## Status of the CKM matrix

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I review the status of Cabibbo-Kobayashi-Maskawa matrix within the Standard Model, with a focus on exclusive $b \to (d,s)\gamma$ transitions and on charm and strange physics.

In the Standard Model (SM), the weak charged-current transitions mix quarks of different generations, which is encoded in the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix. In the case of three generations of quarks, the physical content of this matrix reduces to four real parameters, among which one phase, the only source of CP violation in the Standard Model (the lepton sector can also exhibit similar sources of CP violation once masses, provided by New Physics (NP), are considered). One can define these four real parameters as:

$$\lambda^2 = \frac{|V_{us}|^2}{|V_{ud}|^2+|V_{us}|^2} \qquad A^2\lambda^4 = \frac{|V_{cb}|^2}{|V_{ud}|^2+|V_{us}|^2} \qquad \bar\rho + i\bar\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\,. \tag{1}$$

This parametrisation is exact, unitary to all orders in $\lambda$ and independent of phase conventions. A Wolfenstein-like parametrisation of the CKM matrix can be derived up to an arbitrary power in the Cabibbo angle $\lambda = \sin(\theta_C)$, using the unitarity of the matrix to determine all its elements. A challenge for both experimentalists and theorists consist in extracting information on the underlying mechanism of CP violation from the wealth of data currently available, in the presence of the strong interaction that binds quarks into hadrons. Does the above CKM mechanism describe accurately the data? If yes, what are the values of $\lambda$, $A$, $\bar\rho$ and $\bar\eta$? If no, what is (are) the source(s) of CP violation beyond the Standard Model?

The CKMfitter group follows this program within the Rfit frequentist approach [1]. The likelihood function $\mathcal{L}$ is defined as the product $\mathcal{L}(y_{mod}) = \mathcal{L}_{exp}(x_{exp} - x_{the}(y_{mod})) \cdot \mathcal{L}_{the}(y_{QCD})$ where $x_{exp}$ denote experimental measurements and $x_{the}$ the corresponding theoretical predictions. $x_{the}$ depends on $y_{mod}$ which are either free parameters of the theory (e.g., the CKM matrix parameters) or hadronic quantities (e.g., form factors, decay constants… denoted $y_{QCD}$). Each

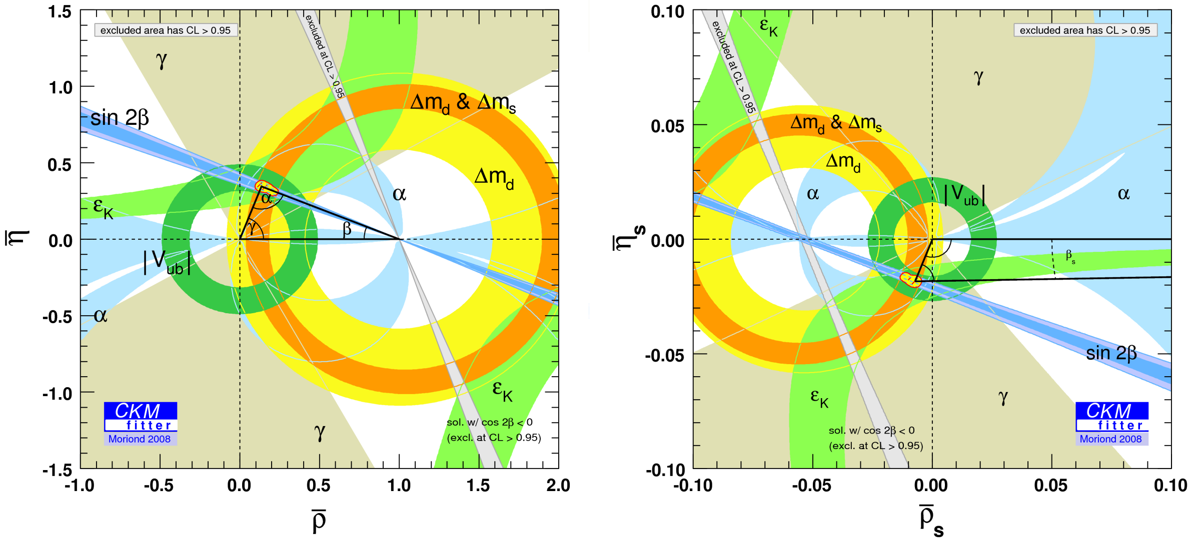


Figure 1: Constraints on the unitarity triangles corresponding to the $B_d$ (left) and $B_s$ (right) mesons.

individual measurement entering $\mathcal{L}_{exp}$ is considered as Gaussian by default (in the case of a non-Gaussian experimental measurement, the exact description of the associated likelihood is directly used in the fit) and correlations, if known, are taken into account. The uncertainties on the theoretical parameters $y_{QCD}$ define the allowed range of values for each parameter: $\mathcal{L}_{the}(y_{QCD}(i))$ is one within the allowed range and zero outside. The fit is performed on all the parameters $y_{mod}$ by minimizing $\chi^2(y_{mod}) \equiv -2\ln(\mathcal{L}(y_{mod}))$. For metrology (assuming a good agreement between data and theory), one splits $y_{mod} = (a, \mu)$, where $a$ are the parameters of interest(e.g., $\bar{\rho}, \bar{\eta}$) and $\mu$ are the remaining parameters. The minimum value $\chi^2_{min;\,\mu}(a)$ is computed for a set of fixed values $a$ while $\mu$ is allowed to vary. The Confidence Level represented on the plots is obtained from the $\chi^2$ difference $\Delta\chi^2(a) = \chi^2_{min;\,\mu}(a) - \chi^2_{min}$.

## 1 The global fit

The global fit involves a large set of constraints. At the time of the conference, recent and significant changes occurred for $|V_{ud}|$ and $|V_{us}|$, which will be discussed below. In addition, BABAR and Belle have presented new determinations of $\gamma$, based on the interference between the colour-allowed $B^- \to D^0K^-$ and colour-suppressed $B^- \to \bar{D}^0K^-$ decays. The accuracy of the method is driven by the size of $r_B = |A_{suppr}|/|A_{favour}| \simeq |V_{ub}V^*_{cs}|/|V_{cb}V^*_{us}| \times O(1/N_c)$ typically of order 0.1-0.2, and the different methods try to improve on this ratio by different choices of $D$ decay channels (GLW: $D$ into CP eigenstates, ADS: $D^{(*)}$ into doubly Cabibbo-suppressed states, GGSZ: $D^{(*)}$ into 3-body state and Dalitz analysis). For the GGSZ analysis, BABAR and Belle have increased their statistics and BABAR includes neutral $D$ into $K^0_SK^+K^-$. There has also been a $DK$ update from BABAR for GLW, and a similar update from Belle for ADS [2].

Combining these results yields $\gamma = (72^{+34}_{-30})^\circ$ (68% CL) which shows a rather mild improvement at 2 and 3 $\sigma$ with respect to combinations showed at previous conferences. The various methods provide values for $\gamma$, but also for the hadronic quantities such as $r_B$ or the relative strong phase $\delta$ between the two amplitudes. The current values for these quantities are not completely consistent among the methods, as illustrated in Fig. 2: the methods yield similar ranges for $\gamma$ but rather different values of the hadronic parameters. In such a situation with sev-

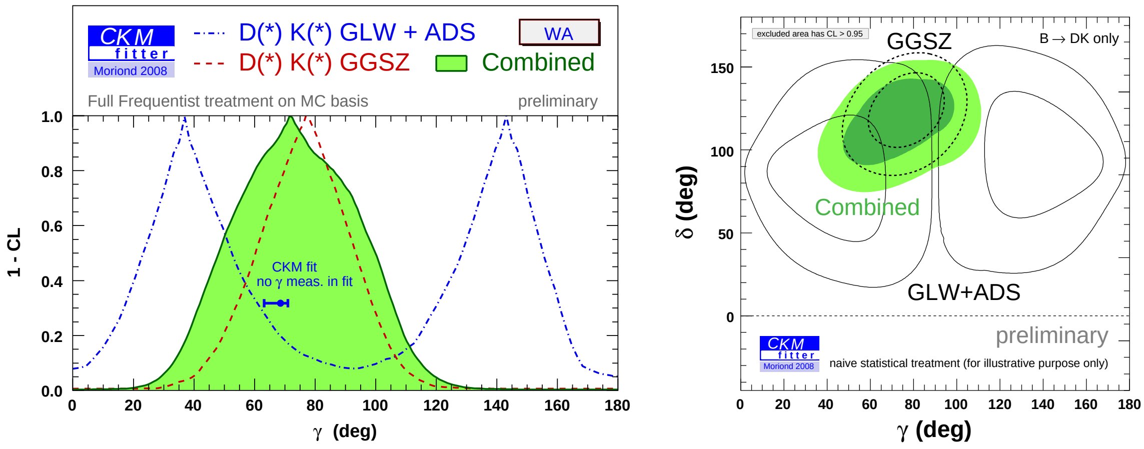


Figure 2: CL profile for $\gamma$ (left) and correlation between $\gamma$ and the strong phase $\delta$ for the different methods based on $B \to DK$ (right).

eral slightly incompatible solutions, the frequentist statistical treatment treats all the solutions on the same footing, leading to a broadening of the confidence intervals for $\gamma$ (a Bayesian analysis would integrate over hadronic parameters, so that different incompatible solutions sharing the same value of $\gamma$ yield an increase degree of belief in this value, reducing the uncertainty in the posterior p.d.f of $\gamma$) [2].

The outcome of the global fit is shown in Fig. 1 in the usual $(\bar\rho, \bar\eta)$ plane

$$A = 0.795^{+0.025}_{-0.015}, \qquad \lambda = 0.2252^{+0.0008}_{-0.0008}, \qquad \bar\rho = 0.135^{+0.033}_{-0.016}, \qquad \bar\eta = 0.345^{+0.015}_{-0.018} \tag{2}$$

but also in the $(\bar\rho_s, \bar\eta_s)$ plane defined as $\bar\rho_s + i\bar\eta_s = -(V_{us}V_{ub}^*)/(V_{cs}V_{cb}^*)$ and more suitable to discuss the CKM mechanism for the $B_s$ sector. The corresponding triangle $(V_{us}V_{ub}^*)/(V_{cs}V_{cb}^*) + 1 + (V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*) = 0$ is squashed, with 2 sides of $O(\lambda^0)$ and 1 side of $O(\lambda^2)$. $\beta_s = \arg[-V_{cs}V_{cb}^*/(V_{ts}V_{tb}^*)]$, the angle opposite to the small side, is related to $B_s$ mixing in the SM. The global fit yields a small and well-predicted value $\beta_s = -0.0183^{+0.0009}_{-0.0008}$ rad, with which recent flavour-tagged $B_s^0 \to J/\psi\phi$ analysis from CDF and D0 present some tension [3]. The two experiments used different assumptions for their analyses (strong phases, width of the $B_s$ meson) and obtained nontrivial likelihoods. It seems sensible to wait for a combined analysis within a common framework and for a larger data sample before claiming a hint of NP in the $B_s$ sector.

## 2 $B \to V\gamma$

It has been known for a long time that the loop processes $b \to (d,s)\gamma$ can give an access to $|V_{t(d,s)}|$ which complement $\Delta m_{d,s}$ in an interesting fashion: we can test penguin versus box diagrams, so that an inconsistency between the two determinations, and with the global fit, would teach us in which direction to look for NP. Inclusive $B \to X_s$ decays have been computed with a high accuracy [4], but one can also consider exclusive $B \to V\gamma$ decays. The first attempts to compute the corresponding amplitudes used a factorisation approach [5,6,7]. It was in particular used to determine

$$R_{\rho/\omega} = \frac{\overline{\mathcal{B}}(\rho^\pm\gamma) + \frac{\tau_{B^\pm}}{\tau_{B^0}}\left[\overline{\mathcal{B}}(\rho^0\gamma) + \overline{\mathcal{B}}(\omega\gamma)\right]}{\overline{\mathcal{B}}(K^{*\pm}\gamma) + \frac{\tau_{B^\pm}}{\tau_{B^0}}\left[\overline{\mathcal{B}}(K^{*0}\gamma)\right]} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \left(\frac{1 - m_\rho^2/m_B^2}{1 - m_{K^*}^2/m_B^2}\right)^3 \frac{1}{\xi^2}\left[1 + \Delta R\right] \tag{3}$$

where $\xi$ is a ratio of form factors and $\Delta R$ is a correction from hadronic physics estimated as $\Delta R = 0.1 \pm 0.1$. This important step left many questions open. What is the dependence of $\Delta R$

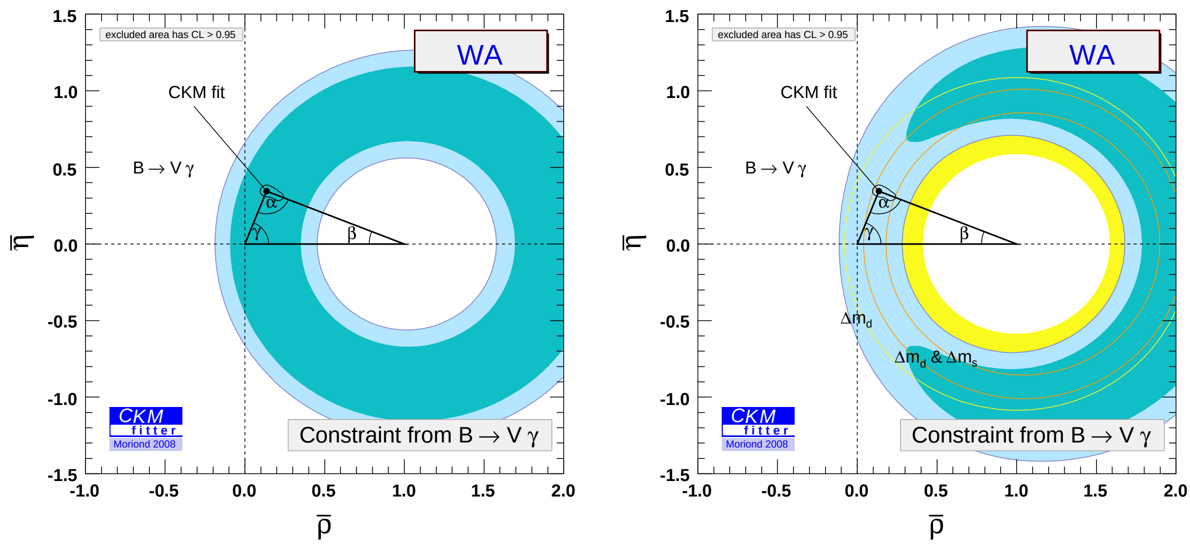


Figure 3: Constraints on the unitarity triangle from $B \to V\gamma$ using the simplified expression of $R_{\rho/\omega}$ (left) and considering the available branching ratios and their theoretical expressions with $1/m_b$ corrections (right).

on the CKM matrix elements? Can one estimate and exploit isospin breaking? How to estimate weak annihilation processes, which for $(\rho,\omega)\gamma$ occurs at tree level and can be large, despite a formal $1/m_b$ suppression. A further step was proposed by estimating $1/m_b$-suppressed terms, missed in QCD factorisation or in SCET, through light-cone sum rules [8]. For each final state, all contributions can be expressed as a factor to the leading amplitude, i.e., the magnetic operator $Q_7 = (e/8\pi^2) m_b \bar{D}\sigma^{\mu\nu}(1+\gamma_5)F_{\mu\nu}\, b$:

$$\bar{A} \equiv \frac{G_F}{\sqrt{2}} \left(\lambda_u^D a_7^u(V) + \lambda_c^D a_7^c(V)\right) \langle V\gamma|Q_7|\bar{B}\rangle \qquad \lambda_U^D = V_{UD}^* V_{Ub}\,, \tag{4}$$

where $D = d, s$ and the coefficient $a_7^U(V) = a_7^{U,\mathrm{QCDF}}(V) + a_7^{U,\mathrm{ann}}(V) + a_7^{U,\mathrm{soft}}(V)$ is the sum of three terms. QCDF denotes the result from QCD factorisation at leading-order in $1/m_b$ and up to $O(\alpha_s)$ corrections, whereas ann and soft correspond to weak-annihilation and soft-gluon contributions. The latter are $1/m_b$-suppressed contributions which can be computed within QCD factorisation, but can be estimated through light-cone sum rules.

In this approach, each decay is described individually and the short- and long-distance contributions of $u$ and $c$ internal loops can be identified (they are note combined in a single correction $\Delta R$). For the evaluation, we followed refs. [6,8] and for the expressions of $a_7$'s and hadronic inputs (form factors, distribution amplitudes), using leading-order Wilson coefficients and the HFAG averages for the branching ratios (in units of $10^{-6}$) [9]:

$$K^{*-}\gamma : 40.3 \pm 2.6, \quad K^{*0}\gamma : 40.1 \pm 2.0, \quad \rho^+\gamma : 0.88^{+0.28}_{-0.26}, \quad \rho^0\gamma : 0.93^{+0.19}_{-0.18}, \quad \omega\gamma : 0.46^{+0.20}_{-0.17},$$

together with the Belle value $B(B_s \to \phi\gamma) = (57^{+18+12}_{-15-11}) \cdot 10^{-6}$. Fig. 3 shows the improvement from the previous treatment. The constraint is not a perfectly circular ring, due to the (previously neglected) sensitivity to other CKM matrix elements in the decay amplitude. The constraints from $B \to V\gamma$ and from neutral $B$ meson mixing have been superimposed to illustrate the compatibility of the two determinations, and their complementarity (we compare box and penguin processes with different theory sources). The study of CP asymmetries should provide further information on the apex of the $B$-meson unitarity triangle.

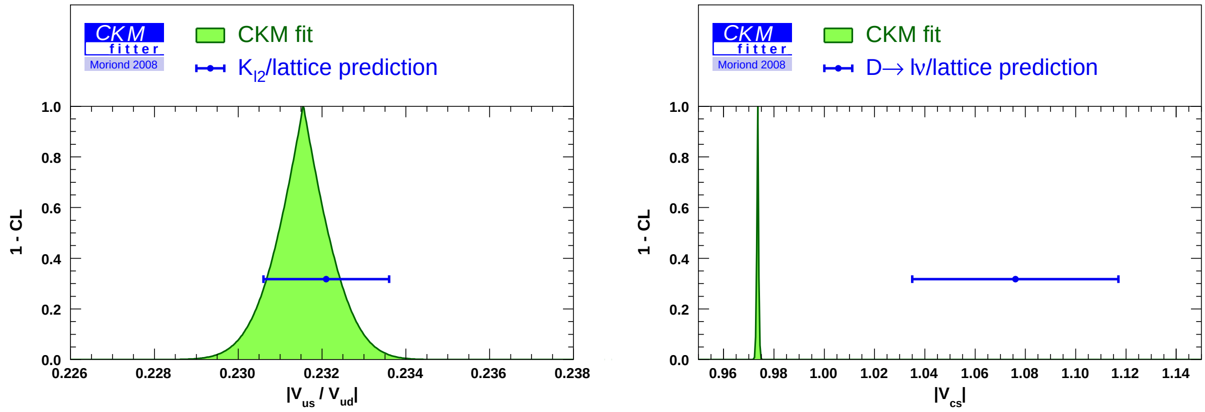


Figure 4: Constraints on $|V_{us}/V_{ud}|$ (left) and $|V_{cs}|$ (right). On each plot, the direct determination is compared to the prediction based on the global fit.

## 3 Lighter quarks and the lattice

The above constraints derived from $b$ transitions can be translated into values of CKM matrix elements involving lighter quark and they can be compared to direct measurements which have recently improved. Indeed, some lattice simulations with three dynamical light quarks (unquenched) are available with astoundingly small systematics, thus reducing QCD uncertainties.

As a first example, $|V_{ud}|$ has benefited from an improved analysis of super-allowed $\beta$ decays of nuclei, whereas $|V_{us}|$ has a shrinking uncertainty due to recent experimental results on $K_{\ell 3}$ and an improved lattice estimate of the relevant form factor $f_+(0) = 0.964(5)$ (domain-wall fermions, UKQCD+RBC) [10,11]. Both values are used in the global fit, but an interesting cross-check consists in comparing the value of $|V_{us}/V_{ud}|$ from the fit with the value obtained by combining the measured ratio of leptonic decays $K \to \ell\nu/\pi \to \ell\nu$ with the lattice ratio of decay constant $f_K/f_\pi = 1.189(7)$ (staggered fermions, HPQCD+UKQCD) [12]. The agreement shown on the left of Fig. 4 is remarkable, $f_K/f_\pi$ being notoriously very difficult to compute on the lattice (it involves only light quarks and the chiral extrapolation can yield large uncertainties).

A second example is the charm sector, which has always been thought of as a favourite place to test lattice QCD, since $m_c$ is close to the typical hadronic scale of 1 GeV. Lattice computations of form factors and decay constants should pin down $|V_{cd}|$ and $|V_{cs}|$ to a high accuracy. We illustrate the current improvement in the field in Fig. 5. The constraints on the nucleon and the kaon provide only a mild constraint, since $|V_{ud}| \simeq |V_{cs}|$ and $|V_{cd}| \simeq |V_{us}|$ only at first non trivial order in $\lambda$ (one needs an input from another sector to fix higher orders). The $B$ sector alone constrains $|V_{cd}|$ and $|V_{cs}|$ tightly and the combination of all indirect constraints turns out to be very powerful. We have also represented the direct constraints for $|V_{cd}|$, from $\nu N$ scattering, and for $|V_{cs}|$ from charmed-tagged $W$ decays (left) and from CLEO-c results on $D \to K\ell\nu$ (right) [13]. The distorted shape of these regions comes from $|V_{cd}|^2 + |V_{cs}|^2 \leq 1$.

These results for lighter quarks seem to confirm both the consistency of the CKM picture and the high accuracy advocated by lattice results. However, a recent result has shattered this beautiful convergence. Indeed, CLEO-c and Belle have both measured the leptonic decay $D_s \to \ell\nu$, whereas a related unquenched lattice result $f_{D_s} = 241 \pm 3$ MeV (staggered, HPQCD+UKQCD) [12]. This yields to $|V_{cs}| = 1.076 \pm 0.041$ in flat disagreement with unitarity and with the fit value $|V_{cs}| = 0.97351^{+0.00020}_{-0.00022}$, as shown on Fig. 4 (right) [14]. This result is quite unsettling since the $D_s$ involves only strange and charm valence quarks and should be an ideal place for lattice simulations, whereas NP is not supposed to play a major role for such mesons. Paradoxically, the much more complicated $f_K/f_\pi$ led to an impressive agreement of experiment

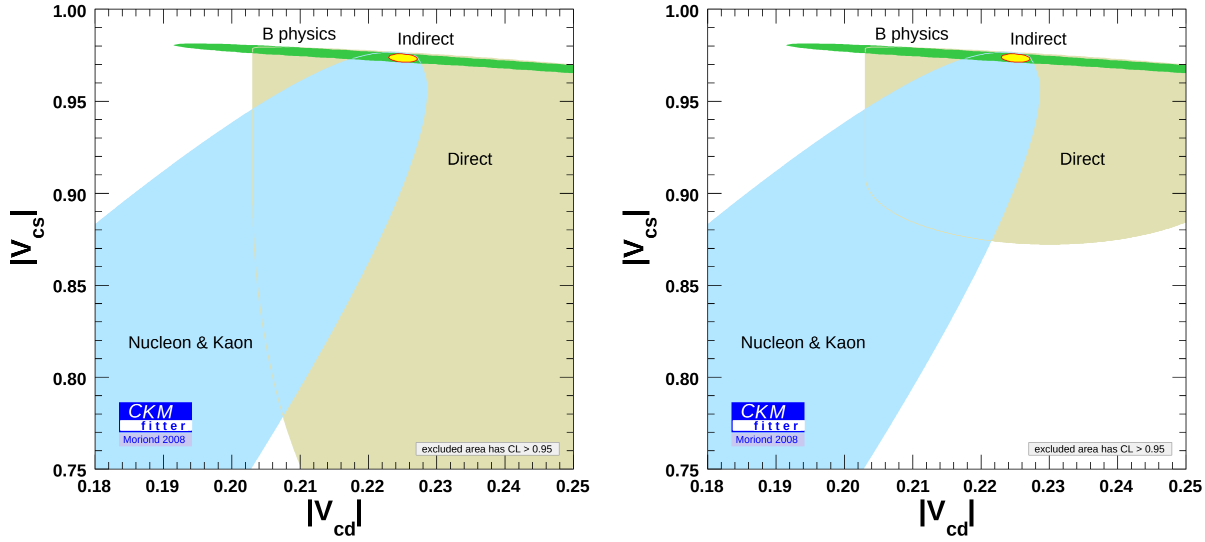


Figure 5: Constraints on $|V_{cd}|$ and $|V_{cs}|$ using PDG 07 (left) and with the new CLEO-c data on $D \to K\ell\nu$ (right).

and theory, while $f_{D_s}$ points towards either uncontrolled systematics in unquenched lattice simulations (due to dynamical quarks?), overlooked systematics in the experimental measurements (radiative corrections?), or NP [15]. In any case, interesting news should come from this sector.

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