A NEW APPROACH TO A GLOBAL FIT OF THE CKM MATRIX

A. Höcker, H. Lacker, S. Laplace and F. Le Diberder

[a]Laboratoire de l'Accélérateur Linéaire, IN2P3-CNRS et Université de Paris-Sud, BP 34, F-91898 Orsay Cedex, France

email: hoecker@lal.in2p3.fr, lacker@lal.in2p3.fr, laplace@lal.in2p3.fr, diberder@in2p3.fr

http://www.slac.stanford.edu/~laplace/ckmfitter.html

INTRODUCTION

The Quark Mixing Matrix $V_{\rm CKM}$

$$V_{\rm CKM} = (V)_{ij} \quad \text{with} : i = u,c,t, \text{ and } j = d,s,b$$
$$= UU'^{\dagger} \quad \text{with } U^{(\prime)} M^{(\prime)} U^{(\prime)\dagger} = \text{diag}\left\{\frac{m_{u(d)}}{m_{t(b)}}, \frac{m_{c(s)}}{m_{t(b)}}, 1\right\}$$
$$\Longrightarrow (V)_{ij}(V)^{\dagger}_{ij} \equiv \text{Id}$$

Unitarity Reduces $V_{\rm CKM}$ from 9 →4 Independent Parameters:

Standard (exact): $V_{\rm CKM} = (V)_{ij}(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$

Wolfenstein (approx.): $V_{\rm CKM} = (V)_{ij}(\lambda, A, \rho, \eta) \; + \; \mathcal{O}(|\lambda|^6)$

The Phase Rotation Invariant *Jarlskog* Parameter:

$$\text{Im}\left(V_{ij}V_{kl}V^*_{il}V^*_{kj}\right) = J \sum_{m,n=1}^{3} \epsilon_{ikm}\epsilon_{jln}$$
$$\det[M, M'] = -2FF'J$$
$$F^{(\prime)} = \frac{1}{m_t^3} \prod_{j<i}^{u(d),c(s),t(b)} (m_i - m_j)$$

⇒ Degenerated masses would remove the CP phase

Statistics . . .

Fit ingredients are characterized by

$$\text{Quantity} = \text{value} \pm \sigma_{\rm stat} \pm \sigma_{\rm sys}$$

Hence, for the statistical part, we can construct the χ^2

$$\chi^2 = \sum_{\rm quantities} \left(\frac{\text{measurement} - \text{theory(CKM elements)}}{\sigma_{\rm stat}}\right)^2$$

. . . and Non-Statistics

Consider the following branching:

Frequentist Approach	Bayesian Approach
(*E.g.*) Allowed interval for systematic error	Assign probability density to systematic error
⇓	⇓
Followed by this analysis (also: 95% CL Scan Method)	*Applied, e.g., by F. Parodi, P. Roudeau, A. Stocchi*

In any case: this is *not* a well defined problem

A New Approach: *Rfit*

Reformulate the problem: for a given point in a parameter space, *e.g.*, $(\bar\rho, \bar\eta)$, search for the best theoretical model

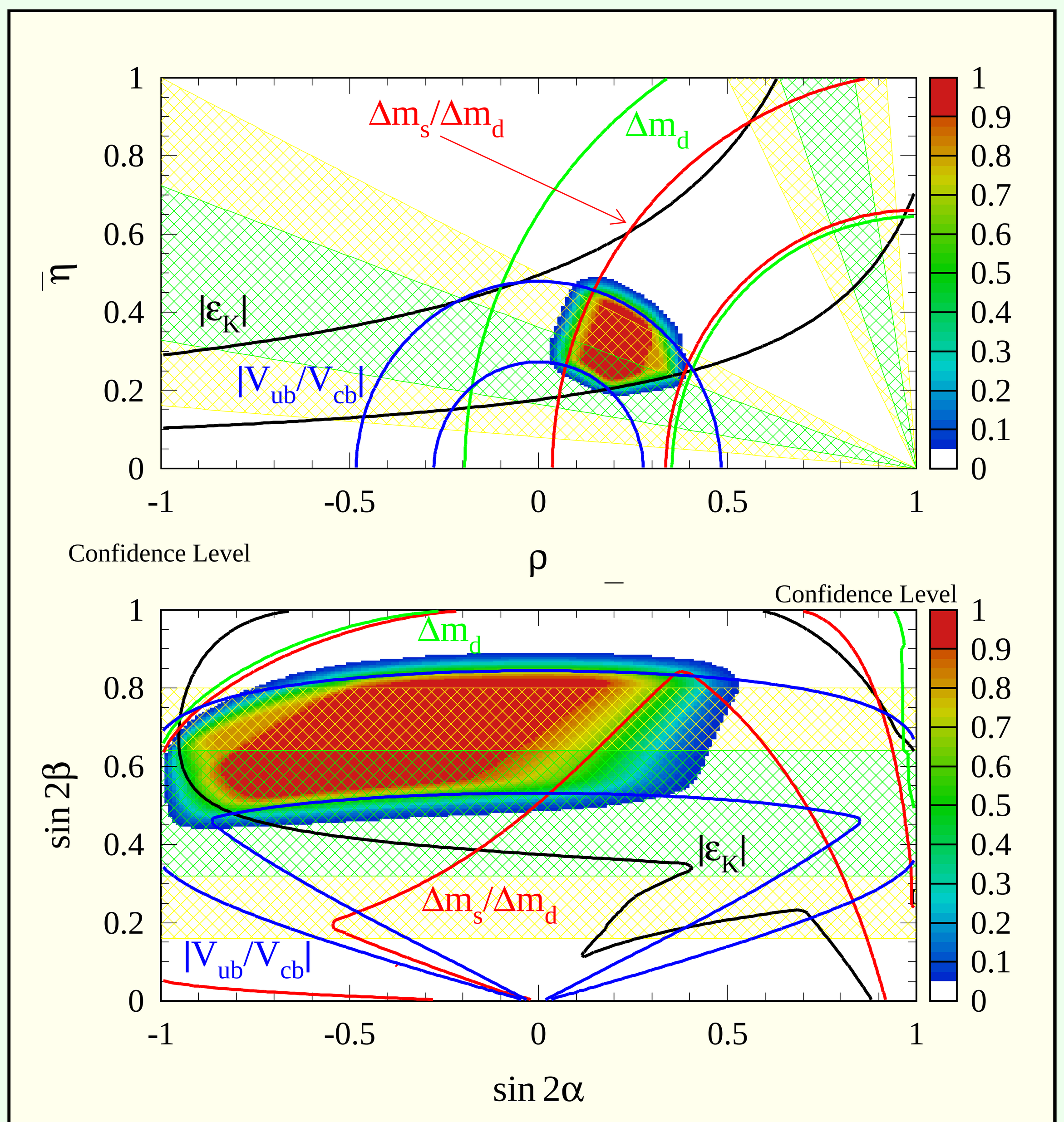
- Perform a discrete scan in the $\bar\rho$, $\bar\eta$ plane and fit minimal $\chi^2(\mu_{\rm min})$, with $(\mu = (\lambda, A, m_t, \kappa)$, for each $(\bar\rho, \bar\eta)$ point
  - κ are parameters varying within their error ranges, *e.g.*, V_{ub}(sys), B_K, $f_{B_d}\sqrt{B_d}$, ξ, η_{cc}, m_c, . . .
  - Smaller errors are added quadratically to the statistical errors
- Probing the Standard Model: find the overall $\chi^2(\mu_{\rm min,\,global})$ in the $\bar\rho$, $\bar\eta$ plane
  - Interpret the confidence level (CL) of the "best" model $\chi^2(\mu_{\rm min,\,global})$ by means of a Monte Carlo simulation
- Metrology: calculate the offset-corrected CL, $\text{Prob}(\chi^2(\mu_{\rm min}) - \chi^2(\mu_{\rm min,\,global}) \;\;, 2)$, for each point in the $\bar\rho$, $\bar\eta$ plane
  - These confidence levels are upper bounds obtained for the best possible combination of theoretical parameters at given $\bar\rho$, $\bar\eta$
- One may replace $\bar\rho$, $\bar\eta$ by any 1D or 2D parameter constellation

METROLOGY

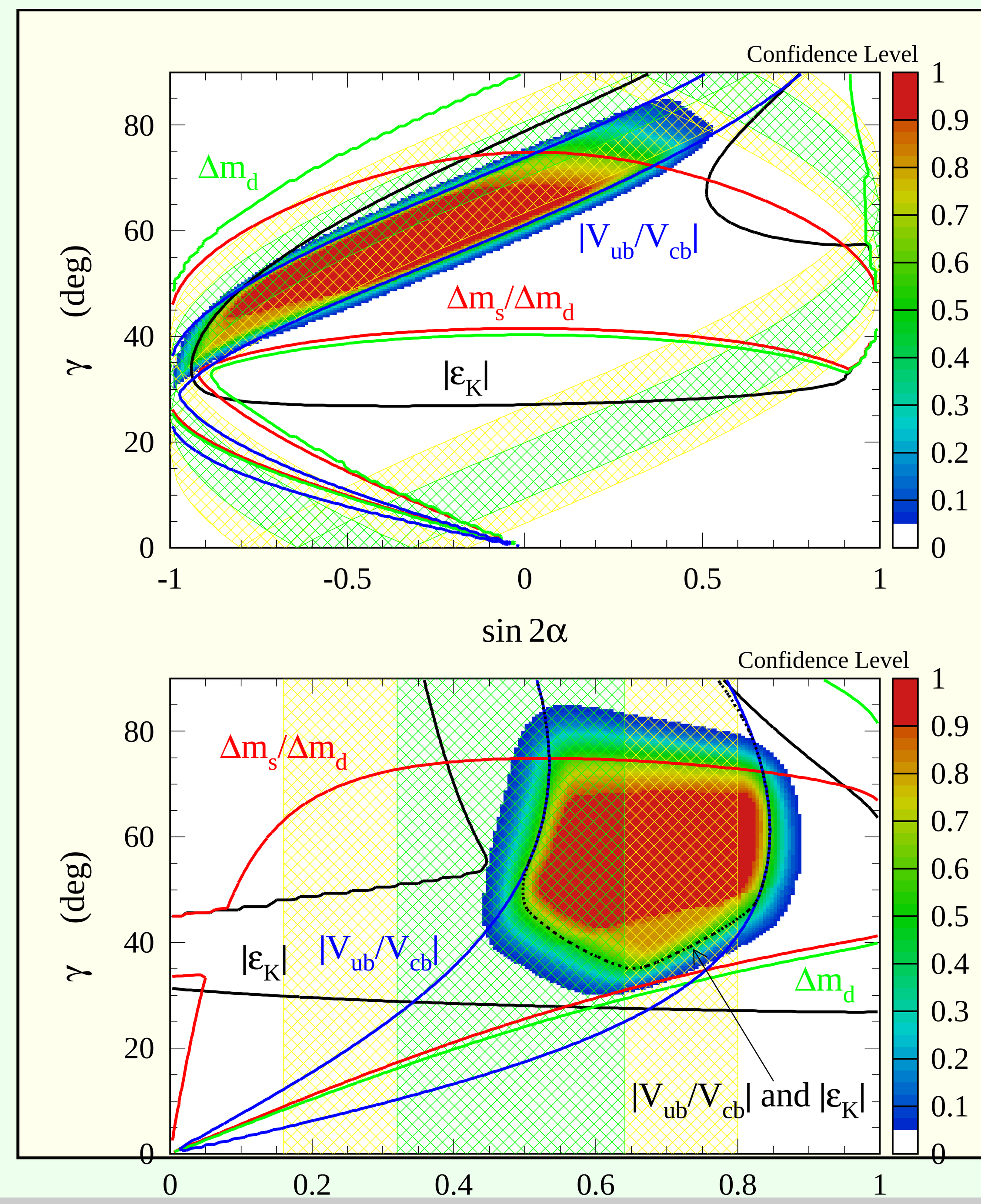
The Fit Ingredients

- Given are unitary CKM parametrizations which ought to be over-constrained in the fit
- Given are observables, the SM predictions of which depend on the CKM matrix elements
- Given are measurements as well as the theoretical parameters that suffer from experimental and systematic uncertainties

Parameter	Value ± Error(s)	Contr. to χ^2	Free	Prop.
$\lvert V_{ud}\rvert$	0.97394 ± 0.00089	*	-	-
$\lvert V_{us}\rvert$	0.2200 ± 0.0025	*	-	-
$\lvert V_{ub}\rvert$	$(3.48 \pm 0.23 \pm 0.55) \times 10^{-3}$	*	-	-
$\lvert V_{cd}\rvert$	0.224 ± 0.014	*	-	-
$\lvert V_{cs}\rvert$	0.969 ± 0.058	*	-	-
$\lvert V_{cb}\rvert$	$(40.75 \pm 0.40 \pm 2.0) \times 10^{-3}$	*	-	-
$\lvert\epsilon_K\rvert$	$(2.271 \pm 0.017) \times 10^{-03}$	*	-	-
Δm_d	(0.487 ± 0.014) ps^{-1}	*	-	-
$\sin 2\beta_{\rm WA}$	0.49 ± 0.23	*	-	-
B_K	$0.87 \pm 0.06 \pm 0.13$	*	*	-
η_{cc}	1.38 ± 0.53	*	*	-
η_{ct}	0.47 ± 0.04	-	-	-
η_{tt}	0.574 ± 0.004	-	-	-
$\eta_B(\overline{\rm MS})$	0.55 ± 0.01	*	*	-
$f_{B_d}\sqrt{B_d}$	$(230 \pm 28 \pm 28)$ MeV	*	*	-
ξ	$1.16 \pm 0.03 \pm 0.05$	*	*	-



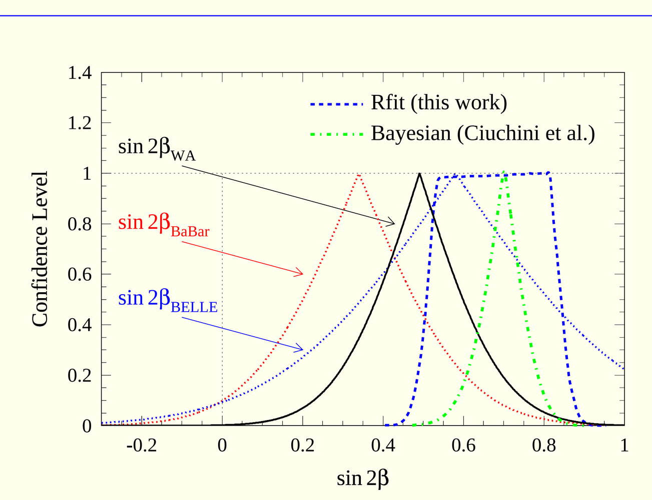
Note: these are *upper bounds* of confidence levels for the best fitting theoretical model at given point in the plane(s)



RESULTS

Selected Numerical Results

Parameter	≥ 5% CL	half width
λ	0.2220 ± 0.0041	
A	0.756 - 0.904	0.074
$\bar\rho$	0.04 - 0.38	0.17
$\bar\eta$	0.20 - 0.48	0.14
J	$(1.6$ - $3.9) \times 10^{-5}$	1.2×10^{-5}
$\sin 2\alpha$	-0.98 - 0.49	0.74
$\sin 2\beta$	0.46 - 0.89	0.22
$\gamma = \delta$	32° - 83°	26°
$\text{BR}(K^0_{\rm L} \to \pi^0\nu\bar\nu)$	$(1.0$ - $4.5) \times 10^{-11}$	1.7×10^{-11}
$\text{BR}(K^+ \to \pi^+\nu\bar\nu)$	$(4.7$ - $10.0) \times 10^{-11}$	2.9×10^{-11}
$\text{BR}(B^+ \to \tau^+\nu_\tau)$	$(3.8$ - $23.8) \times 10^{-5}$	10.0×10^{-5}
$\text{BR}(B^+ \to \mu^+\nu_\mu)$	$(1.5$ - $9.4) \times 10^{-7}$	4.0×10^{-7}



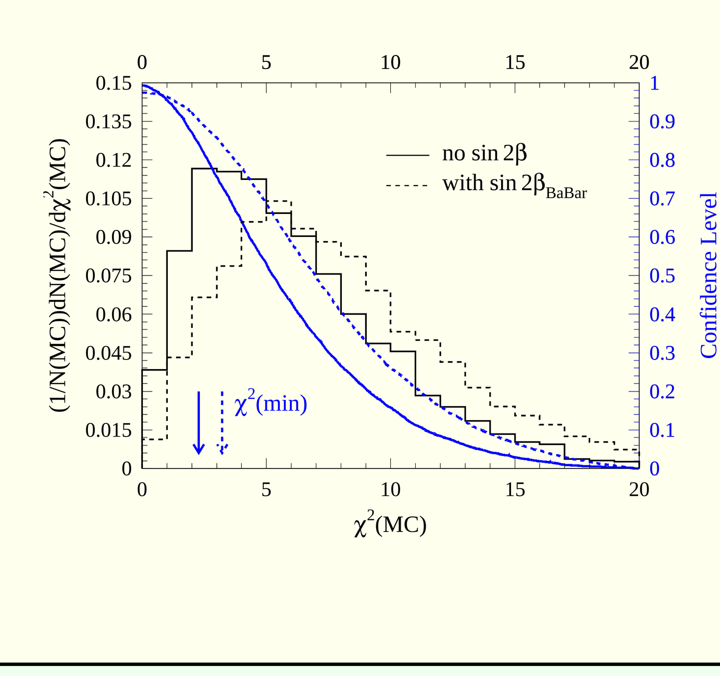
Probing the Standard Model

Probability for the SM to be valid:

$$\mathcal{P}(\text{SM}) = \int_{\chi^2 > \chi^2(\mu_{\rm min,\,global})} \mathcal{F}(\chi^2) d\chi^2$$

where $\mathcal{F}(\chi^2)$ is obtained from toy MC simulation

(replace central values of measurements by theor. predictions $\mu_{\rm min}$ and fluctuate according to stat. errors)



Conclusions

- New frequentist approach to global CKM fits: *Rfit*
- CKM analysis in *Rfit* formally subdivided in
  - Metrological phase
  - Probing the SM: $\text{CL}(\chi^2(\mu_{min,global}))$
  - Probing for New Physics
- Multidimensional Graphical, and numerical results for all relevant CKM parametrizations
- Present Validity of the SM: $\text{CL}(\chi^2(\mu_{min,global})) = 85\%$

- See also talk at "Beyond 10^{34}" workshop: *CKM Fits as a Function of Luminosity (Time)?* by H. Lacker
- *CkmFitter*[a] software package implements *Rfit*, 95% CL Scan Method, *Bayesian* Method

[a]*CkmFitter* is free software.
Please contact the authors to receive the source code and the program driving data cards.