A NEW APPROACH TO A GLOBAL FIT OF THE CKM MATRIX

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We report on a new approach to a global CKM matrix analysis taking into account most recent experimental and theoretical results, in particular, the preliminary sin2β measurements from the B factories. The statistical framework developed advocates formal frequentist statistics. We emphasize the distinction of a model testing and a model dependent, metrological phase in which the various parameters of the theory are determined. Graphical results for confidence levels are drawn in various one and two-dimensional parameter spaces. Numerical results are obtained for all relevant parameters of the theory and predictions for branching ratios of rare K and B meson decays are given.

1 Introduction

The Standard Model (SM) accounts for CP violation by a single phase in the quark mixing CKM matrix. With three generations, the matrix is parameterized by four parameters[a] not predicted by the theory, but which can be constrained by means of a global analysis using sensitive observables like $|V_{us}|$, $|V_{ud}|$, $|V_{cb}|$, $|V_{ub}|$, ϵ_K, $\Delta m_{d|s}$, $\sin 2\beta$, etc. Presentely, the analysis suffers from large theoretical uncertainties, due to the present inability to precisely compute dynamics involving strong interaction quantities.

Analyzing data in a well-defined framework ceases to be a straightforward task when one moves away from Gaussian statistics. The frequentist [b] approach *R*fit LALreport, advocated in this letter, treats systematic theoretical uncertainties without *a priori* knowledge except for the definition of ranges [?,4].

The first goal addressed by a global CKM fit is *Metrology*, that is to find allowed ranges for CKM matrix elements and related quantities, assuming the SM to be correct. Furthermore, one intends to *probe the validity of the SM*, that is to quantify the agreement between the CM and the experimental information. Finally, within an extended theoretical framweork, *e.g.* , Supersymetry, one may search for specific signals of New Physics, by determining the additional theoretical parameters [?].

[a] Unitarity and arbitrary phases reduce the number of independent parameters of the matrix from 18 to 4, including one CP violating phase.
[b] *R*fit is part of a the package **CkmFitter** [2], which provides various statistical approaches such as the 95% CL Scan [?,4] method and a Bayesian [12] approach. Please contact the authors to receive the source code and the program driving data cards of **CkmFitter**.

2 The CKM Matrix

The unitary CKM matrix,[6,7] of three generations, $V_{\rm CKM}$ is introduced in the charged current part of the SM Lagrangian after diagonalization of the mass part. A deviation from unitarity, $V_{\rm CKM}V_{\rm CKM}^\dagger \neq {\rm Id}$ would indicate new generations or couplings (and not a failure of unitarity).

An exact parameterization of the CKM matrix citeCheu (*i.e.* , it strictly satisfies the unitarity relation) uses using the three Euler angles $\theta_{i,j}$ $(i<j=1,3)$ and one phase δ. A useful approximation, proposed by Wolfenstein [9], follows from the observation that the elements of $V_{\rm CKM}$ exhibit a hierarchy in terms of the parameter $\lambda = |V_{us}| \sim 0.22$. The remaining parameters are denoted A, ρ and η, the latter implying CP violation if its value is non-zero.

CP violation can be expressed in a phase-convention independent way using the Jarlskog parameter J given by [10]

$$\mathrm{Im}\left[V_{ij}V_{kl}V_{il}^*V_{kj}^*\right] = J \sum_{m,n=1}^{3} \epsilon_{ikm}\epsilon_{jln} \ , \qquad (1)$$

where ϵ_{ikm} is the total antisymmetric tensor. In the Wolfenstein approximation, J reads:

$$J = A^2\lambda^6\eta \ \sim 10^{-5} \ . \qquad (2)$$

The rescaled unitarity relation (unitarity triangle) between the first and third column,

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0 \ . \qquad (3)$$

can be elegantly displayed in the $(\bar\rho,\bar\eta)$ plane[c].

[c] The replacements $\rho \to \bar\rho = \rho(1-\lambda^2/2)$ and $\eta \to \bar\eta = \eta(1-$

3 Statistical treatment

The interpretation of constraints on CKM matrix elements requires a robust statistical framework which protects against misleading conclusions. Whereas the Bayesian approache[11] assigns *a priori* a probability density function (pdf) to systematic theoretical errors, the frequentist technique *R*fit treats them as "allowed intervals".

The *R*fit analysis proceeds via a χ^2 minimization. The χ^2 receives contributions from the experimental part, which measures the agreement between the experimental measurements and their predictions within the SM, and is given by a Gaussian term:

$$\chi^2_{\rm exp} = \left(\frac{x_{\rm exp} - x_{\rm theo}}{\sigma_{\rm exp}}\right)^2, \tag{4}$$

and the theoretical part expressing our knowledge of the theoretical parameters entering the analysis given by:

$$\begin{aligned} \chi^2_{\rm theo} &= 0 \quad \text{inside the allowed range,} \\ \chi^2_{\rm theo} &= \infty \quad \text{outside the allowed range.} \end{aligned} \tag{5}$$

In practice, a discrete scan is performed in the parameter space, *e.g.* $(\bar\rho, \bar\eta)$, and for each point, the χ^2 is minimized with respect to the set of parameters $\mu = (\lambda, A, m_t, \kappa)$, where κ are parameters with systematic theoretical uncertainties (*e.g.* , $|V_{ub}|/|V_{cb}|_{\rm syst}$, B_K, $f_{B_d}\sqrt{B_d}$, ξ, η_{cc}, m_t, m_c ...), varying freely within their allowed ranges.

In the metrological phase of the *R*fit analysis, one calculates the offset-corrected CL (with respect to the global χ^2 minimum $\chi^2(\mu_{min,global})$) for each point of the N-dimensional parameter space:

$$CL = {\rm Prob}(\chi^2(\mu_{min}) - \chi^2(\mu_{min,global}), N). \tag{6}$$

Since we fit for the best set of theoretical parameters in each point, the CLs thus obtained are interpreted as upper bounds of all possible CLs (this is implicit in the following when invoking the term CL). In particular, CLs should not be interpreted as relative probabilities, *i.e.* inferring equal probabilities from equal shades.

In the SM-probing phase, one interpretes the value of $\chi^2(\mu_{min,global})$ by means of a Monte Carlo simulation: one generates the distribution $\mathcal{F}(\chi^2)$ of $\chi^2(\mu_{\rm min,global})$ by fluctuating the likelihood according to the statistical errors, having set the most likely values of the experimental and theoretical likelihoods to the one obtained with the set μ_{min} of parameters.

$\lambda^2/2$) improve the accuracy of the triangle apex in the Wolfenstein approximation.

The upper bound of the confidence level (CL) for the SM to be valid is then computed by:

$$\mathcal{P}({\rm SM}) \leq \int_{\chi^2 > \chi^2(\mu_{\rm min,\,global})} \mathcal{F}(\chi^2) {\rm d}\chi^2. \tag{7}$$

4 Fit Ingredients

Table 1: Input observables and parameters for the global CKM fit. Quantities that vary freely in the fit are marked as "Free"with a '*'.

Parameter	Value ± Error(s)	Free
$\|V_{ud}\|$	0.97394 ± 0.00089	-
$\|V_{us}\|$	0.2200 ± 0.0025	-
$\|V_{ub}\|$	$(3.48 \pm 0.23 \pm 0.55) \times 10^{-3}$	-
$\|V_{cd}\|$	0.224 ± 0.014	-
$\|V_{cs}\|$	0.969 ± 0.058	-
$\|V_{cb}\|$	$(40.75 \pm 0.40 \pm 2.0) \times 10^{-3}$	-
$\|\epsilon_K\|$	$(2.271 \pm 0.017) \times 10^{-3}$	-
Δm_d	(0.487 ± 0.014) ps^{-1}	-
$\sin 2\beta_{\rm WA}$	0.48 ± 0.16	-
m_c	$(1.3 \pm 0.1_{syst})$ GeV	*
$m_t(\overline{\rm MS})$	(166.0 ± 5.0) GeV	*
m_K	(493.677 ± 0.016) MeV	-
Δm_K	$(3.4885 \pm 0.0008) \times 10^{-15}$ GeV	-
m_{B_d}	(5.2794 ± 0.0005) GeV	-
m_{B_s}	(5.3696 ± 0.0024) GeV	-
m_W	(80.419 ± 0.056) GeV	-
G_F	$(1.16639 \pm 0.00001) \times 10^{-5}$ GeV^{-2}	-
f_K	(159.8 ± 1.5) MeV	-
B_K	$0.87 \pm 0.06 \pm 0.13$	*
η_{cc}	$1.38 \pm 0.53_{syst}$	*
η_{ct}	$0.47 \pm 0.04_{syst}$	-
η_{tt}	$0.574 \pm 0.004_{syst}$	-
$\eta_B(\overline{\rm MS})$	$0.55 \pm 0.01_{syst}$	*
$f_{B_d}\sqrt{B_d}$	$(230 \pm 28 \pm 28)$ MeV	*
ξ	$1.16 \pm 0.03 \pm 0.05$	*

Table 1 gives the values and errors of the measurements and parameters entering the analysis [1]. If not otherwise specified, the first error given is the statistical and accountable systematic error, and the second stands for systematic theoretical uncertainties, the latter being interpreted as an "allowed range" in the fit.

Some of the fit ingredients have been pre-combined, such as the inclusive and exclusive measurements of $|V_{ub}|$ and $|V_{cb}|$ and the $\sin 2\beta$ measurements. The information from $B^0_s - \overline{B^0}_s$ mixing is incorporated in such a way that the information on the sign of $(1 - A)$ (where $A(\Delta m_s)$ is the oscillation amplitude equal to 1 for the true value of Δm_s) is taken into account, since $A > 1$ is at least a hint for oscillation and should lead to a higher CL than $A < 1$. The χ^2 contribution for Δm_s is obtained from [1]:

$$\chi^2_{\Delta m_{B_s}} = \frac{\mathrm{Erfc}^{-1}}{2}\left(\frac{1-A}{\sqrt{2}\sigma_A}\right) \tag{8}$$

5 Metrology

Figure 1 shows the CLs computed as in Eq.(6) in the $(\bar{\rho}, \bar{\eta})$ plane with and without $\sin 2\beta_{WA}$ included in the fit. The 5% CL (upper bound) contours of the constraining measurements are drawn, as well as the 32% and 5% CL contours for $\sin 2\beta_{WA}$. The shaded areas of the global fit output indicate the regions of $\geq 90\%$, $\geq 32\%$ and $\geq 5\%$ CLs. The numerical results for the CKM parameters, as well as branching ratios of some rare K and B meson decays and theoretical quantities are given in Table 2. They are obtained in the same manner as the CLs in the $(\bar{\rho}, \bar{\eta})$ plane, by deriving one dimensional CLs quoting the upper and lower parameter value when CL reaches 5%.

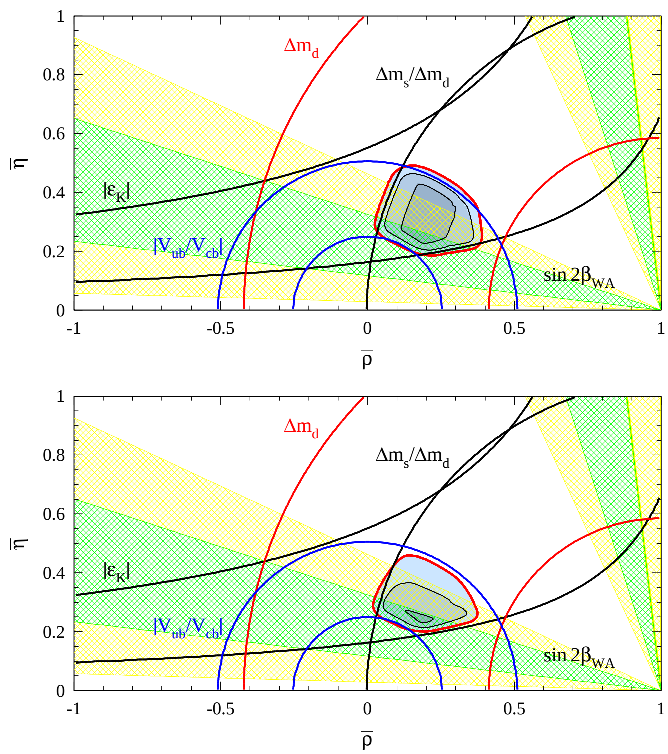

Figure 1: Confidence Levels in the $\rho - \eta$ plane with (upper plot) and without (lower plot) $\sin 2\beta$ included in the fit. The 5% (upper bounds) contours are drawn for the different constraining measurements, as well as the the 32% and 5% CL contours of $\sin 2\beta$. The shaded area of the fit output indicates regions of $\geq 90\%$, $\geq 32\%$ and $\geq 5\%$ CLs.

Figure 2 shows the CLs of the input measurements of $\sin 2\beta$ from Babar and Belle, as well as the output constraints from *R*fit , and a Bayesian approach [12] (using mostly flat pdf's for the systematic theoretical uncertainty). This shows that, depending on the statitstic method used, significiantly different results can be obtained with similar inputs. For the time being, the conclusion wether or not the SM is valid does not depend on the choice of the statistical approach. However, once discrepencies occur, this picture could be altered leading to fondamentally different conclusions.

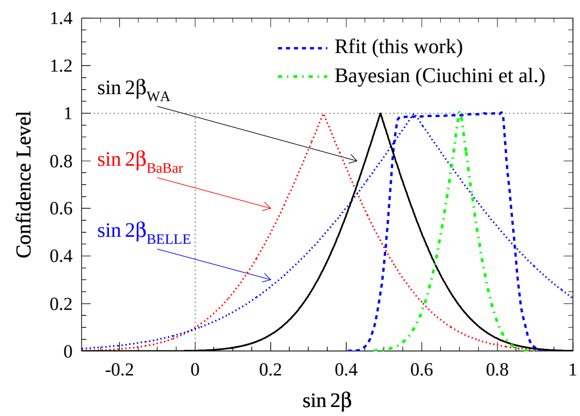

Figure 2: Confidence Levels in the $\sin 2\beta$ plane for the input measurements *sin2beta* from Babar, Belle, and the output constraints by the *R*fit method, and a Bayesian method.

6 Probing the SM

The χ^2 distribution and the associated CL obtained from the MC simulation are shown in Fig.3, with and without the world average value of $\sin 2\beta$ included in the fit. The present validity of the SM (with $\sin 2\beta$ included in the fit) is $\leq 85\%$.

7 Conclusion

A new method to a global fit of the unitary CKM matrix is presented. It is denoted *R*fit and is entirely based on frequentist statistics. The CKM anaysis is subdivided into three phases: a metrological phase where parameteres CLs are calculated; a probing phase for the Standard Model; a probing phase for New Physics [?]. The SM testing phase yields to the upper CL for the SM to be valid of 85%, including the current world average of preliminary sin2β measurements.

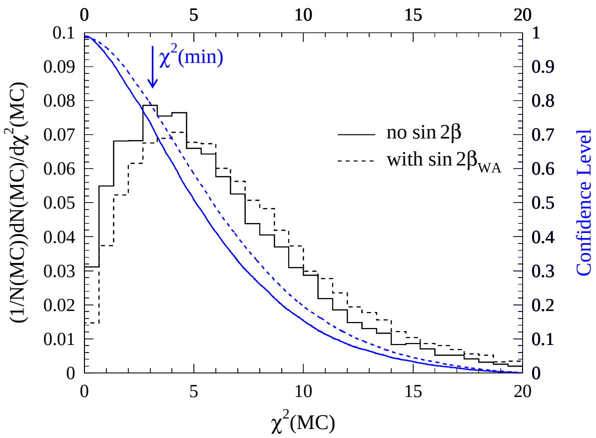

Figure 3: χ^2 distribution and its associated CLs obtained by Monte Carlo (see text). The arrow shows the present value of $\chi^2(\mu_{min})$ when $\sin 2\beta$ is not included in the fit. When $\sin 2\beta$ is included in the fit, it leads to an upper value of the SM CL of 85%.

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Table 2: Fit results for the unitary CKM parameters, the CKM matrix elements, branching ratios of some rare K and B meson decays and theoretical quantities. Ranges are given for the quantities that are significantly limited by systematic theoretical errors.

Parameter	$\geq 5\%$ CL	half width
λ	0.2220 ± 0.0041	
A	0.756 - 0.904	0.074
$\bar{\rho}$	0.04 - 0.38	0.17
$\bar{\eta}$	0.20 - 0.48	0.14
J	$(1.6$ - $3.9) \times 10^{-5}$	1.2×10^{-5}
$\sin 2\alpha$	-0.98 - 0.49	0.74
$\sin 2\beta$	0.46 - 0.89	0.22
α	75° - 129°	27°
β	13.7° - 31.4°	8.9°
$\gamma = \delta$	32° - 83°	26°
$\sin\theta_{12}$	0.2220 ± 0.0041	
$\sin\theta_{13}$	$(2.49$ - $4.52) \times 10^{-3}$	1.02×10^{-3}
$\sin\theta_{23}$	$(38.1$ - $43.5) \times 10^{-3}$	2.7×10^{-3}
$\|V_{ud}\|$	0.97507 ± 0.00094	
$\|V_{us}\|$	0.2220 ± 0.0041	
$\|V_{ub}\|$	$(2.49$ - $4.52) \times 10^{-3}$	1.02×10^{-3}
$\|V_{cd}\|$	0.2219 ± 0.0042	
$\|V_{cs}\|$	0.97320 - 0.97526	0.00103
$\|V_{cb}\|$	$(38.1$ - $43.5) \times 10^{-3}$	2.7×10^{-3}
$\|V_{td}\|$	$(6.2$ - $9.6) \times 10^{-3}$	1.7×10^{-3}
$\|V_{ts}\|$	$(37.6$ - $43.1) \times 10^{-3}$	2.8×10^{-3}
$\|V_{tb}\|$	0.999058 - 0.999280	0.000111
Δm_s	$(14.9$ - $33.2)$ ps^{-1}	9.2 ps^{-1}
$\mathrm{BR}_{K_L^0 \to \pi^0 \nu\bar{\nu}}$	$(1.0$ - $4.5) \times 10^{-11}$	1.7×10^{-11}
$\mathrm{BR}_{K^+ \to \pi^+ \nu\bar{\nu}}$	$(4.7$ - $10.0) \times 10^{-11}$	2.9×10^{-11}
$\mathrm{BR}_{B^+ \to \tau^+ \nu_\tau}$	$(3.8$ - $23.8) \times 10^{-5}$	10.0×10^{-5}
$\mathrm{BR}_{B^+ \to \mu^+ \nu_\mu}$	$(1.5$ - $9.4) \times 10^{-7}$	4.0×10^{-7}
$f_{B_d}\sqrt{B_d}$	(185 - 294) MeV	54 MeV
B_K	> 0.48	
m_t	> 91 GeV$/c^2$	