

## Physique des Saveurs et Matrice CKM : un état des lieux

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pour le groupe CKMfitter.

# OUTLINE

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- ✓ Introduction to unitarity triangle
- ✓ SM metrology : the global CKM fit
  - ✓ Angles
  - ✓ Sides
  - ✓ Summary and CKM mechanism tests
- ✓ Where to search for NP ?
- ✓ Conclusions

# 1. Introduction to unitarity triangle in the Standard Model

The transition between quark flavours by charged current are described in the Standard Model by a unitary complex (nXn) matrix, n being the number of quarks weak isospin doublets.

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

With n=3, the CKM matrix can be written from 3 real parameters and one phase  $\delta_{\text{CKM}}$ . CP violation means  $\delta_{\text{CKM}}$  non zero.

Most popular parametrisation is due to Wolfenstein.

unitarity-exact and convention-independent version of the Wolfenstein parametrization

$$\lambda^2 \equiv \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2}$$

SFP 2007

$$A^2 \lambda^4 \equiv \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2}$$

CKM

$$\bar{\rho} + i\bar{\eta} \equiv -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

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# 1. Introduction to unitarity triangle in the Standard Model

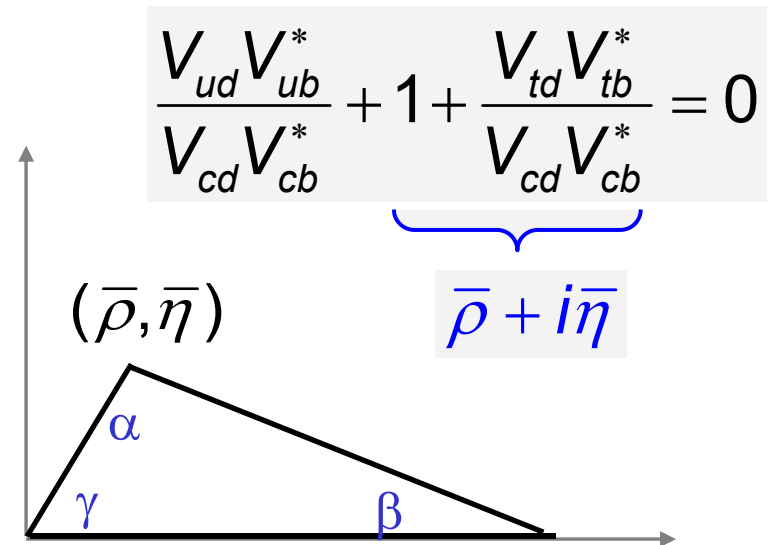
## FOUR UNKNOWNNS

- ✓  $\lambda$  from  $|V_{ud}|$  (nuclear transitions) and  $|V_{us}|$  (semileptonic  $K$  decays) (0.5% precision)
- ✓  $A$  from  $|V_{cb}|$  (incl. and excl. semileptonic  $B$  decays) (2% precision)
- ✓ The 2 other parameters are the subject of the following.

The unitarity relations can be elegantly displayed by triangles in the complex plane. One is not flat:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\left( \propto A\lambda^3 \quad \propto -A\lambda^3 \quad \propto A\lambda^3 \right)$$

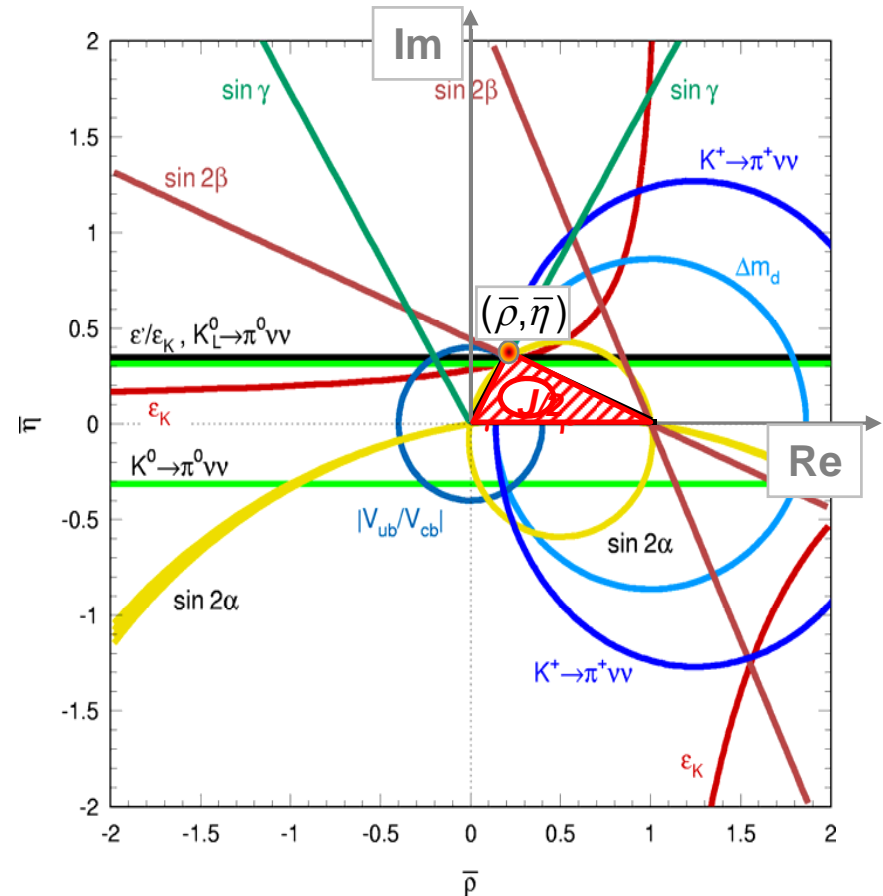


# 1. Introduction to unitarity triangle in the Standard Model

The game consists in over-constraining the SM apex of the triangle w/ as much as possible observables for which a good theory control exists.

If overall agreement (and it is !), then can do the metrology of the Standard Model parameters.

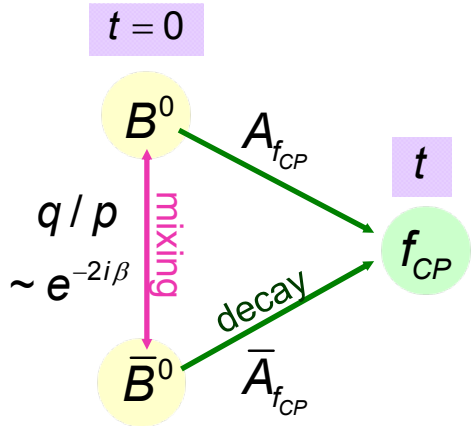
Not a word on kaons in this presentation (covered later).





# 2. The global CKM fit : some details on the inputs

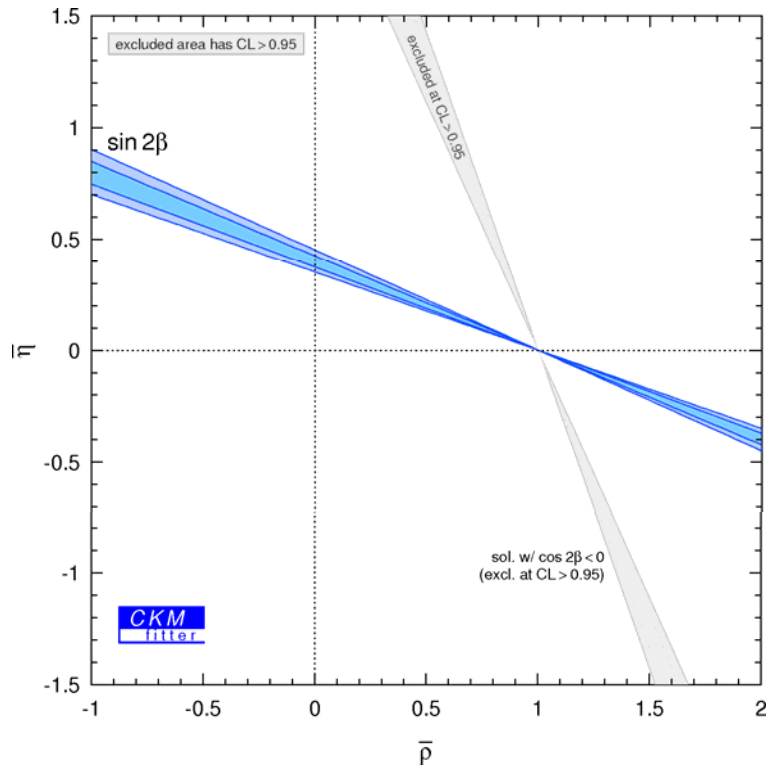
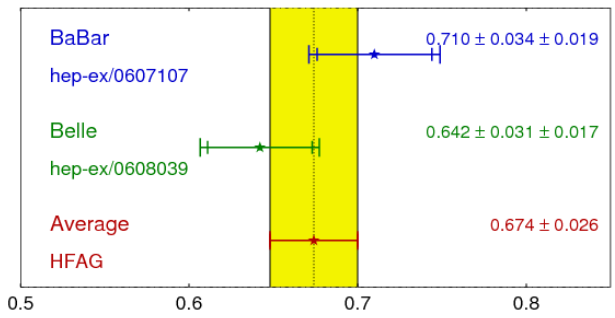
The inputs :  $\sin 2\beta$



$$A_{f_{CP}}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow f_{CP}) - \Gamma(B^0(t) \rightarrow f_{CP})}{\Gamma(\bar{B}^0(t) \rightarrow f_{CP}) + \Gamma(B^0(t) \rightarrow f_{CP})}$$

$$= S_{f_{CP}} \sin(\Delta m_d t) - C_{f_{CP}} \cos(\Delta m_d t)$$

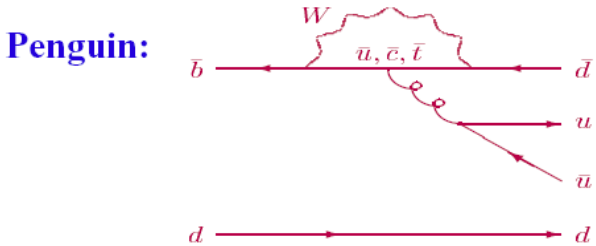
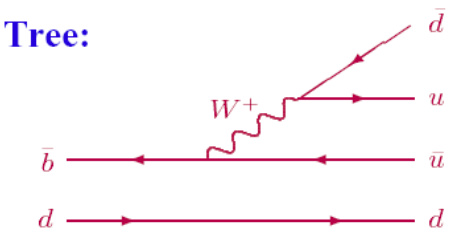
$\sin(2\beta) \equiv \sin(2\phi_1)$  **HFAG**  
ICHEP 2006  
PRELIMINARY



# 2. The global CKM fit : some details on the inputs

The inputs :  $\alpha$  w/  $\pi\pi$  and  $\rho\rho$

Actually  $\alpha = \pi - (\beta + \gamma)$   
 Interference between mixing and decay but significant penguin contributions.

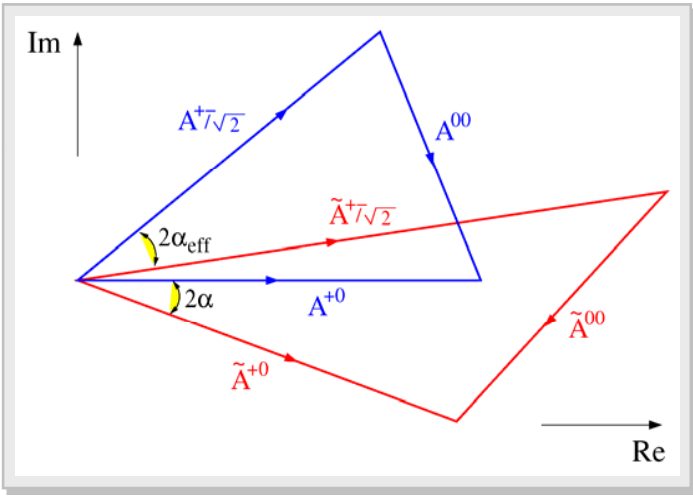


Time-dependent  $CP$  analysis of  $B^0 \rightarrow \pi^+\pi^-$  determines a biased  $\alpha_{\text{eff}}$ . We want  $\alpha$ .

$$A_{h^+h^-}(t) = S_{h^+h^-} \sin(\Delta m_d t) - C_{h^+h^-} \cos(\Delta m_d t)$$

$$= \sqrt{1 - C_{h^+h^-}^2} \sin(2\alpha_{\text{eff}}) \cdot \sin(\Delta m_d t) - C_{h^+h^-} \cos(\Delta m_d t)$$

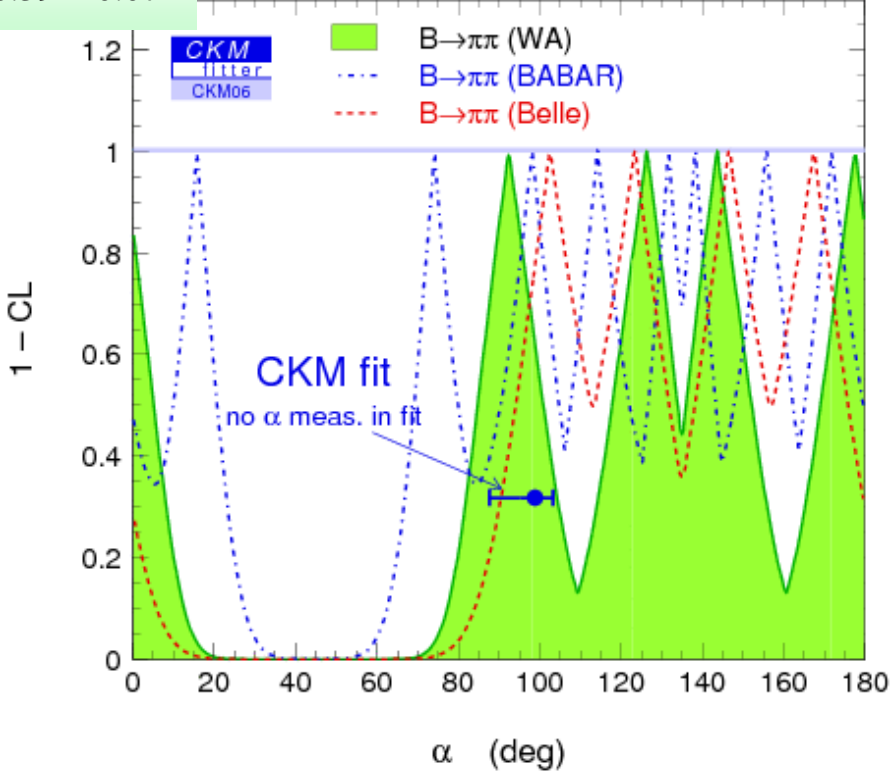
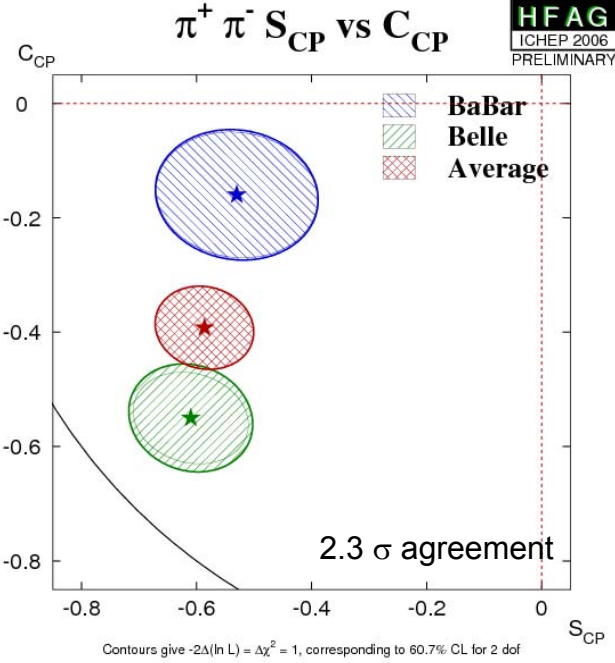
Thanks to  $SU(2)$  symmetry, use all  $\pi\pi$  to resolve  $\alpha$  up to an **8-fold ambiguity** within  $[0, \pi]$ . True also for  $\rho\rho$  provided that mostly longitudinally polarised (true).



# 2. The global CKM fit : some details on the inputs

The inputs :  $\alpha$  the  $(\pi\pi)$  system

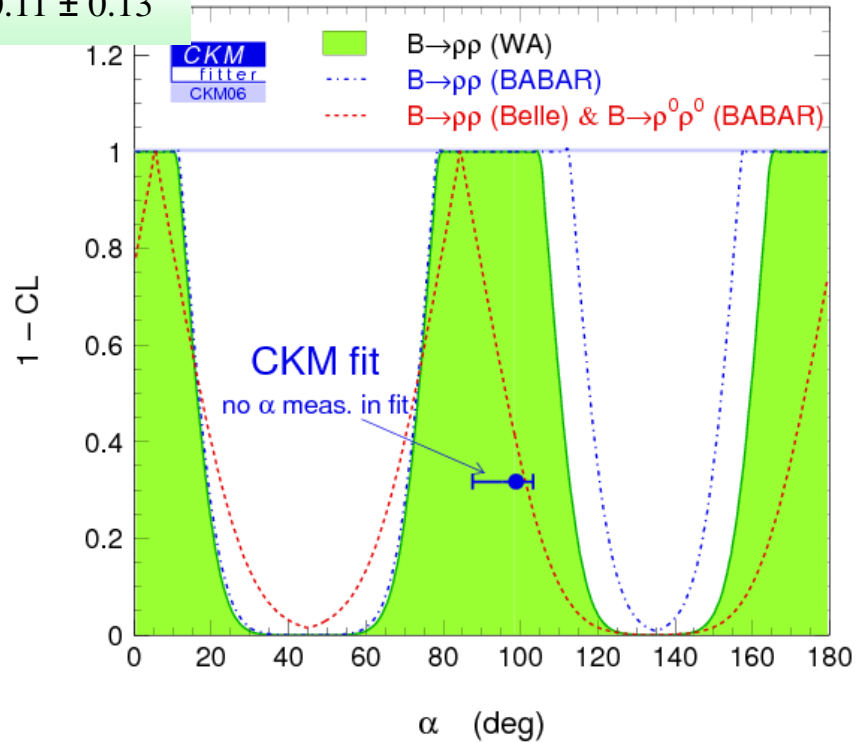
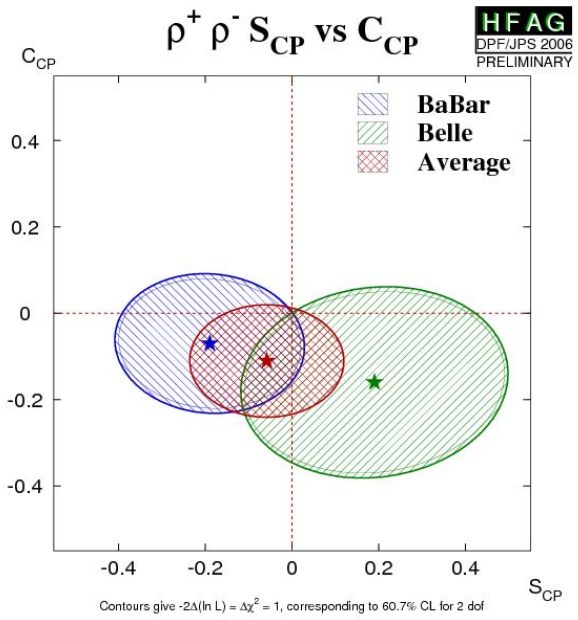
	BABAR (347m)	Belle (532m)	Average
$S_{\pi\pi}$	$-0.53 \pm 0.14 \pm 0.02$	$-0.61 \pm 0.10 \pm 0.04$	$-0.58 \pm 0.09$
$C_{\pi\pi}$	$-0.16 \pm 0.11 \pm 0.03$	$-0.55 \pm 0.08 \pm 0.05$	$-0.39 \pm 0.07$



# 2. The global CKM fit : some details on the inputs

The inputs :  $\alpha$  the ( $\rho\rho$ ) system

	BABAR (347m)	Belle (535m)	Average
$S_{\rho\rho}$	$-0.19 \pm 0.21$	$0.19 \pm 0.30 \pm 0.07$	$-0.06 \pm 0.18$
$C_{\rho\rho}$	$-0.07 \pm 0.15 \pm 0.06$	$-0.16 \pm 0.21 \pm 0.07$	$-0.11 \pm 0.13$

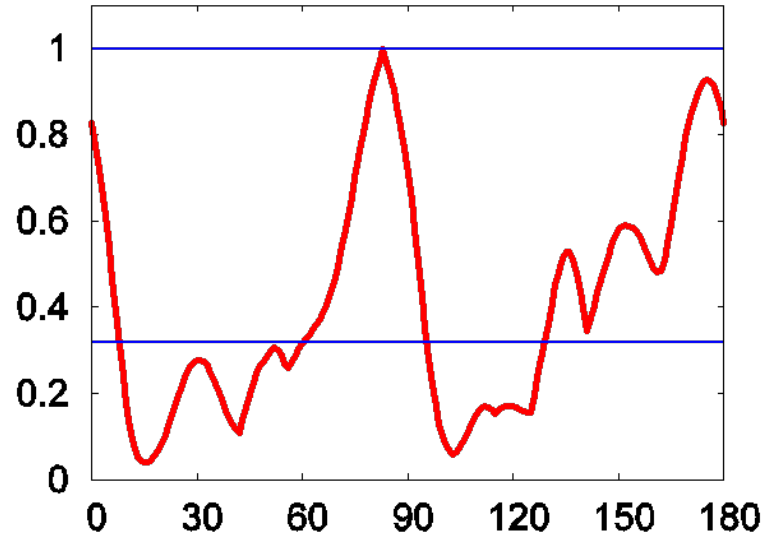
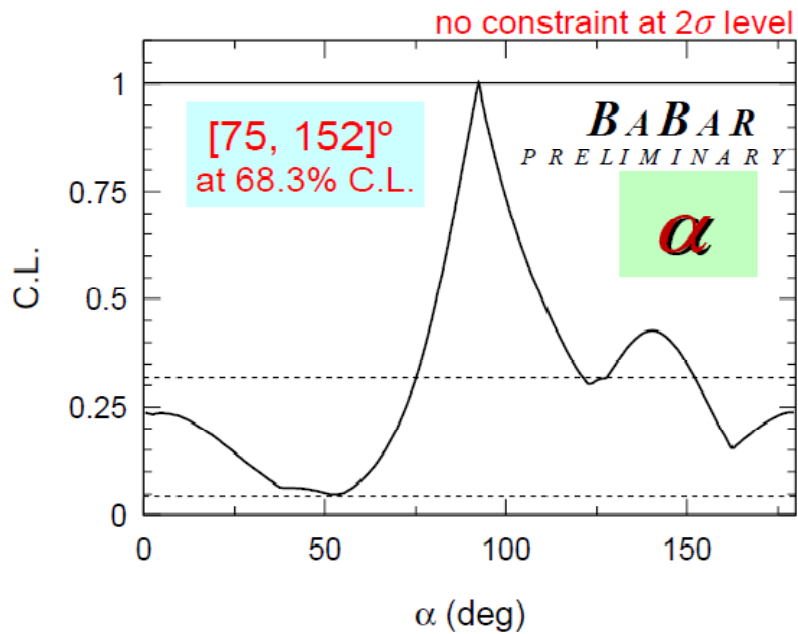


# 2. The global CKM fit : some details on the inputs

The inputs :  $\alpha$  the  $(\rho\pi)$  system

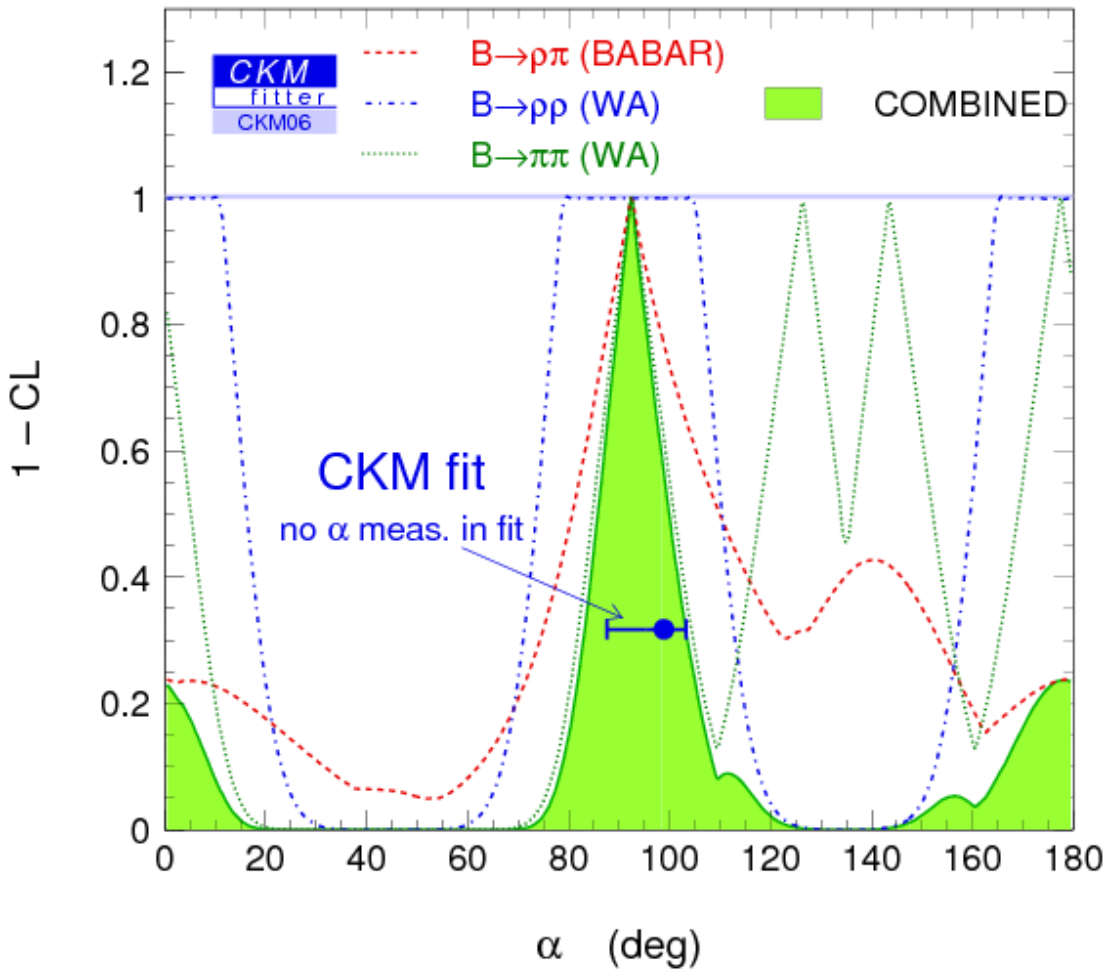
Dominant mode  $\rho^+\pi^-$  is not a  $CP$  eigenstate  
Amplitude interference in Dalitz plot

- ✓ simultaneous fit of  $\alpha$  and strong phases
- ✓ Measure 26 (27) bilinear Form Factor coefficients
- ✓ correlated  $\chi^2$  fit to determine physics quantities



# 2. The global CKM fit : some details on the inputs

The inputs :  $\alpha$  all together



$$\alpha_{\pi\pi} = (92^{+12}_{-10})^\circ$$

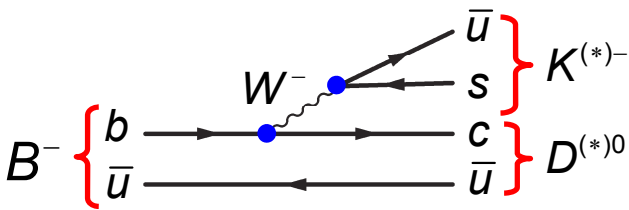
$$\alpha_{\rho\rho} = (92 \pm 21)^\circ$$

COMBINED

$$\alpha = (93^{+11}_{-9})^\circ$$

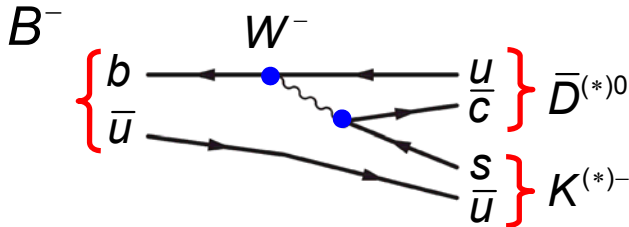
# 2. The global CKM fit : some details on the inputs

The inputs :  $\gamma$



$$\propto V_{cb} V_{us}^*$$

$$\propto \lambda^3$$



$$\propto V_{ub} V_{cs}^*$$

$$\propto \lambda^3 \sqrt{\rho^2 + \eta^2}$$

$$r_B = \frac{|A(b \rightarrow u)|}{|A(b \rightarrow c)|}$$

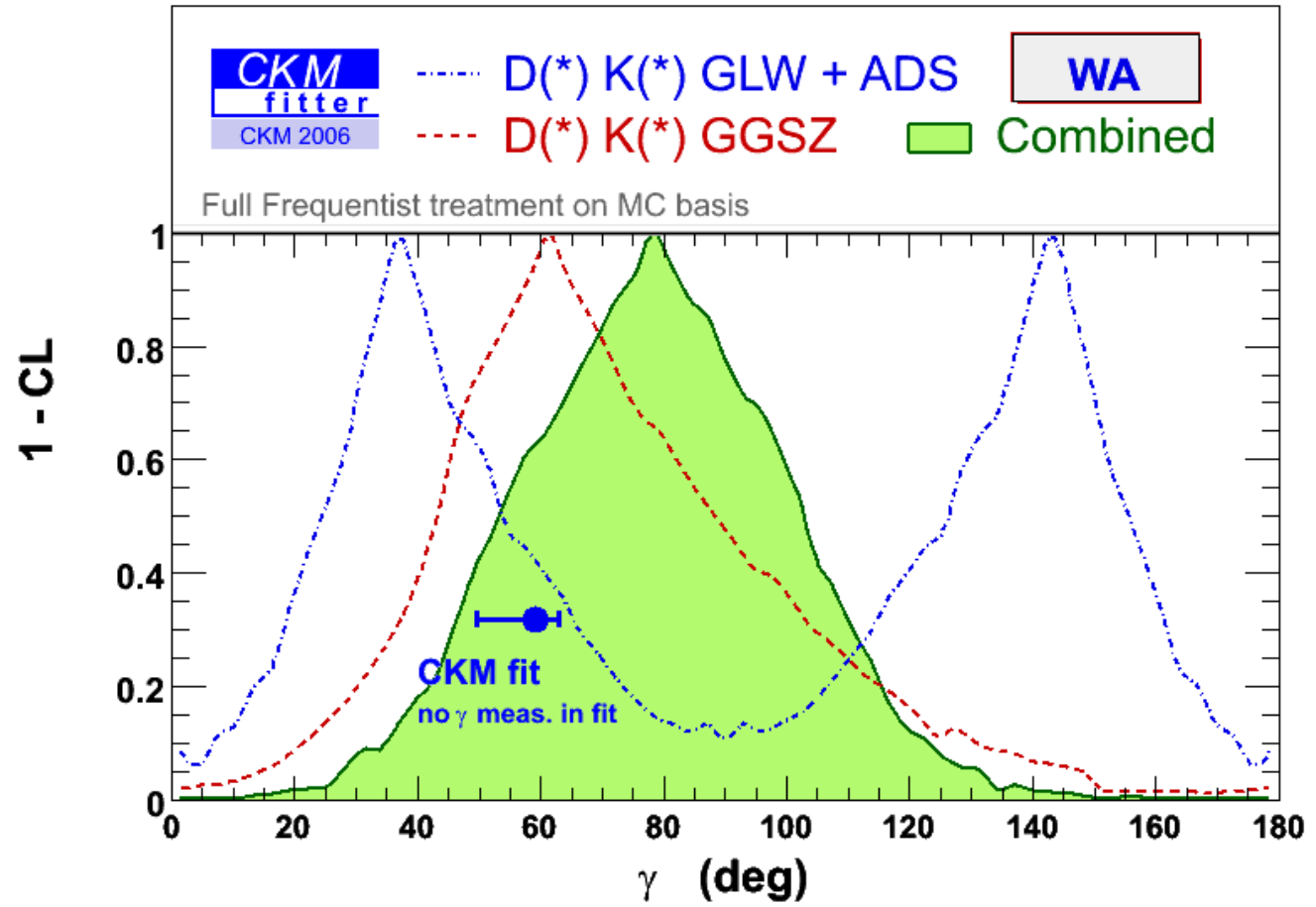
Here lies  $\gamma$

- ✓ **GLW** :  $D^0$  decays into  $CP$  eigenstate
- ✓ **ADS** :  $D^0$  decays to  $K^-\pi^+$  (favored) and  $K^+\pi^-$  (suppressed)
- ✓ **GGSZ** :  $D^0$  decays to  $K_S\pi^+\pi^-$  (interference in Dalitz plot)
  - ✓ **All methods fit simultaneously:  $\gamma$ ,  $r_B$  and strong phase (a priori different).**

$\sigma_\gamma$  depends significantly on the value of  $r_B$ .

# 2. The global CKM fit : some details on the inputs

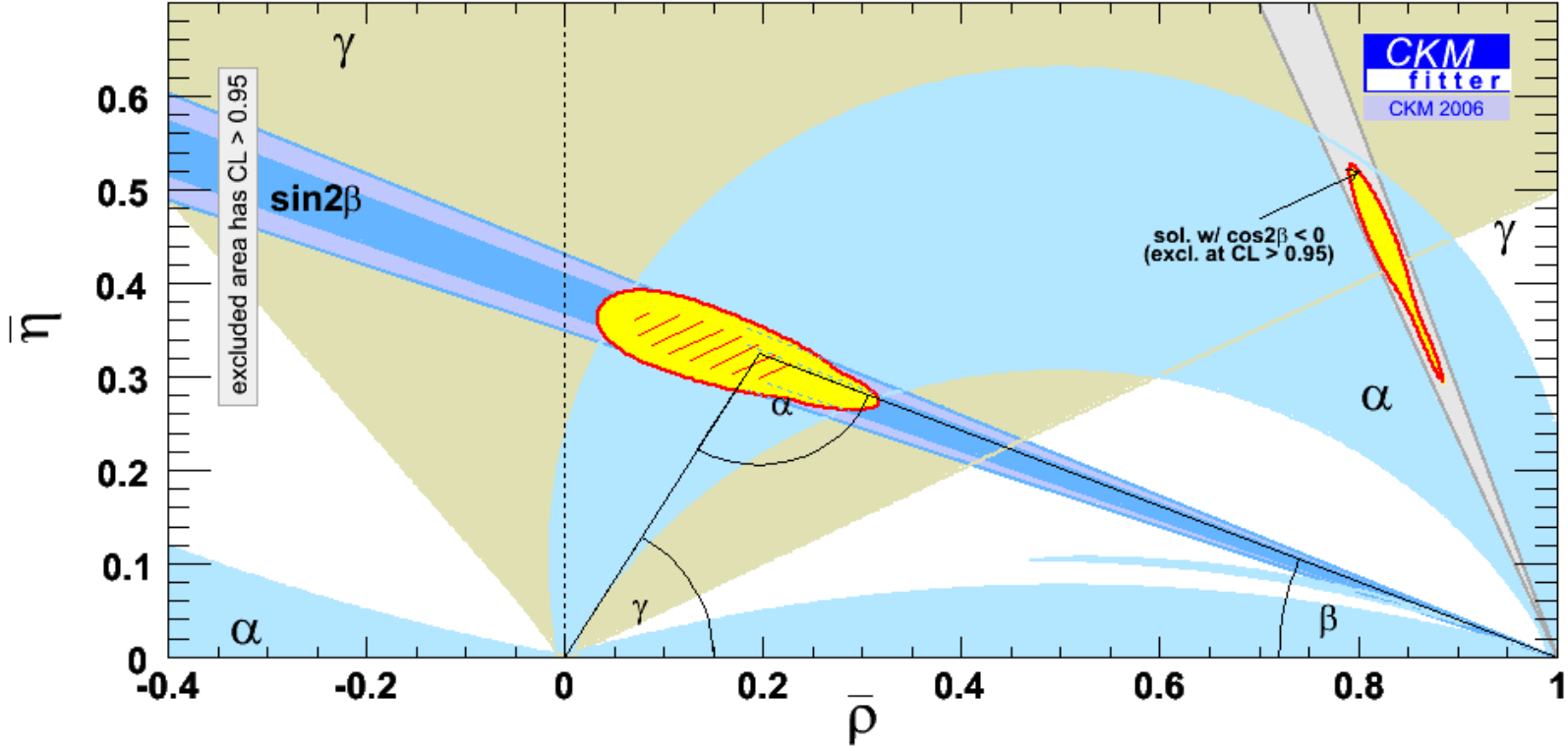
The inputs :  $\gamma$



$$\gamma = (77 \pm 31)^\circ$$

# 2. The global CKM fit

The apex from the angles – remarkable achievement from B-factories



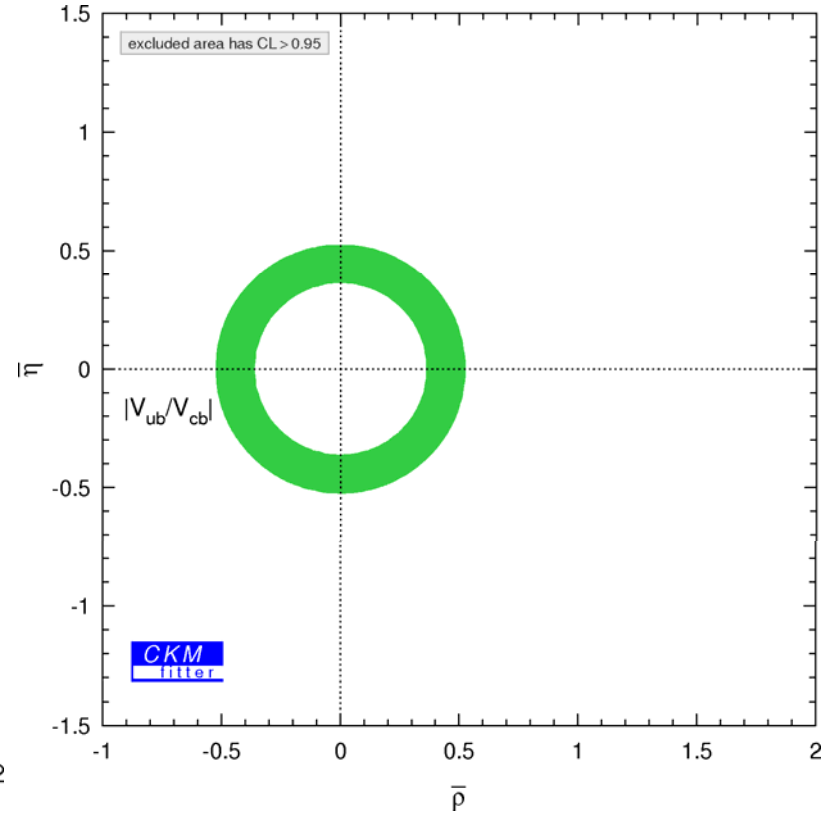
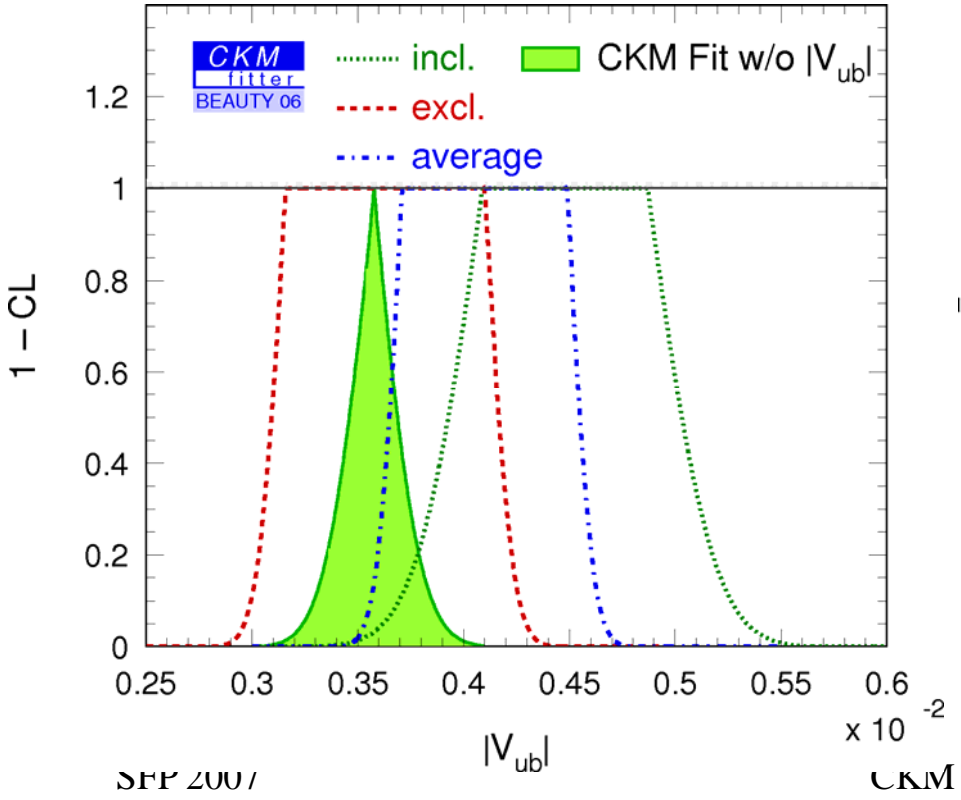
# 2. The global CKM fit : Sides

The inputs :  $|V_{ub}|$  and  $|V_{cb}|$

$$|V_{cb}|_{\text{incl.}} [10^{-3}] = 41.70 \pm 0.70$$

$$|V_{ub}| [10^{-3}] = 4.10 \pm 0.09_{\text{exp}} \pm 0.39_{\text{theo}}$$

Average of inclusive and exclusive measurement from semileptonic decays.  $V_{ub}$  is very difficult to extract both from experimental and theoretical sides.



## 2. The global CKM fit : Sides

The inputs :  $\Delta m_d$  and  $\Delta m_s$

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_{B_d} m_W^2 \eta_B S_0(x_t) f_{B_d}^2 B_d |V_{td} V_{tb}^*|^2 \propto A^2 \lambda^6 [(1 - \bar{\rho})^2 + \bar{\eta}^2]$$

$$\Delta m_s = \frac{G_F^2}{6\pi^2} m_{B_s} m_W^2 \eta_B S_0(x_t) f_{B_s}^2 B_s |V_{ts} V_{tb}^*|^2$$

✓ Large hadronic uncertainties, but

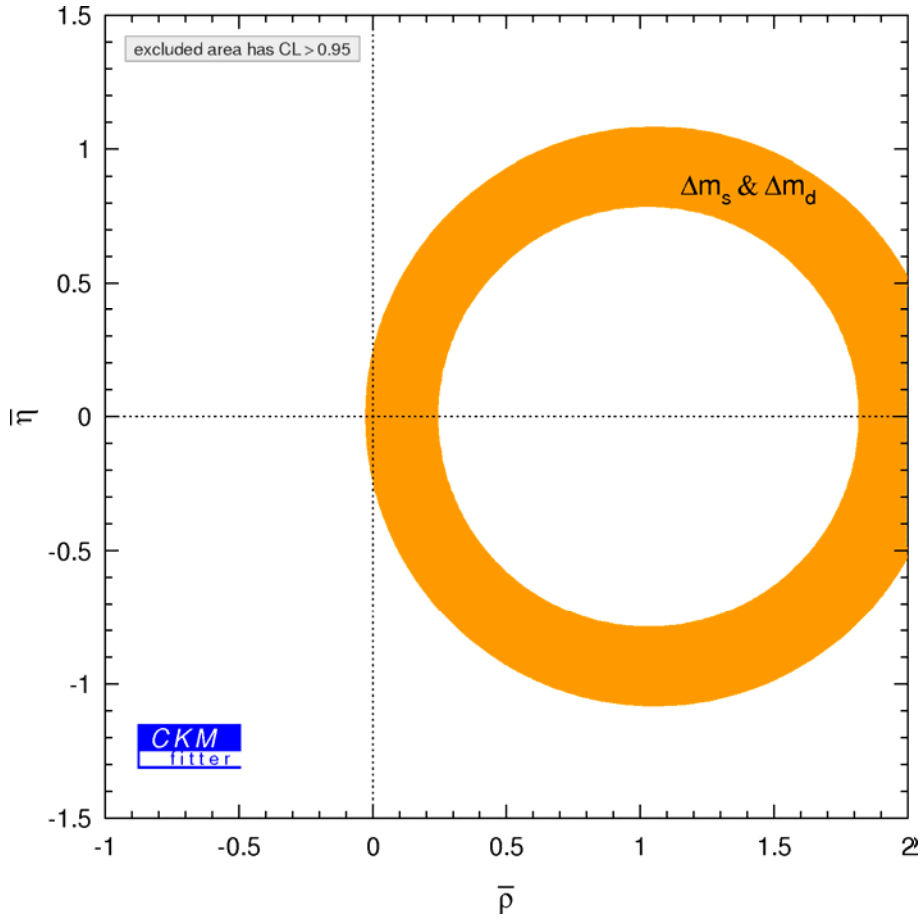
✓ the ratio  $f_{B_s}^2 B_s / f_{B_d}^2 B_d$  is much better known than  $f_{B_d}^2 B_d$  from Lattice QCD

$$\sigma_{rel}(f_{B_d}^2 B_d) \approx 35\% \quad \text{and} \quad \sigma_{rel}\left(\frac{f_{B_d}^2 B_d}{f_{B_s}^2 B_s}\right) \approx 10\%$$

✓ Though  $\Delta m_s$  has very weak dependence on  $\rho$  and  $\eta$ , it's very powerful to improve the  $\Delta m_d$  constraint.

# 2. The global CKM fit : Sides

The inputs :  $\Delta m_d$  and  $\Delta m_s$



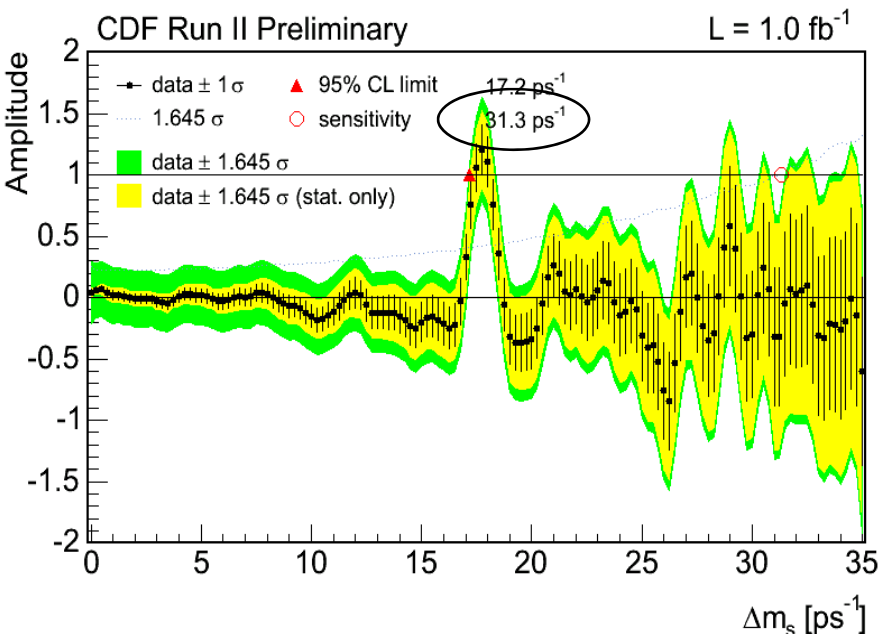
# 2. The global CKM fit : Sides

The inputs :  $\Delta m_d$  and  $\Delta m_s$

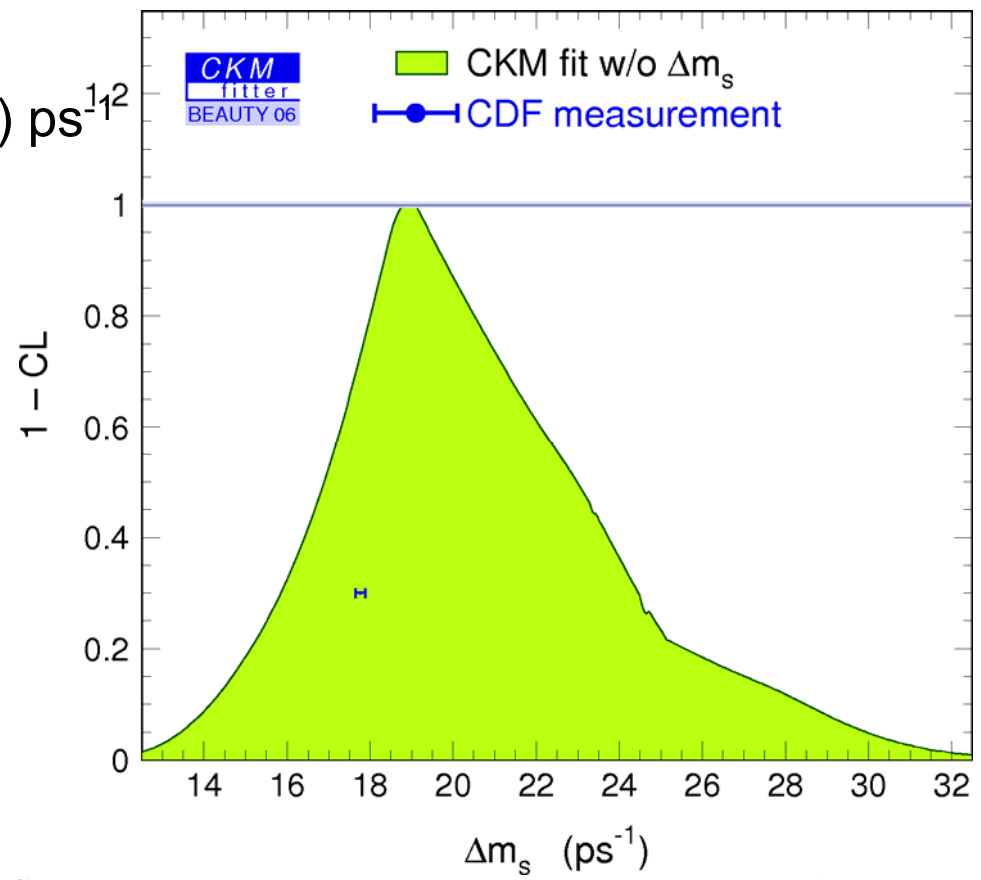
Bs oscillations eventually resolved at CDF – at the value suggested by LEP

## CDF MEASUREMENT (2006)

$\Delta m_s : 17.77 \pm 0.10(\text{stat.}) \pm 0.07 (\text{syst.}) \text{ ps}^{-112}$



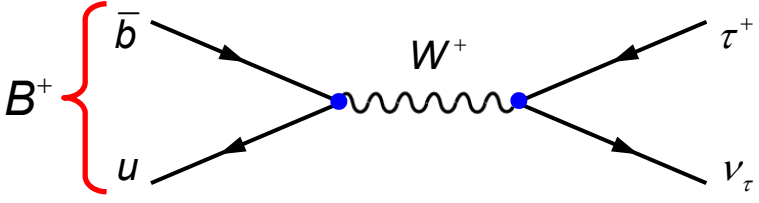
SFP 2007



CKM

# 2. The global CKM fit : Sides

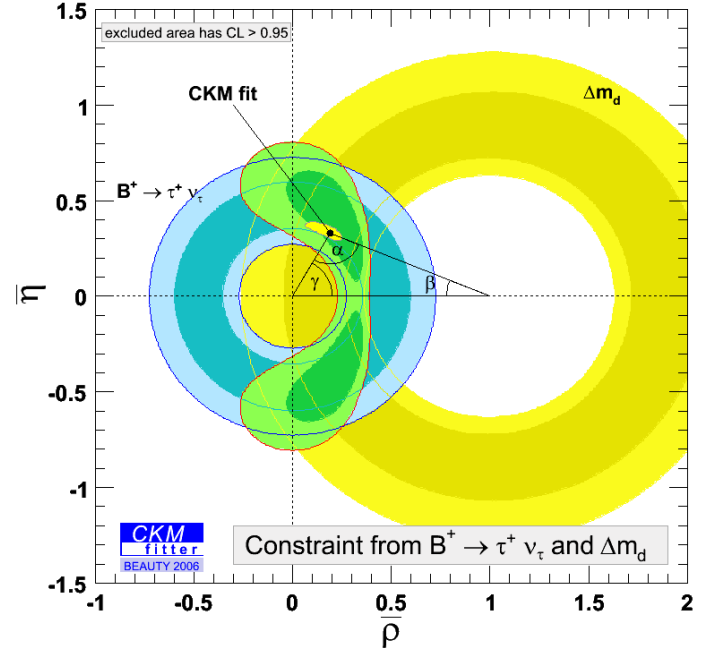
The inputs :  $B^+ \rightarrow \tau^+ \nu_\tau$



$$BR(B^+ \rightarrow \tau^+ \nu) = \frac{G_F^2 m_B \tau_B}{8\pi} m_\tau^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2$$

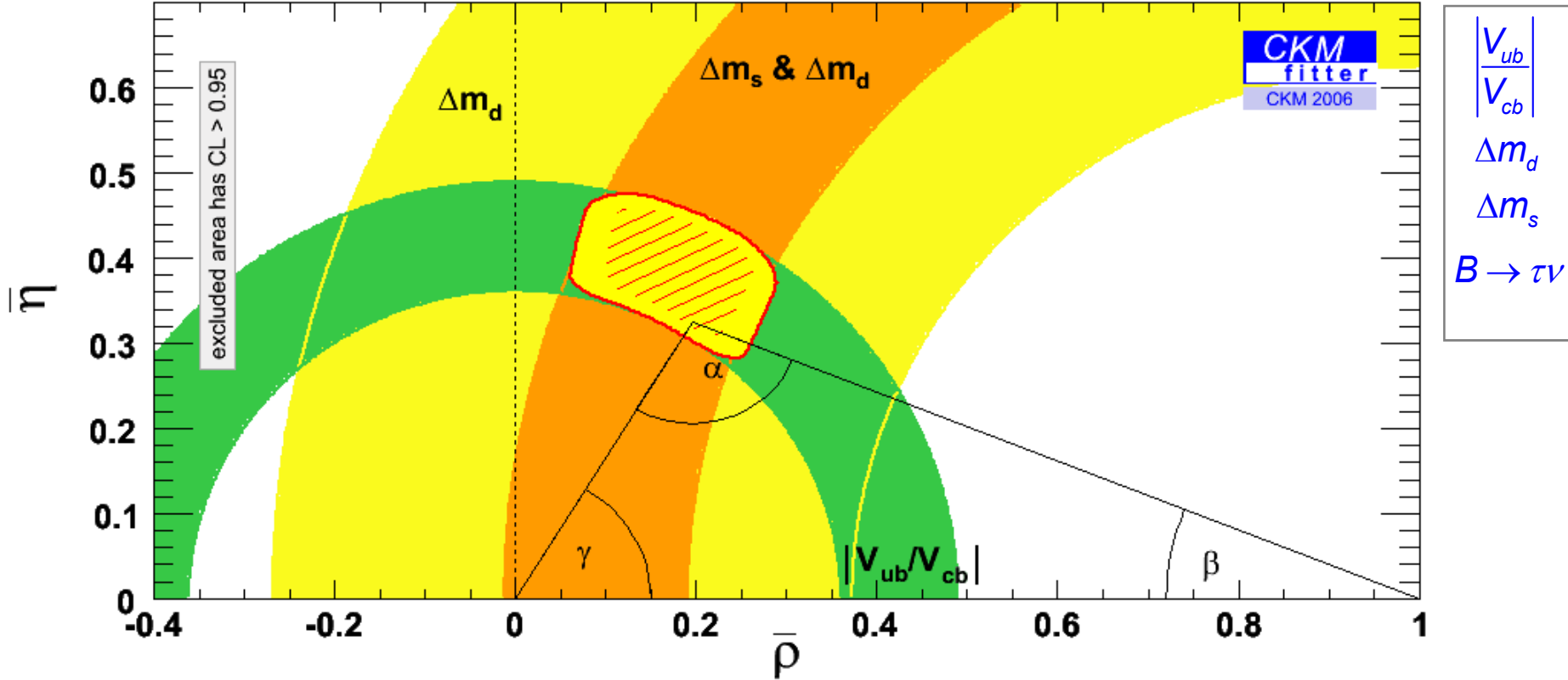
- ✓ Powerful together with  $\Delta m_d$  to remove hadronic uncertainty on  $f_B$
- ✓ Remarkable agreement between measurements and prediction.

	$BF(B^+ \rightarrow \tau^+ \nu_\tau) / 10^{-4}$
BaBar	$1.06^{+0.34}_{-0.28} (stat) {}^{+0.18}_{-0.16} (syst)$
Belle	$1.79^{+0.56}_{-0.49} (stat) {}^{+0.39}_{-0.46} (syst)$
CKM Fit	$0.92^{+0.20}_{-0.19}$

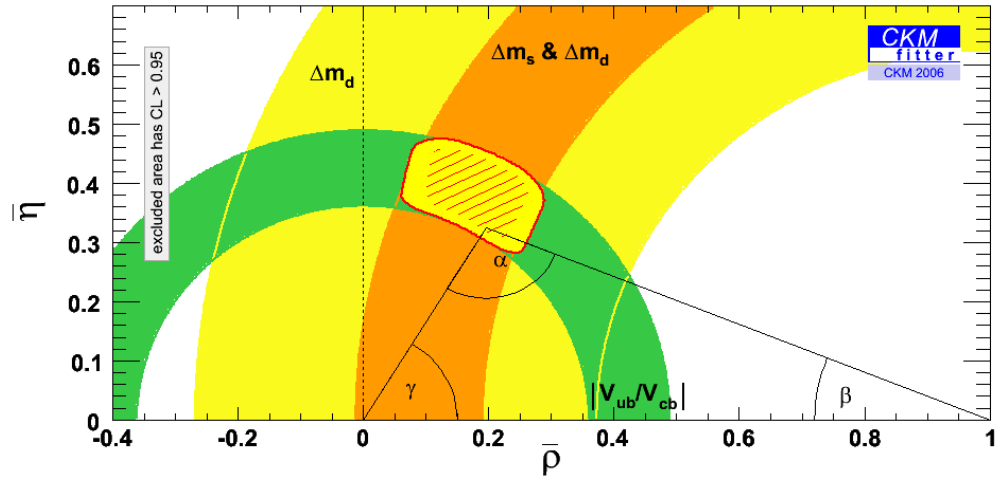


# 2. The global CKM fit : Sides only

## CP CONSERVING OBSERVABLES

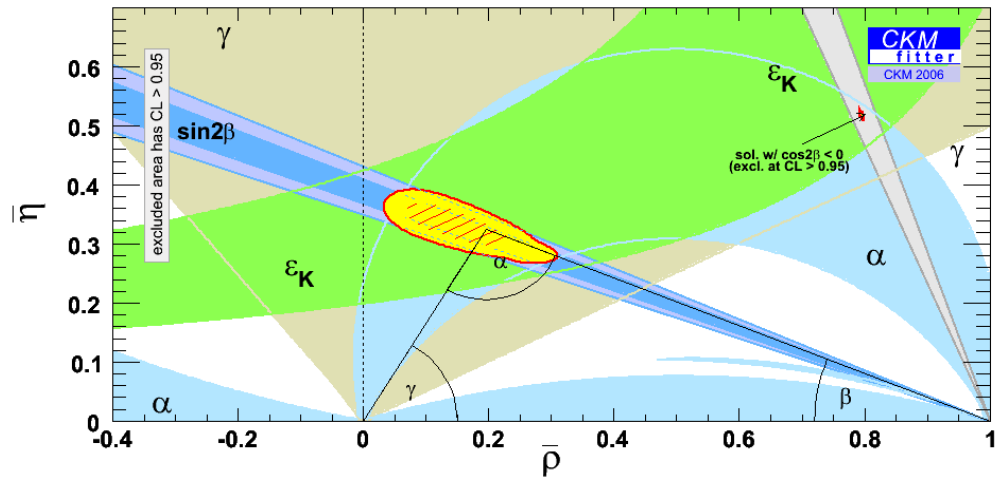


# 2. The global CKM fit : consistency



CP CONSERVING

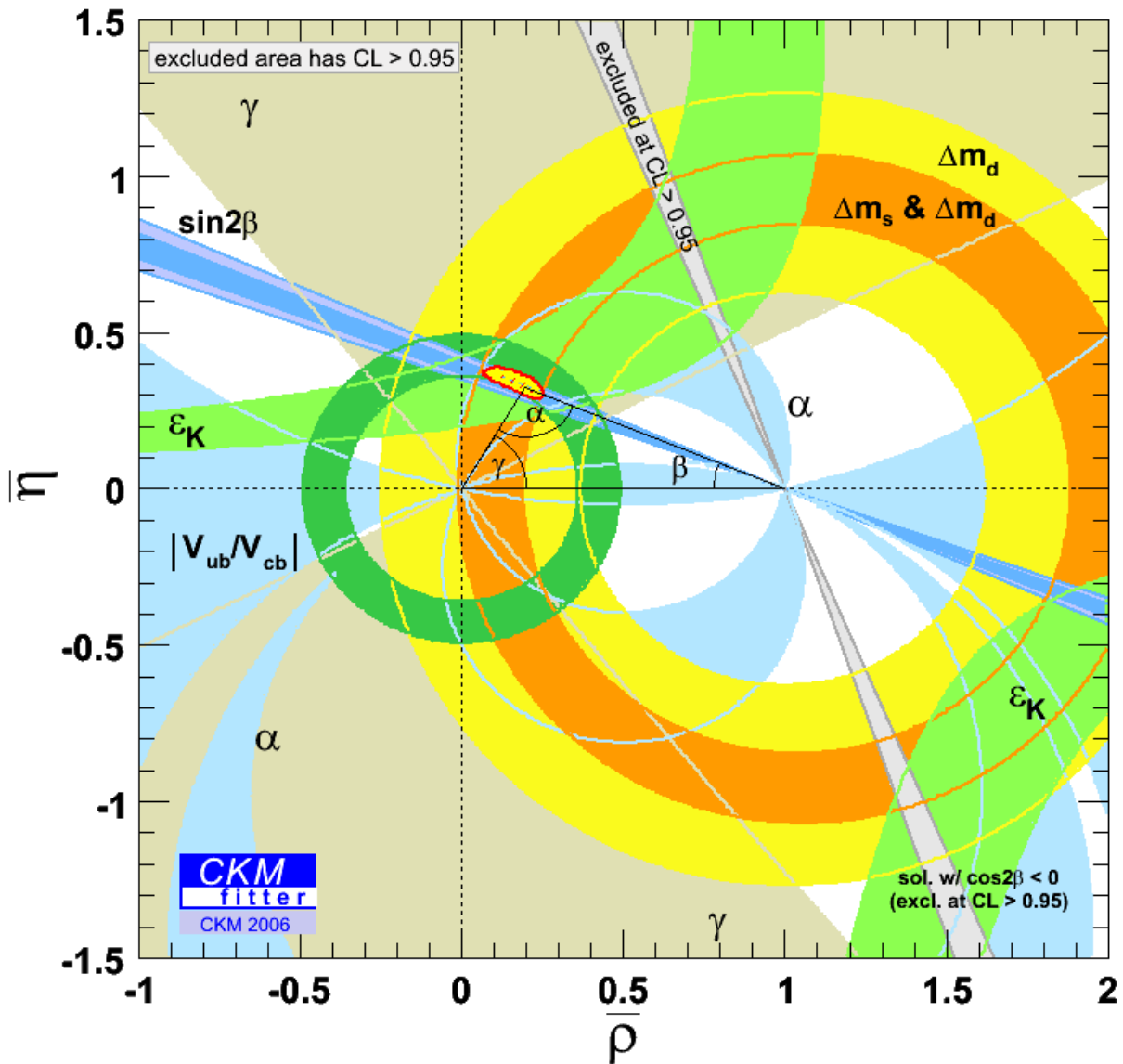
- $\left| \frac{V_{ub}}{V_{cb}} \right|$
- $\Delta m_d$
- $\Delta m_s$
- $B \rightarrow \tau \nu$



CP VIOLATING

- $|\epsilon_K|$
- $\sin 2\beta$
- $\alpha$
- $\gamma$

# 2. The CKM global fit : a conclusion



## 2. The CKM global fit : SM metrology

Wolfenstein parameters:

$$\begin{aligned} A &= 0.806 \pm 0.014 \\ \lambda &= 0.2272 \pm 0.0010 \\ \bar{\rho} &= 0.195^{+0.024}_{-0.067} \\ \bar{\eta} &= 0.326^{+0.032}_{-0.015} \end{aligned}$$

Jarlskog invariant:

$$J = (2.90^{+0.29}_{-0.15}) \times 10^{-5}$$

The UT angles:

$$\alpha + \beta + \gamma = (191^{+33}_{-32})^\circ$$

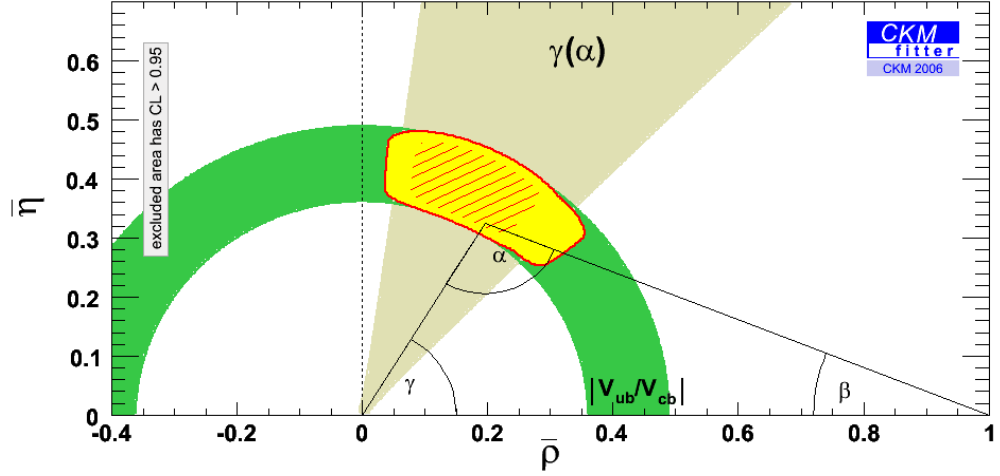
$$\begin{aligned} \alpha &= (98.9^{+4.4}_{-11.3})^\circ, \\ \beta &= (22.00^{+0.69}_{-0.63})^\circ, \\ \gamma &= (59.1^{+11.1}_{-4.1})^\circ. \end{aligned}$$

## 2. The CKM global fit : a selection of SM predictions

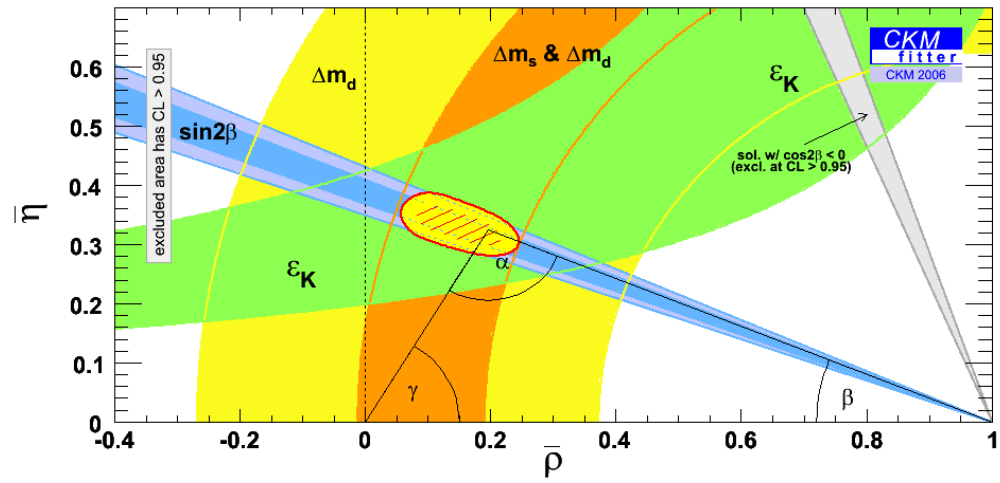
OBSERVABLE	CKM FIT	MEASUREMENT
$\Delta m_s \text{ (ps}^{-1}\text{)}$	$19.2^{+5.1}_{-3.4}$	$17.77 \pm 0.10 \pm 0.07$
$BF(B^+ \rightarrow \tau^+ \nu_\tau) / 10^{-4}$	$0.92^{+0.20}_{-0.19}$	$1.45^{+0.46}_{-0.43}$
$BF(B^+ \rightarrow \mu^+ \nu_\mu) / 10^{-7}$	$4.15^{+0.56}_{-0.64}$	
$\chi \equiv \beta_s \text{ (deg)}$	$1.005^{+0.089}_{-0.045}$	

# 2. The CKM global fit

## TREE



## LOOP



SFP 2007

CKM

$\delta_{CKM}$  is the dominant source of CP violation

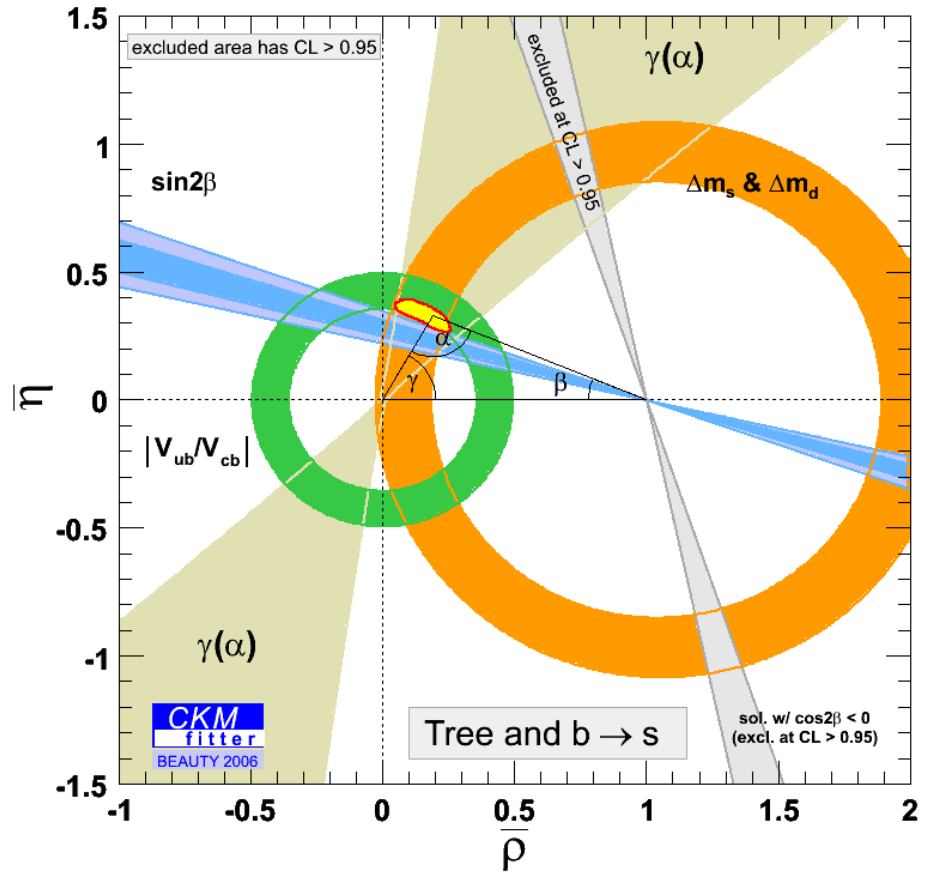
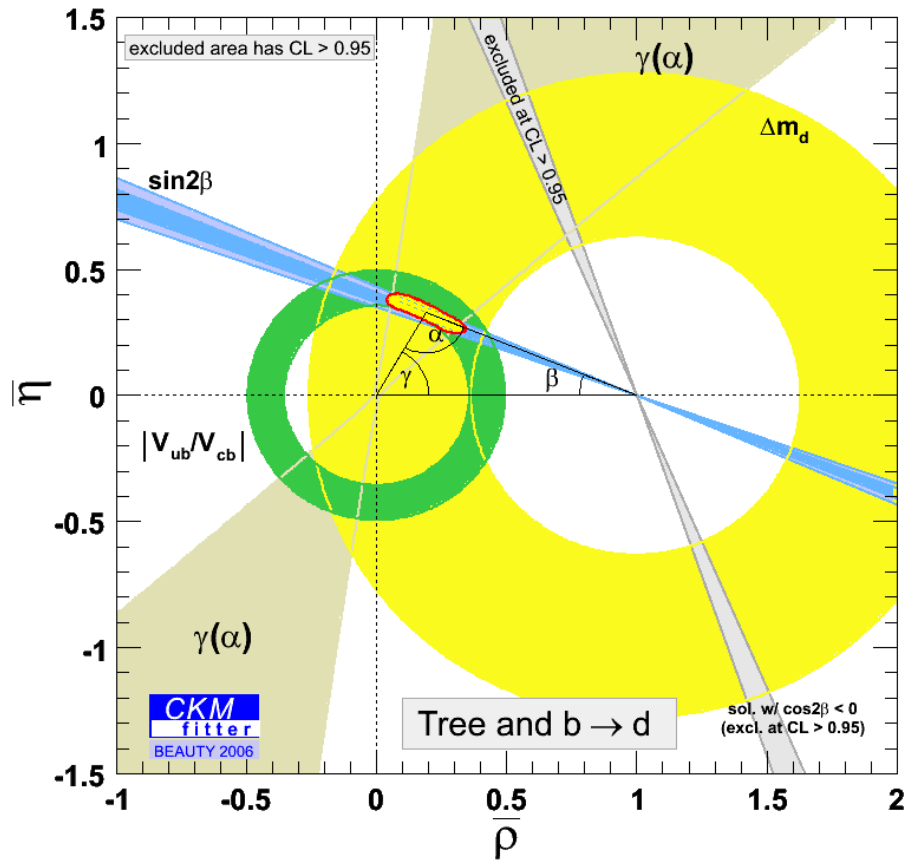
### 3. NP : where to look for ?

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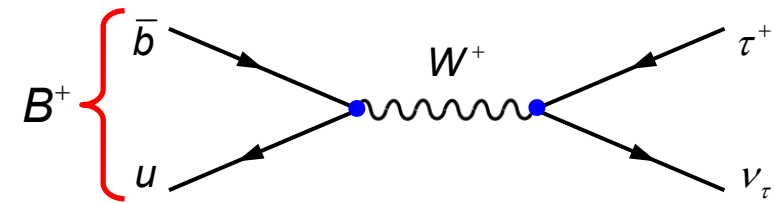
- ✓ Comparison of  $\sin 2\beta$  from  $\Psi K$  and s-penguins diagrams
- ✓  $(b \rightarrow s \ell^+ \ell^-)$
- ✓ Rare decays  $(B_s \rightarrow \mu^+ \mu^-)$
- ✓ Annihilation diagram  $(B^+ \rightarrow \tau^+ \nu_\tau)$
- ✓ NP in  $\Delta F = 2$  transitions

Focus on the last two points

# 3. NP : where to look for ?



### 3. NP : $B^+ \rightarrow \tau^+ \nu_\tau$

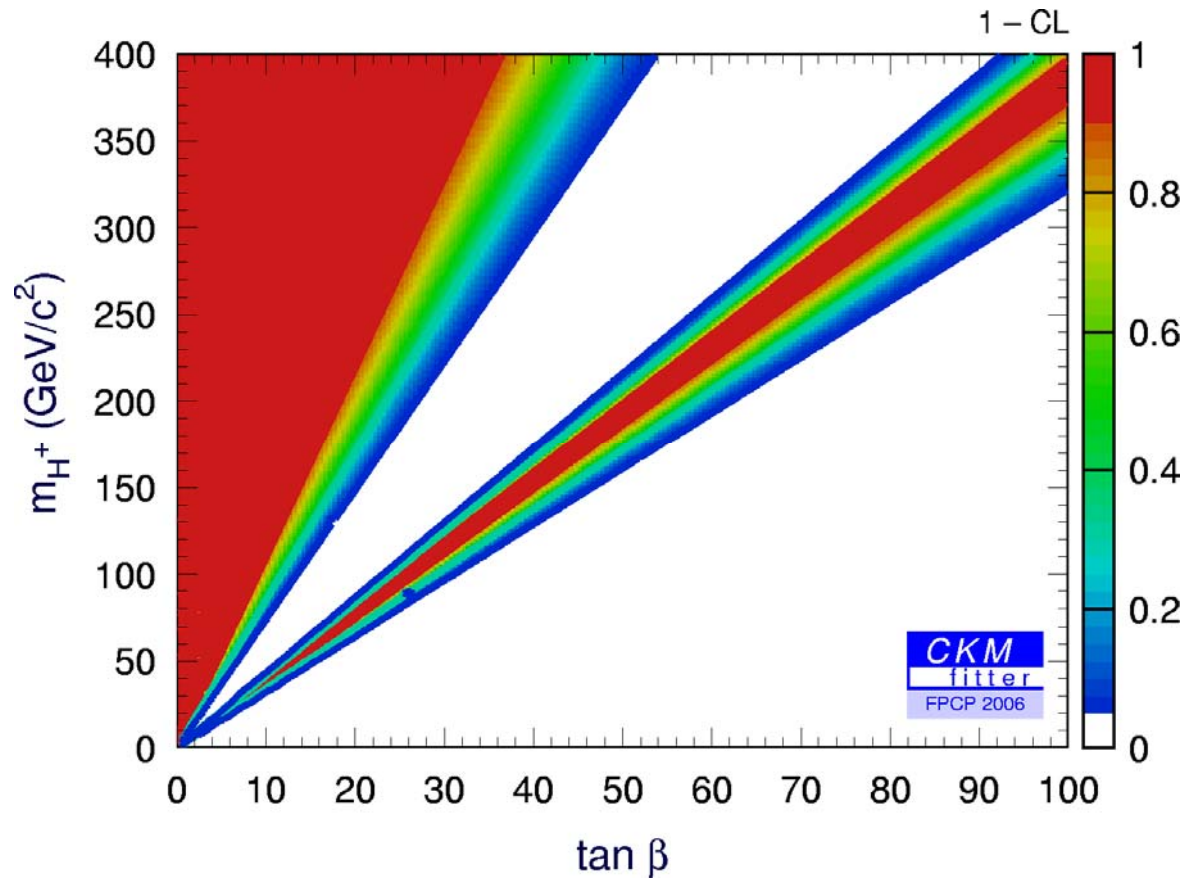


In addition, decay mediated by a charged Higgs.

$$BF(B^+ \rightarrow \tau^+ \nu_\tau) =$$

$$BF(B^+ \rightarrow \tau^+ \nu_\tau)_{SM} \times C_H$$

$$C_H = \left(1 - \frac{m_B^2}{m_H^2}\right) \tan^2 \beta$$



### 3. NP : $\Delta F = 2$ TRANSITIONS

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Assumption :  $V_{ub}/V_{cb}$  and  $\gamma$  are the basic inputs of the analysis since it is tree-mediated.

Introduce NP in  $\Delta F = 2$  model-independantly by means of two additional parameters:

$$r_d^2 \exp(2i\theta_d) = \frac{\langle B^0 | H_{eff}^{full} | \bar{B}^0 \rangle}{\langle B^0 | H_{eff}^{SM} | \bar{B}^0 \rangle}$$

### 3. NP : $\Delta F = 2$ TRANSITIONS $B_d$ - $B_d$ system

$$r_d^2 \exp(2i\theta_d) = \frac{\langle B^0 | H_{eff}^{full} | \bar{B}^0 \rangle}{\langle B^0 | H_{eff}^{SM} | \bar{B}^0 \rangle}$$

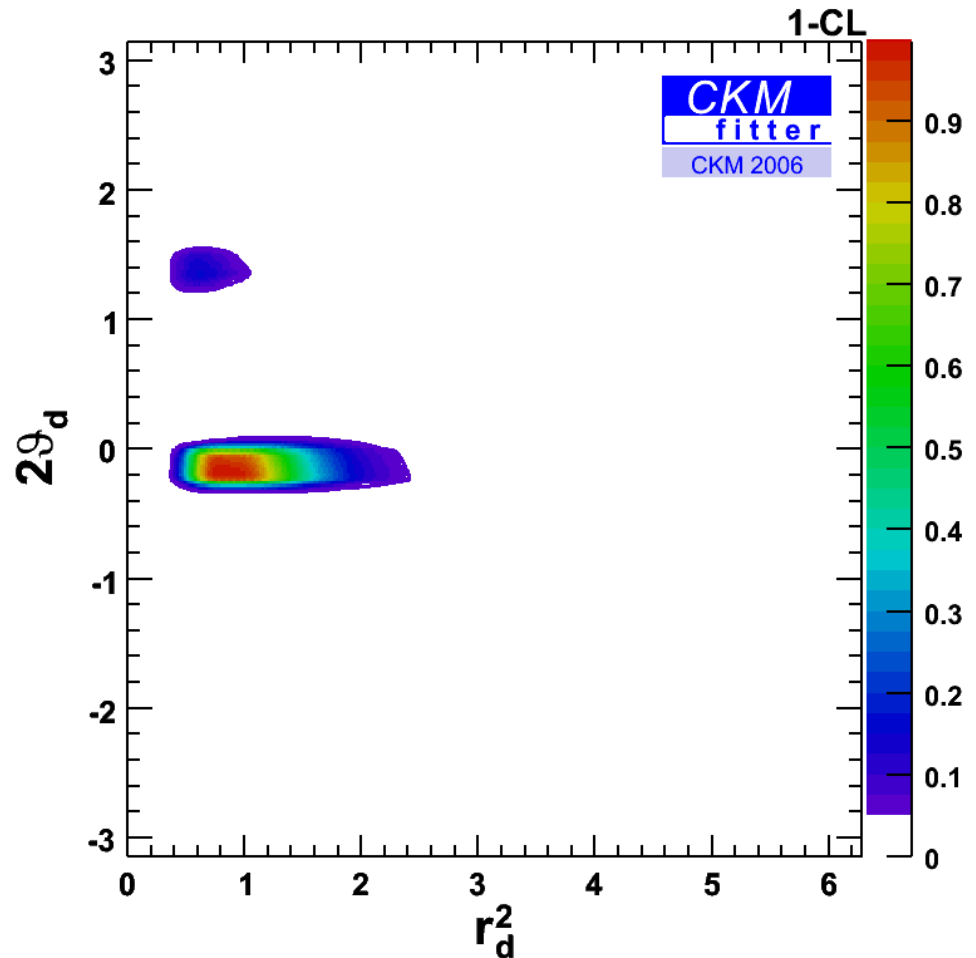
The parameters are modified as :

$$\Delta m_d = (\Delta m_d)^{SM} r_d^2$$

$$\sin(2\beta) = \sin(2\beta - 2\theta_d)$$

CP-violating charge asymmetry in semileptonic B decay is a crucial measurement to remove NP solutions:

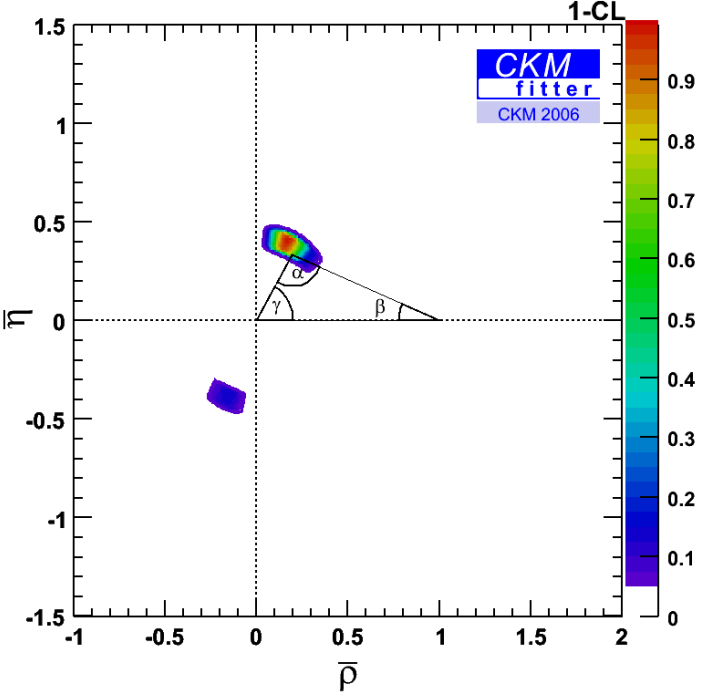
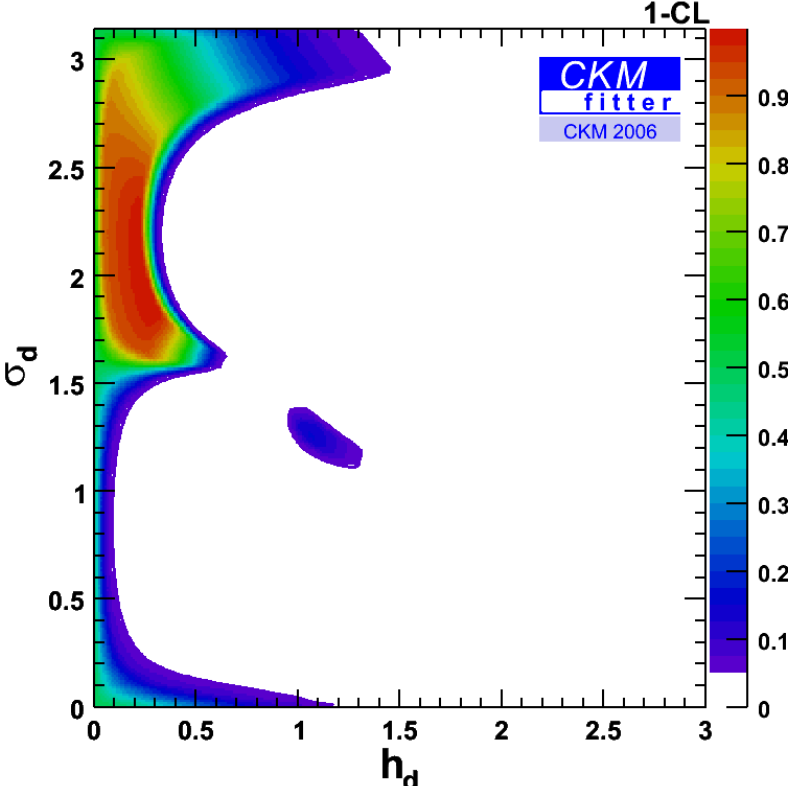
$$A_{SL}^d = -\text{Re}(\Gamma_{12}/M_{12})^{SM} \sin(2\theta_d)/r_d^2 + \text{Im}(\Gamma_{12}/M_{12})^{SM} \cos(2\theta_d)/r_d^2$$



# 3. NP : $\Delta F = 2$ TRANSITIONS $B_d$ - $B_d$ system

Another equivalent parametrisation can be used, displaying explicitly the NP amplitude

$$\frac{\langle B_d^0 | H_{eff}^{SM+NP} | \overline{B}_d^0 \rangle}{\langle B_d^0 | H_{eff}^{SM} | \overline{B}_d^0 \rangle} = r_d^2 e^{i2\theta_d} = 1 + h_d e^{i2\sigma_d}$$



Not much room left for NP in the  $B_d$  system

### 3. NP : $\Delta F = 2$ TRANSITIONS - $B_s$ - $B_s$ system

$$\frac{\left\langle B_s^0 \left| H_{eff}^{SM+NP} \right| \overline{B}_s^0 \right\rangle}{\left\langle B_s^0 \left| H_{eff}^{SM} \right| \overline{B}_s^0 \right\rangle} = r_s^2 e^{i2\theta_s} = 1 + h_s e^{i2\sigma_s}$$

The recent results from Tevatron (CDF and D0) allow to constrain for the first time the NP contribution ( $\Delta m_s$ ,  $\Delta \Gamma_s$ ,  $A_{SL}$ )

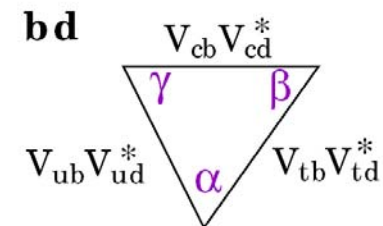
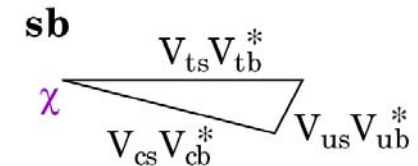
Mass difference:  $\Delta m_s = (\Delta m_s)^{SM} r_s^2$

Width difference:  $\Delta \Gamma_s = (\Delta \Gamma_s)^{SM} \cos^2(2\chi - 2\theta_s)$

Semileptonic asymmetry:

$A_{SL}^s = -\text{Re}(\Gamma_{12}/M_{12})^{SM} \sin(2\theta_s)/r_s^2$

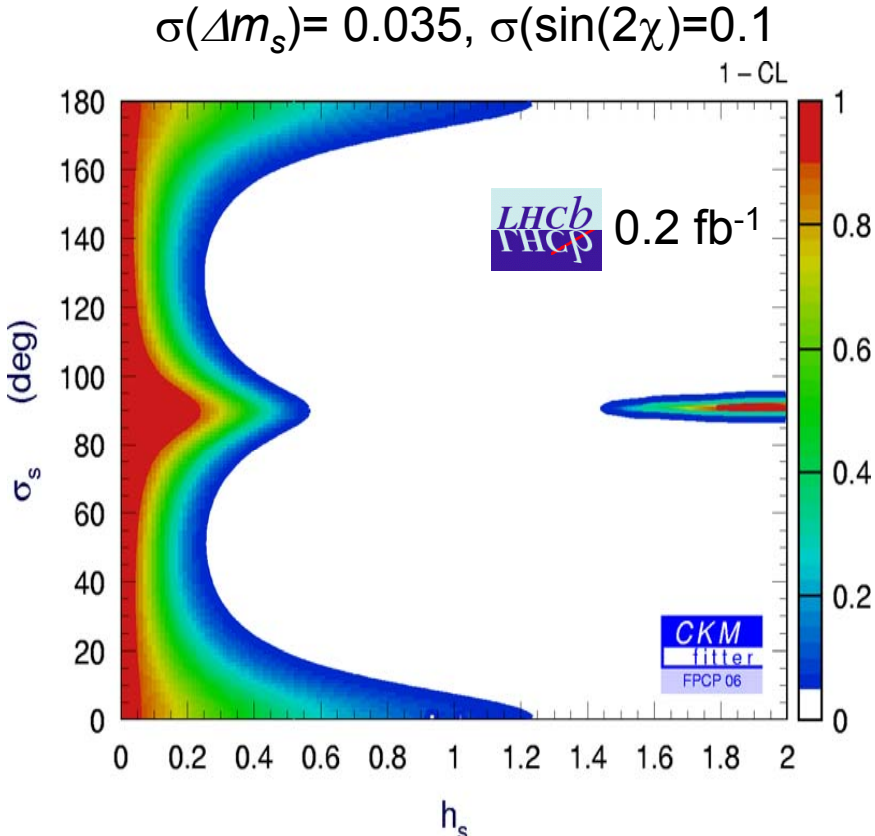
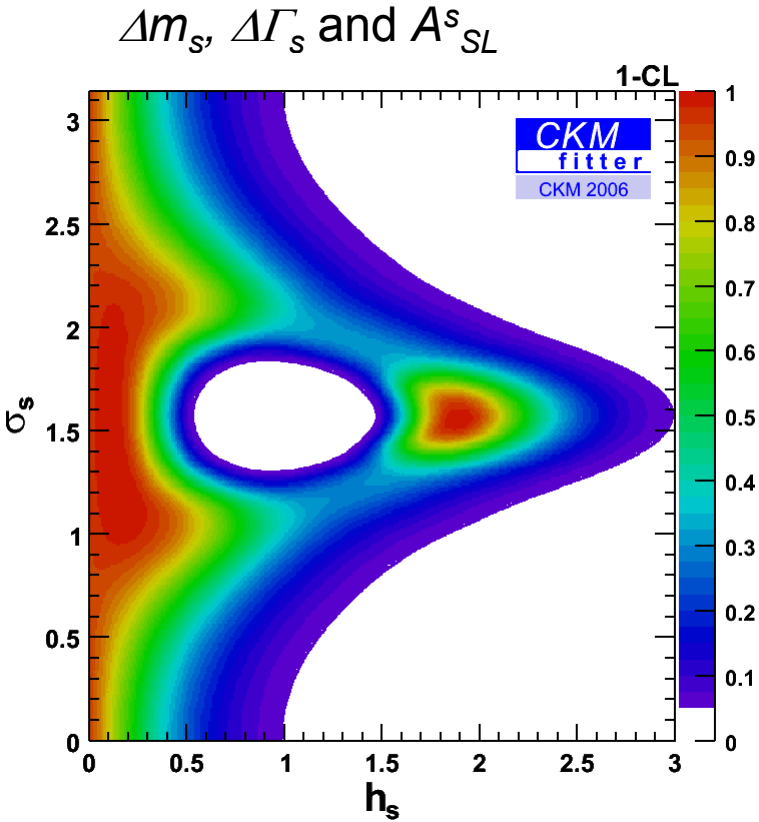
$\sin(2\chi) = \sin(2\chi - 2\theta_s)$



$B_s$  mixing phase very small in SM ( $1^\circ$ ) and very well predicted ( $O(5\%)$ )

$B_s$  mixing: very sensitive probe to NP

# 3. NP : $\Delta F = 2$ TRANSITIONS - $B_s$ - $B_s$ system



$h_s \sim \leq 3$  ( $h_d \sim \leq 0.3, h_K \sim \leq 0.6$ )  
 Tevatron measurement. Still plenty  
 of room for NP.

LHCb shall rapidly say something.

## 4. Conclusions

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- ✓ B-factories provided a fantastic success for the SM
  - ✓ CKM mechanism is at work to describe CP violation
  - ✓ The CKM phase is the dominant source of CP violation in B processes.
- ✓ No evidence so far for corrections to CKM mechanism. Yet, we are just entering a precision era in flavour physics.
- ✓ Major results went last year from the Tevatron, especially in fully hadronic modes. Encouraging for the LHC experiments, singularly for LHCb. The Bs system must be explored. Has to make gamma a precision measurements.