BOX SYNTAX -- A 2-DIMENSIONAL METALANGUAGE

INTRODUCTION

The process of translating a source language \( L_S \) into some other, known, target language \( L_T \) consists of recognizing valid constructs in \( L_S \), and interpreting the meaning of those constructs in \( L_T \). The description of the class of valid constructs of \( L_S \) is called the syntax of \( L_S \). The description of the meaning of those constructs is called the semantics of \( L_S \). The languages in which the syntax and semantics of \( L_S \) are stated are called the syntactic and semantic metalanguages.

Box Syntax is a syntactic metalanguage (abbreviated \( ML_{BS} \)) which combines some features of other metalanguages with a new 2-dimensional format. Its principal advantage is that the 2-dimensional format is easier to scan than previously existing linear formats. Comprehension of the syntax of a language \( L_S \) should be improved by describing it in Box Syntax.

DESCRIPTION OF BOX SYNTAX

(1) Constructs in \( ML_{BS} \) are classified as either terminal or nonterminal constructs.

(2) Terminal constructs in \( ML_{BS} \) are strings of text in \( L_S \). The strings are underlined to indicate their extent. Terminal constructs are the constants of \( ML_{BS} \). (See Fig. 1-c.)

(3) Nonterminal constructs in \( ML_{BS} \) are the variables of \( ML_{BS} \). Each nonterminal construct is denoted by a rectangular box. The box may contain additional information which further characterizes the construct.
(4) There are several ways to indicate the class of values which may be assigned to a given nonterminal construct. One way is to define that construct in terms of other terminal and nonterminal constructs. The defined construct is denoted by a name, consisting of a string of letters and digits and blank spaces, surrounded by the box described in (3) above. (See Fig. 1-a.)

(5) The construct may be defined as the concatenation of several terminal and nonterminal constructs. (See Fig. 1-b.)

(6) A given construct may enclose other constructs within it. The given construct effectively parenthesizes the enclosed constructs. (See Fig. 1-d.)

(7) A given construct may represent an alternative choice among several other constructs. The rectangular box denoting the given construct is segmented vertically (by horizontal lines), and each of the alternative constructs is enclosed in one of the segments. (See Fig. 1-e.)

(8) The syntax of $L_3$ may be such that a construct $B$ in $L_3$ may be repeatedly concatenated with itself. This is denoted in $ML_{B_3}$ by an asterisk following a single replication of the construct $[B]$, which corresponds to $B$. (See Figs. 1-f and 1-g.) The asterisk in Fig. 1-f indicates one or more replications of $[B]$. The asterisk surrounded by a zero in Fig. 1-g indicates zero or more replications of $[B]$. 
(9) Box Syntax also allows representation of repeated concatenation of a construct B in Lₜ, where the replications are separated by some other delimiting construct C. However, the use of box notation requires that the MLₜₜ construct [B] corresponding to B uses at least one box, on which to attach the broken-line box described in (10) below. Therefore, an additional parenthesizing box, as in (6) above, may be needed in order to draw the MLₜₜ construct properly. (See Figs. 1-h, 1-i, and 1-j.)

(10) The repeated construct of (8) above is denoted by a box drawn with solid lines, to which is adjoined a box drawn with broken lines. The broken-line box contains the MLₜₜ construct corresponding to the delimiter C in Lₜ. The combination of solid- and broken-line boxes is followed by either * or \textcopyright, depending on whether one or more replications, or zero or more replications, are permitted in Lₜ.

(11) The syntax of Lₜ is the same regardless of whether the broken-line box of (10) above appears to the left or to the right of the solid-line box. However, the choice of left-to-right association or right-to-left association of the replications can be suggestive of the semantics of Lₜ. (See Figs. 1-h and 1-i.)

(12) The empty construct in MLₜₜ is denoted as in Fig. 2-a. "Empty" is not the same as the string of text in Lₜ consisting of a blank space character.
The semantics involved in translating $L_S$, or in other processing associated with the syntax of $L_S$, optionally may be included in a box along with the corresponding syntactic construct. If so, the box is segmented vertically by a (horizontal) light solid line. The syntax, as described in (1) through (12) above, appears above the light line; the semantic constructs in an appropriate semantic metalanguage appear below the light line. (See Fig. 2-b.) By convention, the semantic constructs are executed after the corresponding syntactic construct has been recognized; that is, after a string of text in $L_S$ has been parsed by successfully matching it with an $ML_{BS}$ construct.

If $L_S$ is context-dependent, then the choice of syntax used by a syntax-recognizing program may be governed by the execution of a semantic construct. Fig. 2-c shows the $ML_{BS}$ notation for a semantics-driven choice of syntax.

**EXAMPLES**

(1) The example in Fig. 3 is taken from the ALGOL 60 Report [3]. I have taken some small liberties in rearranging the definitions and un-naming some of the named ALGOL constructs.

The construct `<for statement>` was transferred from the definition of `<unconditional statement>` to the definition of `<statement>` in the revised ALGOL 60 Report [3].

(2) In Ref. [1], a "triple sequence alarm" is used consistently to demonstrate the appearance and capabilities of a collection of well-known metalanguages. Fig. 4 shows the Box Syntax representation of the triple sequence alarm.
CREDIT WHERE CREDIT IS DUE

Many of the ideas incorporated in Box Syntax were taken from already existing metalanguages.

From Backus Normal Form (Ref. [1]): The use of nonterminal constructs, and the general appearance of their definition. (See Figs. 1-a and 1-b.) Bracketing nonterminal constructs to distinguish them from terminal constructs. (See Fig. 1-c.) The use of an empty construct. (See Fig. 2-a.)

From IBM metalinguistic notation (A good example appears in pp. 51 ff. of Ref. [2]): Parenthesization. (See Fig. 1-d.) The vertical arrangement of alternative constructs. (See Fig. 1-e.)

From Regular Expressions (Ref. [1]): An asterisk to indicate arbitrarily many replications. (See Figs. 1-f and 1-g.)

New notational techniques introduced in Box Syntax: The use of box enclosure. Similar but distinct notations for one or more replications and for zero or more replications. (See Figs. 1-f and 1-g.) A special notation for delimiters. (See Figs. 1-h, 1-i, and 1-j.) Semantic description adjacent to syntactic description. (See Fig. 2-b.) Formalized coupling from semantics to syntax, for context-dependent languages. (See Fig. 2-c.)

WHY A NEW METALANGUAGE?

Each metalanguage has its drawbacks. For example, in Backus Normal Form, iteration is treated as a special case of recursion, without a special notation. I think that the frequency with which iteration occurs in mechanical languages makes it important enough to justify a special notation, such as is found in Regular Expressions. The Box Syntax notation
for iteration in Figs. 1-f and 1-g certainly is more apparent than the equivalent BNF notation and, in the case of Fig. 1-g, requires fewer defined constructs. (Note that the BNF in Fig. 1-g could be compressed into a single definition, but the resulting grammar would have an ambiguous parse.)

As another example, the IBM metalinguistic notation does not include the use of named nonterminal constructs. This eliminates the possibility of "subroutines" of nonterminal constructs; consequently this notation cannot be used to describe recursive languages.

In Box Syntax, I have incorporated what I consider to be the best features of several other metalanguages, with some new features. Chief among the new features is the formalism for context-dependent languages, shown in Fig. 2-c. Almost all mechanical languages in current use have some context-dependent properties, yet no formal notation previously proposed for describing these properties has found widespread acceptance. [5], [6], [7]

Box Syntax, too, is not without its drawbacks (literally). Drawing the boxes such that the resulting diagram is clearly understandable and neat in appearance, requires considerable effort. Box Syntax is best suited to a computer-assisted drafting method, with the aid of a CRT display unit.

One modification of Box Syntax, for CRT display, is due to the inability to generate both light and heavy lines. A suitable equivalent to Fig. 2-b might be a diagonal line to separate syntax from semantics:

```
<syntax here> / <semantics here>
```
GENERALITY

In terms of ability to describe context-free linear languages, Backus Normal Form and Box Syntax are equivalent. For each BNF construct shown in Ref. [1], there is an equivalent Box Syntax construct. For each Box Syntax construct shown in Fig. 1, there is an equivalent BNF construct or set of constructs.

Context-dependent linear languages can be described in Box Syntax, but not in Backus Normal Form.

Box Syntax is not useful in describing most 2-dimensional languages, such as flowcharts, because these languages place no significance on physical juxtaposition. The significant property of flowchart syntax is connectivity, which is topological rather than geographic. Box Syntax describes the concatenation, or geographic placement, of terminal constructs.

There is one relevant 2-dimensional language in which juxtaposition of constructs is significant—Box Syntax itself. A logical extension of Box Syntax is capable of describing unextended Box Syntax. The extension requires the ability to concatenate terminal constructs vertically, as well as horizontally. Since the class of terminal constructs includes boxes, a second color is helpful to distinguish between terminal and nonterminal boxes. Underlining the terminal boxes, in a manner analogous to Fig. 1-c, may substitute for a second color, but the resulting diagram is extremely confusing to read.

ACKNOWLEDGEMENTS

I would like to thank Robert T. Braden for his constructive criticism, Myron Ruderman for his help with the English metametalanguage, and Jerome A. Feldman for his constructive sarcasm.
REFERENCES


### FIG. 1  EQUIVALENT STATEMENTS

<table>
<thead>
<tr>
<th>BACKUS NORMAL FORM</th>
<th>BOX SYNTAX</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ( A ::= B )</td>
<td>( A = B )</td>
<td>METALINGUISTIC CONSTRUCT DEFINITION.</td>
</tr>
<tr>
<td>(b) ( A ::= B \cdot C )</td>
<td>( A = B \cdot C )</td>
<td>CONCATENATION.</td>
</tr>
<tr>
<td>(c) ( \langle \text{THE WORD BOY} \rangle ::= \text{BOY} )</td>
<td>( \text{THE WORD BOY} = \text{BOY} )</td>
<td>TEXT OF THE LANGUAGE BEING DESCRIBED.</td>
</tr>
<tr>
<td>(d) ( \langle A \rangle ::= \langle B \rangle \cdot \langle C \rangle )</td>
<td>( A = \langle B \rangle \cdot \langle C \rangle )</td>
<td>PARENTHESESIZATION.</td>
</tr>
<tr>
<td>(e) ( \langle A \rangle ::= \langle B \rangle</td>
<td>\langle C \rangle</td>
<td>\langle D \rangle )</td>
</tr>
<tr>
<td>(f) ( \langle A \rangle ::= \langle B \rangle</td>
<td>\langle B \rangle \langle A \rangle )</td>
<td>( A = \langle B \rangle</td>
</tr>
<tr>
<td>(g) ( \langle A \rangle ::= \langle C \rangle</td>
<td>\langle \text{EMPTY} \rangle )</td>
<td>( A = \langle C \rangle</td>
</tr>
<tr>
<td>(h) ( \langle A \rangle ::= \langle B \rangle</td>
<td>\langle B \rangle \langle C \rangle \langle A \rangle )</td>
<td>( A = \langle B \rangle</td>
</tr>
<tr>
<td>(i) ( \langle A \rangle ::= \langle B \rangle</td>
<td>\langle A \rangle \langle C \rangle \langle B \rangle )</td>
<td>( A = \langle B \rangle</td>
</tr>
</tbody>
</table>

**Diagram Notes:**
- Box Syntax:
  - B
  - C
  - D

**Comments Examples:**
- **Binary Digit:**
  - UNSIGNED INTEGER = DIGIT

**Punctuation:**
- List = LIST ELEMENT

**Association:**
- Term = FACTOR
### FIG. 1 (CONT'D)

<table>
<thead>
<tr>
<th>BACKUS NORMAL FORM</th>
<th>BOX SYNTAX</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle A \rangle := \langle 0 \rangle \mid \langle \text{EMPTY} \rangle )</td>
<td>[Diagram: ( A = \text{Box} B, C ) ( \otimes )]</td>
<td>ZERO OR MORE REPLICATIONS WITH DELIMITER.</td>
</tr>
<tr>
<td>( \langle D \rangle := \langle \varepsilon \rangle \mid \langle B \rangle \langle C \rangle \langle D \rangle )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### FIG. 2

#### OTHER CONSTRUCTS

| (a) | | EMPTY CONSTRUCT. |
| (b) | \( \langle \text{SYNTAX HERE} \rangle \mid \langle \text{SEMANTICS HERE} \rangle \) | ADJACENT SEMANTICS. SYNTAX ABOVE THE LIGHT LINE, SEMANTICS BELOW. |
| (c) | \( \langle \text{SEMANTIC TEST} \rangle \mid \langle \text{SYNTAX} \rangle \mid \langle \text{SYNTAX} \rangle \) | SEMANTICS-DRIVEN SYNTAX. A SEMANTIC TEST, WRITTEN IN THE SEMANTIC METALANGUAGE, APPEARS INSIDE THE ELLIPSE. |
FIG. 3

ALGOL EXAMPLE
FIG. 4  TRIPLE SEQUENCE ALARM

EXAMPLE

\[
\text{TRIPLE SEQUENCE ALARM} = \begin{array}{c}
0 \\
10 \\
110 \\
\end{array} \quad \otimes \quad \begin{array}{c}
0 \\
1 \\
\end{array}
\]
MULTIVARIATE STANDARD P–P PLOT

Sample Sizes: 100, 100
10-Dimensional Double Exponential Location Difference 3 units
MULTIVARIATE RADIAL P-P PLOT

MULTIVARIATE PLANAR REPRESENTATION.

Sample Sizes: 100, 100
10-dimensional Double Exponential Location Difference 3 units

FIGURE 1b

E = 0.53

Sample Sizes: 100, 100
10-dimensional Double Exponential Location Difference 3 units

FIGURE 1c
MULTIVARIATE STANDARD P–P PLOT

Sample Sizes: 100, 100

FIGURE 2a

10-dimensional double exponential scale difference: 3 units

MULTIVARIATE RADIAL P–P PLOT

Sample Sizes: 100, 100

FIGURE 2b

10-dimensional double exponential scale difference: 3 units
MULTIVARIATE PLANAR REPRESENTATION.

FIGURE 2c

Sample Sizes: 100,100
10-dimensional double exponential scale difference: 3 units

MULTIVARIATE PLANING

FIGURE 3

Iris Data: 4 dimensions
All 3 species
MULTIVARIATE PLANAR REPRESENTATION.

FIGURE 4
Iris Data: 4 dimensions
Versicolor and Virginica

MULTIVARIATE STANDARD P–P PLOT

FIGURE 5a
Sample Sizes: 200, 200
Particle Physics Data
13 Dimensions
MULTIVARIATE RADIAL P–P PLOT

Sample Sizes: 200, 200

Particle Physics Data
13 Dimensions

FIGURE 5b

MULTIVARIATE PLANAR REPRESENTATION.

Sample Sizes: 200, 200

Particle Physics Data
13 Dimensions

FIGURE 5c
MULTIVARIATE PLANAR REPRESENTATION

FIGURE 5d

Central Region

Particle Physics Data

13 Dimensions