The Problem

- Two measurements for quantitity x:
  $x_1 = \langle x_1 \rangle \pm \sigma_1 \pm r_1$
  $x_2 = \langle x_2 \rangle \pm \sigma_2 \pm r_2$

- r_1 and r_2 are ill-defined uncertainties ("ranges")

- How to combine these two measurements and quote a statistical error and uncertainty ("range") on the final result?

- Examples for CKM-Fit:
  1. Combining $|V_{ub}|(incl)$ and $|V_{ub}|(excl)$
  2. Combining $|V_{cb}|(incl)$ and $|V_{cb}|(excl)$
  3. ...

- r_1 and r_2 statistically not well-defined
  $\rightarrow$ There isn't THE method but hopefully
  one or several procedures with reasonable properties

“Range” Method

- Idea: No value within the range is prefered (Democracy)
- Scan x_1 and x_2 within their ranges r_1 and r_2 and calculate for each pair weighted mean

$$\langle x\rangle_\pm = \frac{\frac{\langle x_1\rangle \pm r_1}{\sigma_1^2+r_1^2} + \frac{\langle x_2\rangle \pm r_2}{\sigma_2^2+r_2^2}}{\frac{1}{\sigma_1^2+r_1^2} + \frac{1}{\sigma_2^2+r_2^2}}$$

$$r = \frac{\langle x\rangle_+ - \langle x\rangle_-}{2}$$

$$\langle x\rangle = \frac{\langle x\rangle_+ + \langle x\rangle_-}{2}$$

- Statistical error?

$$\sigma^2 = \frac{1}{\frac{1}{\sigma_1^2+r_1^2} + \frac{1}{\sigma_2^2+r_2^2}}$$

$$\sigma^2 = \frac{1}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$$

- Answer: Neither ... Nor ...

$$\frac{1}{\sigma^2} = \frac{({r_1}^2 \cdot (1 + \frac{2r_1}{r_2}) + {r_2}^2) \cdot \frac{r_2}{{\sigma_1}^2} + ({r_2}^2 \cdot (1 + \frac{2r_2}{r_1}) + {r_1}^2) \cdot \frac{r_1}{{\sigma_2}^2}}{({r_1}^2 + {r_2}^2)^2}$$

- For $r_1 = r_2$: σ should be given by the weighted mean:

$$\frac{1}{{\sigma_{wm}}^2} = \frac{1}{{\sigma_1}^2} + \frac{1}{{\sigma_2}^2}$$

- If one range is much larger than the other one: Squash this measurement but not touch the other one.

$$\frac{1}{{\sigma_{kill}}^2} = \frac{1}{{\sigma_1}^2 \frac{{r_2}^2}{{r_1}^2+{r_2}^2}} + \frac{1}{{\sigma_2}^2 \frac{{r_1}^2}{{r_1}^2+{r_2}^2}}$$

- Weight W to move from one formula to the other depending on $r_1 = r_2$ or not.

$$\frac{1}{\sigma^2} = \frac{1}{{\sigma_{wm}}^2} \cdot (1 - W) + \frac{1}{{\sigma_{kill}}^2} \cdot W$$

Choosing $W = \frac{(r_1 - r_2)^2}{{r_1}^2+{r_2}^2}$ one gets the formula.

Properties

- $r_2 = 0$, $\sigma_1 << \sigma_2 << r_1$:

$$\langle x \rangle = \frac{\sigma_2^2 \cdot x_1 + (\sigma_1^2 + r_1^2) \cdot x_2}{\sigma_1^2 + \sigma_2^2 + r_1^2} \approx x_2$$

$$r = \frac{\sigma_2^2 \cdot r_1}{\sigma_1^2 + \sigma_2^2 + r_1^2} \approx \sigma_2 \cdot \frac{\sigma_2}{r_1}$$

$$\sigma = \sqrt{r_1} \cdot \sigma_2$$

- Example: $|V_{ub}|$

$$|V_{ub}|(excl) = (3.25 \pm 0.29 \pm 0.55_{range}) \cdot 10^{-3}$$

$$|V_{ub}|(incl) = (4.04 \pm 0.63 \pm 0.31_{range}) \cdot 10^{-3}$$

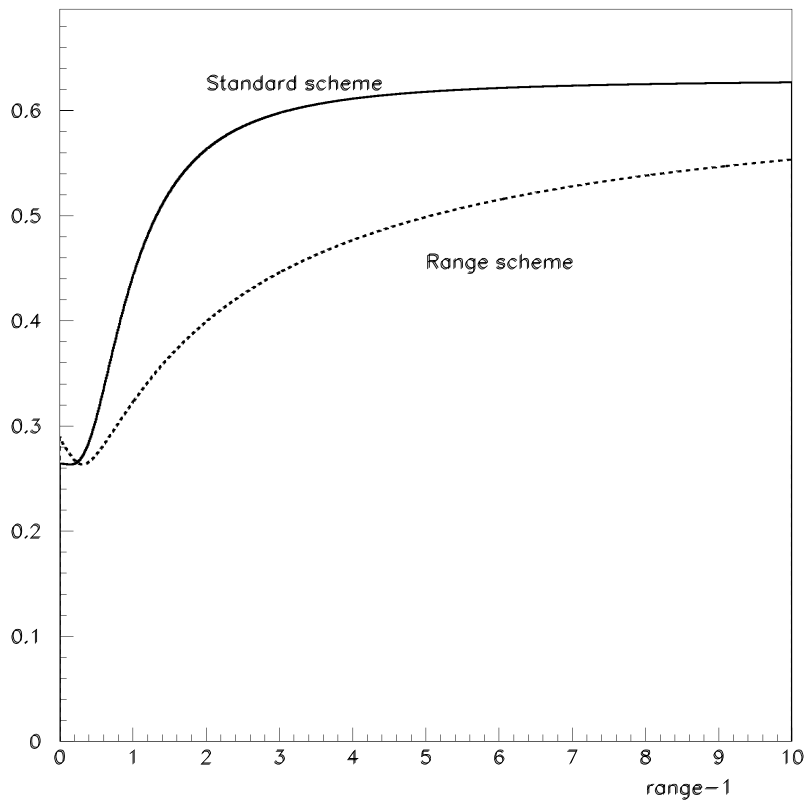
$$|V_{ub}| = (3.60 \pm 0.27 \pm 0.44_{range}) \cdot 10^{-3}$$

- Example: $|V_{cb}|$

$$|V_{cb}|(excl) = (40.6 \pm 1.6 \pm 2.0_{range}) \cdot 10^{-3}$$

$$|V_{cb}|(incl) = (40.76 \pm 0.41 \pm 2.0_{range}) \cdot 10^{-3}$$

$$|V_{cb}| = (40.75 \pm 0.40 \pm 2.0_{range}) \cdot 10^{-3}$$


Standard scheme
Range scheme
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
7
8
9
10
range−1