BABAR Collaboration Meeting
SLAC, December 11-15, 2000

CONSTRAINING THE CKM MATRIX

Andreas Höcker*, Heiko Lacker*, Sandrine Laplace*

and Francois Le Diberder**

***LAL (Orsay) and **LPNHE (Paris)**

(i) THE FRAMEWORK

(ii) STATISTICS AND NON-STATISTICS

(iii) COLORS

(iv) PROBING THE STANDARD MODEL

(v) NEW COLORS ?

(vi) THE TOOLKIT: CKMFITTER

The Quark Mixing Matrix V_{CKM}

$$\begin{aligned} V_{\mathrm{CKM}} &= (V)_{ij} \quad \text{with} : i = u, c, t, \ \text{and} \ j = d, s, b \\ &= UU'^{\dagger} \quad \text{with} \ U^{(\prime)} M^{(\prime)} U^{(\prime)\dagger} = \mathrm{diag}\left\{ \frac{m_{u(d)}}{m_{t(b)}}, \frac{m_{c(s)}}{m_{t(b)}}, 1 \right\} \end{aligned}$$

$$\Longrightarrow \quad (V)_{ij}(V)^{\dagger}_{ij} \equiv \mathrm{Id}$$

Unitarity Reduces V_{CKM} from 9 →4 Independent Parameters:

Standard (exact): $V_{\mathrm{CKM}} = (V)_{ij}(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$

Wolfenstein (approx.): $V_{\mathrm{CKM}} = (V)_{ij}(\lambda, A, \rho, \eta) \ + \ \mathcal{O}(|\lambda|^6)$

Also the Phase Rotation Invariant *Jarlskog* Parameter:

$$\mathrm{Im}\left(V_{ij}V_{kl}V^*_{il}V^*_{kj}\right) = J \sum_{m,n=1}^{3} \epsilon_{ikm}\epsilon_{jln}$$

$$\begin{aligned} \det[M, M'] &= -2FF'J \\ F^{(\prime)} &= (m_{t(b)} - m_{c(s)})(m_{t(b)} - m_{u(d)})(m_{c(s)} - m_{u(d)})/m^3_{t(b)} \\ &\Rightarrow \ \text{Degenerated masses would remove the CP phase} \end{aligned}$$

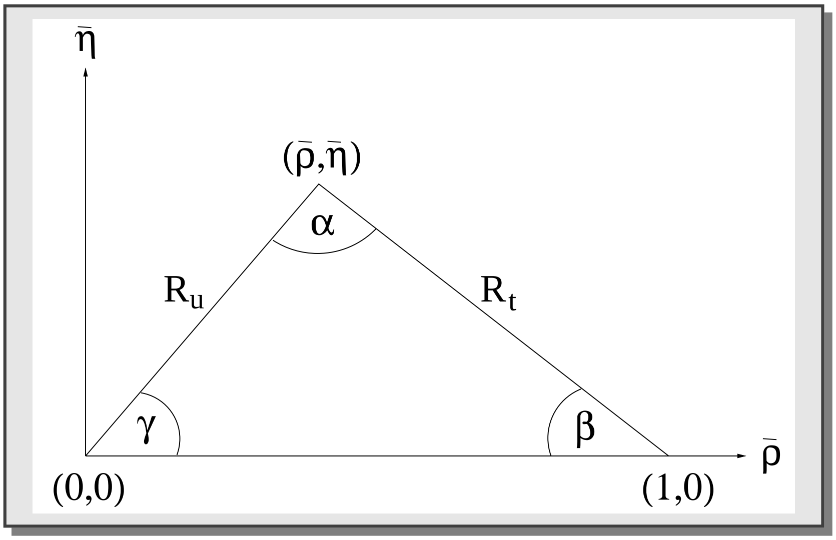
The above parametrizations yield for J:

$$\begin{aligned} J &= c_{12}\, c_{23}\, c^2_{13}\, s_{12}\, s_{23} \sin\delta = A^2\lambda^6\eta + \mathcal{O}(|\lambda|^{10}) \\ &\neq 0 \Longleftrightarrow \text{CP violation} \end{aligned}$$

THE UNITARITY TRIANGLE (UT)

We Exploit the First and Third Column Unitarity Relation:

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$



With, *e.g.*, the Equations ($\bar{\rho}(\bar{\eta}) = \rho(\eta)(1-\lambda^2/2)$):

$$\sin 2\alpha = \frac{2\bar{\eta}(\bar{\eta}^2 - \bar{\rho}(1-\bar{\rho}))}{(\bar{\eta}^2 + (1-\bar{\rho})^2)(\bar{\eta}^2 + \bar{\rho}^2)}$$

$$\sin 2\beta = \frac{2\bar{\eta}(1-\bar{\rho})}{\bar{\eta}^2 + (1-\bar{\rho})^2}$$

$$\tan\gamma = \frac{\bar{\eta}}{\bar{\rho}}$$

In particular: $\gamma \equiv \delta$, and $J = 2 \times |V_{cd}V_{cb}^*|^2 \times$ Surface(UT)

THE FIT INGREDIENTS

- **Given are unitary CKM parametrizations which ought to be (over-)constrained in the fit:**

$$\lambda,\ A,\ \rho,\ \eta$$

$$\theta_{12},\ \theta_{13},\ \theta_{13},\ \delta$$

$$\ldots$$

- **Given are observables, the theoretical predictions of which depend on the CKM matrix elements:**

$b \to u$ and $b \to c$ decay widths

$$\epsilon_K,\ \epsilon'/\epsilon_K$$

$$\Delta m_d,\ \Delta m_s,\ \sin 2\beta\ \ldots$$

- **The measurements as well as the theoretical parameters suffer from experimental and systematic uncertainties:**

$$m_t,\ m_{B_{d,s}},\ \ldots$$

$$B_K,\ \eta_{cc,ct,tt},\ \ldots$$

$$f_{B_d}\sqrt{B_d},\ \eta_B,\ \xi,\ \ldots\ \ldots$$

STATISTICS . . .

Fit ingredients are characterized by

$$\text{Quantity} = \text{value} \pm \sigma_{\text{stat}} \pm \sigma_{\text{sys}}$$

Hence, for the statistical part, we can construct the χ^2

$$\chi^2 = \sum_{\text{quantities}} \left(\frac{\text{measurement} - \text{theory(CKM elements)}}{\sigma_{\text{stat}}} \right)^2$$

. . . AND NON-STATISTICS

Consider the following branching:

Frequentist Approach	Bayesian Approach
(*E.g.*) Allowed interval for systematic error	Assign probability density to systematic error
$\Downarrow$	$\Downarrow$
Followed by this analysis (also: BaBar Book Method)	*Applied, e.g., by F. Parodi, P. Roudeau, A. Stocchi*

! In any case: this is *not* a well defined problem

The BaBar Book Method

S. Plaszczynski, M.H. Schune,

BABAR note #340, 1996

- **Perform a discrete scan over the allowed parameter space defined by systematic-theoretical errors**
  - **Scanned are: V_{ub}(sys), B_K, $f_{B_d}\sqrt{B_d}$, ξ**
  - **Smaller contributions are neglected**

- **Fit minimal $\chi^2(\mu_{\rm min})$ for each scan configuration, with $(\mu) = (\bar\rho, \bar\eta, A, m_t)$**

- **If for a given configuration, the confidence level $\mathrm{Prob}(\chi^2(\mu_{\rm min}), N_{\rm dof}) \geq 5\%$, draw 95% contour in $\bar\rho$, $\bar\eta$,wrt. $\chi^2(\mu_{\rm min})$, *i.e.*,**

$$\mathrm{Prob}(\chi^2(\mu_{\rm contour}) - \chi^2(\mu_{\rm min}), 2) = 5\%$$

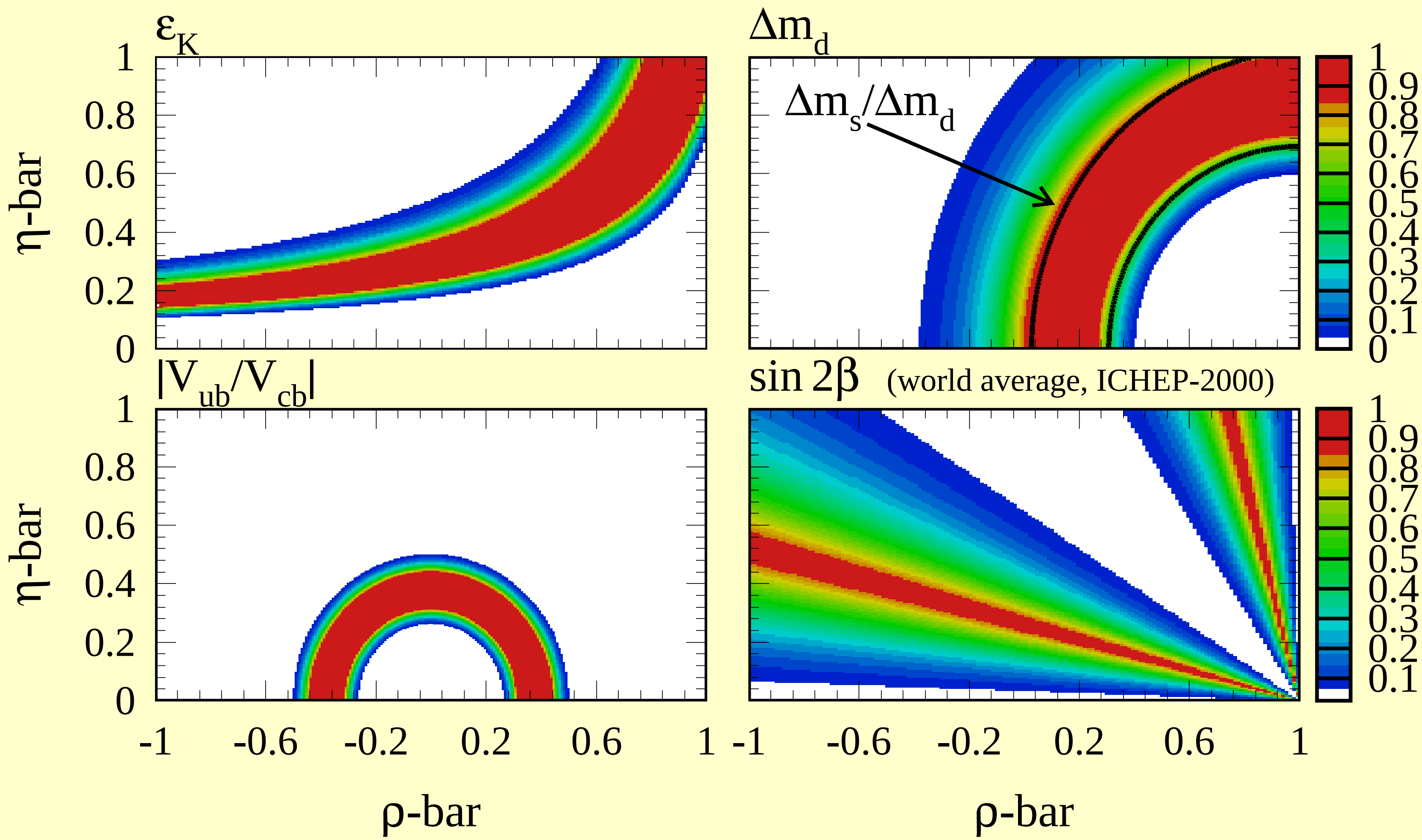
- **One may replace $\bar\rho$, $\bar\eta$ by, *e.g.*, $\sin 2\alpha$, $\sin 2\beta$**

THE NEW APPROACH

Reformulate the problem: for a given point in a parameter space, *e.g.*, $(\bar{\rho}, \bar{\eta})$, search for the best theoretical model

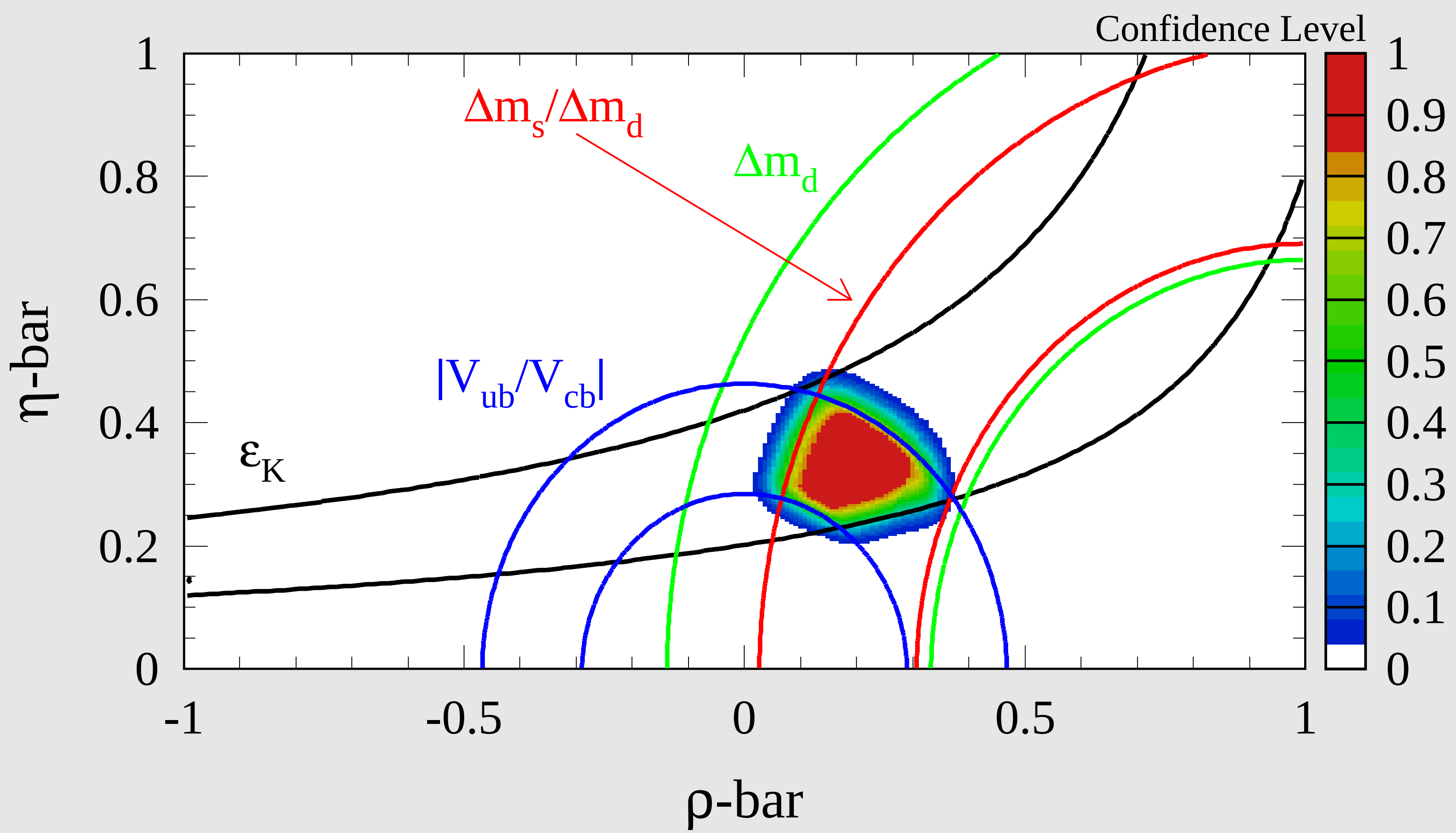
- Perform a discrete scan in the $\bar{\rho}$, $\bar{\eta}$ plane and fit minimal $\chi^2(\mu_{\rm min})$, with $(\mu) = (\bar{\rho}, \bar{\eta}, \lambda, A, m_t, \kappa)$, for each scan point
  - κ are parameters varying within their error ranges, *e.g.*, $V_{ub}(\rm sys)$, B_K, $f_{B_d}\sqrt{B_d}$, ξ, η_{cc}, m_c, ...
  - Smaller error contributions are added quadratically to the statistical errors
- Probing the Standard Model: find the overall $\chi^2(\mu_{\rm min,\,global})$ in the $\bar{\rho}$, $\bar{\eta}$ plane
  - Interpret the confidence level (CL) of the "best" model $\chi^2(\mu_{\rm min,\,global})$ by means of a toy Monte Carlo
- Metrology: calculate the offset-corrected CL, $\mathrm{Prob}(\chi^2(\mu_{\rm min}) - \chi^2(\mu_{\rm min,\,global})\ , 2)$, for each point in the $\bar{\rho}$, $\bar{\eta}$ plane
  - These confidence levels are upper bounds obtained for the best possible combination of theoretical parameters at given $\bar{\rho}$, $\bar{\eta}$
- One may replace $\bar{\rho}$, $\bar{\eta}$ by any 1D or 2D parameter constellation

CONSTRAINTS IN THE $\bar{\rho}$-$\bar{\eta}$ PLANE



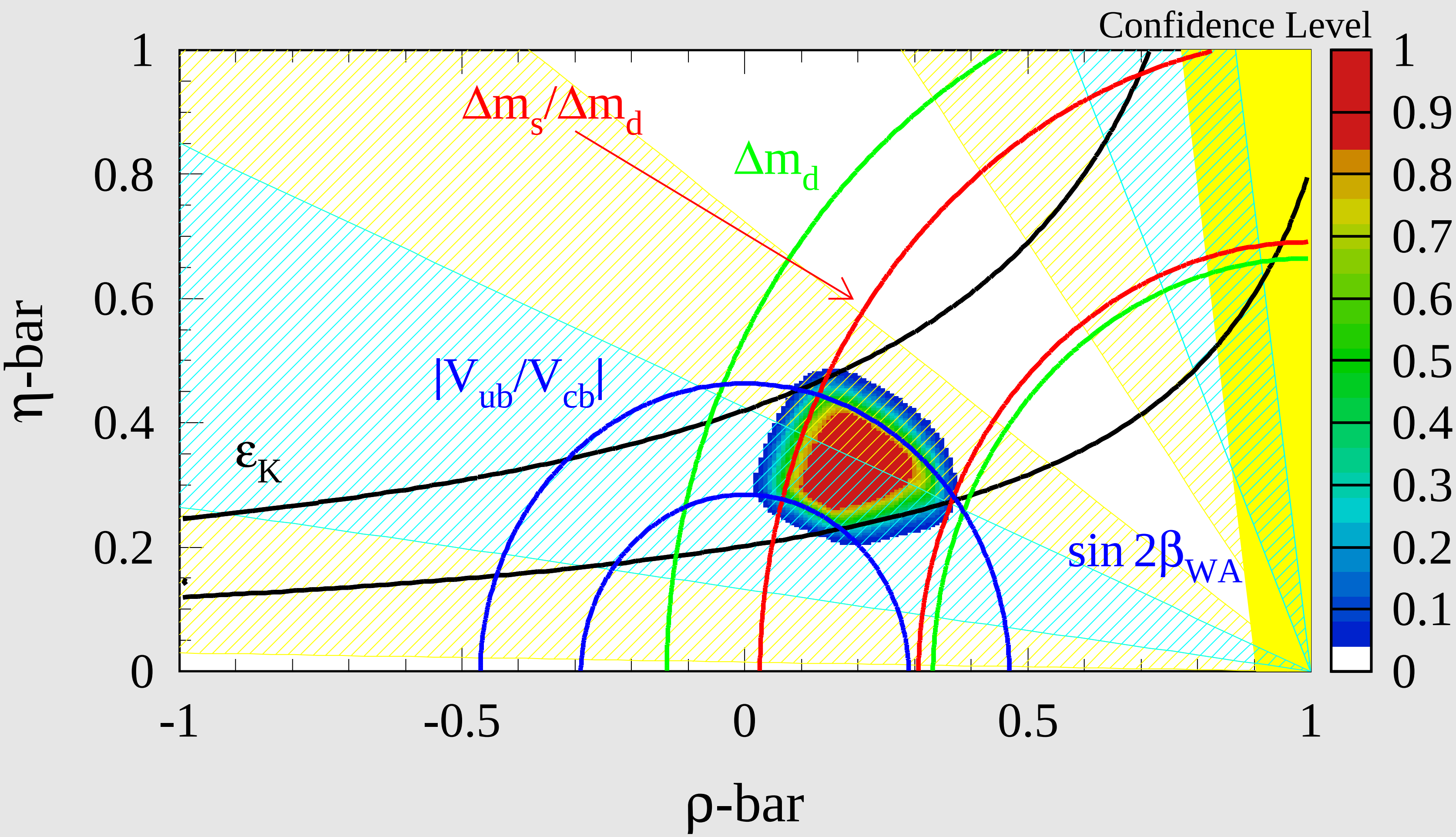
Note: these are *upper bounds* of confidence levels

Constraints in the $\bar{\rho}$-$\bar{\eta}$ Plane

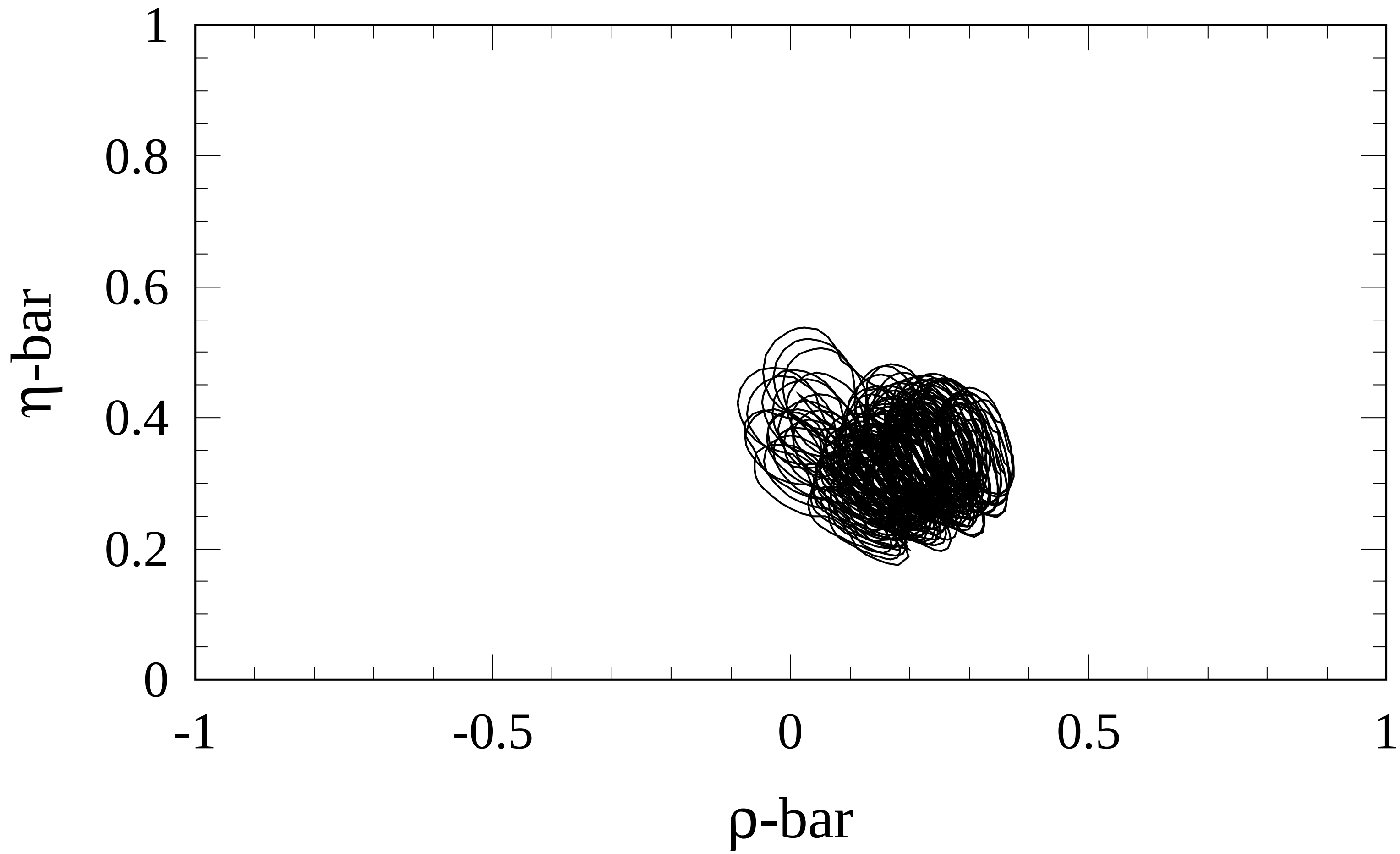


Note: these are *upper bounds* of confidence levels

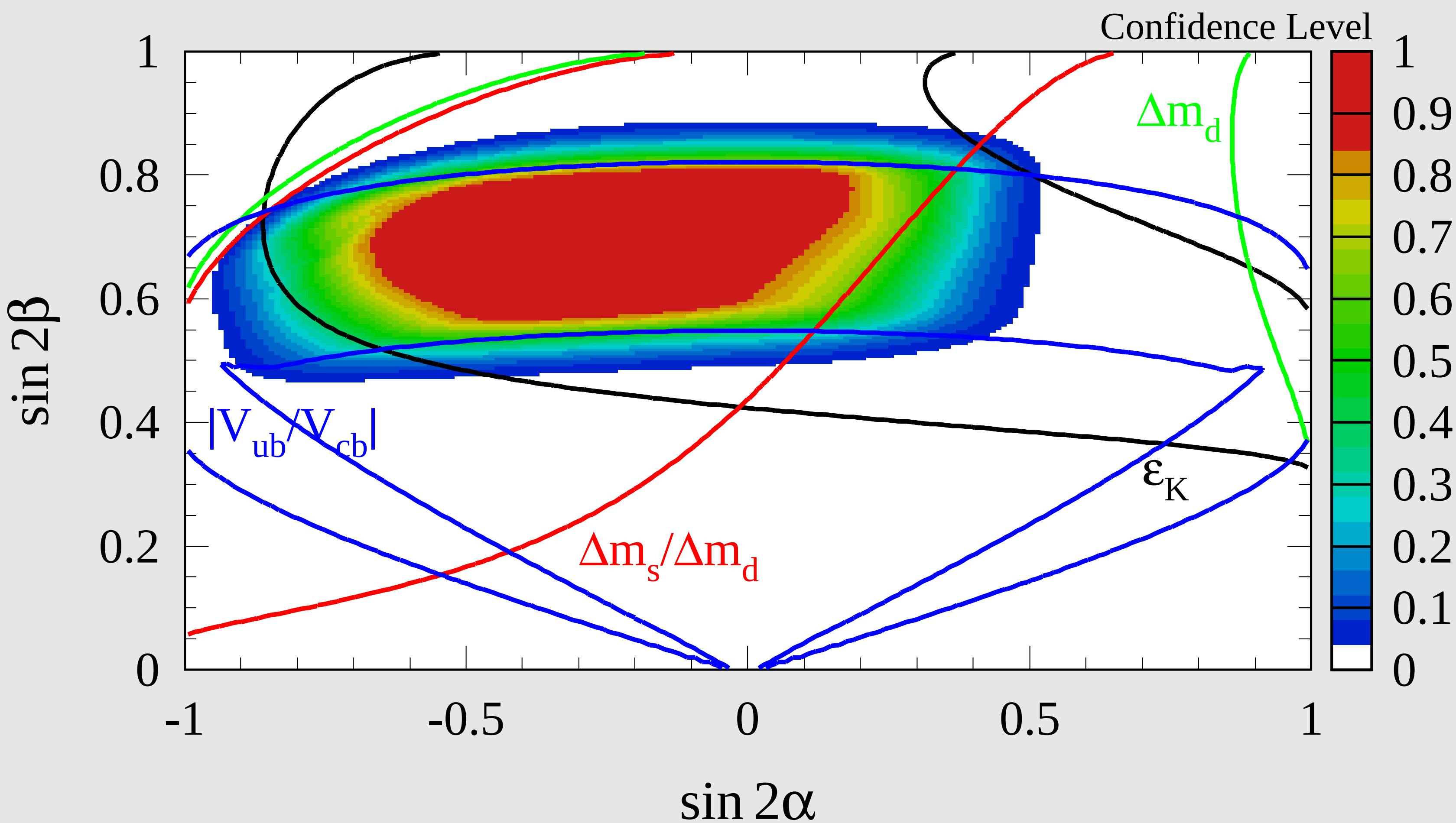
CONSTRAINTS IN THE $\bar{\rho}$-$\bar{\eta}$ PLANE



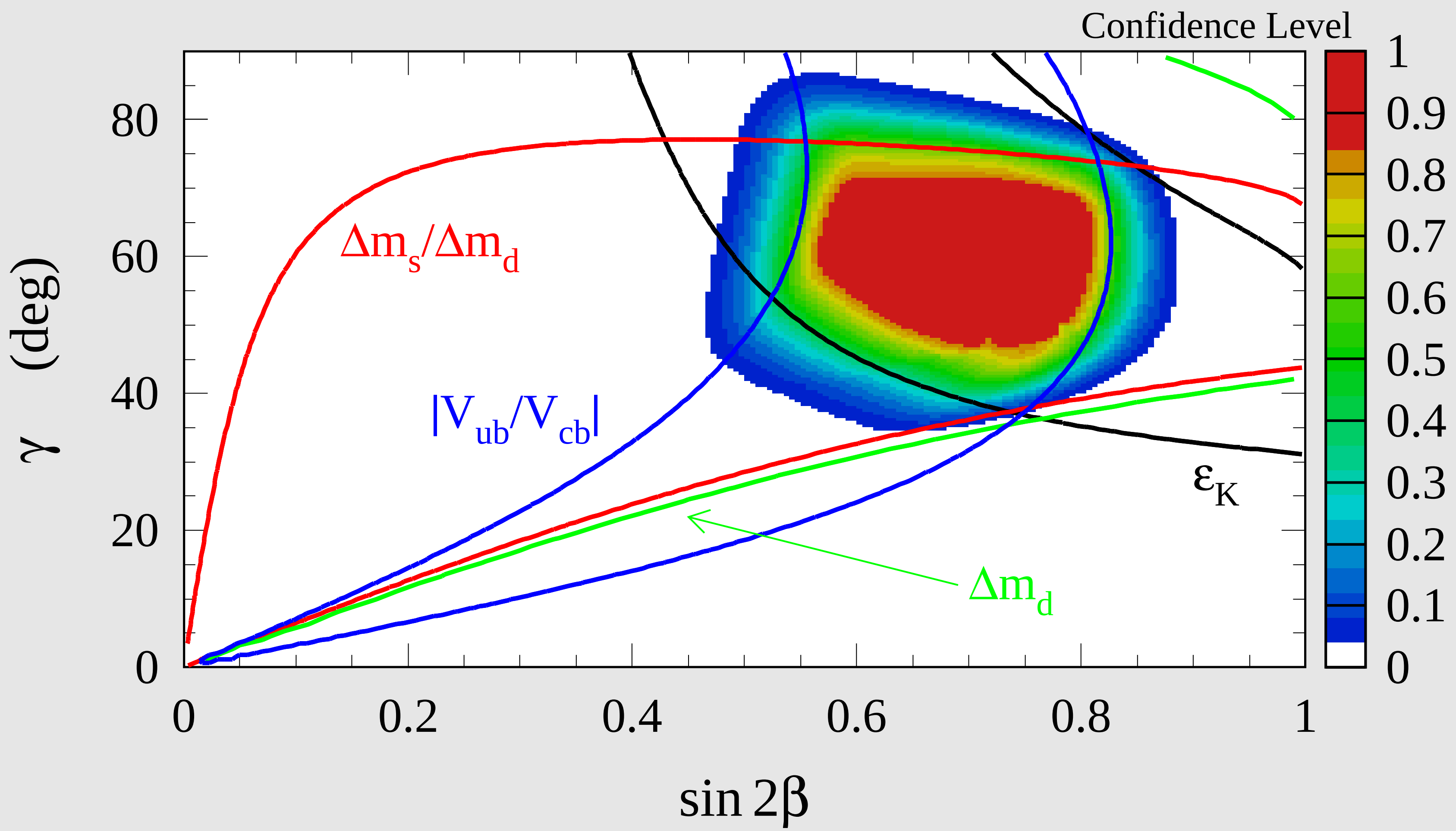
Note: these are *upper bounds* of confidence levels


1
0.8
0.6
0.4
0.2
0
η-bar
-1
-0.5
0
0.5
1
ρ-bar

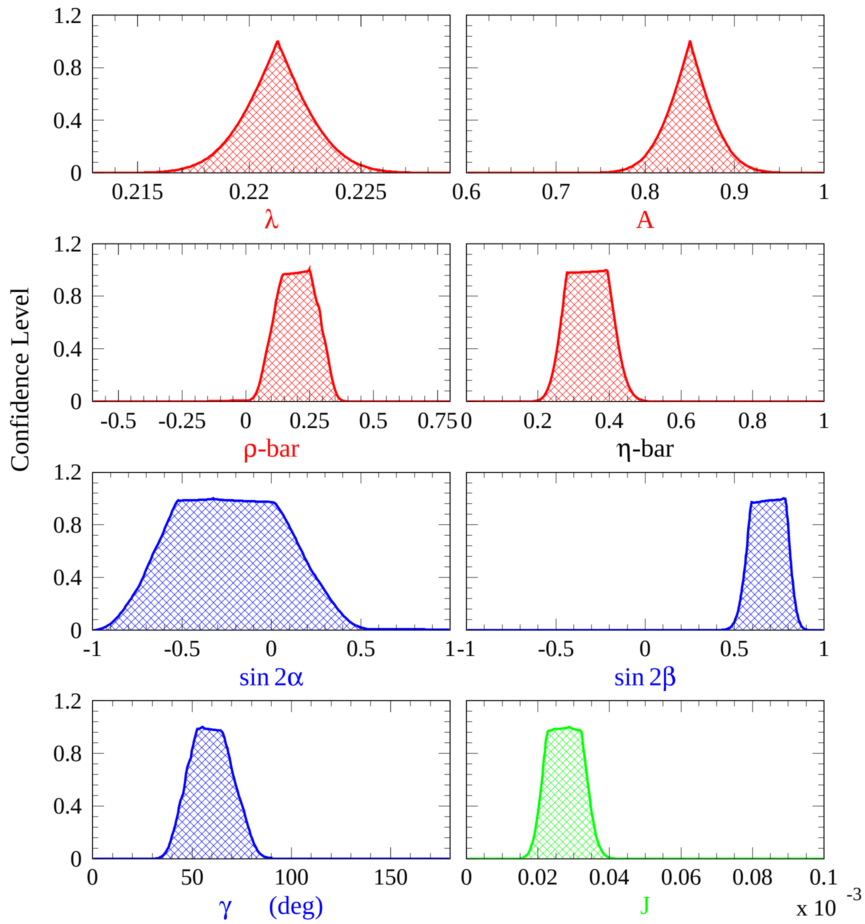
CONSTRAINTS IN THE sin2α-sin2β PLANE



Constraints in the sin2β-γ Plane



FIT RESULTS



Parameter	$\geq$ 32% CL	half width	$\geq$ 5% CL	half width
λ	0.2219 ± 0.0020		± 0.0040	
A	0.851 ± 0.0033		± 0.0066	
$\bar{\eta}$	0.07 - 0.33	0.13	0.04 - 0.36	0.16
$\bar{\rho}$	0.25 - 0.44	0.09	0.22 - 0.47	0.13
$\sin 2\alpha$	-0.74 - 0.30	0.52	-0.90 - 0.46	0.68
$\sin 2\beta$	0.54 - 0.84	0.15	0.50 - 0.88	0.19
γ	43 - 78	18	37 - 84	23
J	2.0 - 3.5 $\times 10^{-5}$	0.8 $\times 10^{-5}$	1.8 - 3.8 $\times 10^{-5}$	1.0 $\times 10^{-5}$

Probing the Standard Model

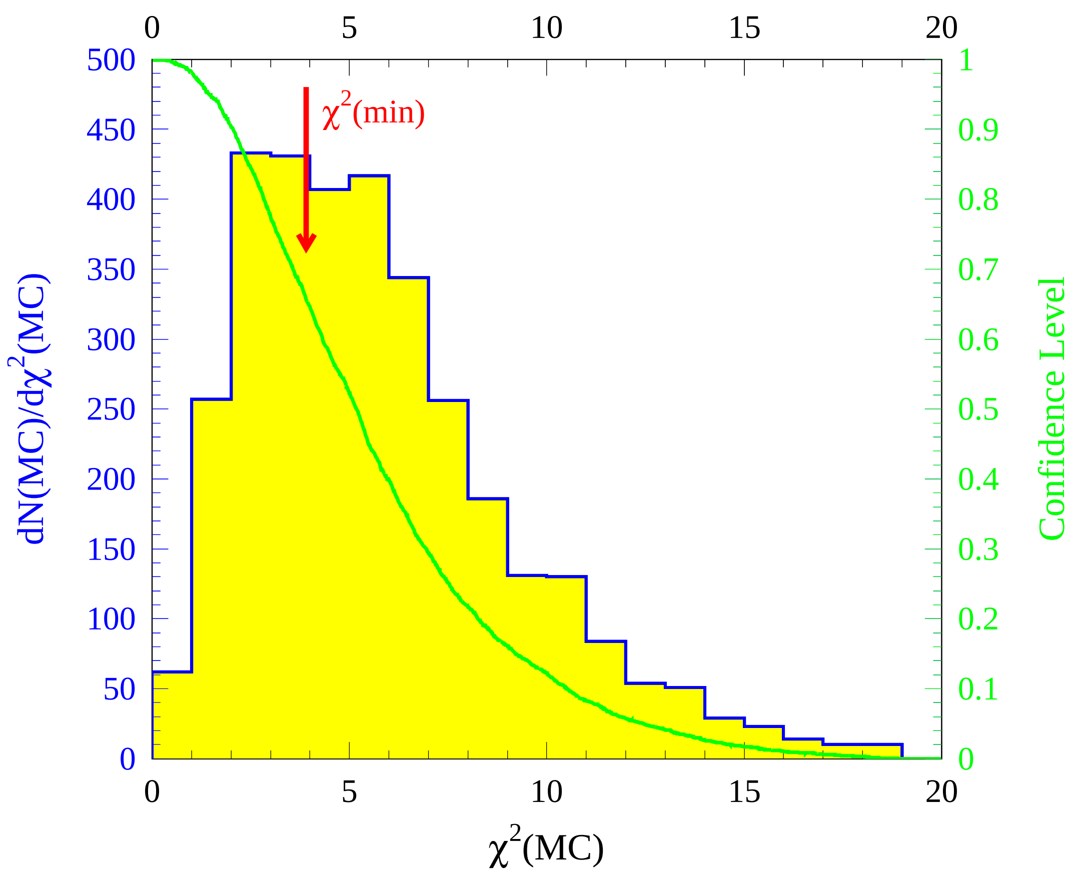
Interpretation of $\chi^2_{\min}(\mu_{\min})$

Probability for the SM to be valid:

$$\mathcal{P}(\mathrm{SM}) = \int_{\chi^2 > \chi^2_{\min}(\mu_{\min})} \mathcal{F}(\chi^2)\mathrm{d}\chi^2$$

where $\mathcal{F}(\chi^2)$ is obtained from toy MC

(replace central values of measurements by theor. predictions $\mu_{\min}$ and fluctuate according to stat. errors)



NEW COLORS ?

We can expect a variety of improvements for the present constraints in the coming years

in particular for sin2β and Δm_s

Improvements are also expected from theory

in particular from lattice QCD

New constraints will appear

sin2α

γ

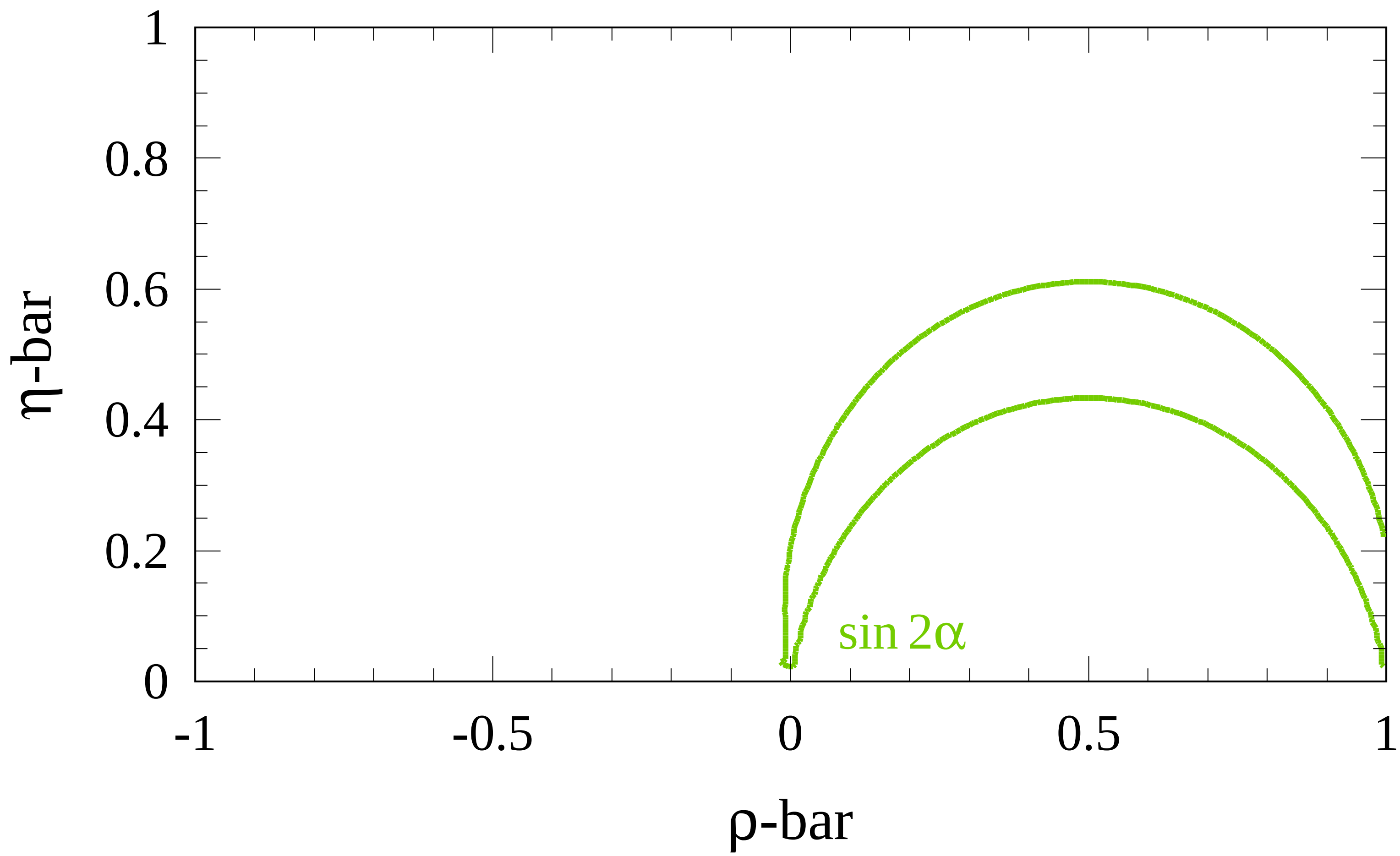
$\mathrm{BR}(K^0_{\mathrm{L}} \to \pi^0 \nu\bar{\nu})$

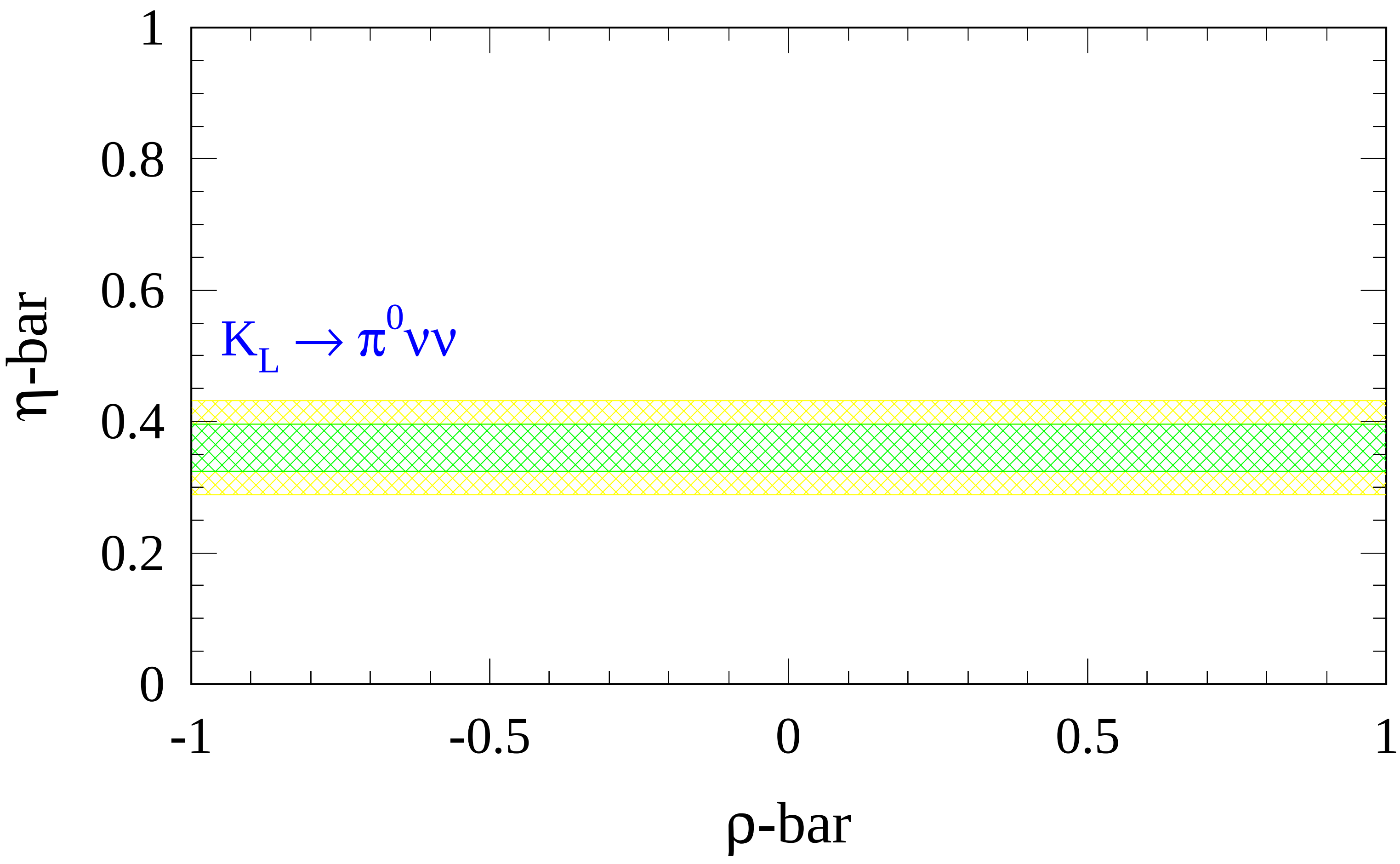
$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})$

$\mathrm{BR}(B^+ \to \tau^+ \nu)$

$b \to s(d)\gamma$ **decays (PRA)**

...


η-bar
ρ-bar
sin 2α
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1


η-bar
ρ-bar
$K_L \to \pi^0 \nu\nu$
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1

CkmFitter

The Toolkit

Designed for CKM Fitting ...

- **... in various parameters spaces:**
  - **2-D: ($\bar{\rho}$-$\bar{\eta}$), (sin2α-sin2β), ...**
  - **1-D: ($\bar{\rho}$), ($\bar{\eta}$), (sin2α), (γ), (J), (B_K), (m_t), ...**
- **... using various statistical approaches:**
  - **The new Frequentist Approach**
  - **The BaBar Book Method**
  - **Bayesian Statistics**
- **... to obtain consistent SM predictions for all related physical observables and theoretical parameters**
  - **UT angles**
  - **Branching ratios of rare K and B decays, ...**
  - **f_{B_d}, B_K, ...**
- **... with straightforward incorporation of physics beyond the Standard Model**
  - **A simple SUSY model being already coded**

A *HowToUse-CkmFitter* page is provided at the location:
http://www.slac.stanford.edu/∼hoecker/ckmfitter/HowToCkmFit.html