| \sigma^{\pm} \tag{2}$$

Note that if $\alpha = \alpha_c$, $\Delta\alpha^+ = \Delta\alpha_c^+$ and $\Delta\alpha^- = \Delta\alpha_c^-$, the correction is dummy (i.e. no adjustments are made for that parameter). If the option `SYMM_ADJU` is on, then the adjustments are performed using symmetrized parameter excursions and systematic uncertainties; in that case, formulae (1) and (2) are replaced by:

$$x_{\mathrm{adj}} = x + \left(\frac{\alpha_c - \alpha}{\Delta\alpha}\right) \sigma \tag{3}$$

and

$$\sigma_{\text{adj}}^{\pm} = \pm \left(\frac{\Delta\alpha_c}{\Delta\alpha} \right) \sigma \tag{4}$$

where

$$\sigma = \text{sign}(\sigma^+) \sqrt{\frac{\max(\sigma^+;\sigma^-;0)^2 + \min(\sigma^+;\sigma^-;0)^2}{2}}\,, \tag{5}$$

$$\Delta\alpha = \sqrt{\frac{(\Delta\alpha^+)^2 + (\Delta\alpha^-)^2}{2}}\,, \quad \Delta\alpha_c = \sqrt{\frac{(\Delta\alpha_c^+)^2 + (\Delta\alpha_c^-)^2}{2}}\,. \tag{6}$$

Corrections associated with different parameters are applied independently to the same measurement, i.e. correlations between the various contributions of the systematic uncertainty on the measurement are ignored.

Of course, systematic uncertainties which are not associated to a parameter don't lead to an adjustment of the central value, and in that case $\sigma_{\text{adj}}^{\pm} = \sigma^{\pm}$.

Note that all corrections are applied, independently of the "lumping" of systematic uncertainties, i.e. the corrections are also applied for parameters which are associated to lumped systematic contributions.

4.4 Lumping of uncertainties

Within a given analysis included in the combination, certain contributions to the systematic uncertainty are merged before the combination procedure is applied. This merging (or lumping) occurs in two different instances:

- when merging all the uncorrelated systematic contributions, to produce an "uncorrelated lump";
- when merging all systematic contributions within a lump as defined on a `LUMP` logical line, to produce a "correlated lump".

The total positive (negative) uncertainty of a lump, σ_{lump}^{+} (σ_{lump}^{-}), is computed according to:

$$\sigma_{\text{lump}}^{+} = +\sqrt{\sum_i (\sigma_{\text{adj},i}^{\max})^2} \qquad \text{and} \qquad \sigma_{\text{lump}}^{-} = -\sqrt{\sum_i (\sigma_{\text{adj},i}^{\min})^2} \tag{7}$$

where the sums run over all systematic contributions i included in the lump and where

$$\sigma_{\text{adj},i}^{\max} = \max(\sigma_{\text{adj},i}^{+}\,;\sigma_{\text{adj},i}^{-}\,;0) \qquad \text{and} \qquad \sigma_{\text{adj},i}^{\min} = \min(\sigma_{\text{adj},i}^{+}\,;\sigma_{\text{adj},i}^{-}\,;0) \tag{8}$$

Note that the lumping of uncertainties defined by the `LUMP` logical lines can be ignored if the `LUMPS` options is turned off by a `SWITCH` logical line.

4.5 Symmetrization of uncertainties

The master routine computes also "symmetric" uncertainties for all contributions (statistical uncertainties, uncorrelated systematic lumps, correlated systematic uncertainties and correlated systematic lumps). This symmetrization is done quadratically and the symmetric uncertainty retains the sign of the positive uncertainty. For example, in case of a systematic contribution i which is not included in a lump, with associated uncertainties $\sigma^+_{\mathrm{adj},i}$ and $\sigma^-_{\mathrm{adj},i}$, the symmetric uncertainty $\sigma_{\mathrm{adj},i}$ is computed as:

$$\sigma_{\mathrm{adj},i} = \mathrm{sign}(\sigma^+_{\mathrm{adj},i}) \sqrt{\frac{(\sigma^{\max}_{\mathrm{adj},i})^2 + (\sigma^{\min}_{\mathrm{adj},i})^2}{2}} \tag{9}$$

where $\sigma^{\max}_{\mathrm{adj},i}$ and $\sigma^{\min}_{\mathrm{adj},i}$ have been defined in Equation (8). Note that if the uncertainties are already symmetric, i.e. $\sigma^+_{\mathrm{adj},i} = -\sigma^-_{\mathrm{adj},i}$, then the above formulae yield $\sigma_{\mathrm{adj},i} = \sigma^+_{\mathrm{adj},i}$.

In the case of a lump, the symmetric uncertainty is computed according to a formula similar to expression (9), which reduces to:

$$\sigma_{\mathrm{lump}} = \sqrt{\frac{(\sigma^+_{\mathrm{lump}})^2 + (\sigma^-_{\mathrm{lump}})^2}{2}} \tag{10}$$

because $\sigma^+_{\mathrm{lump}} \geq 0$ and $\sigma^-_{\mathrm{lump}} \leq 0$.

Note that expressions (9) and (10) used for the symmetrization of the uncertainties are more or less arbitrary and cannot be easily justified. If uncertainties are asymmetric, it is probably just wrong to try to symmetrize them, or if one does so, one would need to know why they are asymmetric to be able to symmetrize them in a reasonable way. The procedure chosen here (i.e. formulae (9) and (10)) corresponds to a "quadratic averaging" of the asymmetric uncertainties. Other possible approaches include a "linear averaging" (less conservative) or "taking the largest of the absolute values of the positive and negative uncertainties" (more conservative).

4.6 The preparation routines

The COMBOS program can be linked with several different preparation routines. Before any combination is performed, one may want to call one or more preparations routines. This is useful in case one wants to perform special adjustments of the individual results before they are combined. Each preparation routine has access to the same data as the combination routines (common block named COMINP and available in `master.inc` as explained in Section 4.7) and is allowed to modify it.

The types and order of the input arguments of the preparation routines are given in Figure 4.

Each preparation routine has to comply to the following rules:

- the preparation routine is not allowed to modify the common blocks available in file `combos.inc`;
- the preparation routine is allowed to overwrite the input data in common block COMINP (the overwritten data will then be used subsequently be the combination routines);
- the routine should print all its output on logical `LUNIT`, if the integer variable `LUNIT` is larger than zero; if `LUNIT` is zero or less, then routine should not print any output;

Note that each single analysis is allowed to have a parameter with a name equal to the name of a combination routine, even if this is not the name of a systematic contribution. This parameter can be used to transmit special analysis-dependent data to the combination routine.

```
*     ===================================
      SUBROUTINE NEW_PREPARATION(N,ICOMB)
*     ===================================
*
*     Input:  N      = number of analyses to combine
*     -----   ICOMB  = array of indexes to analyses
*                    ICOMB(I) = analysis number of Ith analysis to combine
*
*     Output: none
*     ------
*
*     Input data:  available from combos.inc and master.inc
*     -----------
*
*     Output data: written in master.inc
*     ------------
*
*     Note: this routine only writes output to logical unit LUNIT;
*     ----- if LUNIT is zero or less, then this routine should not produce
*           any output
*
      IMPLICIT NONE
      INCLUDE 'master.inc'
*
*     Argument
*
      INTEGER N
      INTEGER ICOMB(N)
*
*     Local variable

*
      IF(LUNIT.GT.0) WRITE(LUNIT,*)
     & 'Dummy routine NEW_PREPARATION called'
      END
```

Figure 4: Template for a preparation routine.

4.7 The combination routines

The COMBOS program can be linked with several different combination routines. These combination routines are called by the master routine. Their input data is provided by the master routine in a common block named COMINP, defined in Fig. 5 and available in a file called `master.inc`. The types and order of the output arguments of the combination routines are given in Figure 6.

Each combination routine has to comply to the following rules:

- if the algorithm can handle asymmetric uncertainties, then the routine should only use the asymmetric uncertainties stored in arrays `STATP`, `STATN`, `USYSP`, `USYSN`, `CSYSP` and `CSYSN`, and ignore the symmetric uncertainties stored in arrays `STAT`, `USYS` and `CSYS`;
- if the algorithm cannot handle asymmetric uncertainties, then the routine should only use the symmetric uncertainties stored in arrays `STAT`, `USYS` and `CSYS`, and ignore the asymmetric uncertainties stored in arrays `STATP`, `STATN`, `USYSP`, `USYSN`, `CSYSP` and `CSYSN`;
- if the algorithm can deal with statistical correlations, then the routine should use the correlation matrix provided in array `STACOR`; otherwise array `STACOR` should be ignored;
- in most cases, the combination routine does not need to use the values and excursions given for the parameters associated with the correlated systematic uncertainties (i.e. the first `NCSYS` elements of arrays `PARA`, `EXCUP`, `EXCUN` and `CHPARA`);
- the "extra" parameters (i.e. the remaining elements of the same arrays) can be used to provide extra input data to the combination routine, for example flags to drive the combination algorithm;
- the combination routine is not allowed to overwrite the input data in common block COMINP (because that same input data may be used subsequently be another combination routine);
- the routine should print all its output on logical `LUNIT`, if the integer variable `LUNIT` is larger than zero; if `LUNIT` is zero or less, then routine should not print any output;
- the combination routine must return an error flag; if that flag has a value of zero, then the combination was successful, and the following results are also returned to the master:
  - a central value for the combined measurement, together with its positive and negative total uncertainties;
  - a confidence level in the range 0–1, for example the probability of the χ^2 of the combination.

The content of the common block COMINP described in file `master.inc` can be dumped on request; to request the dump, one has to specify the name of the routine

```
*  Input for a combination routine:
*  --------------------------------
*
*  CHROUT = name of combination routine; CHCOMB = name of combined analysis
*  CHMEAS = name of measured quantity  ; CHSTEP = name of step variable (or ' ')
*  ISTEP  = step number                ; VSTEP  = step value
*  LUNIT  = logical unit for printout
*
*  NMEAS = number of measurements to combine (I=1,NMEAS)
*  NMEFF = number of measurements which have not been de-weighted (< or = NMEAS)
*  NCSYS = number of correlated systematic uncertainties (J=1,NCSYS)
*  NPARA = total number of parameters (K=1,NPARA)
*
*  For each measurement I=1,NMEAS:
*  CHANAL(I)  = name of analysis producing measurement I
*  MEAS(I)    = central value of measurement I
*  STATP(I)   = absolute value of the positive  statistical uncertainty
*  STATN(I)   = absolute value of the negative  statistical uncertainty
*  STAT (I)   = absolute value of the symmetric statistical uncertainty
*  USYSP(I)   = absolute value of the positive  uncorrelated syst. uncertainty
*  USYSN(I)   = absolute value of the negative  uncorrelated syst. uncertainty
*  USYS (I)   = absolute value of the symmetric uncorrelated syst. uncertainty
*
*  For each pair of measurements I1=1,NMEAS and I2=1,NMEAS:
*  STACOR(I1,I2) = statistical correlation between measurements I1 and I2
*
*  For each measurement I=1,NMEAS and each correlated systematic uncertainty J:
*  CSYSP(I,J) = change in central value of measurement I for positive excursion
*               of the parameter associated to systematic uncertainty J
*  CSYSN(I,J) = change in central value of measurement I for negative excursion
*               of the parameter associated to systematic uncertainty J
*  CSYS (I,J) = symmetric change in central value of measurement I for excursion
*               of the parameter associated to systematic uncertainty J
*
*    Note: the signs of CSYSP(I,J), CSYSN(I,J) and CSYS(I,J) are significant;
*          CSYSP(I,J)=CSYSN(I,J)=CSYS(I,J)=0 if parameter J does not affect
*          measurement I.
*
*  For each parameter K=1,NPARA:
*  PARA(K)    = value of parameter K
*  EXCUP(K)   = absolute value of the positive excursion of parameter K
*  EXCUN(K)   = absolute value of the negative excursion of parameter K
*  EXCU (K)   = absolute value of the symmetric excursion of parameter K
*  CHPARA(K)  = name of parameter K
*
*    Notes: NPARA is always at least equal to NCSYS, and if K is smaller than
*           or equal to NCSYS, K refers to the parameter associated with the
*           correlated systematic uncertainty J=K.
*
      INTEGER MMEAS,MCSYS,MPARA
      PARAMETER(MMEAS=30,MCSYS=50,MPARA=MCSYS+10)
*
      INTEGER ISTEP,LUNIT,NMEAS,NMEFF,NCSYS,NPARA
      DOUBLE PRECISION VSTEP,MEAS(MMEAS),
     & STAT(MMEAS),STATP(MMEAS),STATN(MMEAS),STACOR(MMEAS,MMEAS),
     & USYS(MMEAS),USYSP(MMEAS),USYSN(MMEAS),
     & CSYS(MMEAS,MCSYS),CSYSP(MMEAS,MCSYS),CSYSN(MMEAS,MCSYS),
     & PARA(MPARA),EXCU(MPARA),EXCUP(MPARA),EXCUN(MPARA)
      CHARACTER*80 CHCOMB,CHANAL(MMEAS)
      CHARACTER*16 CHPARA(MPARA),CHROUT,CHMEAS,CHSTEP
*
      COMMON/COMINP/ISTEP,LUNIT,MEAS,STAT,STATP,STATN,STACOR,
     &       USYS,USYSP,USYSN,CSYS,CSYSP,CSYSN,PARA,EXCU,EXCUP,EXCUN,
     &       CHPARA,CHROUT,CHCOMB,CHANAL,CHMEAS,CHSTEP,
     &       VSTEP,NMEAS,NMEFF,NCSYS,NPARA
```

Figure 5: File `master.inc` describing the input available to a combination routine.

```
*     ==================================================
      SUBROUTINE NEW_COMBINATION(CVAL,ERRP,ERRN,CL,IERR)
*     ==================================================
*
*     Input data: available from a common block in file master.inc
*     ----------
*
      INCLUDE 'master.inc'
*
*     Output data: returned in the arguments
*     -----------
*
      DOUBLE PRECISION CVAL ! combined value (or average value)
      DOUBLE PRECISION ERRP ! positive total uncertainty on CVAL
      DOUBLE PRECISION ERRN ! negative total uncertainty on CVAL
      DOUBLE PRECISION CL   ! confidence level of combination (e.g. chi2 prob.)
*                             or -1 if confidence level not available
      INTEGER IERR          ! error flag (non-zero if combination failed)
*
*     Note: this routine should only write output (including error messages)
*     ----  to logical unit LUNIT (variable LUNIT is available in a common block
*           described in master.inc); if LUNIT is zero or less, then this
*           routine should not write any output
*
********
*
*     Add below code to perform the combination.
*     Set IERR to zero if algorithm is successful.
*     For the time being this is a dummy routine, so IERR=-1 is returned.
*
      IERR = -1
      IF(LUNIT.GT.0) WRITE(LUNIT,*)
     & 'Dummy routine NEW_COMBINATION called'
*
*     Return result
*
      IF(IERR.EQ.0) THEN
*       CVAL = ...
*       ERRP = ...
*       ERRN = ...
*       CL   = ...
      ELSE ! combination failed, return dummy values
        CVAL = 0.D0
        ERRP = 0.D0
        ERRN = 0.D0
        CL   =-1.D0
      ENDIF
      END
```

Figure 6: Template for a combination routine.

DUMP_MASTER_INC on a CALL logical line (inside the BEGIN–END block defining a group of analyses to combine). The logical unit used for the dump is the one specified at the end of the CALL logical line (or the logical unit used for the log file output if no unit was specified on the CALL logical line).

4.8 Getting the combined results

To get the results of a combined analysis (at each step), the COMBOS program is calling twice each combination routine that is specified on a CALL logical line:

- a first call is made where all the systematic uncertainties are ignored (technically, the systematic uncertainties stored in the common block described in file `master.inc` are set to zero); the combination routine will then return a combined value $x^{\rm comb}_{\rm stat}$, with uncertainties $\sigma^{\rm comb\pm}_{\rm stat}$;

- a second call is perform without ignoring the systematic uncertainties (the actual systematic uncertainties are stored in the common block described in file `master.inc`); the combination routine will then return a combined value $x^{\rm comb}$, with uncertainties $\sigma^{\rm comb\pm}$.

The COMBOS program then takes, $x^{\rm comb}$ as the central value of the combined measurement, $\sigma^{\rm comb\pm}_{\rm stat}$ as its statistical uncertainties, $\sigma^{\rm comb\pm}_{\rm syst} = \pm\sqrt{(\sigma^{\rm comb\pm})^2 - (\sigma^{\rm comb\pm}_{\rm stat})^2}$ as its total systematic uncertainties, and $\sigma^{\rm comb\pm}$ as its total (statistical + systematic) uncertainties.

In case one does not need/wish to know the statistical uncertainty on the combined result, the following logical line can be specified in the input file:

```
SWITCH QUOTE_STAT OFF
```

In that case, only the second call to each combination routine is performed (case where the systematics uncertainties are taken into account).

After each combination routine has been called for each step, COMBOS will call each termination routine that is specified on a CALL logical line.

A given termination routine is called once per combination routine; COMBOS will make available (by the means of input arguments to that termination routine), all the results obtained with a given combination routine.

The input arguments of a termination routine are described in Figure 7.

4.9 Plotting the results of the combination

If the KUMAC output is turned on by specifying an appropriate logical unit on the OUTPUT logical line, a KUMAC file will be written by the COMBOS program. This file contains, for each combination that the program has performed, information on the individual analyses that have been combined and on the combined results themselves. It can be executed from another KUMAC file (provided by the user) in order to display all or part of this information in nice plots and tables. An example of such a user KUMAC file is given in file `combos.kumac`. Instructions on how to use this example file can be found as comments in the file itself.

```
*     ================================================================
      SUBROUTINE NEW_TERMINATION(NC,NS,STEP,CVAL,ERRP,ERRN,ERR,CL,IERR)
*     ================================================================
*
*     Input:  NC     = number of cases
*     -----            NC=1: means only total error available
*                      NC=2: means also statistical error available
*             NS     = number of steps
*             STEP   = array of step values
*             CVAL   = array of combined values of the measured quantity
*             ERRP   = array of positive errors on CVAL
*             ERRN   = array of negative errors on CVAL
*             ERR    = array of symmetric errors on CVAL
*             CL     = array of confidence levels of CVAL
*             IERR   = array of error codes (0 means OK)
*
*     For step IS=1,NS:
*             CVAL(1,IS) = combined value at step IS (including syst.)
*             ERRP(1,IS) = positive error on CVAL(1,IS)
*             ERRN(1,IS) = negative error on CVAL(1,IS)
*             ERR (1,IS) = symmetric error on CVAL(1,IS)
*             CL  (1,IS) = confidence level of fit performed to get CVAL(1,IS)
*                          (or -1 if not available)
*
*     Other input (available from common block described in master.inc):
*     -----------
*
*             CHROUT = name of combination routine
*                       used to obtain CVAL,ERRP,ERRN,ERR,CL and IERR
*             CHCOMB = name of combined analysis
*             CHMEAS = name of measured quantity
*             CHSTEP = name of step variable
*             LUNIT  = logical unit for printout
*                       produced by this termination routine
*
*     Output:  none
*     -------
*
*     Note: this routine only write output to logical unit LUNIT;
*     ----- if LUNIT is zero or less, then this routine should not produce
*           any output
*
      IMPLICIT NONE
*
*     Arguments
*
      INTEGER MC,NC,NS
      PARAMETER(MC=2)
      DOUBLE PRECISION STEP(NS),CVAL(MC,NS),CL(MC,NS),
     &                 ERRP(MC,NS),ERRN(MC,NS),ERR(MC,NS)
      INTEGER IERR(MC,NS)
*
      INCLUDE 'master.inc'
*
      IF(LUNIT.GT.0) WRITE(LUNIT,*)
     & 'Dummy routine NEW_TERMINATION called'
      END
```

Figure 7: Template for a termination routine.

5 Preparation routines

5.1 ADJU_A_FOR_FBS: special adjustment for the measured B_s oscillation amplitude

This preparation routine is used to perform special adjustments of the statistical uncertainty and/or the central value of the measured B_s oscillation amplitudes. These special adjustments refer only to the adjustments made for parameter `FBS`, which represents the B_s fraction. All other adjustments, performed based on a parameter different from the B_s fraction, are not affected by a call to routine `ADJU_A_FOR_FBS`. Furthermore a call to this routine does not affect any adjustments performed on the systematic uncertainties, even the ones due to the B_s fraction. If this preparation routine is called, the special adjustments are only performed for the analyses where the parameters `ADJU_A_FOR_FBS` and `FBS` are defined; the value of the parameter `ADJU_A_FOR_FBS` indicates the algorithm to be used for the adjustment (the positive and negative errors given for the this parameter are ignored). The value of the parameter must be equal to $z(10n_A + n_{\sigma_A^{\rm stat}})$, where $z = \pm 1$ is a flag to indicate the order in which the adjustments are to be made, n_A a positive integer code defining the algorithm to be used to adjust the central value of the measured amplitude A and $n_{\sigma_A^{\rm stat}}$ a positive integer code defining the algorithm to be used to adjust the statistical uncertainty on the measured amplitude, $\sigma_A^{\rm stat}$. Both n_A and $n_{\sigma_A^{\rm stat}}$ cannot exceed 4, but only one may exceed 3. The meaning of these two codes are the following:

- a value of 0 indicates that a default value should be picked from the combined analysis (i.e. from parameter `ADJU_A_FOR_FBS` as defined in the combined analysis); if this default value is 0 or if parameter `ADJU_A_FOR_FBS` is not defined in the combined analysis, then the default value in effect is 2 (in the case where $n_A = 0$, then default values for both n_A and z are picked from parameter `ADJU_A_FOR_FBS` as defined in the combined analysis);

- a value of 1 indicates that no adjustment should be performed because of a change in the B_s fraction, not even the regular adjustment that COMBOS would normally do if routine `ADJU_A_FOR_FBS` was not called;

- a value of 2 indicates that the regular adjustment should be done, i.e. the adjustment that COMBOS would normally do if routine `ADJU_A_FOR_FBS` was not called;

- a value of 3 indicates that the adjustment should be performed assuming a scaling like $1/f_{B_s}$, where f_{B_s} is the fraction of B_s as given by parameter FBS; this adjustment consist of multiplying the central value or the statistical uncertainty by $f_{B_s}^{\rm old}/f_{B_s}^{\rm new}$, where $f_{B_s}^{\rm old}$ is the B_s fraction specified in the analysis to be combined and $f_{B_s}^{\rm new}$ is the B_s fraction specified in the combined analysis (as values of parameter `FBS`); this adjustment is performed instead of the regular adjustment that COMBOS would normally do if routine `ADJU_A_FOR_FBS` was not called;

- a value of 4 indicates that the adjustment for the new B_s fraction should be performed in such a way that the ratio $A/\sigma_A^{\rm stat}$ is kept unchanged.

The sign of parameter `ADJU_A_FOR_FBS` has the following meaning:

- a value $z = -1$ indicates that the all the adjustments performed for the new value of the B_s fraction (parameter `FBS`) should be performed before any other adjustments performed by COMBOS for the new values of other parameters;
- a value $z = +1$ indicates that the all the adjustments performed for the new value of the B_s fraction (parameter `FBS`) should be performed after any other adjustments performed by COMBOS for the new values of other parameters.

Note that the sign of the value is relevant only in the case where n_A is equal to 3 or 4; in this case the order of the adjustment on A matters, since the adjustment due to `FBS` is a multiplicative constant whereas the adjustments due to the other parameters are additive constants.

5.2 NEW_PREPARATION: a template for a new preparation routine

A dummy routine called `NEW_PREPARATION` (see Figure 4) is also linked with COMBOS. This allows any member of the LEP B oscillation working group to program her/his favourite preparation algorithm and try it out.

6 Combination algorithms

In this section, the various algorithms available to combine results inside COMBOS are described. Each algorithm is presented in a separate section with a title corresponding to the name of the combination routine which implements it in COMBOS. The main characteristics of the various combination routines are summarised in Table 5.

6.1 SINGLE: Dummy combination routine

Routine `SINGLE` performs a dummy combination of a single measurement. It does not work if more than one measurement is to be combined. It may be needed to transfer correct asymmetric uncertainties to termination routines.

6.2 BLUE: Best Linear Unbiased Estimator to combine statistically and systematically correlated measurements with symmetric uncertainties

Given a set of n unbiased measurements X_i of a physical quantity Q and their error matrix $\mathbf{M}$, it can be seen that the estimator

$$X = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} X_i(\mathbf{M}^{-1})_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n}(\mathbf{M}^{-1})_{ij}} \tag{11}$$

Combination routine	Asymmetric errors used	Correlation matrix used	Mandatory parameters	Optional parameters
SINGLE	YES	—	—	—
BLUE	NO	YES	—	—
BLUE_ASYM	YES	YES	—	—
CHI2_SYM_INDEP	NO	NO	—	—
CHI2_SYM	NO	YES	—	—
CHI2_SYM_MIN	NO	YES	—	MINUIT_PRTLEV
CHI2_ASYM_MIN	YES	YES	—	MINUIT_PRTLEV (+ MINOS logical line)
CHI2_SYM_CIRC	NO	YES	DMD, TAUBD, CHIBAR, FLAMB	CHID_UPSILON4S, CHIS, FBS_FROM_BR, CORR_FLAMB_FBS_ MINUIT_PRTLEV
CHI2_SYM_CIRC_G	NO	YES	DMD, TAUBD, CHIBAR, FLAMB	CHID_UPSILON4S, CHIS, TAUBU/TAUBD, TAUBS/TAUB, TAULAMB/TAUB, FBS_FROM_BR, CORR_FLAMB_FBS_ MINUIT_PRTLEV
COMBY_LIFETIME	YES*	NO	—	—
NEW_COMBINATION	*template for a new combination routine*			

* Only statistical uncertainties and uncorrelated uncertainties not associated to any parameter

Table 5: List of combination routines.

is a linear combination of the individual measurements providing an unbiased estimate of Q with the minimum possible variance

$$\sigma^2 = \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{n}(\mathbf{M}^{-1})_{ij}} \tag{12}$$

(Best Linear Unbiased Estimator) [1]. The error matrix $\mathbf{M}$ includes all the error contributions, including the correlated systematics uncertainties:

$$M_{ij} = \rho_{ij}\,\sigma_i^{\mathbf{stat}}\sigma_j^{\mathbf{stat}} + \delta_{ij}\,\sigma_i^{\mathbf{syst}}\sigma_j^{\mathbf{syst}} + \sum_{\alpha=1}^{m}\Delta_{\alpha i}\Delta_{\alpha j} \tag{13}$$

where ρ_{ij} is the statistical correlation coefficient between measurements i and j, $\sigma_i^{\mathbf{stat}}$ ($\sigma_i^{\mathbf{syst}}$) is the statistical uncertainty (uncorrelated systematic uncertainty) on measurement i, $\Delta_{\alpha i}$ is the change on measurement i when the correlated systematic parameter α is moved from its central values and δ_{ij} is the Kronecker symbol.

The χ^2 is then obtained by :

$$\chi^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\Big([X_i - X]\,(\mathbf{M}^{-1})_{ij}\,[X_j - X]\Big)\,. \tag{14}$$

In the case of neglected sytematic correlations, BLUE is expected to give exactly the same results as CHI2_SYM.

The `BLUE` routine was provided by Paolo Checchia (DELPHI). This is the algorithm used in DELPHI for combinations.

6.3 BLUE_ASYM: Best Linear Unbiased Estimator to combine statistically and systematically correlated measurements with asymmetric uncertainties

Routine provided by Paolo Checchia (DELPHI). No description available yet. The result produced by `BLUE_ASYM` has symmetric uncertainties. Needs testing.

6.4 CHI2_SYM_INDEP: a χ^2 method to combine systematically correlated and statistically independent measurements with symmetric uncertainties

The CHI2_SYM_INDEP algorithm is the one that was used to combine the ALEPH $\Delta m_{\rm s}$ results for the Warsaw conference in summer 1996. It takes into account statistical uncertainties, uncorrelated systematic uncertainties and correlated systematic uncertainties in a global χ^2 fit. Only symmetric uncertainties can be handled and the measurements are assumed to be statistically independent.

The method, developed in ALEPH [2], assumes that one has several measurements X, which are statistically independent but may have systematic correlations. In addition the systematic dependencies are related to a set of parameters P, which are the same in all of the measurements. This may be rephrased as stating that one has measured M which is very close to X, but has (small) dependencies on P as well. To extract X one needs the values of M and P as well.

For n measurements, a χ^2 is constructed as follows:

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{M_i - F_i(X, P_1, \ldots, P_m)}{\sigma_{Mi}} \right)^2 + \sum_{\alpha=1}^{m} \left(\frac{P_\alpha - \bar{P}_\alpha}{\delta_\alpha} \right)^2 \tag{15}$$

where $\bar{P}_\alpha$ are independently determined values of P_α, δ_α their uncertainties and where the functions F_i relate the physics parameters to the measured quantity M_i (Roman indices are used to denote measurements and Greek indices systematics). The quantities σ_{Mi} are the error on the values M_i coming from purely internal sources (i.e. statistical and systematic errors not related to the parameters P).

Normalising the parameters in the following way:

$$Y_\alpha = \frac{P_\alpha - \bar{P}_\alpha}{\delta_\alpha} \tag{16}$$

and linearising the functions F_i leads to:

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{X_i + \sum_{\alpha=1}^{m} \Delta_{\alpha i} Y_\alpha - X}{\sigma_i} \right)^2 + \sum_{\alpha=1}^{m} Y_\alpha^2 \tag{17}$$

where now the X_i are the measurements of X, σ_i the uncorrelated systematic and statistical errors on X_i, and $\Delta_{\alpha i}$ are the correlated systematics from parameter P_α and measurement i, determined as the change in the measured quantity when the parameter value is changed from $\bar{P}_\alpha$ to $\bar{P}_\alpha + \delta_\alpha$. All one needs are the values of X_i, σ_i, and $\Delta_{\alpha i}$.

The χ^2 problem is linear and can be solved using matrix inversion. Defining $Y_0 \equiv X$, the result is:

$$Y_\alpha = \sum_{\beta=0}^{m} (A^{-1})_{\alpha\beta} B_\beta \qquad \text{for } \alpha = 0, 1, \ldots, m \tag{18}$$

where:

$$B_\beta = \begin{cases} \sum_{k=1}^{n} X_k / \sigma_k^2 & \text{if } \beta = 0 \\ \sum_{k=1}^{n} -X_k \Delta_{\beta k} / \sigma_k^2 & \text{if } \beta \neq 0 \end{cases} \tag{19}$$

$$A_{\alpha\beta} = A_{\beta\alpha} = \begin{cases} \sum_{k=1}^{n} 1/\sigma_k^2 & \text{if } \alpha = \beta = 0 \\ \sum_{k=1}^{n} -\Delta_{\alpha k}/\sigma_k^2 & \text{if } \alpha = 0 \text{ and } \beta \neq 0 \\ \sum_{k=1}^{n} -\Delta_{\beta k}/\sigma_k^2 & \text{if } \alpha \neq 0 \text{ and } \beta = 0 \\ \delta_{\alpha\beta} + \sum_{k=1}^{n} \Delta_{\alpha k}\Delta_{\beta k}/\sigma_k^2 & \text{if } \alpha \neq 0 \text{ and } \beta \neq 0 \end{cases} \tag{20}$$

$\delta_{\alpha\beta}$ being the Kronecker symbol.

The method has to be used with care in the case that the dependencies on the parameters P are large, i.e. when the correlated uncertainties are large compared to the uncorrelated ones. For example, if we apply this method to average the following two measurements:

$$1.2 \pm 0.1(stat) \pm 0.2(syst) \quad (21)$$

$$0.8 \pm 0.1(stat) \pm 0.5(syst) \quad (22)$$

and assume that the systematic uncertainties are fully correlated, one gets 1.38 ± 0.17.

6.5 CHI2_SYM and CHI2_SYM_MIN: a χ^2 method to combine systematically correlated measurements with symmetric uncertainties

This algorithm is an extension of the CHI2_SYM_INDEP algorithm where possible statistical correlations between the various measurements are taken into account.

Generalising Eq. (17), the χ^2 to be minimised has now the following expression:

$$\chi^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\left[X_i + \sum_{\alpha=1}^{m}\Delta_{\alpha i}Y_\alpha - X\right](\mathbf{M}^{-1})_{ij}\left[X_j + \sum_{\beta=1}^{m}\Delta_{\beta j}Y_\beta - X\right]\right) + \sum_{\gamma=1}^{m} Y_\gamma^2 \quad (23)$$

where the X_i are the n measurements, $\mathbf{M}$ their error matrix, and $\Delta_{\alpha i}$ their changes when the correlated systematic parameters are moved from their central values (Roman indices are used to denote measurements and Greek indices systematics). The error matrix $\mathbf{M}$ includes statistical uncertainties and uncorrelated systematic uncertainties and its elements are equal to:

$$M_{ij} = \rho_{ij}\,\sigma_i^{\mathbf{stat}}\sigma_j^{\mathbf{stat}} + \delta_{ij}\,\sigma_i^{\mathbf{syst}}\sigma_j^{\mathbf{syst}} \quad (24)$$

where ρ_{ij} is the statistical correlation coefficient between measurements i and j, $\sigma_i^{\mathbf{stat}}$ ($\sigma_i^{\mathbf{syst}}$) is the statistical uncertainty (uncorrelated systematic uncertainty) on measurement i, and δ_{ij} is the Kronecker symbol.

Again, the problem is to minimise the χ^2 with respect to X and the m parameters Y_α. The problem can be handled easily if a formalism with vectors (indicated by an arrow "$\rightarrow$" above symbols) and matrices (indicated by bold type symbols) is introduced. Let $\ell = m + 1$ and:

$$\vec{X} = \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix} = n \times 1 \text{ vector of measurements} \quad (25)$$

$$\mathbf{\Delta} = \begin{pmatrix} \Delta_{11} & \Delta_{12} & \cdots & \Delta_{1n} \\ \Delta_{21} & \Delta_{22} & \cdots & \Delta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{m1} & \Delta_{m2} & \cdots & \Delta_{mn} \\ -1 & -1 & \cdots & -1 \end{pmatrix} = \ell \times n \text{ matrix of changes} \quad (26)$$

$$\vec{V} = \begin{pmatrix} Y_1 \\ \vdots \\ Y_m \\ X \end{pmatrix} = \ell \times 1 \text{ vector of unknowns} \tag{27}$$

With these notations, and since $\Delta_{\ell i} = -1$, the χ^2 can be re-written as:

$$\chi^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\left[X_i + \sum_{\alpha=1}^{\ell}\Delta_{\alpha i}Y_\alpha\right](\mathbf{M}^{-1})_{ij}\left[X_j + \sum_{\beta=1}^{\ell}\Delta_{\beta j}Y_\beta\right]\right) + \sum_{\gamma=1}^{m}Y_\gamma^2 \tag{28}$$

$$= \left(\vec{X} + \mathbf{\Delta}^T\vec{V}\right)^T \mathbf{M}^{-1}\left(\vec{X} + \mathbf{\Delta}^T\vec{V}\right) + \vec{V}^T\vec{V} - V_\ell^2 \tag{29}$$

where the notation "^{T}" is used to indicate transposition. When the χ^2 is minimum, one has:

$$\frac{\partial\chi^2}{\partial\vec{V}} = 0 \iff \mathbf{\Delta M}^{-1}\left(\vec{X} + \mathbf{\Delta}^T\vec{V}\right) + \mathbf{Q}\vec{V} = 0 \tag{30}$$

where $\mathbf{Q}$ is a diagonal $\ell \times \ell$ matrix defined by:

$$Q_{\alpha\beta} = \delta_{\alpha\beta}(1 - \delta_{\alpha\ell}) \tag{31}$$

The solution of Eq. (30) is:

$$\vec{V} = -\mathbf{S\Delta M}^{-1}\vec{X} \tag{32}$$

where $\mathbf{S}$ is the $\ell \times \ell$ error matrix on $\vec{V}$ obtained from the second derivatives of the χ^2:

$$\mathbf{S} = \left(\frac{1}{2}\frac{\partial^2\chi^2}{\partial\vec{V}^2}\right)^{-1} = \left(\mathbf{\Delta M}^{-1}\mathbf{\Delta}^T + \mathbf{Q}\right)^{-1} \tag{33}$$

The combined measurement X and its total uncertainty $\sigma(X)$ are therefore given by:

$$X = V_\ell \tag{34}$$

$$\sigma(X) = \sqrt{S_{\ell\ell}} \tag{35}$$

The `CHI2_SYM` routine supersedes the `CHI2_SYM_INDEP` routine, because it is more general. It gives exactly the same results as `CHI2_SYM_INDEP` in case of independent measurements or when the statistical correlations are ignored because the `STAT_CORR` option was turned off by a `SWITCH` logical line in the input file.

The `CHI2_SYM_MIN` routine performs the same job as the `CHI2_SYM` routine, except that it uses MINUIT for the χ^2 minimisation instead of solving the problem directly using matrix inversion. It also uses the values of the parameter `MINUIT_PRTLEV` (if available) to define the MINUIT printout level.

6.6 CHI2_ASYM_MIN: Combining systematically correlated results with asymmetric errors

The combination method CHI2_SYM_MIN is able only to combine results which have symmetric errors, and to this end the experimental results which are used are first symmetrized. A fully correct handling of asymmetric errors requires a model for their source, which for mixing results (unlike combinations of lifetime results where there is a simple physical explanation for the asymmetries) is not understood. In the method presented here no attempt is made to model the physics behind the asymmetry, instead a simple linear function of the relevant parameter and the given positive and negative errors is used. This prescription is similar to that used by the particle data group when they average results which have asymmetric errors. It should be noted that this is not strictly speaking correct either, but should provide results closer to the true value than those determined from symmetrization of the errors.

In the symmetric methods the errors on the parameters are taken as constant. In this procedures the error used for MINUIT is taken as a function of the original parameter value, the fitted value of that parameter and the positive and negative errors.

$$\sigma = \begin{cases} \sigma^+ & \text{if } x > x_0 + \sigma^+ \\ \sigma^- & \text{if } x < x_0 - \sigma^- \\ \sigma^{\mathrm{comb}} & = \sigma^- + (x - x_0 + \sigma_n)\frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \quad \text{otherwise} \end{cases}$$

where $\sigma^\pm$ are the positive and negative errors on the parameter with experimental value x_0 and x is the value chosen for it by MINUIT's variation procedure. This returns positive and negative errors for values of x far from x_0, and a linear combination of the σ^+ and σ^- for intermediate values.

The method is otherwise identical to that used for the routine `CHI2_SYM_MIN`.

By default the routine performs the combination using the asymmetric errors on the input parameters, but the final answer has symmetric errors. If the `MINOS` card is also included, the output errors are asymmetric as calculated with routine `MINOS`.

One more step remains to be implemented. The adjustment of parameters to new ones, is done in a symmetric fashion. In principle this should also be done asymmetrically, and the calculation of the error should take the adjustment into account.

6.7 CHI2_SYM_CIRC: a χ^2 method to combine systematically correlated Δm_{d} measurements

This algorithm is an extension of the CHI2_SYM_MIN algorithm where the problem due to the "$\chi_d \to$ b-meson fractions $\to \Delta m_{\mathrm{d}}$ circularity" is solved according to a method originally suggested by Hans-Günther Moser and presented at the LEP B oscillation working group meeting on December 3, 1996. This problem comes from the fact the b-meson fractions, namely $f_{\mathrm{B_s}}$ (fraction of $\mathrm{B_s}$) and $f_{\mathrm{B_d}} = f_{\mathrm{B_u}}$ (fraction of $\mathrm{B_d}$ or $\mathrm{B_u}$), can be determined using integrated mixing measurements, χ_s, χ_d and $\bar{\chi} = f_{\mathrm{B_s}}\chi_s + f_{\mathrm{B_d}}\chi_d$, as well as an estimate of the b-baryon fraction, $f_{\Lambda_{\mathrm{b}}}$. If such determinations are used in an analysis which measures Δm_{d}, the result of the measurement (once combined with the $\mathrm{B_d}$ lifetime) might not yield a value of χ_d which is consistent with the input of the b-meson fraction calculation.

The b-meson fractions and χ_d are therefore special parameters that need to be treated in a different way. Without loss of generality, let us assume that all such special parameters used in the various analyses correspond to the first λ parameters P_α. For example, if only

$f_{\mathrm{B_s}}$ is considered as a systematic parameter in the various analyses, one could have:

$$P_1 = f_{\mathrm{B_s}} \quad , \quad \lambda = 1 \tag{36}$$

or, if both $f_{\mathrm{B_s}}$ and $f_{\mathrm{B_d}} = f_{\mathrm{B_u}}$ are considered:

$$P_1 = f_{\mathrm{B_s}}, P_2 = f_{\mathrm{B_d}} = f_{\mathrm{B_u}} \quad , \quad \lambda = 2 \tag{37}$$

Another case is when the b-meson fractions are expressed in terms of χ_s, χ_d and $\bar{\chi} = f_{\mathrm{B_s}}\chi_s + f_{\mathrm{B_d}}\chi_d$ inside the analyses, in such a way that $f_{\mathrm{B_s}}$ and $f_{\mathrm{B_d}} = f_{\mathrm{B_u}}$ are not external parameters anymore; in that case χ_d is normally used as an external systematic parameter (i.e. there is a systematic uncertainty associated to χ_d) and one can assume:

$$P_1 = \chi_d \quad , \quad \lambda = 1 \tag{38}$$

The calculation of χ_d and the b-meson fraction is performed according to the following functions f_χ, f_s and f_d

$$\begin{aligned}
\chi_d &= f_\chi(P_{\lambda+1}, P_{\lambda+2}, \ldots, P_m, P_\ell, P_{\ell+1}, P_{\ell+2}, \ldots, P_p) && (39)\\
f_{\mathrm{B_s}} &= f_s(P_{\lambda+1}, P_{\lambda+2}, \ldots, P_m, P_\ell, P_{\ell+1}, P_{\ell+2}, \ldots, P_p) && (40)\\
f_{\mathrm{B_d}} = f_{\mathrm{B_u}} &= f_d(P_{\lambda+1}, P_{\lambda+2}, \ldots, P_m, P_\ell, P_{\ell+1}, P_{\ell+2}, \ldots, P_p) && (41)
\end{aligned}$$

which depend on the $(m-\lambda)$ remaining parameters used in the systematic studies, the combined measurement $\Delta m_{\mathrm{d}} = X \equiv P_\ell$, and $p-\ell$ additional quantities P_α ($\alpha = \ell+1, \ldots, p$) which were not used in the systematic studies but are needed for the calculation. The arguments of the functions f_x ($x = \chi, s, d$) must obviously include Δm_{d}, $\tau_{\mathrm{B_d}}$, $\bar{\chi}$, and $f_{\Lambda_{\mathrm{b}}}$; these last three parameters in this list may or may not have been used as systematic parameters in the individual analyses to combine, i.e. they may correspond to an index α which is either above or below ℓ (although in practice $\tau_{\mathrm{B_d}}$ is likely to have an index below ℓ and $\bar{\chi}$ an index above ℓ). Arguments of the functions f_x may also include other quantities like a measurement of χ_d at the $\Upsilon(4\mathrm{S})$ and an estimate of $f_{\mathrm{B_s}}$ from branching ratio measurements.

Having redefined the normalised parameters as

$$\begin{aligned}
Y_\alpha &= \frac{f_x(P_{\lambda+1}, \ldots, P_p) - \bar{P}_\alpha}{\delta_\alpha} && \text{for } \alpha = 1, \ldots, \lambda && (42)\\
Y_\alpha &= \frac{P_\alpha - \bar{P}_\alpha}{\delta_\alpha} && \text{for } \alpha = \lambda+1, \ldots, p && (43)
\end{aligned}$$

where f_x stands for f_χ, f_s or f_d as appropriate, and where

$$\bar{P}_\ell = 0 \qquad \text{and} \qquad \delta_\ell = 1 \tag{44}$$

the expression of the new χ^2 to minimise is the following:

$$\chi^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\left[X_i + \sum_{\alpha=1}^{\ell}\Delta_{\alpha i}Y_\alpha\right](\mathbf{M}^{-1})_{ij}\left[X_j + \sum_{\beta=1}^{\ell}\Delta_{\beta j}Y_\beta\right]\right) + \sum_{\gamma=\lambda+1}^{m} Y_\gamma^2 + \sum_{\gamma=\ell+1}^{p} Y_\gamma^2 \tag{45}$$

which is similar to expression (28), except for the following differences:

- the b-meson fractions $f_{\mathrm{B_s}}$ and $f_{\mathrm{B_d}} = f_{\mathrm{B_u}}$ have been replaced by functions f_s and f_d of other parameters, i.e. the quantities $Y_1, \ldots, Y_\lambda$ have been redefined;
- the constraints on the b-meson fractions, i.e. the terms $Y_1^2, \ldots, Y_\lambda^2$, have been removed;
- new constraints on the additional quantities needed to evaluate the functions f_s and f_d, i.e. the terms $Y_{\ell+1}^2, \ldots, Y_p^2$, have been added.

The present implementation of this algorithm in routine CHI2_SYM_CIRC requires only parameters Δm_{d}, $\tau_{\mathrm{B_d}}$, $\bar{\chi}$, and $f_{\Lambda_{\mathrm{b}}}$ (specified as DMD, TAUBD, CHIBAR, and FLAMB). If parameter χ_s (specified as CHIS) is provided, its value and uncertainty are also used, otherwise a value of $\frac{1}{2}$ is assumed without uncertainty. In case no external constraints on χ_d and $f_{\mathrm{B_s}}$ are given (i.e. in case no parameter CHID_UPSILON4S or FBS_FROM_BR is given), the functions f_x have the following expressions:

$$\chi_d = f_\chi(\Delta m_{\mathrm{d}}, \tau_{\mathrm{B_d}}) = \frac{1}{2}\frac{(\Delta m_{\mathrm{d}}\,\tau_{\mathrm{B_d}})^2}{(\Delta m_{\mathrm{d}}\,\tau_{\mathrm{B_d}})^2+1} \tag{46}$$

$$f_{\mathrm{B_s}} = f_s(\Delta m_{\mathrm{d}}, \tau_{\mathrm{B_d}}, \chi_s, \bar{\chi}, f_{\Lambda_{\mathrm{b}}}) = \frac{2\bar{\chi} - (1-f_{\Lambda_{\mathrm{b}}})\,\chi_d}{2\chi_s - \chi_d} \tag{47}$$

$$f_{\mathrm{B_d}} = f_{\mathrm{B_u}} = f_d(\Delta m_{\mathrm{d}}, \tau_{\mathrm{B_d}}, \chi_s, \bar{\chi}, f_{\Lambda_{\mathrm{b}}}) = \frac{1}{2}(1 - f_{\Lambda_{\mathrm{b}}} - f_{\mathrm{B_s}}) \tag{48}$$

In case of external constraints $\chi_d^{\Upsilon(4S)}$ on χ_d and $f_{\mathrm{B_s}}^{\mathrm{BR}}$ on $f_{\mathrm{B_s}}$ (specified as CHID_UPSILON4S and FBS_FROM_BR) the functions f_x are modified in the following way:

$$\chi_d^{\Delta m} = \frac{1}{2}\frac{(\Delta m_{\mathrm{d}}\,\tau_{\mathrm{B_d}})^2}{(\Delta m_{\mathrm{d}}\,\tau_{\mathrm{B_d}})^2+1} \tag{49}$$

$$\chi_d = f_\chi(\Delta m_{\mathrm{d}}, \tau_{\mathrm{B_d}}, \chi_d^{\Upsilon(4S)}) = \mathcal{W}(\chi_d^{\Delta m}, \chi_d^{\Upsilon(4S)}) \tag{50}$$

$$f_{\mathrm{B_s}}^{\chi} = \frac{2\bar{\chi} - (1-f_{\Lambda_{\mathrm{b}}})\,\chi_d}{2\chi_s - \chi_d} \tag{51}$$

$$f_{\mathrm{B_s}} = f_s(\Delta m_{\mathrm{d}}, \tau_{\mathrm{B_d}}, \chi_d^{\Upsilon(4S)}, \chi_s, \bar{\chi}, f_{\Lambda_{\mathrm{b}}}, f_{\mathrm{B_s}}^{\mathrm{BR}}) = \mathcal{W}(f_{\mathrm{B_s}}^{\chi}, f_{\mathrm{B_s}}^{\mathrm{BR}}) \tag{52}$$

$$f_{\mathrm{B_d}} = f_{\mathrm{B_u}} = f_d(\Delta m_{\mathrm{d}}, \tau_{\mathrm{B_d}}, \chi_d^{\Upsilon(4S)}, \chi_s, \bar{\chi}, f_{\Lambda_{\mathrm{b}}}, f_{\mathrm{B_s}}^{\mathrm{BR}}) = \frac{1}{2}(1 - f_{\Lambda_{\mathrm{b}}} - f_{\mathrm{B_s}}) \tag{53}$$

where $\mathcal{W}(x, y)$ denotes the weighted average of x and y. The program can also cope with the cases where only one of the two external constraints is provided.

If a value $f_{\mathrm{B_s}}^{\mathrm{BR}}$ if given (parameter FBS_FROM_BR), it will by default be assumed uncorrelated with $f_{\Lambda_{\mathrm{b}}}$ (parameter FLAMB). No mixing information should be used to determine this fraction of $\mathrm{B_s}$ mesons; traditionally, $f_{\mathrm{B_s}}^{\mathrm{BR}}$ is determined from the measurement of the $\mathrm{B_s} \to \mathrm{D_s}\ell\nu$ branching ratio (hence the name). More recent techniques have been developed to measure the fraction of weakly decaying b-hadrons at LEP. This results in an estimate of $f_{\mathrm{B_s}}$ (independent of mixing information) which may have a non-zero correlation with $f_{\Lambda_{\mathrm{b}}}$, the fraction of b-baryons. In that case, the χ^2 of expression (45) has to be modified; the sum of the two terms $Y_{\gamma_{\mathrm{s}}}^2$ and $Y_{\gamma_\Lambda}^2$, where the indexes γ_{s} and γ_Λ correspond to the parameters $f_{\rm B_s}^{\rm BR}$ and $f_{\Lambda_{\rm b}}$, have to be replaced by

$$\frac{Y_{\gamma_{\rm s}}^2 + Y_{\gamma_\Lambda}^2 - 2\rho Y_{\gamma_{\rm s}} Y_{\gamma_\Lambda}}{1-\rho^2}\,, \tag{54}$$

where ρ is the correlation coefficient between the values of $f_{\rm B_s}^{\rm BR}$ and $f_{\Lambda_{\rm b}}$ used as constraints in the fit. This is implemented in routine `CHI2_SYM_CIRC` since version 2.51. The correlation ρ is taken from parameter `CORR_FLAMB_FBS_`; if that parameter is not defined, then $\rho = 0$ is assumed.

Routine `CHI2_SYM_CIRC` also uses the values of the parameters `MINUIT_PRTLEV` (if available) to define the MINUIT printout level. In addition to the MINUIT output, this routine prints the final values and errors for $\Delta m_{\rm d}$ and all the parameters, including χ_d and the b-meson fractions. In case the $\chi_d^{\Upsilon(4S)}$ constraint is used, an "extended average" of $\Delta m_{\rm d}$ is also computed, which includes $\chi_d^{\Upsilon(4S)}$ (after proper conversion to an oscillation frequency using $\tau_{\rm B_d}$), taking properly into account all correlations (this constraint produces indeed a small correlation between the LEP and Υ(4S) results).

6.8 CHI2_SYM_CIRC_G: a χ^2 method to combine systematically correlated $\Delta m_{\rm d}$ measurements

The algorithm here is similar to the one used for routine `CHI2_SYM_CIRC`. The difference is that the input value of `CHIBAR` is assumed to represent the quantity

$$\bar{\chi}_g = g_{\rm B_s}\chi_s + g_{\rm B_d}\chi_d \tag{55}$$

rather than

$$\bar{\chi} = f_{\rm B_s}\chi_s + f_{\rm B_d}\chi_d \tag{56}$$

where $g_{\rm B_s}$ and $g_{\rm B_d}$ are the fractions of $\rm B_s$ and $\rm B_d$ mesons among semileptonically decaying b-hadrons, $f_{\rm B_s}$ and $f_{\rm B_d}$ being, as before, the fractions among all weakly decaying b-hadrons.

Assuming that the semileptonic partial widths of the various b-hadron species are equal, one has:

$$g_\alpha = f_\alpha R_\alpha \quad , \qquad \text{with } R_\alpha = \frac{\tau_\alpha}{\langle\tau_b\rangle} \tag{57}$$

where α stands for each species $\rm B_s$, $\rm B_d$, $\rm B_u$, or $\Lambda_{\rm b}$, and where $\langle\tau_b\rangle$ is the average lifetime of b-hadrons produced in Z decays. Note that one assumes $\sum_\alpha f_\alpha = \sum_\alpha g_\alpha = 1$ and $f_{\rm B_d} = f_{\rm B_u}$, implying $g_{\rm B_d} \neq g_{\rm B_u}$ if $\tau_{\rm B_d} \neq \tau_{\rm B_u}$.

In this algorithm, the function f_s (and therefore also f_d) depends on $\bar{\chi}_g$ instead of $\bar{\chi}$, and on three additional parameters, $r = R_{\rm B_u}/R_{\rm B_d}$, $R_{\rm B_s}$, and $R_{\Lambda_{\rm b}}$. These parameters can be specified as `TAUBU/TAUBD`, `TAUBS/TAUB`, and `TAULAMB/TAUB` respectively; if they are not specified, a value of 1 without uncertainty is assumed for each of them. The functions f_x have the following expressions (in the case that external constraints $\chi_d^{\Upsilon(4S)}$ on χ_d and $f_{\rm B_s}^{\rm BR}$ on $f_{\rm B_s}$ are specified as `CHID_UPSILON4S` and `FBS_FROM_BR`):

$$\chi_d = f_\chi(\Delta m_{\rm d}, \tau_{\rm B_d}, \chi_d^{\Upsilon(4S)}) \;\; = \;\; \mathcal{W}(\chi_d^{\Delta m}, \chi_d^{\Upsilon(4S)}) \tag{58}$$

$$
\begin{aligned}
f_{\mathrm{B_s}} = f_s(\Delta m_{\mathrm{d}}, \tau_{\mathrm{B_d}}, \chi_d^{\Upsilon(4S)}, \chi_s, \bar{\chi}_g, r, R_{\mathrm{B_s}}, R_{\Lambda_{\mathrm{b}}}, f_{\Lambda_{\mathrm{b}}}, f_{\mathrm{B_s}}^{\mathrm{BR}}) &= \mathcal{W}(f_{\mathrm{B_s}}^{\chi}, f_{\mathrm{B_s}}^{\mathrm{BR}}) \quad (59)\\
f_{\mathrm{B_d}} = f_{\mathrm{B_u}} = f_d(\Delta m_{\mathrm{d}}, \tau_{\mathrm{B_d}}, \chi_d^{\Upsilon(4S)}, \chi_s, \bar{\chi}_g, r, R_{\mathrm{B_s}}, R_{\Lambda_{\mathrm{b}}}, f_{\Lambda_{\mathrm{b}}}, f_{\mathrm{B_s}}^{\mathrm{BR}}) &= \frac{1}{2}(1 - f_{\Lambda_{\mathrm{b}}} - f_{\mathrm{B_s}}) \quad (60)
\end{aligned}
$$

where

$$
\begin{aligned}
\chi_d^{\Delta m} &= \frac{1}{2}\frac{(\Delta m_{\mathrm{d}}\,\tau_{\mathrm{B_d}})^2}{(\Delta m_{\mathrm{d}}\,\tau_{\mathrm{B_d}})^2 + 1} \quad (61)\\
f_{\mathrm{B_s}}^{\chi} &= \frac{1}{R_{\mathrm{B_s}}}\frac{(1+r)\bar{\chi}_g - (1 - f_{\Lambda_{\mathrm{b}}} R_{\Lambda_{\mathrm{b}}})\,\chi_d}{(1+r)\chi_s - \chi_d} \quad (62)\\
& \quad (63)
\end{aligned}
$$

Like routine CHI2_SYM_CIRC, routine CHI2_SYM_CIRC_G can also cope with the cases where one or both external constraints are missing, can also deal with a correlation between the fractions $f_{\mathrm{B_s}}^{\mathrm{BR}}$ and $f_{\Lambda_{\mathrm{b}}}$ if parameter CORR_FLAMB_FBS_ is provided, and is also sensitive to parameter MINUIT_PRTLEV if provided.

6.9 COMBY_LIFETIME: a method to combine lifetime measurements

This algorithm [3] is the same as the one used in the COMBY program used by the LEP b-lifetime working group to combine lifetime measurements.

The implementation in routine COMBY_LIFETIME has been provided by Hans-Günther Moser. It can handle asymmetric statistical uncertainties, and asymmetric uncorrelated uncertainties not associated to any parameter (it uses the symmetrized uncertainties for the correlated systematics). However, it can only handle measurements that are statistically independent.

6.10 DUMP_MASTER_INC: a routine to print the input to a combination routine

A special routine called DUMP_MASTER_INC is provided to dump the content of the common block described in file master.inc. This "combination" routine does not perform any combination (and will therefore always "fail" to combine the results). Such a routine is useful for debugging purposes.

6.11 NEW_COMBINATION: a template for a new combination routine

A dummy routine called NEW_COMBINATION (see Figure 6) is also linked with COMBOS. This allows any member of the LEP B oscillation working group to program her/his favourite combination algorithm, try it out and compare the results with other combination routines available in COMBOS.

7 Termination routines

7.1 FINAL_FRACTIONS: calculation of the final b-hadron fractions

This routine can be called after $\Delta m_{\rm d}$ measurements have been combined to compute the fraction of B_s using mixing information only, the final b-hadron fractions (using all information), and the $\Delta m_{\rm d}$ average including the $\Upsilon(4S)$ information. The full error calculation (including correlation coefficients) is performed, assuming only symmetrized uncertainties. The computation can take into account the correlation between the B_s and b-baryon fractions determined without mixing information (parameter `CORR_FLAMB_FBS_`). It is based on the formulae given in Section 6.8 describing combination routine `CHI2_SYM_CIRC_G`.

7.2 SET_DMS_LIMIT and KUMAC_DMS_LIMIT: routines to determine excluded values of $\Delta m_{\rm s}$, a lower limit on $\Delta m_{\rm s}$ and the $\Delta m_{\rm s}$ sensitivity

These routines should only be called in case $\Delta m_{\rm s}$ amplitude measurements have been combined at various values of $\Delta m_{\rm s}$ (`DMS` being the step variable). They determine the $\Delta m_{\rm s}$ ranges where the combined amplitude is excluding $A = 1$ or $A = 0$ (A being the $\Delta m_{\rm s}$ oscillation amplitude). They also determine the "lower limit" on $\Delta m_{\rm s}$, defined as the smallest value of $\Delta m_{\rm s}$ such that $A = 1$ is excluded at any lower $\Delta m_{\rm s}$ value. The "sensitivity" (defined in the same way as the lower limit, except that the combined amplitude are set to 0 at each value of $\Delta m_{\rm s}$) is also determined.

If the parameter `CONFIDENCE` is given a value on a `PARAMETER` logical line, then the routines will use the specified value as the (1-sided) confidence level at which values of $\Delta m_{\rm s}$ are excluded. If no confidence level is given, it defaults to 0.95 (i.e. exclusion will be done at 95% CL).

These two routines are identical except for the printout that they produce: `SET_DMS_LIMIT` produces a user friendly (i.e. easy to read) output, whereas `KUMAC_DMS_LIMIT` produces an output which can be executed inside PAW (KUMAC file). A PAW macro which can be used to execute the output of the `KUMAC_DMS_LIMIT` routine is provided in file `plot_amplitude_vs_dms.kumac`.

7.3 AVG_AMPLITUDE: a routine to determine the weighted average the combined amplitudes over the $\Delta m_{\rm s}$ range under study

This routine should only be called in case $\Delta m_{\rm s}$ amplitude measurements have been combined at various values of $\Delta m_{\rm s}$ (`DMS` being the step variable). It computes the weighted average of the combined amplitudes over the whole $\Delta m_{\rm s}$ range under study. This calculation takes into account the statistical (and systematic) correlations between the combined amplitudes at different values of $\Delta m_{\rm s}$.

WARNING: This routine is still under development and might not be correct. It will hopefully be finalised for the next version of COMBOS.

7.4 NEW_TERMINATION: a template for a new termination routine

A dummy routine called `NEW_TERMINATION` (see Fig. 7) is also linked with COMBOS. This allows any member of the LEP B oscillation working group to program her/his own termination routine.

References

[1] See for example L. Lyons et al., Nucl. Instr. and Methods **A270** (1988) 110 and references therein.

[2] D. Brown, ALEPH internal note, ALEPH 94–144, PHYSIC 94–125, September 1994

[3] D. Forty, CERN–PPE/94–114, September 1994