INPUT PARAMETERS FOR A GLOBAL CKM-FIT

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- CKM Matrix & Wolfenstein Parametrisation
- The Unitarity Triangle
- Present Knowledge of the CKM Parameters
- Constraints on the Unitarity Triangle
- Prospects

CKM Matrix & Wolfenstein Parametrisation

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Wolfenstein Parametrization of $V_{\rm CKM}$:

$$\begin{aligned} s_{12} &\rightarrow \lambda \\ s_{23} &\rightarrow A\lambda^2 \\ s_{13}e^{-i\delta} &\rightarrow A\lambda^3(\rho - i\eta) \end{aligned}$$

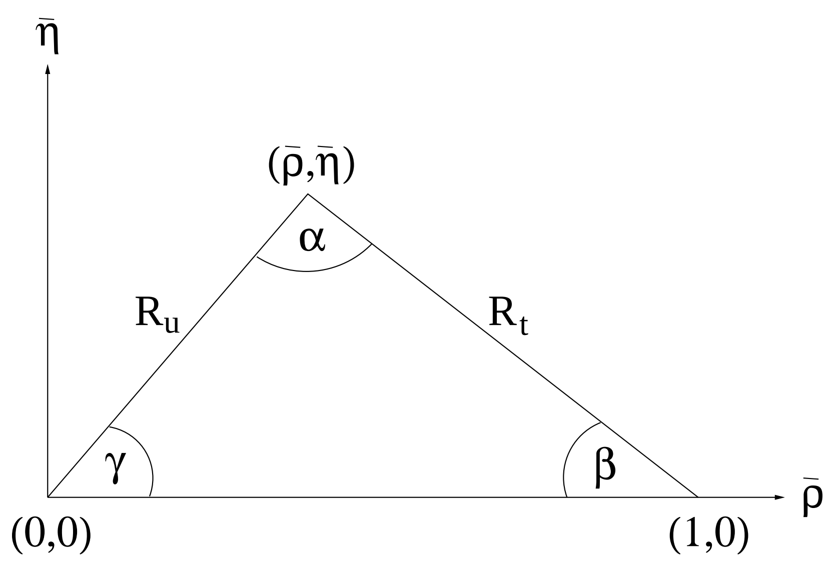
$$V_{\rm CKM}(O(\lambda^4)) =$$

$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda\left[1 + A^2\lambda^4\left(\rho + i\eta - \frac{1}{2}\right)\right] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3\left[1 - (\rho + i\eta)\left(1 - \frac{1}{2}\lambda^2\right)\right] & -A\lambda^2\left[1 + \lambda^2\left(\rho + i\eta - \frac{1}{2}\right)\right] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

The Unitarity Triangle

$$\begin{aligned} V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* &= 0 \\ \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} &= \mathrm{R_u} + 1 + \mathrm{R_t} = 0 \\ R_u &= \sqrt{\bar{\rho}^2 + \bar{\eta}^2} \\ R_t &= \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} \end{aligned}$$

$$\bar{\rho} = \rho(1 - \lambda^2/2); \bar{\eta} = \eta(1 - \lambda^2/2)$$
$$\gamma = atan(\tfrac{\bar{\eta}}{\bar{\rho}}) = \delta$$



Present Knowledge of the CKM Matrix I

- $V_{\rm ud}$
  - Super-allowed nuclear β-decays ($0^+ \to 0^+$)
    $|V_{\rm ud}| = 0.9740 \pm 0.0001_{exp} \pm 0.00048_{theo}$ (0.0010_{theo})
  - Neutron β-decay ($\tau_{\rm n}$, $g_{\rm A}/g_{\rm V}$)
    $|V_{\rm ud}| = 0.9749 \pm 0.0024$
    (recent measurements not included)
  - Pion β-decay ($\pi^+ \to \pi^0 {\rm e}^+ \nu_{\rm e}$)
    $|V_{\rm ud}| = 0.967 \pm 0.0160_{exp} \pm 0.0009_{theo}$
    no nuclear structure effects but not competitive
    $(BR = (1.025 \pm 0.034) \cdot 10^{-8})$
  - $|V_{\rm ud}| = 0.9741 \pm 0.0009$

- $V_{\rm us} = \lambda$
  - Semi-lept. Hyperon-Decays:
    $|V_{\rm us}| = 0.21 - 0.24$ $(SU(3))$
  - Semi-lept. $K^+_{\rm e3}$-Decays:
    $K^+ \to \pi^0 {\rm e}^+ \nu_{\rm e}$, $K_{\rm L} \to \pi^- {\rm e}^+ \nu_{\rm e}$
    $|V_{\rm us}| = 0.2196 \pm 0.0017 \pm 0.0018_{theo}$

- $V_{\rm ud}^2 + V_{\rm us}^2 + V_{\rm ub}^2 = 0.9969 \pm 0.0014$

Present Knowledge of the CKM Matrix II

- $V_{\rm cd}$
  - Charm production in νN-, $\bar{\nu} N$-DIS:
    $|V_{\rm cd}| = 0.224 \pm 0.014$

- $V_{\rm cs}$
  - Charm production in νN-, $\bar{\nu} N$-DIS:
    $|V_{\rm cs}| = 1.04 \pm 0.16$
  - Semi-lept. $D^{+}_{\rm e3}$-Decays:
    $|V_{\rm cs}| = 1.04 \pm 0.05_{exp.} \pm 0.14_{theo.}$
  - Unitarity for three families:
    $|V_{\rm cs}| = 0.97 \pm 0.09_{exp} \pm 0.07_{sys.}$

$$R_c = \frac{\Gamma(W^+ \rightarrow c\bar{q})}{\Gamma(W^+ \rightarrow hadrons)} = \frac{\sum_{i=d,s,b} |V_{ci}|^2}{\sum_{i=d,s,b;j=u,c} |V_{ji}|^2} = 1/2$$

$|V_{\rm cs}| = 0.989 \pm 0.016$

$$\frac{\Gamma(W^+ \rightarrow hadrons)}{\Gamma(W^+ \rightarrow leptons)} = \sum_{i=d,s,b;j=u,c} |V_{ji}|^2 \cdot (1 + \frac{\alpha_s(m_W)}{\pi})$$

Present Knowledge of the CKM Matrix III

- $V_{\rm ts}$
  $|V_{ts}V_{tb}/V_{cb}|$ from $\frac{\Gamma(b\to s\gamma)}{\Gamma(b\to c\ell^-\bar{\nu}_\ell)}$
  $|V_{ts}V_{tb}/V_{cb}|^2 = 0.93 \pm 0.14 \pm 0.08$

- $V_{\rm tb}$
  Semi-lept. top-decays: $\frac{|V_{\rm tb}|^2}{\sum_{i=d,s,b}|V_{ti}|^2}$
  $|V_{\rm tb}| = 0.99 \pm 0.15$

Present Knowledge of the CKM Matrix IV

- $V_{\rm cb} = A \cdot \lambda^2$

- $V_{\rm cb}$ (exclusive)
  - $B \to D^{(*)} \ell \nu_\ell$ ($F(w)$ Isgur-Wise function)
  - Extrapolation to point of zero recoil: $|V_{cb}| \cdot F_{D^*}(w = 1)$
  - $F_{D^*}(1) = 1 +$ HQET corrections

$$
\begin{aligned}
|V_{cb}| \cdot F_{D^*}(1) &= (33.8 \pm 0.9 \pm 1.9) \cdot 10^{-3} (LEP) \\
|V_{cb}| \cdot F_{D^*}(1) &= (42.4 \pm 1.0 \pm 1.9) \cdot 10^{-3} (CLEO) \\
|V_{cb}| \cdot F_{D^*}(1) &= (38.1 \pm 1.0 \pm 1.9) \cdot 10^{-3} (av.) \\
|V_{cb}| &= (42.8 \pm 3.0 \pm 2.4_{theo}) \cdot 10^{-3}
\end{aligned}
$$

$F_{D^*}(1) = 0.89 \pm 0.05$ Bigi et al.
$F_{D^*}(1) = 0.91 \pm 0.03$ Neubert et al.

- $V_{\rm cb}$ (inclusive)
  Inclusive semi-leptonic BR

$$|V_{cb}| = 41.1 \cdot 10^{-3} \sqrt{\frac{BR(B \to X_c \ell \nu_\ell)}{0.105} \cdot \frac{1.55\text{ps}}{\tau_b}}$$

$$(1 - 0.024 \frac{\mu_\pi^2 - 0.5\text{GeV}^2}{0.2\text{GeV}^2})$$

$$\cdot(1 \pm 0.030_{pert.} \pm 0.024_{1/m_b^3} \pm 0.020_{m_b})$$

$|V_{cb}| = (40.76 \pm 0.41 \pm 2.0_{theo.}) \cdot 10^{-3}$(LEP)
$|V_{cb}| = (40.0 \pm 1.0 \pm 2.0_{theo.}) \cdot 10^{-3}$(CLEO)

- $V_{\rm cb}$ (average)

$$|V_{cb}| = (41.1 \pm 1.6) \cdot 10^{-3}$$

Constraining the Unitarity Triangle I: $V_{\rm ub}$

- $V_{\rm ub}$ (exclusive)
  $B^0 \to \rho^-(\pi^-)\ell^+\nu_\ell$ experimentally easier ($\Upsilon(4S)$)
  than inclusive measurements but: model dependent
  $|V_{ub}| = (3.25 \pm 0.14^{+0.21}_{-0.29}{}_{exp.} \pm 0.55_{mod.}) \cdot 10^{-3}$ (CLEO)

- $V_{\rm ub}$ (inclusive)
  Lepton end-point spectrum of $b \to u\ell^-\bar{\nu}_\ell$
  Large errors due to extrapolation to small momenta
  Inclusive semi-leptonic BR (hadronic invariant mass M_X)

$$|V_{ub}| = 4.45 \cdot 10^{-3}\sqrt{\frac{BR(B \to X_u\ell\nu_\ell)}{0.002} \cdot \frac{1.55\text{ps}}{\tau_b}}$$
$$\cdot(1 \pm 0.010_{pert.} \pm 0.030_{1/m_b^3} \pm 0.035_{m_b})$$

$$|V_{ub}| = (4.04^{+0.41}_{-0.46}(stat. + det.)^{+0.43}_{-0.48}(b \to c)^{+0.24}_{-0.25}(b \to u)$$
$$\pm 0.02(\tau_b) \pm 0.19(OPE)) \cdot 10^{-3}(LEP)$$

- $|V_{ub}| = (3.48 \pm 0.23 \pm 0.55_{theo.}) \cdot 10^{-3}$

Constraining the Unitarity Triangle II: $\epsilon_{\rm K}$

$$|\epsilon_{\rm K}| = (2.271 \pm 0.017) \times 10^{-3}$$

$$|\epsilon_{\rm K}| = \frac{G_F^2 m_W^2 m_K f_K^2}{12\sqrt{2}\pi^2 \Delta m_K} B_K \Big(\eta_{cc} S(x_c, x_c) \mathrm{Im}\big[(V_{cs}V_{cd}^*)^2\big] + \eta_{tt} S(x_t, x_t) \mathrm{Im}\big[(V_{ts}V_{td}^*)^2\big] + 2\eta_{ct} S(x_c, x_t) \mathrm{Im}\big[V_{cs}V_{cd}^* V_{ts}V_{td}^*\big] \Big)$$

Besides top- also charm-quark contribution important

$$S(x_c, x_c) : S(x_c, x_t) : S(x_t, x_t) \approx 10^0 : 10^1 : 10^4$$

$$\mathrm{Im}\big[(V_{cs}V_{cd}^*)^2\big] : \mathrm{Im}\big[V_{cs}V_{cd}^* V_{ts}V_{td}^*\big] : \mathrm{Im}\big[(V_{ts}V_{td}^*)^2\big] = 1 : 1 : \lambda^4$$

$$\eta_{cc} = 1.38 \pm 0.53, \eta_{tt} = 0.574 \pm 0.004, \eta_{ct} = 0.47 \pm 0.04$$

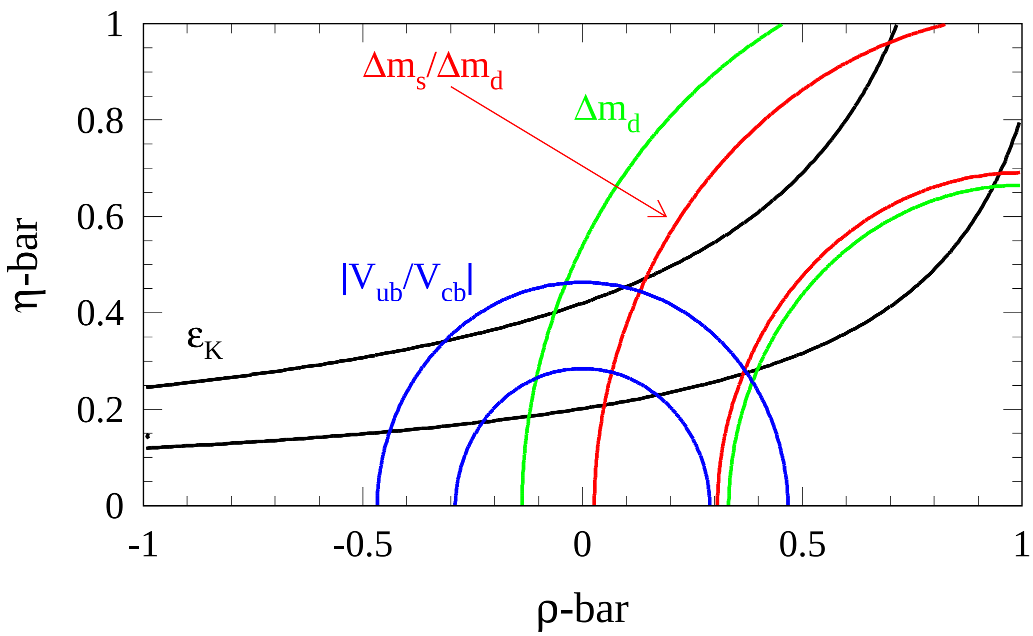
Determination of Hadronic Matrix element by the Vacuum insertion approximation:

$$\langle \bar{K^0} | (\bar{s}\gamma^\mu (1-\gamma^5) d)^2 | K^0 \rangle = \frac{8}{3} B_K \langle \bar{K^0} | \bar{s}\gamma^\mu \gamma^5 d | 0 \rangle \langle 0 | \bar{s}\gamma^\mu \gamma^5 d | K^0 \rangle$$

$$= \frac{8}{3} m_K^2 f_K^2 B_K$$

$$\langle \bar{K^0} | \bar{s}\gamma^\mu \gamma^5 d | 0 \rangle = f_K p^\mu, \; K^+ \to \mu^+ \nu_\mu : \langle K^+ | \bar{s}\gamma^\mu \gamma^5 u | 0 \rangle$$

$B_K = 0.87 \pm 0.06_{stat.+sys.} \pm 0.13_{quench.+SU(3)}$ (Lattice QCD)



Constraining the Unitarity Triangle II: ϵ'/ϵ

- $\propto \eta$
- Inconsistency in the data (E731,NA31,NA48,KTeV)
- Calculation of hadronic matrix elements (B_6, B_8)

Constraining the Unitarity Triangle III: Δm_d

$$\Delta m_d = (0.487 \pm 0.014)\mathrm{ps}^{-1}$$

$$\Delta m_d = \frac{G_F^2}{6\pi^2} \cdot \eta_B \cdot m_{B_d} \cdot f_{B_d}^2 \cdot B_d \cdot m_W^2 \cdot S(x_t) \cdot \left|V_{td}V_{tb}^*\right|^2$$

$\eta_B = 0.55 \pm 0.01$ (QCD), f_{B_d} and B_d from Lattice QCD:

$$\begin{aligned} f_{B_d}(quench.) &= (175 \pm 20_{stat.+sys.})\mathrm{MeV} \\ f_{B_d} &= (200 \pm 23_{stat.+sys.}{}^{+27}_{-17,quench.+SU(3)})\mathrm{MeV} \\ B_d &= 1.30 \pm 0.12_{stat.+sys.} \pm 0.13_{quench.+SU(3)} \\ f_{B_d}\sqrt{B_d} &= (230 \pm 28 \pm 28)\mathrm{MeV} \end{aligned}$$

First partially unquenched calculations published
Test of Lattice QCD: measuring f_{D_s}, f_D and f_{B_d}

- $B^+ \to \ell^+\nu_\ell$: $|V_{ub}| \cdot f_{B_d}$
  $B^+ \to \mu^+\nu_\mu$: 10 events in $30 - 300 fb^{-1}$ (helicity suppression)
  $B^+ \to \tau^+\nu_\tau$: much larger BR but experimentally difficult
- $\Psi(3770) \to D^+D^-$: f_D

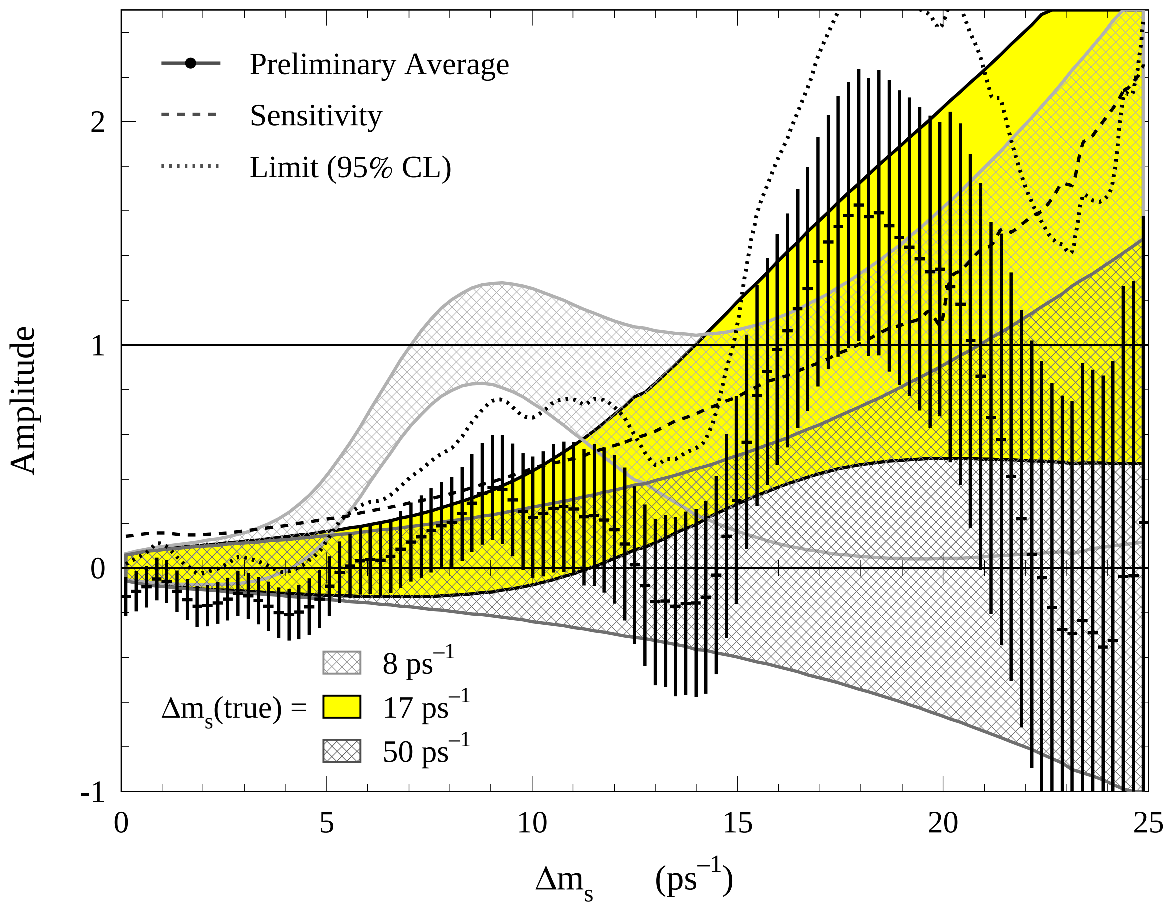
Constraining the Unitarity Triangle IV: $\frac{\Delta m_{\mathrm{s}}}{\Delta m_{\mathrm{d}}}$

$$\Delta m_s = \frac{G_F^2}{6\pi^2} \cdot \eta_B \cdot m_{B_s} \cdot f_{B_s}^2 \cdot B_s \cdot m_W^2 \cdot S(x_t) \cdot \left|V_{ts} V_{tb}^*\right|^2$$

- Δm_{s}: Weak bound in $\bar{\rho}$—$\bar{\eta}$-plane
  $\frac{\Delta m_s}{\Delta m_d} = \frac{m_s}{m_d} \frac{f_{B_s}\sqrt{B_s}}{f_{B_d}\sqrt{B_d}} |\frac{V_{ts}}{V_{td}}|^2$ is quite strong:
  Dependence on m_t and V_{cb} cancel
  $\xi = \frac{f_{B_s}\sqrt{B_s}}{f_{B_d}\sqrt{B_d}} = 1.16 \pm 0.03_{stat.+sys.} \pm 0.05_{quench.}$ (Lattice)

- Amplitude method: $P(t) = \frac{\Gamma}{2} e^{-\Gamma t} (1 \pm A \cdot \cos(\Delta m_s \cdot t))$
  combines results from LEP, SLD, CDF:
  $\Delta m_s > 15.0 \mathrm{ps}^{-1}$ (95% C.L., $A + 1.645\sigma_A < 1$)

- CDF expectation (Run II): %-level



$$\chi^2 = \left(\frac{1-A}{\sigma_A}\right)^2$$

$$y_{meas} = \left(\frac{1-A}{\sigma_A}\right)$$

$$CL = Prob(y > y_{meas}) \rightarrow \chi^2$$

$$\chi^2 = Inv\left(\frac{1}{\sqrt{2\pi}\sigma_A}\int_{1-A}^{\infty} e^{-\frac{1}{2}\cdot(\frac{1-x}{\sigma_A})^2} dx\right)$$

Constraining the Unitarity Triangle IV: $\sin 2\beta$

- $\sin 2\beta$ from $B \to J/\Psi K_{\mathrm{S}}$

$$
\begin{aligned}
0.79 &\pm 0.43\,(CDF)\\
0.93 &\pm 0.91\,(ALEPH)\\
3.2 &\pm 2.0\;\;(OPAL)\\
0.12 &\pm 0.37\,(BaBar)\\
0.45 &\pm 0.44\,(Belle)
\end{aligned}
$$

- $\sin 2\beta = 0.49 \pm 0.23$ (weighted mean)

Prospects: γ from $B \to K\pi$

- $B^\pm \to \pi^\pm K^0$: dominated by strong penguin P
  $B \to \pi^\pm K^\mp$: $P - e^{i\gamma} e^{i\delta} T$
  Strong Penguin Contribution $r = \frac{T}{P}$

- Fleischer-Mannel Bound(no FSI, no EW penguins)
  Neubert-Rosner Bound
  $R = \frac{\Gamma(B \to \pi^\mp K^\pm)}{\Gamma(B^\pm \to \pi^\pm K^0} = 1 - 2 \cdot r \cdot \cos\delta \cos\gamma + r^2$
  $R > 1 - \cos^2\gamma \cos^2\delta$
  FM-bound: $\cos\delta = 1 \to \sin^2\gamma < R$

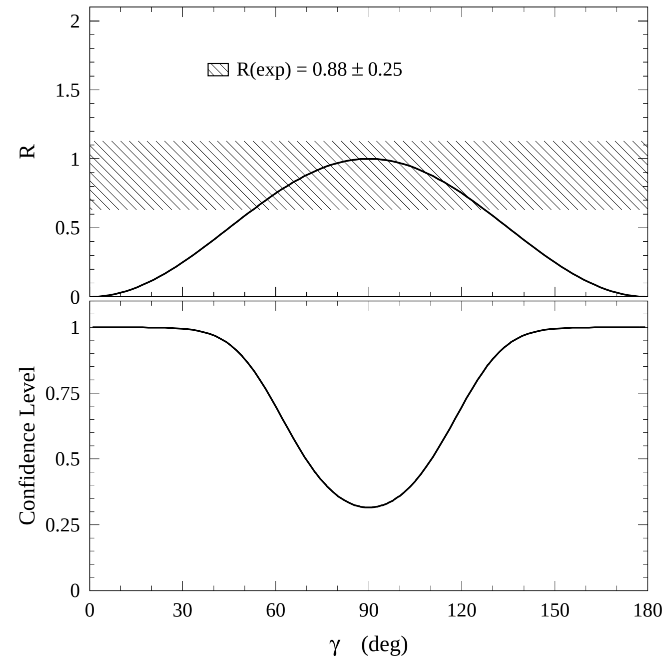
- Factorisation Approximation (Beneke, Neubert et al.)
  Strong phases + FSI; EW penguin contribution $\to \gamma$

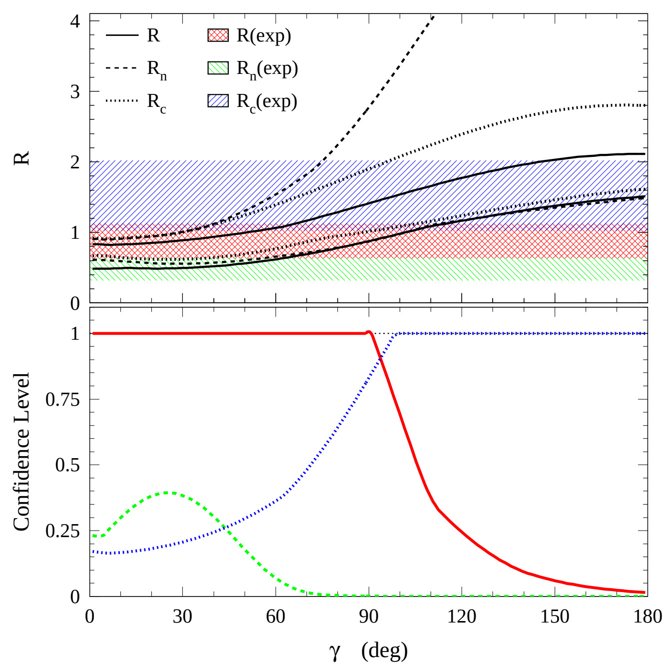
$$R = \frac{BR(B \to \pi^\mp K^\pm)}{BR(B^\pm \to \pi^\pm K^0)} = 0.88 \pm 0.25$$

$$R_n = \frac{1}{2} \frac{BR(B \to \pi^\mp K^\pm)}{BR(B \to \pi^0 K^0)} = 0.48 \pm 0.16$$

$$R_c = \frac{BR(B^\pm \to \pi^0 K^\pm)}{BR(B^\pm \to \pi^\pm K^0)} = 1.52 \pm 0.50$$

Comparison with bounds from Δm_s: New Physics?


R(exp) = 0.88 ± 0.25
R
Confidence Level
0
0.5
1
1.5
2
0
0.25
0.5
0.75
1
0
30
60
90
120
150
180
γ (deg)


R
R_n
R_c
R(exp)
R_n(exp)
R_c(exp)
R
Confidence Level
0
1
2
3
4
0
0.25
0.5
0.75
1
0
30
60
90
120
150
180
γ (deg)

Prospects: Rare Kaon Decays

- $K^+ \to \pi^+ \nu \bar{\nu}$
  top- and charm-contribution important
  Theoretical uncertainty: $\approx 7\%$ (from c-quark contribution)
  Hadronic matrix element from $K^+ \to \pi^0 \mathrm{e}^+ \nu_\mathrm{e}$
  $BR(\mathrm{SM}) = (8.2 \pm 3.2) \cdot 10^{-11}$
  E787 (BNL): one event $\to BR = (1.5^{+3.4}_{-1.2}) \cdot 10^{-10}$
  E949 (BNL): $N_{exp} \approx 5 - 10$
  CKM (FNAL): $N_{exp} \approx 100$ (Starting 2005)

- $K_L \to \pi^0 \nu \bar{\nu}$
  Direct CP violation ($|K_L > \approx \frac{1}{\sqrt{2}}(|K^0> - |\bar{K^0}>)$)
  Only top-quark contribution relevant: $Im(V_{ts}^* V_{td})^2 \propto \eta^2$
  Theoretically very clean: %-level
  $BR(\mathrm{SM}) = 4.08 \cdot 10^{-10} \mathrm{A}^4 \eta^2 = (3.1 \pm 1.3) \cdot 10^{-11}$
  KOPIO (BNL): $N_{exp} \approx 60$
  KAMI (FNAL): $N_{exp} \approx 120$ (Starting 2005)